A TRANSIENT ANALYSIS OF AN IMPEDANCE TRANSFORMING DEVICE

(The Quarter Wave Transformer)

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Dedication

This work is dedicated to Sukru Cafer Durusel, who gave me the book by Goldman and was a very dear friend and mentor. Sukru was born in Russia with the surname Rustamberghof, fled to Turkey during the time of the revolution, was educated at the University of Warsaw and at the Sorbonne in Paris. He spoke Russian, Polish, French, Turkish and English. He was my mentor from late 1960 until early 1965. He died suddenly in July 1965. I am forever indebted to him.

The Quarter Wave Transformer

Reference (a): "Transformation Calculus and Electrical Transients", Stanford Goldman.

OBJECTIVE:
To analyze the behavior of the system under startup transient conditions with a view towards understanding the nature of the impedance transformation that takes place, at Point X, in a quarter wave line (Line 2) during the transient conditions.

SYSTEM DESCRIPTION:

```
            Line 1    Point X   Line 2
                          |
GEN_______________________|_____________LOAD
                          |

50 ohm Gen  50 ohm Line    150 ohm line  450 ohm load
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Generator: 50 ohm Generator (Develops 1 watt into matched load)
Line 1: 50 ohm Line (Zo = Ro = 50 ohms) [The discussion to follow assumes Line 1 to be of sufficient length that no reflections reach the generator until the transient phase is for all practical purposes completed.]
Point X: Transition from Line 1 to Line 2
Line 2: 150 ohm Line (Zo = Ro = 150 ohms) – a lossless and distortionless line of length one quarter wave in length at the frequency of interest.
Termination: 450 ohm load

TRANSIENT BEHAVIOR OF TRAVELING WAVES:
It has been shown in Reference (a) that signals introduced to a lossless transmission line that is terminated in a non-reactive source and load (schematic above), will propagate without change in waveform and with a velocity of $\frac{1}{\sqrt{LC}}$. That is, the voltage at an arbitrary distance x along the direction of propagation is exactly similar to the voltage at x = 0, with no attenuation or change of wave form, except that there is a time lag of $x\sqrt{LC}$. In this paper we assume that in order to properly assess the effects of waves traveling on the line in question, it is only necessary to account for those waves that are physically present at the point of interest. For example, a wave-train that is traveling to or away from the point of interest is of no concern if it is completely past the point or has not yet arrived at the point of interest. This assumption is of particular importance when summing the instantaneous voltages or currents of all waves present in order to calculate the

impedance at a point. For purposes of determining the direction of energy flows, the incident wave is considered to be **positive** energy flow, while the reflected wave is **negative** energy flow.

Whereas a rigorous transient analysis should be presented in terms of an arbitrary function of time as the excitation function, this analysis will be restricted to a simple harmonic function of time in the form of a sinusoidal voltage at a single frequency. Also, we assume that prior to and at t = 0, there is no voltage or current in the system. This was assumed by Goldman in order to eliminate the initial conditions and simplify the mathematical model.

As a result of our simplifications we take the liberty of assuming that we can use the peak values and their RMS equivalents of the sinusoidal components for the purpose of discussing power levels and impedances, just as if we were dealing with phasors instead of instantaneous voltages and currents.

VOLTAGES AND CURRENTS:
All voltages and currents in this paper will be expressed in **peak** values rather than RMS values. By carefully choosing our points of observation, we will avoid phase angle differences other than multiples of Pi. Therefore, Power levels will be computed as peak voltage times peak current divided by 2. Impedances will be calculated using peak voltage divided by peak current.

REFLECTION COEFFICIENTS:
We will use reflection coefficients M and N as they are defined by Goldman. See Rule III. The classical reflection coefficient ρ will be used where appropriate. In each case reflection coefficient used will be clearly indicated so as to avoid any ambiguity.

EVENT 1:
At time t = 0, the Generator begins generation of a sinusoidal wave (from quiescent conditions) at the frequency for which Line 2 is one quarter wavelength. The generator sees an SWR of 1:1.

EVENT 2:
The sine wave arrives at Point X and is partially reflected by the impedance discontinuity that exists at that point (see Note 3). The impedance discontinuity is defined by the reflection coefficient N.

Line 1 has a characteristic impedance of 50 ohms, and at this particular event, Point X presents an impedance of 150 ohms (see Note 3), and as a consequence of that impedance mismatch, the reflection coefficient N = -0.5. (See Rule III)

A reflected wave now originates in Line 1 in the region just to the left of Point X and coexists with the incident wave in that region. In that region where both waves exist, the incident wave stands at a level of 1.0 watts and the reflected wave is at 0.25 watts. This satisfies all constraints relative to the SWR of 3:1, the reflection coefficient of -0.5 and the net energy flow into Line 2 of 0.75 watts. In Line 2, an incident wave originates at a level of 0.75 watts. (See Rule III). The generator still sees an SWR of 1:1.

The following diagram illustrates the conditions that exist after Event 2 and prior to Event 3:

