Gaining on the Decibel

Part 1: Would you say the bel, like the henry and the farad, is a unit of measure? If you answered yes (or if you answered no), you must read this article.

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If Alexander Graham Bell were alive today, he'd be turning over in his grave. And not only because we hams are forever taking his name in vain. After all, how would you feel if your life's work went largely ignored for over a hundred years, and instead you achieved fame for something which happened almost accidentally?

I am referring, of course, to this business of the telephone. Professor Bell, it seems, was first and foremost a teacher of the deaf. His greatest gift was training other teachers in helping the hearing impaired to function in a society of sounds. In fact, years after his now-famous invention, he told family members that it was really for his work with the deaf that he wanted to be remembered. Things seldom work out according to plan.

How exactly did the telephone come about? It's evident that Bell was trying to develop a device to amplify human speech. In a sense, he succeeded. Although Dr. Lee DeForest's audion tube (circa 1907) is most likely the first all-electronic amplifier, "the carbon microphone of telephony is an electromechanical amplifier, whose electrical power output can be a thousand times greater than its mechanical voice-power input."[1]

Dr. Bell realized that the human ear, like the "guess meter" of your favorite shortwave receiver, responds logarithmically to its input stimulus. And this logarithmic relationship, quantified by Bell and other early acoustical experimenters, has given us our standard tool for describing the behavior of nearly all electronic communications systems.

Reviewing Logarithms

The logarithm of a number is simply the power to which a specific base (or radix) must be raised to equal that number. The two most familiar bases are ten (the base of so-called *common* logarithms), and *e*, which represents the base of the *Napierian*, or *natural*, system of logarithms.[2] In this paper, with but a single exception (which I will clearly identify), we will be dealing exclusively with the radix 10. For example, since the number 1000 can be expressed as the radix 10 raised to the power three (10^3), we can say that the common logarithm of 1000 is 3. Simple, isn't it?

One advantage of logarithms is that they reduce rather cumbersome numbers to manageable proportions. Another is that their use reduces multiplication and division problems (always a challenge for me) to simple addition and subtraction (which even I can handle). But logarithmic response, be it of the ear or the calibrated meter, means that a large change in the applied stimulus results in a significantly smaller change in the output parameter, or response.

Because the ear responds logarithmically to applied acoustical power, a logarithmic unit, the bel (after Alexander Graham), can be used to express *changes* in power, or power *ratios*. Mathematically,

$$\text{bel} = \log_{10}(\text{Ap}) \qquad \text{(Eq 1)}$$

where

$\log_{10}$ represents the common (base 10) logarithm

(Ap) represents a given power ratio

I must emphasize this is the *only* meaningful definition of the bel, and any attempt to apply the term to anything other than power ratios will get you into trouble. A recent article by Gruchalla did an excellent job of clarifying the reasons for such rigid standardization.[3] I won't repeat that presentation here, but highly recommend Gruchalla's paper to anyone interested in applying what comes next, the decibel. Go read it now!

Introducing the Decibel

Done reading? Good. Now the bel, like the farad, the henry, the ampere and the watt, is a basic unit of respectable magnitude, one which we might care to subdivide. The Greek alphabet provides us with ample prefixes to indicate subdivision of a quantity, and we can apply these to the bel to increase our resolution without going too far to the right of the decimal point. The millibel (mB), microbel (μB), nanobel (nB), picobel (pB), femtobel (fB) and tacobel (tB) are all viable designations for thousandth, millionth, billionth, trillionth, quadrillionth of a bel, and lunchtime, respectively.

Of course, you might argue that millibel sings on the Grand Ole Opry, and a microbel relays many multiplexed telephone signals at extremely short wavelength. Fig 1 depicts a typical μB relay station.

The most convenient subdivision of the bel for our purposes, one that affords us ample resolution for electronic applications, is simply the 10th of a bel. The Greek prefix for 10th is deci, hence the unit decibel, or dB.

Since there are 10 decibels in each bel, we can expand Eq 1:

$$(\text{number of dB}) = 10 \times (\text{number of bels})$$

or

$$\text{dB} = 10 \times (\log_{10} \text{Ap}) \qquad \text{(Eq 2)}$$

which is the fundamental, and only valid, definition of the decibel. Easy as falling off a log!

