SMOOTHING CIRCUITS:

(1) Resistance - Capacitance

How to Calculate the Best Number of Sections

By "CATHODE RAY"

MANY readers have asked me to write about filters, with particular reference to smoothing circuits. This I have hesitated to do, because the subject of filters is so involved that most people like to leave it to a specialist. Most of the general books on radio take care not to embark on it at all seriously, while the books devoted specially to electrical networks almost invariably plunge the reader into a morass of hyperbolics, where he is likely to lose sight of all physical realities.

The orthodox manner of complying with the above request would be to start by dealing with the general theory of filters. That would take several months (at least), by the end of which there would be few survivors to take an interest in the application of that theory to smoothing circuits. I have therefore decided to reverse the order and start with smoothing circuits—which are of practical interest to nearly everybody—and use that familiar ground as an approach to filters in general.

Nobody ought to be encouraged to hope that he can become a proficient filter designer without prolonged study and much practice. But I do think that anybody seriously interested in radio ought to have at least a clearly defined and reliable skeleton of information on filters in place of the vague and mysterious ghost that too often haunts him.

Greatly daring, I shall attempt to construct such a skeleton without going more mathematical than usual. But if you propose to go through with it I should say beforehand that you ought to be entirely familiar with the meanings of reactance and impedance, and the standard relationships:

$$\omega = 2\pi f$$
$$X_L = \omega L$$
$$X_C = 1/\omega C$$

from which

$$X_L X_C = L/C$$

and $X_L/X_C = \omega^2 LC$ (= 1 at the resonant frequency $\omega_r/2\pi$)

$$Z = \sqrt{R^2 + X^2}$$

These formulæ for X and Z are "magnitudes" only; to take account of phase angle one has to bring in j.

Whenever I am reckoning with circuits I always try to have by me one of the charts or abacs connecting L, C, f, and X. Of the several kinds, the one I prefer has four parallel vertical scales which can be connected anywhere by a stretched thread or a celluloid ruler. Among other things it shows the frequency at which any given L and C resonate, or conversely the L and C required to resonate at any given frequency. Its main use is to indicate the reactance of any L or C without any of that 2π arithmetic in which the decimal point so often gets into the wrong place.

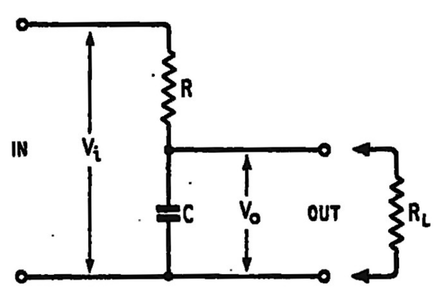


Fig. 1. The R-C filter is simply a potential divider in which the ratio varies with frequency

But it is time we finished with introductory remarks and got down to smoothing circuits.

The simplest of all is the combination of one resistance with one capacitance (Fig. 1). It is the commonest type of "decoupler," and when the current is small or the drop in voltage doesn't rule it out it is a convenient form of rectified a.c. smoother. It appears in almost every detector circuit, to filter out the r.f., and in every a.g.c. circuit, to filter out the a.f.

It works, of course, as a simple potential divider in which the impedance of the element across which the output is taken (C) is less for high frequencies than for low.

As with all potential dividers, the loss of voltage depends not only on the ratio of its two impedances, but also on the impedance connected across its output terminals—the load impedance. Unless we say otherwise, we shall assume that the load impedance is a resistance, denoted by R_L. Whenever R_L is very large compared with the impedance of the part of the potential divider it comes across (in this case X_C) it makes calculations much easier, because then the ratio of input to output voltage (V_i/V_0) is equal simply to the ratio of the whole impedance ($\sqrt{R^2 + X_C^2}$) to X_C.

We shall call V_i/V_0 the attenuation, and denote it by α. If α is 3, for example, it means that only one third of the input voltage reaches the output.* In Fig. 1, then, we have

$$\alpha = \frac{\sqrt{R^2 + X_C^2}}{X_C} \quad \cdots \quad (1)$$

One should always scrutinize equations to see if they are in the most convenient form. In (1) the part that depends on frequency appears twice; so to see more clearly how it affects α and to avoid needless duplication of

* You can, of course, if you prefer, reckon α in decibels, but they would have to be turned back into ratios to fit our equations.

Smoothing Circuits—

effort when evaluating α it is beneficial to divide above and below by X_C, with the result

$$\alpha = \sqrt{\frac{R^2}{X_C^2} + 1} \quad .. \quad (2)$$

It can be made to look even tidier, and at the same time to express the relationship more distinctly, by using any convenient symbol, say p, to denote the ratio of resistance to reactance. Then we have

$$\alpha = \sqrt{p^2 + 1} \quad .. \quad .. \quad (3)$$

Using this we can draw a curve of α against p that will hold good for the Fig. 1 class of circuit in general (Fig. 2). Incidentally, this illustrates last month's story about generalized graphs.

Still more of a simplification can be made so long as we are aiming at a fairly large attenuation, so that α is at least several times greater than 1. For then the 1 can by comparison be neglected and we get

$$\alpha \simeq p \quad .. \quad .. \quad .. \quad (4)$$

Up to the present I have not been able to think of any further simplification. On the contrary, in case you forget what p stands for I suggest the fuller version

$$\alpha \simeq \frac{R}{X_C} = 2\pi f CR \quad .. \quad (5)$$

Remember that for (5) to be reasonably accurate it is necessary for

(a) R_L to be much greater than X_C.

(b) R to be at least several times greater than X_C (i.e., α to be several times greater than 1).

We shall call (a) the shunting assumption and (b) the vector assumption. As an example, suppose R is 4 times X_C; then (5) gives α as 4, which can be compared with the correct value given by (2) or (3), 4.12. The 3% error is quite negligible in this kind of work. The dotted line in Fig. 2 is drawn to equation (5), so that you can see how the error becomes imperceptible as p increases.