```
            Line 1     Point X    Line 2
                          |
GEN_WWWWWWWWWWWWWWWWWWWWWW|WWWWWWW______LOAD
                   WWWWWWW|

50 ohm Gen  50 ohm Line    150 ohm line  450 ohm load

WWW   Incident Wave
WWW   Reflected Wave
```

EVENT 3:
At Event 2 plus 90 electrical degrees (see Note 1 below) the wave traveling on Line 2 arrives at the 450 ohm termination and is partially reflected with a reflection coefficient N = -0.5. As a result of the partial reflection occurring at the 450 ohm Termination, there will again be a distribution of the power with 75% of the incident power being dissipated in the 450 ohm termination and 25% being reflected. In the region immediately to the left of the 450 ohm termination a reflected wave originates toward Point X.

In the region of Line 2 where both the incident wave and the reflected wave coexist, the incident energy flow is .75 watts and the reflected wave is 0.1875 watts. As in Line 1 above, all constraints relative to an SWR of 3:1 and a reflection coefficient of -0.5 are met by these power levels, including a net energy flow of 0.5625 watts into the 450 ohm load. The generator still sees an SWR of 1:1.

The following diagram illustrates the conditions that exist after Event 3 and prior to Event 4:

```
            Line 1     Point X    Line 2
                          |
GEN_WWWWWWWWWWWWWWWWWWWWWW|WWWWWWWWWWWWWLOAD
          WWWWWWWWWWWWWWWW|          WWW

