Chapter 1

Resonant Matching Networks

1.1 Introduction

Frequently power from a linear source has to be transferred into a load. If the load impedance may be adjusted, the **maximum power theorem** states that maximum power is developed in the load when its impedance is equal to the complex conjugate of the source impedance. In practice, we are not usually able to adjust the load impedance and instead have to design a matching network to power match a particular source to a particular load.

However, useful power transfer systems (audio amplifiers, mains wiring) are rarely maximum power matched. Usually, we just wish to transform the reactive load into a pure resistance of known value such as $50\,\Omega$, which is the designed load of high-frequency *rf* power amplifiers.

If the transformation is required over a broad band of frequencies, one approach is to use a transformer consisting of inductively coupled windings on a ferrite or iron core. At higher frequencies, these are lossy and difficult to design. If a broad-band response is not required and at frequencies above about 1 MHz, resonant matching networks are almost invariably used instead. They are said to be resonant because precise matching occurs at only **one** frequency. If the matching network consists of only ideal capacitors and inductors, no power is dissipated within the network, and so we have the situation in Figure 1.1 where a linear source and load are matched by a **lossless** network. For simplicity, we start with a resistive source and load, but later relax it to estimate power losses in "real" networks.

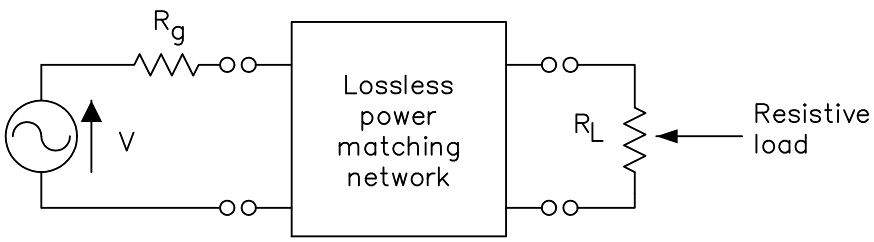

Figure 1.1: Resistive linear source and load matched by a lossless network.

1.2 L Matching networks

Figure 1.2 shows the simplest such network, a reactive L section, having series and shunt reactance elements X_s and X_p respectively. (Remember that reactances are **signed** quantities, inductive reactance is positive and capacitative reactance is negative). Here the series reactance is next to the **source**. We shall see that this is appropriate when the source resistance is **less** than the load resistance.

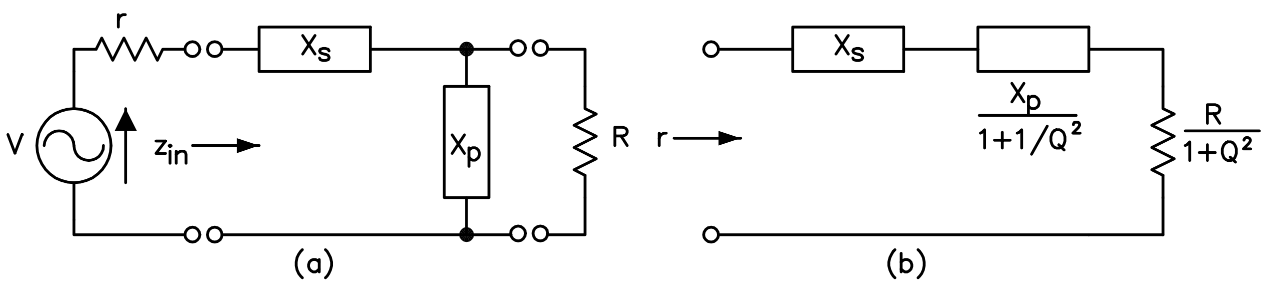

Figure 1.2: (a) A resonant L matching section. (b) Reactive load transformed using Bold's equations.

Converting the parallel components to their series equivalent using **Bold's equations**[1] gives figure 1.2 (b) and equating the resistive components gives one way of deriving the results, and this is left as a problem at the end of the chapter. However, we'll do it directly, from figure 1.2 (a).

The impedance seen by the source, z_{in}, must be resistive, and equal to r. Thus,

$$r = jX_s + \frac{jX_pR}{jX_p + R} \quad (1.1)$$

$$(r - jX_s)(R + jX_p) = jRX_p \quad (1.2)$$

$$rR + jrX_p - jRX_s + X_pX_s = jRX_p \quad (1.3)$$

Equating reals and imaginaries gives

$$X_pX_s = -rR \quad \text{(reals)} \quad (1.4)$$

$$rX_p - RX_s = RX_p \quad \text{(imaginaries)} \quad (1.5)$$

$$\text{rearranging equation 1.5, } \frac{X_s}{X_p} = -\frac{R-r}{R} \quad (1.6)$$