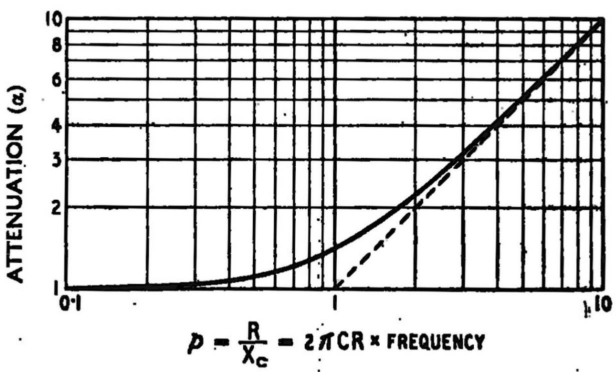


Fig. 2. Curve of attenuation against p (ratio of resistance to reactance) for a single section. This is an example of a generalized frequency curve, for $p = 2\pi CR$ times frequency. The dotted line shows the result of making the "vector" assumption.

As we saw last month, this type of circuit doesn't discriminate sharply between different frequencies. Its characteristic curve (Fig. 2) shows it to have a slope that approaches 6 db. per octave at the high-frequency end. In other words, doubling the frequency only halves the output, at best. If it is necessary for the lower of two frequencies to be nearly 100% preserved, the reduction of a voltage at double the frequency is much less still, owing to the gradualness of the bend in the region of $p = 1$ or less. The effect of R_L not being relatively very large reduces the discrimination still more, because it cuts down the output at low frequencies without making much difference at high.

In smoothing filters, however, the only frequency to be passed is zero, so one can hardly have too much a.c. attenuation. At the very least, the lowest a.c. frequency present should come on to the main slope, and preferably as far up it as is required for the purpose in view. At the same time, R must fit into the d.c. requirements.

For example, suppose an attenuation of at least 40 is required from a half-wave 50 c/s rectifier to supply 5 mA at 1,000 V., and the output of the rectifier across the reservoir capacitor is 1,300 V. at 5 mA. The value of R is fixed at once by the voltage drop; it is $(1300 - 1000)/5 = 60\text{k}\Omega$ (remembering that mA and kΩ are "consistent" units). For α as large as 40 we can use equation (5), and, as the lowest frequency is 50 c/s, we find $C = 40/(2\pi \times 50 \times 60{,}000)$ F $= 2\mu$F approximately. R_L is $1000/5 = 200\text{k}\Omega$, so the shunting assumption is justified. The harmonics present are reduced 2, 3, etc., times as much, so it might seem that so long as the lowest frequency is sufficiently attenuated it won't matter about any of the others. And that is quite so when the objectionable effects of the ripple depend only on its magnitude. But it must be remembered that the sensitiveness of the ear to hum increases more rapidly with frequency than the attenuation shown in Fig. 2, so in sound-producing equipment it is necessary to do better than reduce the *lowest* frequency to inaudibility.

Distribution of R and C

It should be clear by now that the effectiveness of the Fig. 1 circuit depends solely on p, which is $2\pi fCR$, and therefore at a given frequency depends solely on the product CR. (This is not necessarily true where the effect of R_L is appreciable, but for the time being we are continuing the shunting assumption.) A specified α can therefore be obtained by any C and R which give the right figure when multiplied. But, as we have seen, R is usually dictated by the required or allowable d.c. drop, in which case C is also decided. If a very smooth output is needed, the value of C found in this way may be disconcertingly large. So we may well ask whether the Fig. 1 circuit makes the best use of C and R.

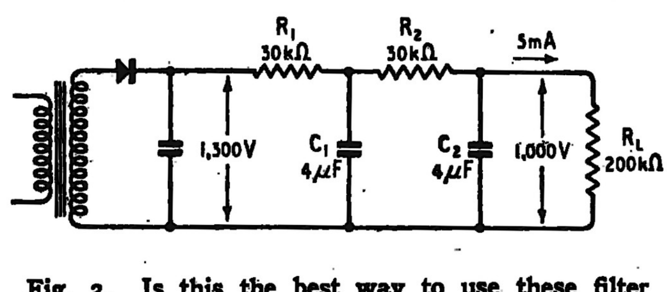


Fig. 3. Is this the best way to use these filter components?

Any number of units or "sections" like Fig. 1 can be used in cascade (i.e., one feeding into another), and, so long as our two assumptions apply to every section, the a of the whole combination will be the product of the a's of the separate sections. A second section like the first in the example just taken would reduce the 50 c/s content another 40 times, making the overall

$$a = 40 \times 40 = 1600$$

To keep the total voltage drop the same, the R in each section would have to be halved, so to preserve the original CR in each section the capacitances would have to be doubled (Fig. 3). A single section having the same total CR would have an a at 50 c/s of $2\pi \times 50 \times 8 \times 60{,}000 \times 10^{-6} = 160$, or only one tenth that of the two sections. In this case the advantage of splitting up is obvious enough. But is it always an advantage? And, if so, into how many sections?

Never having seen definite answers to these questions, I tackled them as follows:

Making the shunting and vector assumptions, the total a due to a given CR used all in one section (call it a_1) is p, which is ωCR. But if this CR is divided equally into two sections, say by halving both C and R, their product is CR/4. So the a of each section is $p/4$. The total a (call it a_2) is therefore $p/4 \times p/4 = p^2/16$. If you try a few values for p you will find that with low values a_1 is greater than a_2, but with high values it is the other way round. So that is the answer to the first question—it is *not* always an advantage to sectionalize; it depends on the value of p.

If you want to find the value of p (and hence CR) that gives the same total a whether in one section or two, you just put $a_1 = a_2$:

$$p^2/16 = p$$
$$\therefore p = 16.$$

So 16 is the critical value of p, below which one section is better than two. Let us call it p_1, to indicate that there is no sense in going beyond one section until p exceeds that value.