Capitalization

Ever notice how we abbreviate everything in electronics and that there are

[1]Notes appear on page 22.

only 26 letters in the alphabet? Fortunately, the Greek alphabet affords us a few more characters, but sooner or later we're bound to run out of unique literals with which to define a quantity unambiguously. One way out of the dilemma is through the use of capitalization. For example, what's the difference between an mB and an MB? The former obviously represents millibel (lower case m), a very small unit. MB, on the other hand, starting with a capital M, most likely stands for Ma Bell, a very large unit to be sure, which is why the government had to break it up.[4]

Capitalization standards are no less important when we get to the decibel. The abbreviation for bel, being derived from a proper noun, should of course be capitalized. And remember that deci, a tenth, is a *small* prefix and thus should be written lower case. That gives us dB, but certainly *not* DB (which would mean Decabel, 10 bels, which differs from what we intend by a factor of only a hundred). And most emphatically not Db, which stands for Dumbbel!

The Voltage Decibel

This next problem will be solved using Ohm's law, so you might want to go back and review first. Ready?

Consider an electromotive force (voltage) applied across a *fixed and unchanging* resistance. If the applied potential is increased by, say, a factor of two, what change, if any, will occur to the resulting kinetic energy, or current? If you said the current will double, give yourself an A for the day.

Now the hard part. What change, if any, occurred to the total *power* dissipated in our resistor? Since potential energy (measured in volts) and kinetic energy (measured in amperes) both doubled, and since power is the product of potential and kinetic energy, power gain was the product of voltage gain and current gain, or four.

In our example, the voltage gain and current gain were the same. This occurs only in fixed-resistance (or more generally, fixed-impedance) cases. But when (and only when) the impedance across which we are measuring is *constant,* we can develop the following relationships:

$$Ap = Av \times Ai$$

where Ap, Av and Ai represent power, voltage and current ratios (or gains), respectively. Now, since in our special case

$$Av = Ai$$

we can combine the two above equations. Thus

$$Ap = Av \times Av = Av^2 \qquad \text{(Eq 3)}$$

or

$$Ap = Ai \times Ai = Ai^2 \qquad \text{(Eq 4)}$$

These relationships hold only if Av and Ai are equal. And when does that occur? *Only* in a constant-impedance system.

You may have seen in textbooks an equation that looks something like this:

$$dB = 20 \log_{10} (Av)$$

Perhaps, from the preceding discussion, you can guess where it came from. In a constant-impedance, or matched-impedance situation, since $Ap = Av^2$, you could combine Eqs 2 and 4, thus:

$$dB = 10 \log_{10} (Av)^2$$

This can be simplified by recalling that a logarithm is simply an exponent. Therefore

$$\log (A^x) = x \log (A)$$

By moving the exponent out front, we get

$$dB = 2 \times 10 \log_{10} (Av)$$

which becomes

$$dB = 20 \log_{10} (Av)$$

Please notice that the foregoing works only when impedances are constant and is a *derivation,* not a definition, of dB.

Introducing the Neper

We have established that when impedance is constant, voltage ratio and current ratio are equal. We now come to the exception I mentioned earlier, in which we will be dealing with logarithms to a base other than 10.

The Neper (a misspelling of Napier) is a convenient way of expressing voltage or current ratios logarithmically. It is defined as

$$N = \log_e (Av) \qquad \text{(Eq 5)}$$

or

$$N = \log_e (Ai) \qquad \text{(Eq 6)}$$

There are two noteworthy points in these relationships. One is that, unlike the bel, the Neper is based on natural, or Napierian, logarithms, which certainly seems appropriate. The other is that the Neper is defined in terms of either voltage ratio or current ratio. Obviously, this makes sense only if the two ratios are the same, and that happens only when impedance is constant. So a significant constraint on Nepers is that there are indeed both "voltage" Nepers and "current" Nepers, but they bear a meaningful relationship one to the other *only* in an impedance-matched system. The dB, on the other hand, is *defined* only for an impedance-matched system!