50 ohm Gen  50 ohm Line    150 ohm line  450 ohm load

WWW   Incident Wave
WWW   Reflected Wave
```

In Line 1, the reflected wave is still traveling from Point X towards the generator.

We should now pause to look more closely at the coexistence of the reflected wave and the incident wave in Line 2. The reflected energy flow at each point subtracted from the incident energy flow at each point equals the net energy flow at that point. The consequences of this are that at each point along Line 2, the impedance seen at that point is completely consistent with the sum of the instantaneous voltages and the sum of the instantaneous currents of all traveling waves at that point (see Notes 2 and 4). Additional analysis of these power levels, given that we know the characteristic impedance of the line, also allows us to calculate the impedance seen at any point on Line 2. The impedance calculated is the "impedance seen at a point" - a function of the characteristics of the line, the distance to the termination and the impedance of the termination. (See Rule III)

Contrast this with the advance of an incident wave which is traveling towards an impedance discontinuity but has not yet reached it. In that case, the traveling wave sees only the characteristic impedance of the line, which appears to be of infinite length or terminated in its characteristic impedance. (See Rules I & II)

Returning to Line 1, the incident and reflected power levels in that line are dependent upon whether we are looking at a portion of Line 1 in which there is a reflected wave, or not. Immediately to the left of Point X there is a reflected wave traveling toward the generator. Meanwhile, the generator continues to deliver 1.0 watts to Line 1. The reason that the power entering Line 1 is still 1.0 watts is that the generator does not yet see an SWR that would cause a power reduction - the reflection from Point X has not yet reached the generator. The generator still sees an SWR of 1:1. Until the reflected energy arrives at the generator, the generator sees a 50 ohm load and continues to deliver 1.0 watts to the line. Eventually, that situation will change as the reflected energy from Point X arrives at the generator.

EVENT 4:
At Event 3 plus 90 electrical degrees (180 degrees since Event 2), the reflected wave traveling to the left on Line 2 arrives at Point X. At this same time the reflected energy on Line 1 is still traveling towards the generator but has not yet arrived.

The impedance seen looking to the right anywhere on Line 2 is now seen to have changed by virtue of the presence of the reflected wave all along that line. The impedance at Point X is no longer 150 ohms. The new impedance at that point can be determined in either of two ways - a) by combining the incident and reflected waves at any point of interest and computing the ratio of total voltage to total current , or b) by using a Smith Chart or an impedance transformation formula based on length of line, terminating impedance and characteristic impedance.

The energy flow from Line 1, passing into Line 2, is 0.75 watts. The reflected voltage wave in Line 2 has a magnitude of 7.5 volts peak.

$$E = \sqrt{PZ_0} = \sqrt{(0.1875)(150)} = 5.3v.rms = 7.5v.peak$$

The reflected current wave is 0.05 amps peak. Note that this satisfies the constraint that the Voltage to Current ratio in Line 2 must be150 ohms for the reflected wave. Also for that same incident power level, the incident voltage wave in Line 2 has a magnitude of 15 volts peak, and the incident current wave in Line 2 is .0.1 amps peak. This satisfies the constraint that the incident voltage/current ratio in Line 2 must be150 ohms.

The incident and reflected waves combine to produce a (summed voltages/summed currents) ratio representing the impedance at that point, provided that the relative phase angles of each wave are properly accounted for (see Note 4 and Rules I through IV). This will be covered in detail under the sub-topic “Combining the Wave Energies on Line 2”.

However, something else happens at Event 4 – the reflected voltage wave arriving at point X sees an impedance discontinuity at point X due to the fact that the main line impedance is 50 ohms.

Therefore, the reflection coefficient for a voltage wave arriving from the load M = 0.5, which causes a re-reflection voltage wave directed incident to the 450 ohm load, and that wave has a voltage of 3.75 volts peak and a current of 0.025 amps peak. Note that the power level in the reflected wave is 0.18725 watts (7.5 volts peak times 0.05 amps peak / 2). Note also that due to the reflection coefficient at point X seen by the arriving reflected wave, that the power level passing through point X from the load side and in the direction of the generator will be only three fourths of that, or 0.1404 watts. The generator still sees an SWR of 1:1.

COMBINING THE WAVE ENERGIES ON LINE 2:

In order to properly combine all of the waves present on Line 2 after Event 4, it is necessary to consider the phase angles for the waves on Line 2, which are a function of the total distance traveled in electrical degrees. At any point to the right of the leading edge of the reflected wave, the phase angle of the reflected wave and the phase angle of the incident wave will be a function of the distances traveled to that point. The phase difference is a minimum of 0 degrees (directly at the 450 ohm termination, at which point the two waves have been traveling as one) and is a maximum of 180 degrees (immediately to the right of Point X, at which point the incident wave and the reflected wave have travel distances different by twice the length of Line 2).