Multiplying equations 1.4 and 1.6 we obtain

$$X_s^2 = r(R-r) \quad (1.7)$$

Since X_s must be real (a complex reactance has no meaning), it is necessary that $R > r$. If this is true, then there are 2 possible solutions for X_s:

$$X_s = \pm\sqrt{r(R-r)} \quad (1.8)$$

Either sign can be chosen. Having done this, rearranging equation 1.4 gives X_p in terms of X_s:

$$X_p = -\frac{Rr}{X_s} \quad (1.9)$$

The matching and realizability equations, presented for easy reference, are then

$$X_s = \pm\sqrt{r(R-r)} \quad (1.10)$$

$$X_p = -\frac{Rr}{X_s} \quad (1.11)$$

$$R > r \quad (1.12)$$

[1]See *Linear Steady-State Network Theory*, Bold and Earnshaw. The latest edition is in the University Bookshop.

The negative sign in equation 1.11 indicates that the two reactances must be of **opposite** signs, that is, one must be an inductor, the other a capacitor. Either one can be chosen as the inductor.

In Figure 1.3(a), we show a maximum power matching situation where the positions of the source and load have been reversed. If we write down the condition that the impedance seen by the new source be R, we find that the **same** matching equations result.

Thus the **same** matching network will match a load r whose resistance is smaller than the source resistance R.

Hence the rule for finding the correct orientation of the L network: Associate the **series** reactance with the **smaller** of the two resistances. In fact, it is conceptually simpler to ignore the position of the ideal voltage source altogether and to think of the L network as "power matching two resistors" as in Figure 1.3 (b). The matching network transforms the resistance values so that each resistor "looks" into an impedance equal to itself.

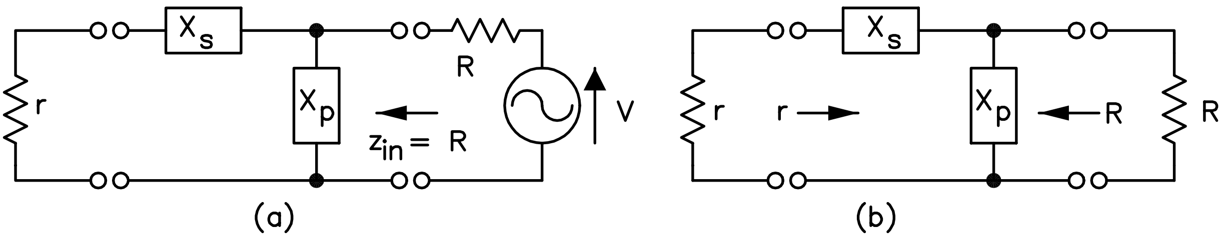

Figure 1.3: (a) Matching a high resistance source to a low resistance load. (b) "Sourceless" network matching two resistances.

1.2.1 Example 1.1

Match a 50 Ω linear source to a load resistance of 1000 Ω at a frequency of 10 MHz.

The required configuration is shown in Figure 1.4 (a).

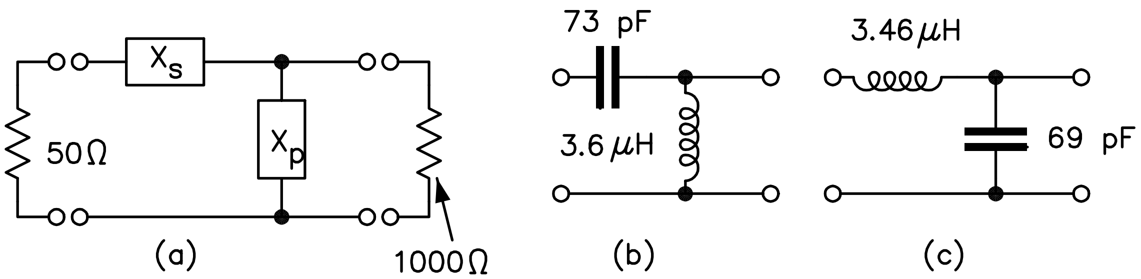

Figure 1.4: (a) Configuration for Example 2.1. (b) High-pass version. (c) Low-pass version

Given that $r = 50\,\Omega$ and $R = 1000\,\Omega$, we find that

$$X_s = \pm\sqrt{r(R-r)} = \pm\sqrt{50 \times 950} = \pm 218\,\Omega \quad (1.13)$$

$$\text{and } X_p = -\frac{Rr}{X_s} = -\frac{1000 \times 50}{X_s} = \mp 229\,\Omega. \quad (1.14)$$

There are two possible configurations:

- **High-pass:** X_s is chosen negative (a capacitor), and X_p positive (an inductor), as in figure 1.4 (b). Then

$$
\begin{aligned}
X_s &= -218\,\Omega = \frac{1}{2\pi f C} && (1.15)\\
\text{whence } C &= 73 \times 10^{12}\ \text{F} \ = 73\ \text{pF} && (1.16)\\
X_p &= +229\,\Omega = 2\pi f L && (1.17)\\
\text{whence } L &= 3.64 \times 10^{-6}\ \text{H} \ = 3.46\,\mu H && (1.18)
\end{aligned}
$$

- **Low-pass:** If X_s is an inductor and X_p a capacitor, as in figure 1.4 (c), we similarly show that

$$
\begin{aligned}
L &= 3.64\,\mu H && (1.19)\\
C &= 69\ \text{pf} && (1.20)
\end{aligned}
$$

Note that the values of L and C required are quite different in each case! Figure 1.4 (b) is a high-pass filter because as $f \to 0$, no power can be transmitted into the load through the series capacitor. Similarly, figure 1.4 (c) is a high-pass filter because as $f \to \infty$, the series inductive reactance also tends to infinity, blocking power into the load.

Both configurations are useful, depending on whether we want to reduce the effects of unwanted low-frequency (like hum, DC drifts) or high-frequency (like harmonic distortion) signals respectively.

1.3 Disadvantages of L Matching Networks

L networks are simple to design, but they have problems.

1. Both the inductor and capacitor have to be the **exact** values required for matching. Since it is impossible in practice to wind an inductor to an exact value, some trial and error is necessary.
2. If the two resistances to be matched have widely different values, the frequency range over which a match can be obtained is very narrow.
3. A different topology is required depending on whether the load resistance is higher or lower in value than the source resistance. (That is, the **series** element always has to be associated with the **lower** resistance).

For these reasons, **Pi** or **Tee** matching sections are usually chosen rather than **L** sections.

1.4 Pi Matching Networks

Figures 1.5(a) and (b) show low-pass and high-pass versions of the **Pi** matching network. The high-pass version, having two inductors, is never used in practice, since inductors are inherently lossy, and we prefer to have as few as possible.

Pi network design commences by considering the **Pi** to be two L networks back-to-back, as in Figure 1.5(c). The left-hand end of the **Pi** is to be matched to resistance R_1 and hence the resistance seen looking into the left-hand end of the **Pi** should also be R_1. Similarly the right-hand end of the **Pi** is to be matched to resistance R_2, so that the resistance looking into the right-hand end of the **Pi** should be R_2. The series reactance of the **Pi** network is composed of the series elements X_{s1} and X_{s2} of the two L sections in series.

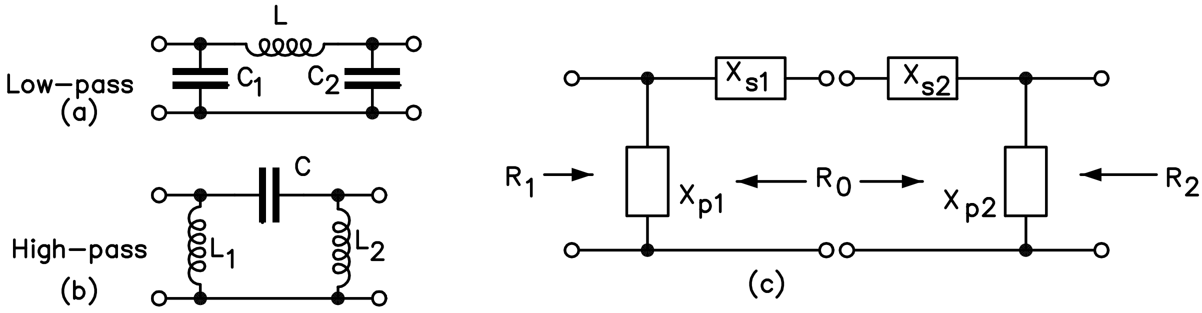

Figure 1.5: (a) Low-pass **Pi**. (b) High-pass **Pi**. (c) Decomposition into two L networks.

We design the left and right L sections separately. Each is matched to a "central, virtual" resistance R_0, which we shall call the **common or "characteristic" impedance**. This is a **free parameter**. The only limitation on R_0 is that it must be **smaller** than **both** R_1 and R_2. This is in order that the realizability condition given above can be satisfied for both L sections. The required reactance values can be written by inspection, using the L matching design expressions.

The design procedure for a **Pi** network to match resistances R_1 and R_2 thus is

1. Choose R_0 so that $R_0 < R_1$ and $R_0 < R_2$.
2. Calculate $X_{s1} = \pm\sqrt{R_0(R_1 - R_0)}$ and $X_{s2} = \pm\sqrt{R_0(R_2 - R_0)}$, using the **same** sign for both.
3. Calculate $X_{p1} = -R_0R_1/X_{s1}$ and $X_{p2} = -R_0R_2/X_{s2}$.
4. The series reactance of the **Pi** network is $X_s = X_{s1} + X_{s2}$.