Now that we have two logarithmic units to choose from, we can establish some operating guidelines. When dealing with voltage ratios or current ratios, use the Neper when a logarithmic unit is required. If it is power ratio you're interested in, use the decibel. And in either case, apply the units across a fixed and constant impedance. These constraints will prevent the confusion which often results from trying to use power Nepers, or voltage dB.

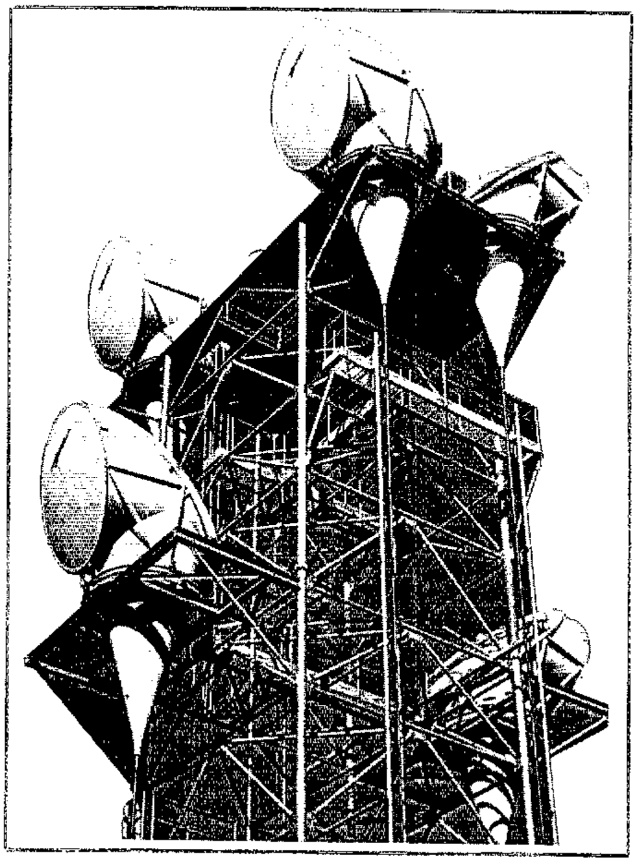

Fig 1—Antenna tower, the most visible element of a μB relay station (see text).

What's Wrong with Voltage Decibels?

Is there something wrong with using voltage dB? Plenty. It tends to imply that the dB numbers for power and voltage are somehow different (they use different equations, don't they?). And that, in turn, leads to the most common mistakes people make with dB, the ones Gruchalla relates. See note 3.

Let's consider our previous example, in which we doubled the potential energy applied across a fixed resistance. By squaring the quantity (twice the voltage) and multiplying its common logarithm by 10, we find that doubling voltage results in a gain of 6 dB. You will recall that the current also doubled, so we can square the quantity (twice the current), take 10 times its log, and conclude that doubling current results in a gain of 6 dB.

The problem starts when we try to compute power gain. Power gain is clearly voltage gain times current gain, and didn't we say that you can multiply two numbers by adding their logarithms? Well, 6 dB plus 6 dB (related to the log of voltage gain and current gain, respectively, remember?) implies a power gain of ... 12 dB!

The fallacy in this is that there *is* no such thing as "voltage dB" or "current dB." The decibel is a logarithmic expression of power ratio *only*. The two 6-dB figures we got, by squaring voltage ratio and current ratio, are *both* actually power gains (because that, by definition, is what decibels are).

Back in the early days of my career, I lost a consulting job over that one. My client had characterized a filter I was going to incorporate into a receiver I was designing, and my task was to design in enough gain to overcome its insertion loss. Recalling the different "dB formulas" I had been taught in school, I asked my employer, "Is that 8 dB voltage or 8 dB power loss?" He didn't renew my contract, and I knew then that someday I'd have to write this article.

Absolute vs Relative Power

Since dB expresses a power ratio, or change in power, it is a relative measure. But if we use dB to compare a particular power level to a *specified* reference power, an absolute measure results. The most widely accepted standard power levels for comparison are the milliwatt or, for high-power transmitter applications, the watt. A logarithmic expression of any power level, as it compares to the reference levels 1 milliwatt and 1 watt, would be written in dBm (decibels compared to a milliwatt) and dBW (decibels compared to a watt), respectively.