There is one additional consideration, and that is that when a line is terminated with an impedance *greater* than its characteristic impedance, the reflected *current* wave suffers a 180 degree phase shift at the point of reflection. Conversely, when the terminating impedance is *less* than the characteristic impedance, the reflected *voltage* wave suffers a 180 degree phase shift at the point of reflection (See Rule VI). Remembering that Line 2 is a 150 ohm line terminated with a 450 ohm termination, it is readily determined that immediately to the right of Point X, the incident and reflected voltages sum to (15 - 7.5 + 3.75 = 11.25) volts peak, while the incident and reflected currents sum to (0.1 + .05 + .025 = 0.175) amps, indicating an impedance of 11.25 / 0.175 = 64.3 ohms.

Note that Line 1 is now terminated in an impedance much closer to its characteristic impedance than during the time between Events 1 and 4 (See Rule III). The consequences of this are significant.

COMBINING THE WAVE ENERGIES ON LINE 1:

With Event 4, a reflected wave in Line 1 might be expected to continue traveling to the left of Point X and in the direction of the generator. Were it not for the dramatic change in conditions that occurred precisely at the moment the reflected wave arrived at Point X, the level of reflected energy in Line 1 might continue indefinitely. However, the reflection that was caused by the mismatch at Point X is dramatically reduced by Event 4. That level of reflection existed only for the period of time from Event 1 until immediately prior to Event 4, because during that period and that period only, the impedance seen at Point X was *not* equal to the characteristic impedance of Line 1. With Event 4 the impedance seen at Point X instantaneously changes with the arrival of the reflected wave from 150 ohms to 64.3 ohms, and the reflection of power from Point X reduces accordingly at that instant.

Adding to that reflected energy level however, is the pass-through of reflected energy from the load that is now arriving at point X at a power level of 0.18725 watts, of which 0.1404375 watts continues on through point X towards the generator with the difference of .0468125 watts being re-reflected toward the load.

Also remember that up until this time (Event 4) the incident power level in Line 1 was 1 watt and that 0.25 watts were being reflected back toward the generator from the impedance discontinuity seen at point X. If we now combine the voltages and currents of the wave energies in Line 1 immediately to the left of point X, and subsequent to Event 4, we obtain the following:
Combined voltages = 10 + 5 – 3.75 = 11.25 volts peak (No phase shifts apply to the 3rd wave other than its travel distance of 180 degrees)
Combined currents = 0.2 - 0.1 + 0.075 = 0.175 amps peak (Wave 2 is phase reversed at point X. Wave 3 has one phase reversal at the load which is cancelled out by its travel distance of 180 degrees)

Note that while these are different voltages and different currents at two different points on the two different lines immediately to the left and right of point X, the impedances calculated for those two different but adjacent points are essentially identical at 64.3 ohms.

In summary, the arrival of the first reflected energy at point X has caused the following:

- Dramatic reduction in the reflection of energy at the interface point X in Line 1
- Addition of a small level of energy reflected from the load and passed through the interface at point X and into Line 1.
- Abrupt reduction of the mismatch at point X.
- Addition of a small amount of energy incident to the load in Line 2 as a result of reflected energy being re-reflected at point X.
- Unless the reflected energy from point X has reached the generator, the generator still sees an SWR of 1:1.

OTHER EVENTS SUBSEQUENT TO EVENT 4:
It should be clear from the above sequence of events that a similar sequence will continue in which the small amount of re-reflected energy will eventually arrive at the load, be reflected and ultimately arrive again, significantly diminished, at point X, and that such sequential, stepwise events will cause the impedance seen looking toward the load at point X to asymptotically approach 50 ohms.

In fact, as shown in the summation below, after adding in the next arriving wave energy and also creating the concomitant re-reflection of that wave at point X, the impedance seen immediately to the right of point X is 53.22 ohms:
Combined voltages = 15 - 7.5 + 3.75 – 1.875 + 0.9375 = 10.3125 volts peak.
Combined currents = 0.1 + 0.05 + 0.025 + 0.0125 +0.00635 = 0.19375 amps peak.
10.3125 / 0.19375 = 53.22 ohms.

If this were a simple generator of 50 ohms with a load of 53.22 ohms, then the mismatch would be so negligible that 99.9% of the maximum available power would be absorbed in the load.

It can be shown that the same impedance value would be obtained using the voltage and current summation on the left side of point X.

As can be seen from the results of summing the first 5 terms of the series (3 incident wave and two reflected waves), the combined voltage is rapidly reducing to a limit of 10 volts peak while the combined current is rapidly rising to a limit of 0.2 amps peak for a net power level at steady state of 1 watt.

Please note that the summation of voltages and currents on each side of point X must be performed differently for each side of the discontinuity. In the case of the source side of point X, the voltages and currents are all different from their counterparts on the load side of point X, because the line impedance is 50 ohms in Line 1 and is 150 ohms in Line2. The summation that we have just performed immediately above was on the 150 ohm side of point X. This is easy to verify by simply dividing any voltage term by its corresponding current term and confirming that the ratio is 150 ohms.