1.4.1 Example 1.2

Design a low-pass **Pi** matching network to match a linear source of $50\,\Omega$ to a resistance of $1000\,\Omega$ at a frequency of 10 MHz.

Suppose that we choose $R_0 = 40\,\Omega$. Since we want a low-pass filter, X_{s1} and X_{s2} should both be chosen to be positive in order to be inductors. We find that

$$X_{s1} = \sqrt{40(50-40)} = 20\,\Omega \tag{1.21}$$

$$X_{s2} = \sqrt{40(1000-40)} = 196\,\Omega \tag{1.22}$$

$$X_{p1} = -\frac{40 \times 50}{20} = -100\,\Omega \tag{1.23}$$

$$X_{p2} = -\frac{40 \times 1000}{196} = -204\,\Omega \tag{1.24}$$

$$\text{and } X_s = X_{s1} + X_{s2} = 216\,\Omega \tag{1.25}$$

Calculating the element values at the operating frequency using $X = 2\pi fL$ for an inductor and $X = -1/(2\pi fC)$ for a capacitor, we find that $L = 3.44\,\mu$H, $C_1 = 159\,$pF and $C_2 = 78$ pF.

Note that there are now an **infinite** number of solutions, corresponding to different possible choices of R_0. This makes **Pi** matching networks very versatile, since in practice it allows one element, the inductor, to be (within reason) **fixed**, while the other two are varied to obtain a match.

In practice the inductor is usually variable in steps, with a series of taps along it enabling any number of sections between taps to be selected. Matching is then achieved using continuously variable capacitors for C_1 and C_2, and adjusting each in turn until a match is obtained. As a result,

- An inductor having an accurately specified value is **not** required,
- the **same** configuration can be used to achieve a match to both lower and higher values of load resistance.
- Different values of R_0 correspond to matching conditions having different degrees of selectivity. In practice this gives some control over the "reasonable" matching bandwidth, as often a "close" match is good enough. As R_o decreases, the bandwidth also decreases.

1.5 Power lost in a Pi Matching Network

The elements of matching networks are designed as pure reactances in order to have no power dissipation within the matching network. However real reactive components are lossy, and so will dissipate some power. Inductors are typically one or two orders of magnitude more lossy than capacitors, and it is therefore usual to assume that all lost power is dissipated in the inductor(s). We shall consider the power lost in the inductor of a low-pass **Pi** network.

If the proportion of power lost in the matching network is small (say less than 10 %) we can estimate it using a **pertubation** method. In order to do this we

1. Design the network as usual, and estimate the *rms* current flowing in the (lossless) inductor.
2. Assume that this current is essentially unchanged by the insertion of a small loss resistance in series with the inductor.
3. Find the power dissipated by this current flowing through the actual loss resistance of the inductor.

Consider Figure 1.6 (a), which shows the **left-hand** L section of a low-pass **Pi** matching network which is connected to a linear source on the left-hand (input) side. Recall that the network has been designed so that the right-hand L section of the **Pi** network and the load resistor can be replaced by the common impedance, R_0, without changing the conditions in the left-hand L section, and so we can use Figure 1.6(a) to find the current through the inductor.

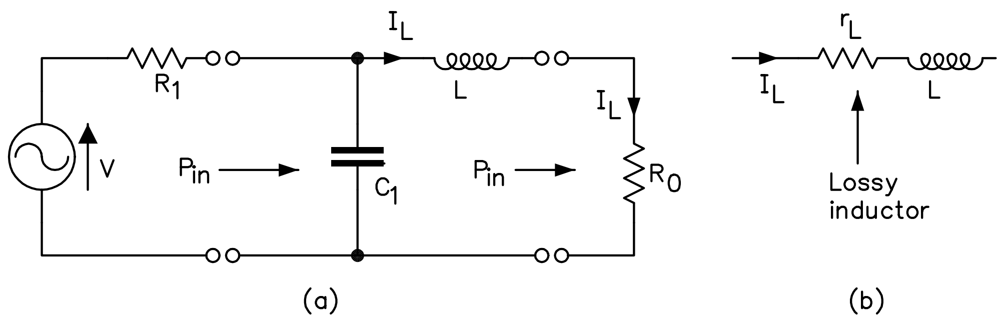

Figure 1.6: (a) The left-hand section of a low-pass **Pi** matching section. (b) The lossy inductor.