Inadvertent mixing of "dB" with "dBm" can produce some interesting results. For example, you can increase your transmitter power by a certain number of dB, or dBm, and either can be correct, depending on your intentions. Increasing power 3 dB will double your output, whereas increasing power by 3 dB*m* adds an additional 2 *milliwatts* to your signal. Are the two equivalent? Only if your transmitter was putting out 2 milliwatts to begin with!

In this part of the article we have traced the history of the decibel, or dB, and introduced some standards for its proper use. In Part 2, we will explore a number of applications of the dB to electronic communications in general, and ham radio in particular. Until then, I invite you to reflect on the advice of John Donne, "... and therefore never send to know for whom the bel tolls; it tolls for Power."

Notes

[1]Dr. Robert A. Chipman, *Transmission Lines* (New York: McGraw-Hill Book Co, 1968), p 5.

[2]After John Napier, Laird of Merchinston, the Scottish mathematician who published the first system of logarithms in 1614.

[3]Michael Gruchalla, "Defining the Decibel," *Ham Radio*, Feb 1985, p 51.

[4]Small letter, small unit seems to work for everything except kilo, which for some unknown reason is properly abbreviated with a lower case k.

An Extra Class ham first licensed in 1961, Paul has the distinction of being one of the few hams in the world to be operational in all 18 ham bands from 1.8 MHz to 10 GHz. His main interest is microwaves, and his nearly three dozen articles on circuit design and construction have appeared extensively in 73, Ham Radio, Microwaves, Microwave Systems News, *the* Radio Handbook *and* IEEE Transactions on Microwave Theory and Techniques *during the past 10 years. Paul has operated moonbounce, meteor scatter, sporadic E and tropo scatter, as well as all the ham satellites since OSCAR 6. He frequently leads his division in ARRL VHF and UHF contests.*

Paul currently serves on the board of directors of Project OSCAR Inc, as an Assistant Director of the ARRL and as a member of the League's VHF/UHF Advisory Committee. He has been a featured technical speaker at numerous West Coast and Central States VHF Conferences, the IEEE International Microwave Symposium, WESCON, the first three Satellite Private Terminal Seminars and various ARRL Division and national conventions.

Professionally, Paul is an aerospace engineer and educator. He currently heads the Microwave Technology program at San Jose City College and serves as Professor of Aeronautics at San Jose State University. His consulting engineering activities have included the design of biomedical telemetry systems, satellite remote sensing equipment and the world's first commercial home satellite TV receiver.

Paul's chief nonelectronics interest is aviation. He is a commercial pilot and flight instructor, and was founding chairman of the Santa Clara County Airport Commission. He is listed in Who's Who in Aviation and Aerospace, *and* Who's Who in California.

Strays

I would like to get in touch with...

☐ anyone with information or suggestions to help me convert a Drake L4B linear amplifier for operation on the 160-m band. James Garis, W9SKO, 824 Henrietta St, Pekin, IL 61554.

☐ anyone with a manual/full-size schematic for a Hallicrafter SX-42 receiver. Will George, W4LHJ, 1731 Country Club Dr, Tullahoma, TN 37388.

☐ any hams interested in tuning into the AACS Alumni Net, which meets at 2200Z Fridays on 14,287. Jules Wenglare, W6YO, 1416 7th Ave, Delano, CA 93215, tel 805-725-8707.

☐ other Amateur Radio real-estate brokers who have had experience in dealing with antenna location restrictions. Bob Crockett, W1LF, 2 Ripley La, Weston, MA 02193.

☐ anyone with information on the Clegg FM 28. Tom Popovic, KI3R, 1008 Monroe, McKeesport, PA 15133.

☐ anyone with information on using the Macintosh computer on RTTY and AMTOR. Robert A. Winters, KD7P, 5633 123rd Ave, SE, Snohomish, WA 98290.

☐ anyone with a manual or schematics for a DuMont 304A oscilloscope or information on its filament current regulator. Arthur Katz, W2NJW, 7804 Haymarket La, Raleigh, NC 27609.

☐ anyone with a schematic or manual for a Bendix TA12 G aircraft transmitter. Dick Moore, VE3LRB, 235 Epworth Ave, London, ON N6A 2M2.