INCIDENT AND REFLECTED TOTALS IN LINE 2:
With a total of six wave energies in Line 2, we would have the necessary data at hand to compute the sums of the first three incident wave energies and the first three reflected wave energies, and we could expect that with only those terms the result should be very close to steady-state conditions. To that end, we add the sixth term to the equations given above and compute, as follows:

Summing only the **incident** waves at point X and using the same individual wave energies from above and the added term we find that the total incident voltage, current and power are as follows:
Combined voltages = 15 + 3.75 + 0.9375 = 20 volts peak.
Combined currents = 0.1 + 0.025 +0.00635 = 0.13333 amps peak.
Multiplying peak voltage by peak current and dividing by 2 gives 1.3333 watts incident power.

Likewise, using only the **reflected** waves at point X, we find that the total reflected voltage, current and power are as follows:
Combined voltages = -7.5 - 1.875 - 0.46875 = -9.84375 volts peak.
Combined currents = 0.05 + 0.0125 + 0.003125 = 0.065625 amps peak.
Multiplying peak voltage by peak current and dividing by 2 gives -0.323 watts reflected power.

This solution is very close to the expected values, considering that we are using only 6 terms of the infinite series. The negative sign on the reflected voltage and power is consistent with the fact that we have declared our energy flow convention such that incident and reflected energy flows carry positive and negative signs in accordance with their direction of flow in the line.

The classical reflection coefficient may be calculated from the forward and reflected power levels using the formula $\rho = \sqrt{\frac{P_R}{P_F}} = \sqrt{\frac{0.323}{1.333}} = 0.492$.

The SWR may then be calculated from $SWR = \frac{1+|\rho|}{1-|\rho|} = 2.937$, which is close to the expected, steady-state value of 3:1

WHAT HAPPENS WHEN THE REFLECTED ENERGIES IN LINE 1 REACH THE GENERATOR?
In the discussion of Events 3 and 4 we noted that the reflected energies traveling toward the generator in Line 1 had not yet arrived at the generator. When those energies reach the generator terminals, the impedance seen by the generator changes instantly to a value consistent with the forward and reflected wave energies at the generator terminals. In other words, the generator sees an impedance that is determined by the total wave energies present at the generator terminals but is unaffected by wave energies that have not yet arrived. With a forward energy level of 1.0 watts and

a reflected wave energy level of 0.25 watts, the generator sees a reflection coefficient and SWR, as follows:

$$\rho = \sqrt{\frac{P_R}{P_F}} = \sqrt{\frac{0.25}{1.0}} = 0.5$$

$$SWR = \frac{1+|\rho|}{1-|\rho|} = 3.0$$

The generator reacts accordingly by developing a terminal voltage and current flow consistent with a load of 16.666 ohms (we are assuming, for purposes of discussion that the generator is an odd number of quarter wavelengths distant from point X, exactly). Under these conditions the generator's output falls to 5 volts peak and 0.3 amps peak or 0.75 watts. If the generator were an even number of quarter wavelengths distant, the output would be 15 volts peak and 0.1 amps peak for a total of 0.75 watts. Either way the generator no longer delivers the power into Line 2 that our simple model assumes. The effect of these transient conditions being seen by the generator is to slow the build up of energy flows in Line 2. However, it is clear that these transient effects on the generator are temporary, since the SWR on Line 1 is a temporary aberration lasting approximately one complete 360 degree cycle, depending on the actual length of Line 1.

The actual effect on the overall system is very dependant upon the length of Line 1. The longer Line 1, the more likely it is that the transient phase in Line 2 has completed by the time the generator output dips in reaction to the SWR seen. Conversely, a shorter Line 1 means that the generator reacts with less delay. In either case, such interactions cannot be modeled without significant increase in complexity. However, it will be instructive to play out this scenario to its logical conclusion. Again, assuming Line 1 is very long, the generator will see the 3:1 SWR for exactly 180 degrees of the electrical cycle. At the end of that period generator will see an abrupt and dramatic reduction in the SWR. The reason the effect lasts 180 degrees is the it takes that long for the first wave of energy in Line 2 to travel the length of the quarter wave transformer and back, which is the point at which the impedance at point X changed to 64.3 ohms for an SWR of 1.28:1. Therefore, at the end of the 180 degree period of SWR=3:1, the generator will see an SWR of only 1.28:1 and that condition will change until the effects of that change are seen at point X, which is another 180 degrees. In other words, after only two periods of 180 degrees each the SWR seen at the generator has changed from its original value of 1:1 to 3:1 and 180 degrees later to 1.28:1 and at the end of that 180 degree period the generator will see an even lower SWR such that it can be said that the generator is again seeing an SWR that is nearly 1:1.