Assume initially that the network is lossless and suppose that the power fed into the network from the linear source is P_{in} as shown. This will be equal to the power dissipated in the common impedance, R_0. Since the **same** *rms* current, I_L, flows in the series inductance as in R_0, we see that

$$P_{in} = I_L^2 R_0 \tag{1.26}$$
$$\text{so } I_L^2 = \frac{P_{in}}{R_0} \tag{1.27}$$

Although we have evaluated this current using the left-hand L section alone, this is **also** the current flowing through the total central inductor, as constituent inductors L_1 and L_2 are connected in series.

We now insert a small loss resistance, r_L, in series with the inductor, as shown in Figure 1.6(b), and assume that I_L is essentially **unchanged** by this insertion. If the two capacitors are lossless, this resistance will be the only source of power loss. Thus,

$$P_{lost} \approx I_L^2 r_L \tag{1.28}$$

Let Q_L denote the Q factor of the inductor, defined by $Q_L = X_L/r_L$, then $r_L = X_L/Q_L$. Substituting this into Eq. (1.28) yields

$$P_{lost} \approx \frac{I_L^2 X_L}{Q_L} = \frac{P_{in} X_L}{R_0 Q_L} \tag{1.29}$$

Now express X_L in terms of the terminating resistances R_1 and R_2:

$$X_L = X_{s1} + X_{s2} = \sqrt{R_0(R_1 - R_0)} + \sqrt{R_0(R_2 - R_0)} \tag{1.30}$$
$$\text{so } P_{lost} \approx \frac{P_{in}}{R_o Q_L}\left[\sqrt{R_0(R_1 - R_0)} + \sqrt{R_0(R_2 - R_0)}\right] \tag{1.31}$$
$$= \frac{P_{in}}{Q_L}\left[\sqrt{\frac{R_1}{R_0} - 1} + \sqrt{\frac{R_2}{R_0} - 1}\right] \tag{1.32}$$

In practice, this expression is always pessimistic (i.e., it errs on the conservative side, can you see why?) and is usually within 5 % of the correct answer, which can be easily determined by running a numerical simulation with a modelling program such as `LTspice`.

1.5.1 Example 1.3

Estimate the power lost in the matching network of example 1.2, assuming that the inductor $Q = 100$, and 200 Watt is being fed into the matching network from the source.

For this problem, $R_1 = 50\,\Omega$, $R_2 = 1000\,\Omega$ and $R_0 = 40\,\Omega$.

Hence

$$P_{lost} \approx \frac{P_{in}}{Q_L}\left[\sqrt{\frac{R_1}{R_0} - 1} + \sqrt{\frac{R_2}{R_0} - 1}\right] \tag{1.33}$$
$$= \frac{200}{100}\left[\sqrt{\frac{50}{40} - 1} + \sqrt{\frac{1000}{40} - 1}\right] = 10.8 \text{ Watt} \tag{1.34}$$

1.6 Tee matching networks

These have the high-pass and low-pass configurations shown in Figure 1.7(a) and (b) respectively. A **Tee** is obtained by placing two L sections back-to-back, as in Figure 1.7(c). Again we define a common impedance, R_0, as that impedance in the middle to which each of the two L sections match. By the realizability condition for the L networks, R_0 must be **greater than** both R_1 and R_2. Once again, the design can be done using the L section results.

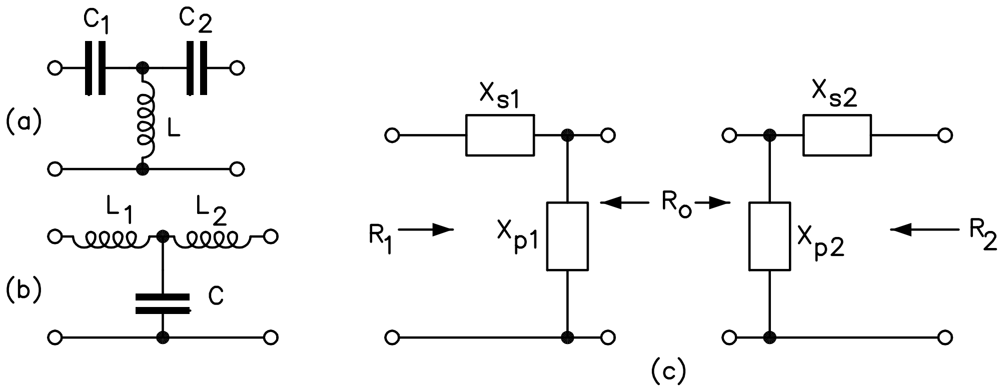

Figure 1.7: (a) High-pass and (b) low-pass **Tee** matching sections. (c) Decomposition of the **Tee** into two back-to-back L sections.