To answer the second question we have to find another value—p_2—above which it pays to use three sections instead of two; and so on. In general, we want p_n, the value of p at which the best number of sections changes over from n to $n + 1$. The calculation is made along exactly the same lines as for p_1 (for details see the Appendix) and the results are given in Table 1 above.

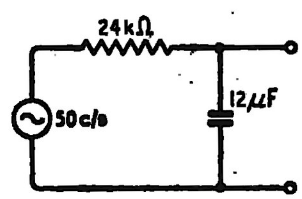
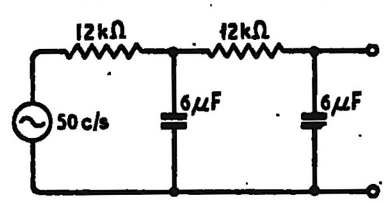
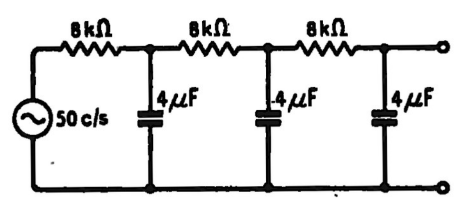
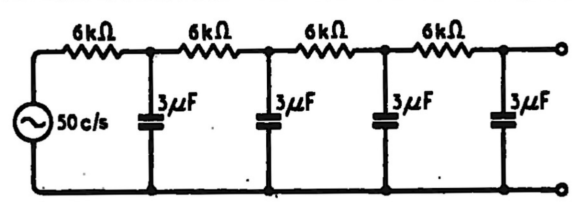
TABLE I.

n	p_n	a_n	RC in kΩ-μF *per section*, when f=100c/s	$p_{s(n+1)}$
1	16	16	25.5	4
2	45.5	129	18	5.06
3	90	1,000	16	5.62
4	149	7,540	15	5.86
5	223	56,600	14	6.19
6	311	416,000	13.7	6.35

Suppose you have a certain total capacitance and want to know the best way of connecting it. For example, the d.c. drop requirement gives you 20k Ω, and you have 16μF. It is a 50 c/s full-wave rectifier, so the lowest ripple frequency is 100 c/s. So your total p is $2\pi \times 100 \times 20 \times 16/1000 = 200$. The table shows this is more than p_4 but not more than p_5; so 5 is the best number of sections. The total a (third column) lies between 7,540 and 56,600; the actual value is $(p/n^2)^n$, in this case $(200/25)^5 = 32{,}770$. Whether it is convenient to divide your 16μF into five equal sections is another matter, of course; you may have to make it four, at some slight sacrifice of a. But at least the table gives you something more to go on than pure guesswork.

Or you may want to find the minimum C and R for a specified attenuation, say 1000. The table shows that you can get it with three (or four) sections having sufficient total C and R to make $p = 90$, and (fourth column) that

Fig. 4. Summary of attenuation measurements made on filters having the same total C and R, nominal values as shown.

CIRCUIT	n	CALCULATED α	MEASURED α
	1	—	100
	2	$\left(\frac{100}{4}\right)^2$ = 625	690
	3	$\left(\frac{100}{9}\right)^3$ = 1,375	1,430
	4	$\left(\frac{100}{16}\right)^4$ = 1,530	2,000

each section should be made up of 16 kilohm-microfarads.

You may wonder why I have included the last column, which shows the p per section on changing over to the next higher number of sections. For example, we saw that one section is best up to $p = 16$, and changing over to two sections at that point has no effect on the total attenuation, but it would reduce the p per section to 4. The last column, therefore, shows the lowest p you would ever get if you followed the table strictly. The point is that if this figure dropped to something like 2 or less, the vector assumption would be unjustified and the table would be invalid. But we see the least p is 4, with a vector error of 3%, which is not serious even when multiplied by 2. The other errors are still smaller, so are not serious even when multiplied by the larger number of sections. Such error as there is tends to make the true a_n larger and reduce p_n. p_1, for example, would be 15 instead of 16.

What about the shunting error ? Even if R_L is infinitely large, so that the last section is unshunted, the last section shunts the last but one, and so on. It is difficult to

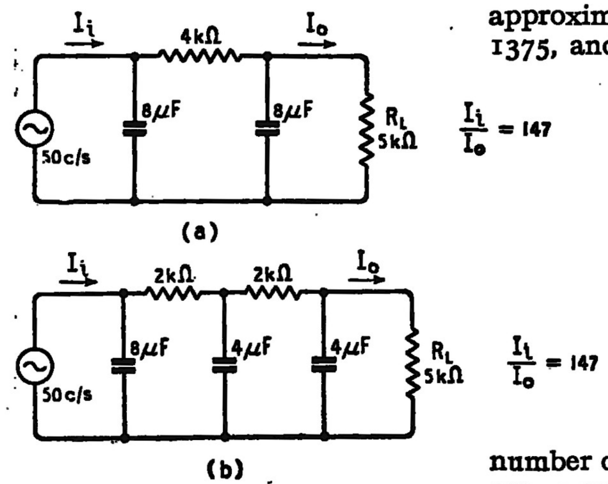


Fig. 5. Another comparative test.

work out the size of the error exactly when there are several sections, and anyway it depends on the load resistance; but the $p_s(_{n+1})$ figures show that it ought not to be very serious, and again it would tend to make the actual attenuation and also the best number of sections higher than in the table. But since the cost of the filter generally goes up with the number of sections, the fact that the table puts the change-over point higher than the theoretical ideal is all to the good.

Another point to be considered is that the attenuation increases in proportion to the nth power of the frequency, so in sound-reproducing equipment, at least, it is advisable to make n not less than 2.

To check the theory and to see how far its assumptions were justified I did a few measurements. I aimed at $p = 90$, to check the middle lines in the table, but had to use the components available, so it worked out at about 100. Fig. 4 shows the nominal values, which were subject to commercial tolerances. Assuming that the measured p in one section (100) was divided exactly into 2, 3 and 4 sections, the attenuations predicted by the approximate theory were 625, 1375, and 1530 respectively. The measured attenuations (subject to possibly ± 10% error of measurement) were 690, 1430, and 2000. These results show that

(a) The theoretical table is good enough for design purposes.