Having characterized the SWR changes seen by the generator as having lasted at least 360 degrees before returning to a near 1:1 value, we should concern ourselves next with the question of how the generator reacts to that transient disturbance. Again, for purposes of discussion and in order to avoid complex interaction between various components of the system, we consider that this entire scenario plays out before any secondary effects are seen in the power being delivered to point X by the generator. The immediate effect of an increase in SWR seen by the generator is for the net generator output to decrease significantly, and that is exactly what must transpire during the first 180 degrees of the transient seen at the generator. Since the generator delivers 1 watt into a matched load, it can only deliver 0.75 watts net power to a load with SWR=3:1. In the subsequent 180 degree period, however, the SWR falls to 1.28 and the effect of that upon the generator is that it can deliver

0.984 watts to that load, and so on. For all practical considerations the transient has lasted for more than 360 degrees, but is essentially over at 540 degrees.

Next, we could study the effect on Line 2 when those generator anomalies reach Line 2. However, our premise was that Line 1 is long enough that such effects as are coupled from Line 2 back to the generator are also delayed to the point that they cannot in return affect Line 2 while the transient phase in Line 2 is ongoing. Therefore, when the dip in generator output finally reaches Line 2 and effectively causes everything in Line 2 to dip proportionately and for the same length of time as the generator output, it is of no concern to us in regard to our study of the transient phase in Line 2, because that phase is essentially completed already. So, through the simple artifice of making Line 1 sufficiently long, we can study the transient effects in the two lines in isolation.

If it were necessary to study such effects on a shorter line wherein there is significant coupling between the transient phase in Line 2 and the resulting effects on the generator, a computer simulation using numerical methods would be a viable approach. However, with the Goldman equations, we have a model in which Line 1 is shortened to an infinitesimally short length (zero), and both the transient phase solution and the steady-state solution are found for the same model using infinite convergent series.

A SIMPLIFIED MODEL (THE GOLDMAN EQUATIONS):
Although it may be possible to model the behavior of the more general system, that requires a significant increase to the complexity of the model. It is dramatically more practical to model a system with an infinitesimally short length for Line 1. If we make Line 1 so short that there is essentially no delay between the generator terminals at one end of Line 1 and point X at the other end of Line 1, the model degenerates to the very model that is treated by Goldman in Reference (a), Section 10-4. Once we make that simplification, we will obtain a solution that matches our step-by-step approach except that we need not be concerned any difference between what happens at point X and what the generator sees. There is no delay between the impedance changes at point X and the impedance seen by the generator.

Let us further compare the Goldman model against the long source line model to see what significant differences there may be. In the long source line model, the generator sees a perfect match and delivers its full output (1.0 watt) until Event 4 plus. In that same model, Line 2 initially operates at an incident power level of 0.75 watts and builds up to the full 1.0 watts delivered to the load at steady state. In the Goldman model, the generator initially delivers only 0.75 watts, since it initially sees a 3:1 SWR, and it builds up to the full 1.0 watt output at steady-state. In other words, insofar as the analyses of the transient phase in Line 2, the two models give the same result.

Goldman has provided us with the complete solution, and that solution is a convergent power series with the exact same values of voltages and currents in Line 2 as were generated in our intuitive, step by step analysis above. The reader should satisfy himself as to the validity of our results above by using the Goldman equations from Section 10.4 with a simple harmonic function of time (a sine wave) as the driving function. Goldman also develops the steady-state solution in his Appendix C.

STEADY-STATE:
The steady-state condition has a 1 watt net power level throughout the system, an SWR of 3:1 in Line 2, and Line 1 has an SWR of 1:1. Goldman's steady-state solution is also an infinite series. The primary difference in the two solutions is that the steady-state solution is able to employ

phasors, while the transient solution results in functions for instantaneous voltage and instantaneous current for any excitation function of time.

Note 1: TIME, VELOCITY AND LINE LENGTH

The relationships between velocity, time and electrical degrees for a lossless line is $\frac{\omega}{\beta} = c$ where ω is radian frequency, β is the phase shift constant for the line in radians per unit length of line, and c is the velocity of propagation, roughly 300,000,000 meters per second. For example, a line of approximately 10 meters length at 7.5 MHz (47,000,000 radians per second) has an electrical length of 1.57 radians, or 90 electrical degrees, and the signal travels this distance in 33 1/3 nanoseconds.

Note 2: IMPEDANCE SEEN AT A POINT ON THE LINE