The design procedure for a **Tee** network to match resistances R_1 and R_2 is:

1. Choose R_0 so that $R_0 > R_1$ and $R_0 > R_2$.
2. Calculate $X_{s1} = \pm\sqrt{R_1(R_0 - R_1)}$ and $X_{s2} = \pm\sqrt{R_2(R_0 - R_2)}$, using the **same** sign for both.
3. Calculate $X_{p1} = -R_0 R_1 / X_{s1}$ and $X_{p2} = -R_0 R_2 / X_{s2}$.
4. The parallel reactance of the **Tee** network is then $X_p = X_{p1} \parallel X_{p2} = X_{p1}X_{p2}/(X_{p1} + X_{p2})$.

1.6.1 Example 1.4

Design a high-pass **Tee** section to match a 50 Ω linear source to a 1000 Ω load at a frequency of 10 MHz.

Since R_0 must be greater than R_1 and R_2, we can choose $R_0 = R_1 + R_2 = 1050\,\Omega$. Then choosing negative signs for the series reactances so that they represent capacitors,

$$X_{s1} = -\sqrt{50(1050 - 1000)} = -223.6\,\Omega \tag{1.35}$$

$$X_{s2} = -\sqrt{1000(1050 - 1000)} = -223.6\,\Omega \tag{1.36}$$

$$X_{p1} = -\frac{50 \times 1050}{-223.6} = 234.8\,\Omega \tag{1.37}$$

$$X_{p2} = -\frac{1000 \times 1050}{-223.6} = 4687\,\Omega \tag{1.38}$$

$$\text{and } X_p = X_{p1} \parallel X_{p2} = 223.6\,\Omega. \tag{1.39}$$

Notice that all reactances have the **same magnitude**, a remarkable result! We shall persue this later. The component values required for the design are found from

$$-\frac{1}{2\pi f C_1} = -\frac{1}{2\pi f C_2} = -223.6 \tag{1.40}$$

$$\text{giving } C_1 = C_2 = \frac{1}{2\pi \times 10^7 \times 223.6} = 71\text{pF} \tag{1.41}$$

$$2\pi f L = 223.6 \tag{1.42}$$

$$L = \frac{223.6}{2\pi \times 10^7} = 3.6\,\mu\text{H} \tag{1.43}$$

1.7 Power lost in a Tee network

The **high-pass Tee** network is usually preferred in practice since it has only one inductor. A perturbation method can also be used to estimate the power lost in this inductor. We proceed as follows:

1. Design the **Tee** assuming lossless components.
2. Consider the **Tee** decomposed into its two L sections. The right-hand L section and the load is now replaced by the common impedance R_0.
3. The voltage across the inductor V_L is equal to the voltage that would be developed across R_0. Since the power developed in R_0 is equal to the input power, $P_{\text{in}} = V_L^2/R_0$.
4. Assume that the voltage across the inductor is unchanged when it is replaced with the lossy inductor (this is where the approximation comes in).
5. Using this estimated voltage, calculate the resulting current through the inductor.
6. Find the power dissipated by this current passing through r_L, the inductor loss resistance.

$$\text{let } V_o = \text{the voltage developed across } R_o\text{, and hence the inductor.} \tag{1.44}$$

$$P_{in} = \frac{V_o^2}{R_o} \tag{1.45}$$

$$\text{so } V_o^2 = P_{in} R_o \tag{1.46}$$

$$I_o = \text{the resulting current flowing in the inductor } X_L. \tag{1.47}$$

$$\text{then } I_o = \frac{V_o}{X_L} \tag{1.48}$$

$$P_L \approx I_o^2 r_L = \frac{V_o^2}{X_L^2}\frac{X_L}{Q_L} = \frac{V_o^2}{X_L Q_L} = \frac{P_{in} R_o}{X_L Q_L} \tag{1.49}$$

$$\text{where } Q_L = \text{the quality factor of the inductor.} \tag{1.50}$$

$$\frac{1}{X_L} = \frac{1}{X_{C1}} + \frac{1}{X_{C2}} = \frac{X_{s1}}{R_o R_1} + \frac{X_{s2}}{R_o R_2} \tag{1.51}$$

$$= \frac{\sqrt{R_1(R_o - R_1)}}{R_o R_1} + \frac{\sqrt{R_2(R_o - R_2)}}{R_o R_2} \tag{1.52}$$

$$= \frac{1}{R_o}\left[\sqrt{\frac{R_o}{R_1} - 1} + \frac{1}{R_o}\sqrt{\frac{R_o}{R_2} - 1}\right] \tag{1.53}$$

$$\text{substituting, } P_L \approx \frac{P_{in}}{Q_L}\left[\sqrt{\frac{R_o}{R_1} - 1} + \sqrt{\frac{R_o}{R_2} - 1}\right] \tag{1.54}$$

This is the same expression as for the **Pi** network except that the fractions under the square roots are **inverted**. It is easy to remember which is which, since in both cases the fractions must be greater than one in order to make the square roots real. For the **Tee** network, R_0 is greater than both R_1 and R_2 and so R_0 should appear in the numerators. For the **Pi** network, R_0 is in the denominator since it is less than both R_1 and R_2.