(b) It tends to underestimate the results.

(c) The discrepancy increases with the number of sections. (This is what one would expect, because the p per section is less with more sections, so the assumptions are less justifiable.)

To get still nearer to working conditions I did a test using a typical value of reservoir capacitor (8µF) and load resistance (5kΩ), as in Fig. 5. The reservoir, without any other filtration, would reduce the hum current passing into the load. Adding one filter section having a p of about 10 made the total measured current attenuation 147. Splitting this filter section into two, the attenuation was the same. According to the approximate theory, the p for which this would happen would be 16 (or 15, correcting the vector error). But the actual p per section in Fig. 5 (b) is only 2½, so one would expect the approximate theory to be thrown out somewhat by shunting. Taking into account the higher cost with two sections, one would almost certainly want p to be at least 16 before sectionalizing.

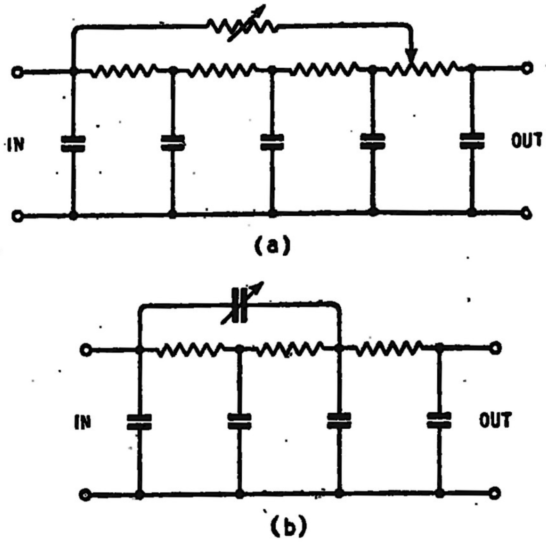


Fig. 6. Two methods of balancing out one ripple frequency completely—usually the lowest.

There are at least two methods (Fig. 6(a) and (b)) for eliminating the most troublesome ripple frequency altogether. They take advantage of the fact that its phase shifts as one moves along the filter. If the right amount of ripple from the input is fed to a point where there is a 180° phase difference, the two will cancel one another out. There may be some occasions when these devices are worth while (such as when only a small d.c. drop is allowable), but there are several objections. One is that success depends on both magnitude and phase being correctly adjusted. Another is that only one frequency is cancelled; for the others the attenuation of the original filter (admittedly relatively high for the higher frequencies) is actually reduced. It is necessary to use several sections to get the required phase shift, and the table has shown us that quite a large a can be obtained with several sections straightforwardly, without going beyond reasonable limits for CR. Still another disadvantage is that the necessary design information would increase the length of this article, and it is already full size. Calculation of actual hum voltage, and inductance-capacitance filters must wait until next month.

APPENDIX

(Showing how to calculate the table giving the number of sections for maximum attenuation).

$$p = 2\pi f \times \text{total } CR.$$

If CR is divided into n equal sections, the attenuation per section is approximately p/n^2 (assuming that it is at least several times greater than 1, so that the simplifying assumptions apply). The total attenuation, α_n, is therefore

$$\alpha_n \simeq \left(\frac{p}{n^2}\right)^n$$

If the same CR were divided into $n + 1$ equal sections the attenuation would be

$$\alpha_{n+1} \simeq \left(\frac{p}{(n+1)^2}\right)^{n+1}$$

If p_n is the value of p that makes $\alpha_n = \alpha_{n+1}$, then approximately

$$\left(\frac{p_n}{n^2}\right)^n = \left(\frac{p_n}{(n+1)^2}\right)^{n+1}$$

$$\therefore \quad \frac{p_n^{\,n+1}}{p_n^{\,n}} = p_n = \frac{(n+1)^{2(n+1)}}{n^{2n}} \quad \text{.. (A)}$$

$$\text{So } \alpha_n \simeq \left(\frac{p_n}{n^2}\right)^n = \left[\frac{(n+1)^{2(n+1)}}{n^{2n} \times n^2}\right]^n$$

$$= \left(\frac{n+1}{n}\right)^{2n(n+1)} \quad \text{.. (B)}$$

And $p_{s(n+1)}$ (the p per section when p_n is divided into $n + 1$ sections) is

$$p_{s(n+1)} \simeq \sqrt[(n+1)]{\alpha_n} \simeq \left(\frac{n+1}{n}\right)^{2n} \quad \text{.. (C)}$$

SMOOTHING CIRCUITS:

(2) Inductance-Capacitance

How to Calculate the Hum Voltages

By "CATHODE RAY"

LAST month we began the study of smoothers by considering the very simple combination shown in Fig. 1(a). Alternatively (as they say in law) if we didn't (or did, and have forgotten it all), it should be quite easy to pick it up, because this month's circuit (Fig. 1(b)) is to be tackled along exactly the same lines, making only the changes necessitated by the fact that L takes the place of R.

To be more strictly correct it is the inductive reactance (X_L; equal to ωL, or $2\pi f L$) that takes the place of R, because a reactance, like a resistance, is a particular kind of impedance, and it is the impedances in the filter that determine its effectiveness as a smoother. A convenient standard by which to reckon such effectiveness is the attenuation, which we have been denoting by the symbol a and defining as input-voltage/output-voltage, V_i/V_0 (at the particular frequency being considered).

We found that if we could make two assumptions the whole thing became extraordinarily easy. The attenuation, in fact, became practically equal to what we denoted by p—the ratio of resistance to capacitive reactance, R/X_c. And X_c, of course, is $1/\omega C$, so a longer but more directly useful form of p is ωCR (or $2\pi fCR$). This shows that the attenuation is directly proportional to frequency and to C and R. Or, rather, that it would be approximately if our assumptions were justified. These assumptions are:

(a) The "shunting" assumption, that the load impedance is so high compared with X_c that it makes no appreciable difference to a. If we can assume this it is a tremendous relief, not only because it vastly simplifies the calculations but because we might not even know at first exactly what the load impedance was going to be.