Perhaps the most striking characteristic of transmission lines is the concept of impedance at a point on the line being a function of the distance to and the value of the terminating impedance - all of which is a direct result of the fact that the electrical parameters of a transmission line are distributed rather than lumped, and the fact that the velocity of signal propagation on transmission lines is finite. The impedance seen at a point on a transmission line is defined specific to a direction of *net* energy flow or a direction of view. In order to look into a line in a specific direction, the line is "virtually" cut at the point of interest and the impedance is defined as that impedance which would be seen looking into the line from the point of the virtual cut. The impedance seen at that point is determined by the line characteristics and the terminating load in the direction of observation. A Smith Chart may also be used for computing impedance seen at a point on the line. The sum of all traveling wave voltages, incident and reflected, at the point of interest divided by the sum of all traveling wave currents at that point will also define the impedance seen at that point on the line. As a consequence, measurements of actual voltages and currents on real transmission lines, whether using voltmeters, ammeters or oscilloscopes, will correlate or agree with the "impedance seen at a point". Contrast this with the concept of the impedance seen by each individual traveling wave as described in Note 3, below. Strictly speaking, impedance is not defined except at steady-state. In this paper a reference to an impedance seen at a point on the line presumes to be the ratio of PEAK voltage to PEAK current, taking into account the phase angle between them, just as if they were true phasor quantities.

Note 3: ENERGY FLOW AND IMPEDANCE SEEN BY INDIVIDUAL TRAVELING WAVES

The energy in a traveling wave arriving at an impedance discontinuity will be partially passed through and partially reflected as a function of the reflection coefficient. Both an incident and a reflected wave are created at an impedance discontinuity. The voltage/current relationship of an individual traveling wave will automatically conform to the characteristic impedance of the line at every point. Note that we are referring to the characteristic impedance of the line, not the "impedance seen at a point" discussed in Note 2, above. In other words, for a given flow rate of energy entering a line, the voltage and current are determined solely by energy flow rate (power) and the characteristic impedance of the line at that point. It should also be noted that while the individual traveling wave conforms to the characteristic impedance of the line in which it travels, it is also quite oblivious to the termination or source impedances. Contrast this concept with Note 2, above.

Note 4: PHASORS

In a transient analysis we are forced to deal with the instantaneous values of sine waves. Once steady-state conditions exist, then the voltages and currents can be represented as phasors, but until then, we are forced to consider the voltages and currents as functions of time rather than in their complex exponential phasor representations. For example, the function of time, $e(t) = A\cos(\omega \cdot t + \theta)$ is the Real component of the complex exponential phasor form. The sine wave has a radian frequency of $\omega = 2\pi f$ and a relative phase angle, theta, relative to some appropriate reference. To the extent that all sinusoidal functions in the system are at the same radian frequency, and the sinusoidal functions are all phase coherent, the phasor representations can be manipulated as complex quantities either in Cartesian or Polar coordinates. Recognizing that phasor representations are valid only at steady-state, Goldman conducted his anaysis of the transient phase in terms of instantaneous voltages and currents rather than using phasors. Nonetheless, in this paper, as was stated in Note 2, references to impedances during the transient phase will presume PEAK voltages divided by PEAK currents, taking into account the phase angle between them, just as if they were true phasor quantities.

DEFINITIONS:

Impedance Discontinuity - a junction or transition point joining a transmission line with a source, load or another transmission line. The discontinuity is trivial if the two entities have the same characteristic impedance.

Standing Waves or Combination of Traveling Waves - the sum of incident and reflected waves at a point on a line.

Energy Flow Balance - The energy flows into (+) and out of (-) a point or defined region not containing an energy source or sink must always sum to zero. (See also Rule IV).

RULES (For either transient or Steady-State Conditions):

Rule I. The Zo of the line determines the voltage/current ratio for an individual traveling wave (in either direction), i.e., $\frac{E}{I} = Z_0$.

Rule II. The Zo of the line determines the voltage and the current for a given energy flow in a traveling wave (of either direction). I.e., $E = \sqrt{PZ_0}$, or $I = \sqrt{\frac{P}{Z_0}}$

[This does not attempt to define which is the "cause" and which is the "effect" - energy flow (power), or voltage or current.]

Rule III. The load reflection coefficient $N = \frac{Z_0 - Z_T}{Z_0 + Z_T}$ and the reflection coefficient in the direction of the source $M = \frac{Z_0 - Z_G}{Z_0 + Z_G}$.

Rule IV. The sum of the energy flows into a point must equal the sum of the energy flows exiting that point. (Algebraic sum of the energy flows is zero). For example, if the incident power is 1 watt and the reflected power is 1/4 watt, then there will be 3/4 watt continuing through the junction. This rule is based on the constraint of no energy source or energy sink at the point.

Rule V. If the impedance discontinuity or transition is the connection of a line to a "load", then that "load" will follow the same rules as would a transmission line insofar as reflection coefficient, energy flow balance, etc.

Rule VI. 180 degree phase reversals occur in the reflected wave at an impedance discontinuity in accordance with the following criteria: Current is phase reversed when Zx > Zo and Voltage is phase reversed when Zx < Zo. [Zx is the impedance discontinuity terminating the Zo line in the direction of observation]