1.8 The "Equal reactance" Design Algorithms

In Example 1.4 we found that it was possible to design a **Tee** network where the three reactances had equal absolute values when the common impedance R_0 was chosen equal to $R_1 + R_2$. This is true in general, and the value of the reactances for this design is

$$|X_s| = |X_p| = \sqrt{R_1 R_2} \tag{1.55}$$

Even more surprisingly, the same choice of reactances works for designing a **Pi** network, provided that we choose R_0 to be equal to $R_1 \parallel R_2$. Figures 1.8(a) and (b) show the resulting low-pass **Pi** and high-pass Tee networks where $X = \sqrt{R_1 R_2}$.

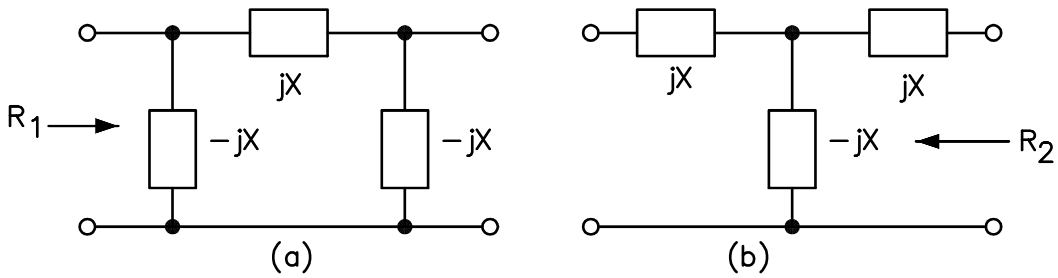

Figure 1.8: (a) The simplest **Pi** and (b) **Tee** matching sections designed using the equal reactance matching algorithm.

This gives a simple method of designing matching networks since we don't have to choose R_0. We now prove this for the **Tee** circuit. Proving the **Pi** result is left as a problem at the end of the chapter.

Consider figure 1.8 (b). We require

$$R_1 = jX - \frac{jX(jX + R_2)}{jX - jX + R_2} \tag{1.56}$$

$$= jX - \frac{(jX)^2 - jXR}{R_2} \tag{1.57}$$

$$= jX + \frac{X^2}{R_2} - jX = \frac{X^2}{R_2} \tag{1.58}$$

$$\text{whence } X = \sqrt{R_1 R_2} \tag{1.59}$$

1.9 Matching to Reactive Loads

It's unusual (in fact incredible) to be presented with a completely resistive load. Furthermore, a small change in frequency will often change the load reactance markedly while having little effect on its resistance. The classical example is a resonant radio antenna, which can be designed to be completely resistive (normally $5\,\Omega - 500\,\Omega$) at the design frequency. Below this frequency the antenna has negative reactance (looks like a lossy capacitor) and above it has positive reactance. Radio systems designed for operation over a **band** of frequencies must therefore include a matching system which allows this reactive component to be cancelled and adjusted over small frequency changes.

Fortunately, matching to reactive loads is simple with both **Pi** and **Tee** systems. The procedure is:

1. When designing a **Pi** matching section, represent the load by its **parallel** equivalent circuit (i.e., a resistance in parallel with a reactance). When designing a **Tee** matching section, use the **series** equivalent circuit.

2. Design the matching network to match the **resistive** component of the load.

3. Incorporate the **reactive** portion of the load by combining it with the **output reactance** of the matching section.

1.9.1 Example 1.5

Design a high-pass **Tee** matching section to match a $1\,\Omega$ linear source to two loads having impedance operators (a) $Z_2 = 4 + j$ and (b) $Z_2 = 4 - j$.

Writing the load impedance as a series combination $Z_2 = R_2 + jX_2$, we use the equal reactance algorithm to match $R_1 = 1\,\Omega$ to the resistive part of the load $R_2 = 4\,\Omega$.
This gives $X_{s1} = X_{s2} = -\sqrt{1 \times 4} = -2\ \Omega$ and $X_p = \sqrt{1 \times 4} = 2\ \Omega$. From Figure 1.9 we see that instead of setting $X_{C1} = X_{s1}$, we must set the **sum** of X_{C2} and X_2 to X_{s2}. Hence

$$X_{C2} = X_{s2} - X_2$$

In both cases, $X_{C1} = -2\ \Omega$ and $X_L = 2\ \Omega$. For load (a), $X_2 = 1$ and so $X_{C2} = -3\ \Omega$. Similarly for load (b), $X_2 = -1$ and so $X_{C2} = -1\ \Omega$.