(b) The "vector" assumption, that R is at least several times greater than X_c at all the frequencies concerned. This allows us to say that the whole impedance of the smoother (from the input side) is approximately equal to R, instead of having to use the correct value, $\sqrt{R^2 + X_c^2}$.

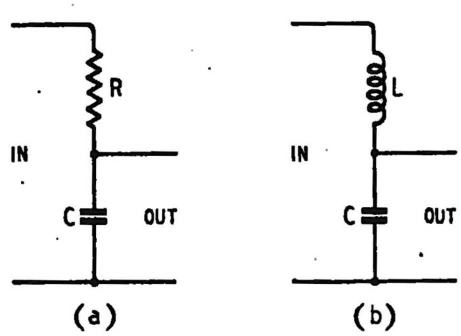


Fig. 1. Comparison between RC smoother discussed last month (a), and the LC smoother (b).

We found that fortunately these assumed conditions do actually apply, unless the smoother is a very poor one with an a of, say, 3 or less. Using several Fig. 1 sections in cascade complicates the shunting assumption, admittedly, because each section shunts the one in front of it; but after we had worked out the best number of sections to give any required attenuation we found that the worst error due to this shunting was not likely to be enormous and in any case was on the right side—the actual smoothing was better than that calculated by the simplified theory.

Coming now to Fig. 1(b), we are as grateful as ever to avail ourselves of the shunting assumption. There is not quite so much point in using the vector assumption, however, because X_L and X_c directly subtract instead of having to be combined under a square root sign. And we shall see that because of this the error due to using it is considerably greater.

One other assumption we shall make, most of the time, is that the resistance of the inductor is negligible.

Treating Fig. 1(b) (as we did (a), by virtue of the shunting assumption) as a potential divider, the attenuation is equal to the whole impedance divided by the impedance of C alone:

$$a = \frac{X_L - X_c}{X_c}$$

$$= \frac{\omega L - 1/\omega C}{1/\omega C}$$

and multiplying above and below by ωC

$$= \omega^2 LC - 1$$

$$= (2\pi f)^2 LC - 1 \quad .. \quad (1)$$

Making the vector assumption just knocks off the 1, which, at the higher frequencies at least, is generally small compared with $\omega^2 LC$. This $\omega^2 LC$, by the way, is the ratio of X_L to X_c, taking the place of R to X_c in Fig. 1(a). We found it convenient to denote R/X_c by p, and with the seam idea we shall denote the corresponding quantity in the inductive smoother, X_L/X_c, by q. So, making the shunting (but not the vector) assumption,

$$a = q - 1 \quad .. \quad .. \quad (2)$$

Before going on to find the best number of sections into which to divide the LC smoother, it may be a good thing to note some points of contrast between it and the RC type.

The first is that using L to provide the high series impedance for reducing the unwanted a.c. avoids having to have the same impedance in series with the wanted d.c. Even if, to reduce the d.c. voltage, we actually need resistance, the amount of it that is right for that purpose is unlikely to be the best choice for smoothing purposes. And often the less resistance the better. Of course even with an inductor it isn't possible *economically* to obtain unlimited series impedance with negligible resistance. But at least there is much more scope than with resistance only.

Next, the inductive smoother is double-acting; not only does the impedance of C decrease as the frequency rises, but the impedance of L increases. So instead of being proportional to frequency, a is proportional to frequency-squared. This steepens the cut-off slope, as shown in Fig. 2 (top right-hand corner and beyond). In figures, LC gives 12 db per octave, compared with 6 for RC. As it happens, the sensitiveness of the ear to weak sounds in the 50–500 c/s region also increases at about 12 db per octave, so for sound-making apparatus this is another reason for preferring an inductive smoother to a single-section resistive type.

While the frequency curve of a single-section LC smoother is practically the same as that of a 2-section RC smoother in the useful or "assumption" region, where a is substantial, it is decidedly different at the foot of the bend. Whereas the RC curve slides smoothly down to $a = 1$ (i.e., output voltage as big as input) at zero frequency, the LC curve dips well below that level, meaning that the output ripple-voltage is actually greater than the input. Where $X_L = X_c$ (which happens when $q = 1$) series resonance occurs, and according to equation (1) a would be 0 and the output ripple infinitely large! In practice, of course, there are several reasons why it doesn't get quite as bad as that; but at least it is a situation to be avoided. The way to avoid it is to see that ω^2LC is well above 1 at the lowest frequency to be suppressed.

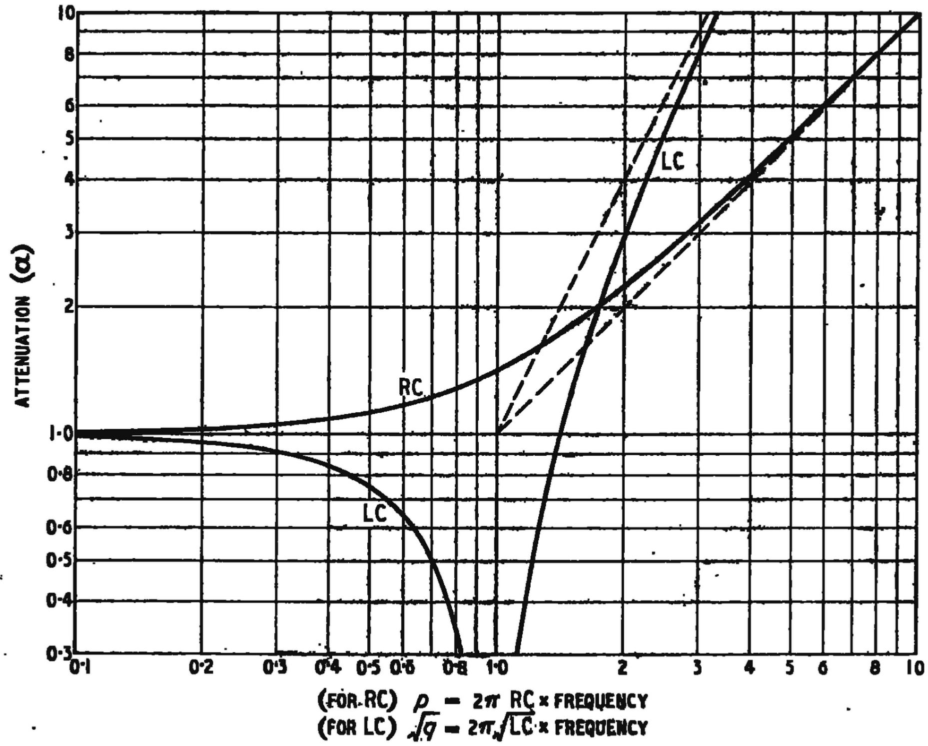


Fig. 2. The performance of single RC and LC smoother sections is compared here. The dotted lines show the results of making the "vector" assumption—the discrepancy becomes very large near $\sqrt{q} = 1$, because L and C resonate. But the full-line curve itself is highly theoretical here, as it neglects the resistance of the choke and the effect of the load resistance. The useful part of the characteristic lies mainly off the diagram, beyond the top right corner.

Inductive Coupling

Whereas most of the features of the inductive smoother are in its favour, it must be admitted that an inductor is generally larger, heavier, and much more expensive than the corresponding resistor, and is liable to generate hum inductively in nearby audio windings, to say nothing of humming itself if the stampings are not tight.

In calculating the number of sections that gives the greatest attenuation for a given total LC, I have assumed that there is no inductive coupling between sections. Although the vector assumption gave results that were near enough with the RC smoother the errors with LC would be too much, so I have used equation (1). This makes the calculation of the table slightly more complicated, but the general idea is this: q is our abbreviation for ω^2 times the total LC in the smoother. If it is all used to make one section, the attenuation of that section (a_1) is equal to $q - 1$. But if it is divided equally into two sections, both L and C have to be halved, so the q per section is $q/4$ and the attenuation per section is $q/4 - 1$, and a_2 (the attenuation of the two sections) is $(q/4 - 1)^2$. In the

n	q_n	a_n	LC per section when f = 100 c/s
1	23.5	22.5	60 henry-microfarads
2	67	248	42.5 ,, ,,
3	136	2,800	38 ,, ,,
4	234	34,500	37 ,, ,,
5	362	450,000	37 ,, ,,

same way the attenuation of the smoother when divided into n sections is

$$a_n = \left(\frac{q}{n^2} - 1\right)^n$$

To find the critical value of q, which we call q_n, such that the same attenuation is obtained when the same total L and C are divided among $n + 1$ sections, we put $a_n = a_{n+1}$. Doing this for $n = 1$, 2, etc., up to 5, we get the figures given in the table on the preceding page.

Compared with the corresponding RC figures, these show a

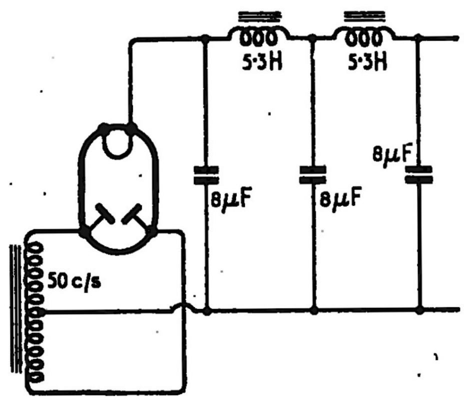


Fig. 3. Example of suitable values for typical requirements.

tendency for fewer sections to be needed. So I haven't gone as far as $n = 6$, because 5 gives more than all the a anybody is likely to want. Nor have I included the column to show the lowest possible q per section, because it is obviously more than in the RC table, and even that was sufficient to justify the shunting assumption.

The reason for the higher q_n figures is the fact that the a per section is 1 less than q, so q has to be larger to make sectionalizing worth while.

To take an example, suppose a total attenuation of 2800 at 100 c/s is about right. This being a_n for $n = 3$, the best number of sections is 3. (Four would give approximately the same attenuation for the same total L and C, but would presumably cost more). The table also shows that q_n is 136, so putting $(2\pi f)^2$ LC = 136 we get LC = $136/(200\pi)^2$ = 345 henry-microfarads. If we happen to have available a good line of 6H chokes, one for each section, the total C to go with them should be 345/18 = 19μF or roughly 6μF per section. Doubling C, of course, would allow us to halve L.

The last column by-passes most of this calculation by showing directly the LC *per section*, corresponding to total attenuation and number of sections (a_n and n respectively), for a ripple frequency of 100 c/s, which is the lowest from a properly balanced full-wave 50 c/s rectifier. An interesting point is that with, say, 8μF capacitors in each section, the best value of choke lies within the narrow limits of 4.6 to 7.5 H. Present practice, it seems, tends to use too few chokes of too high inductance.

Naturally everybody is out to reduce costs as much as possible, so the results of the foregoing investigation are more than welcome in so far as they indicate that smoothing chokes need not have such a large inductance as is usually supposed. The same conclusion helps in minimizing choke resistance, too. But the idea of using a whole string of chokes—even small ones—is not quite so attractive. Designers may be reluctant to go beyond two.