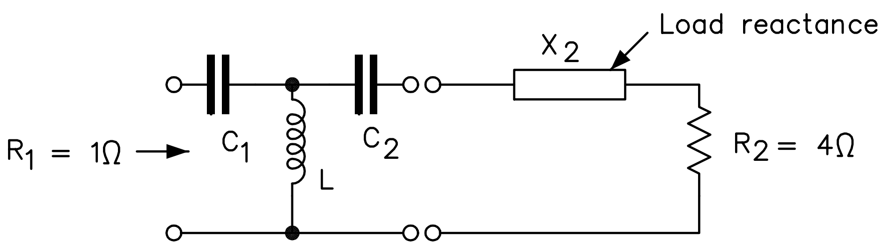

Figure 1.9: Matching a high-pass **Tee** section to a reactive load.

To design a **Pi** matching section, we use a similar procedure. However, since X_{p2} is in **parallel** with the load, it is easier to combine the load and output reactances in parallel. Hence if the load impedance is given in series form, it must first be converted to its equivalent parallel form.

In both cases, if the load reactance is the **same sign** as the output section reactance, there is an upper limit to the load reactance which can be matched, $X_{lim} = X_{s2}$. Above this limit, the output section reactance must be changed to one of opposite sign (a capacitor becomes an inductor). Alternatively, it may be possible to use a different design based on a different common impedance to avoid replacing the output reactive element with one of opposite sign.

There are no simple analytic algorithms for designing **lossy** matching sections. In practice, we design a **lossless** section and make empirical adjustments, usually small, until matching is acceptable.

1.10 Problems on Chapter 1

1. A Thevenin source having output resistance R_1 is to be power matched to a resistive load R_2. Show that reactive **Tee** and **Pi** networks can accomplish this matching by choosing

$$\begin{aligned} R_0 &= R_1 + R_2 \text{ (for the \textbf{Tee} network)} \\ R_0 &= R_1 \parallel R_2 \text{(for the \textbf{Pi} network)} \end{aligned}$$

   The resulting networks can then be constructed with **all** elements having reactances of the same magnitude, given by

$$X = |X_s| = |X_p| = \sqrt{R_1 R_2}.$$

2. Use this "equal reactance" algorithm to find the component values of a low-pass **Pi** network which will match, at a frequency of 3.5 MHz, a 50 Ω resistive source to

   (a) a 2000 Ω resistive load,

   (b) a 5 Ω resistive load,

   (c) a load impedance of $100 + j100\ \Omega$.

3. The input power to the above networks is 100 Watt. If inductors can be wound having $Q_L = 100$, find the power lost in the network in each case.

4. Figure 1.10 shows an "**SPC transmatch**", developed for matching high frequency radio transmitters to antennas. Both capacitors C_2 on the output side are ganged on a common shaft, and therefore have the same value, C_2. Show that the design equations can be written

$$\begin{aligned} X_{C1} &= -\sqrt{R_1(R_0 - R_1)} \qquad X_{L1} = -\frac{R_1 R_0}{X_{C1}} \\ X_{C2} &= -\sqrt{R_2(R_0 - R_2)} \qquad X_{L2} = -\frac{X_{C2}}{2 - (R_2/R_0)} \\ X_L &= X_{L1} \parallel X_{L2} \qquad \text{where } R_0 > R_1 \text{ and } R_0 > R_2/2 \end{aligned}$$

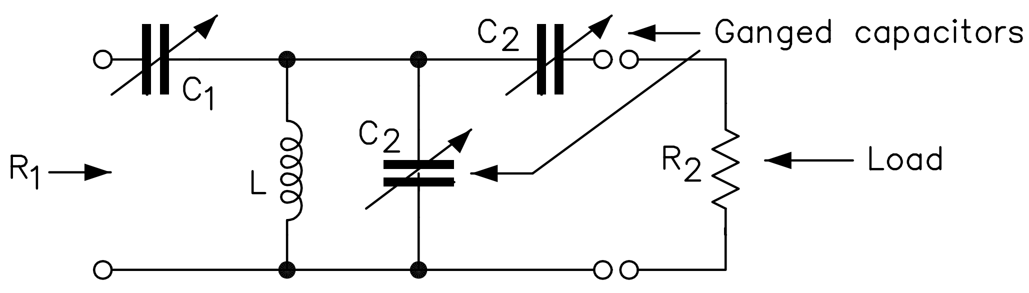

Figure 1.10: The **SPC** transmatch

5. Derive the L matching equations using the transformed circuit of figure 1.2(b).