So it is worth seeing what two can do. Fig. 3 shows a 2-section smoother based on the table, using the customary 8μF for each section and also for the reservoir. The inductance of each choke is only 5.3H, and the total attenuation at 100 c/s is 248 by the table. (That doesn't count the smoothing due to the reservoir, of which more anon). And it increases as the 4th power of the frequency, so the attenuation of the 400 c/s harmonic (for example) will be 256 times as much as at 100 c/s, or 65,300. That ought to be good enough there, but if the loudspeaker reproduces 100 c/s strongly (though the makers usually try to keep the cone resonance off it) the relatively strong ripple at that frequency may be

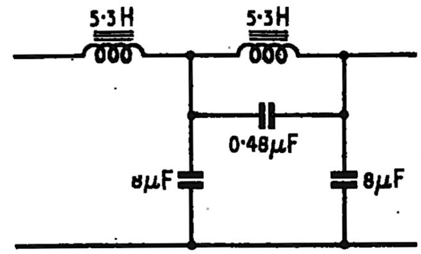


Fig. 4. A device for tuning out the main hum frequency.

troublesome. What is the answer? Use three sections? ("No!" says the designer firmly).

In cases like this, where the smoothing at all frequencies except one seems adequate, it may pay to use the trick shown in Fig. 4. It is one that will reappear, under the curious name of an "m-derived" section, when we consider filters. At the moment it looks like just what it is—a rejector circuit tuned to the offending frequency. At the higher frequencies this section will be markedly inefficient, because it will tend to act as a capacitance

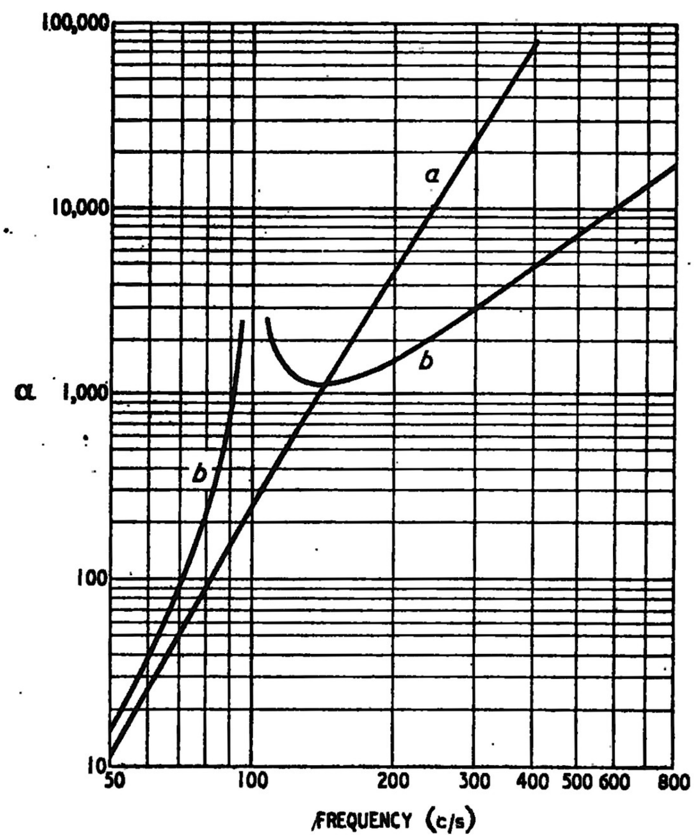


Fig. 5. Curve *b* shows how the Fig. 4 circuit affects the low- and high-frequency hum, as compared with Fig. 3 (curve *c*).

potential divider having an attenuation of only about 18. One therefore relies heavily on the first section to deal with them.

In Fig. 5 the performance of the modified system (curve *b*) can be compared with the original one (curve *a*). (If you intend to study filters you ought to take particular note of the shapes of these two curves). The resonance peak at 100 c/s is not completely drawn in because its height depends on the resistance of the choke, which we don't know.

With reasonably good judgment (or luck), the large reserve of high-frequency a in the normal type of section will be enough and the 100 c/s (or other troublesome low frequency) attenuation will be brought up to the required standard without having to resort to wasteful brute force.

Tuning Difficulties

The fact that this device is not more used may suggest to the cautious reader that there are some snags. One is that a rather odd value of capacitance may be needed for tuning the choke. Another is the wide tolerance in the inductance of commercial chokes. A third is that the inductance depends largely on the d.c. carried, so that even if it is right for one loading it is not for another. Owing to the flatness of the tuning, however, these snags don't amount to as much as might appear. Even if the d.c. milliamps are liable to vary over a wide range, it is usually satisfactory to make the tuning correct at or near the maximum current, where the effectiveness of the first section is least.

Choke-tuning is a dodge worth remembering if you have an ordinary 2 (or more) section smoother that is not quite good enough, and you don't want any drastic alterations.

As with last month's treatise. I thought that undiluted theory might be considered somewhat bald and unconvincing, so hastened to take a few readings on an actual smoother. Fig. 6 shows the test circuit. The two chokes were marked 9H 0.1A, and (unlike most of their kind) their rated inductance at the rated current turned out to be reasonably correct. With $8\mu F$ total capacitance this gave a total q at 100 c/s of 57.5, which according to the table would give of its best in two sections. In one section, (a), the hum voltage viewed on the oscilloscope seemed to be mainly 100 c/s, but with appreciable 50 c/s. The latter was unimportant at this stage, but later became more obvious and was balanced out by inserting some resistance in series with one of the rectifier anodes. When this was done, the hum was nearly pure 100 c/s, and amounted to 0.33V r.m.s.

With the same L and C connected as two sections (b) the voltage was reduced to 0.14V.

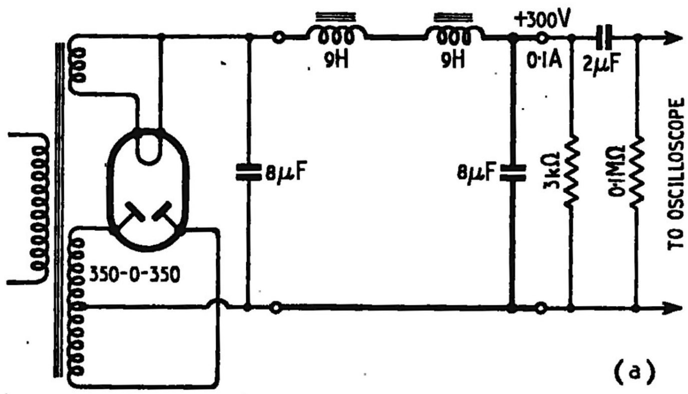
Next, capacitance was connected across the second choke to tune it to 100 c/s, (c). It was then, with the predominant 100 c/s removed, that the 50 c/s became obvious. It amounted to 0.12V; but after balancing the rectifier the residue of hum was only about 0.03V. The value of capacitance required to tune the choke confirmed the inductance rating.

Lastly, the first choke was tuned instead of the second, (d). The results were much as before, but there seemed to be rather more high-frequency ripple. Possibly there was some intermodulation due to the relatively large amplitude in the first choke.

Resistance Balance

One conclusion to draw is that although tuning is very effective in reducing the main ripple, it is by no means safe to assume what the books tell us, that 50 c/s is absent from the output of a full-wave rectifier. Most of the centre-tapped power transformers I have come across are very lopsided as regards resistance, however well balanced they may be for voltage.

Another thing to notice is that the 2-section arrangement is rather less than $2\frac{1}{2}$ times as good as the single section, whereas $(57.5/4-1)^2$ is over 3 times more than $57.5-1$. This discrepancy is opposite to what we had with resistive smoothers. But whereas one resistive section shunted across the capacitance of the previous one tends to reduce the impedance and thereby improve the smoothing, an inductive section shunted across capacitance tends to increase its impedance (by going some way towards forming a rejector circuit), with the opposite result. So while the RC table tends to underestimate the value of sectionalizing, the LC table tends to overestimate it. And taking into account the



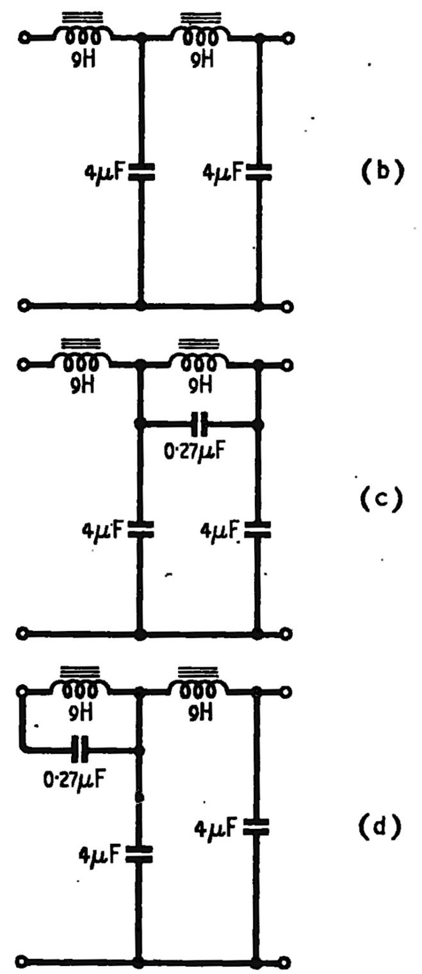


Fig. 6. Test circuit used for comparing various smoothing arrangements.

greater cost of divided chokes and capacitors, the conventional values may not be so unsuitable after all.

The final thing to be done is to see how to calculate the actual hum voltage. With the capacitor-input or reservoir type of circuit, which is the only kind we are considering, it is easier

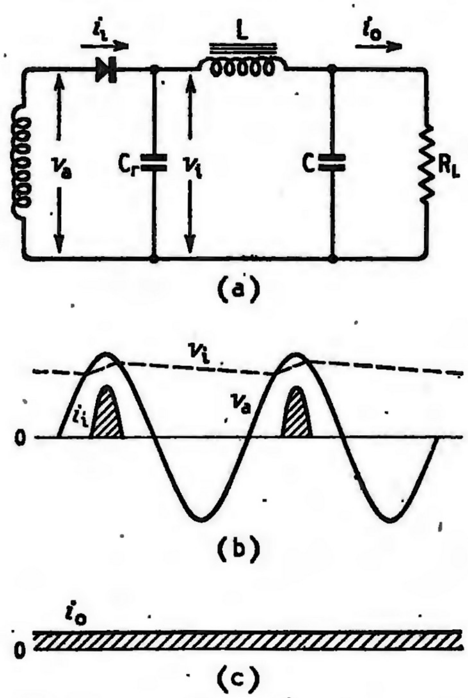


Fig. 7. Showing how the formula is obtained for calculating the hum voltage.

than might appear. That is because it is a fair assumption that the current flows through the rectifier into the reservoir in the form of pulses. Fig. 7 is a reminder of how this comes about. The alternating voltage is denoted by v_a. Current (i_i) can flow through the rectifier only when v_a exceeds v_i, and diagram (b) shows how this happens only at the peaks of v_a. Now although this current is far from steady it is d.c. of a kind (in the sense that it is unidirectional), and exactly the same amount of current, on the average, must come out somewhere, namely, into the load, after having been ironed out by the smoother. So if we know the load current (i_0 at (c)) we know the average value of i_i—it is the same. Diagrammatically, the shaded area in (b) is the same size as that in (c).

Provided that the reservoir capacitance is large enough for its job, it is not far wrong to assume that the whole of i_i flows while v_a is at its peak. That being so, investigation of pulse waveforms shows that the peak fundamental alternating component of i_i is very nearly $2i_0$. (If i_i really did all occur exactly at the peak of v_a, so that the pulse was infinitely narrow, all the harmonics would also be equal to $2i_0$; but owing to the finite width of the current pulse the harmonic amplitudes fall off at the higher frequencies). So if we take the *r.m.s.* value of the ripple current of any one frequency from the rectifier into the reservoir (denoted by I_i) as $\sqrt{2}$ times the load current, we shall be nearly correct for the lowest frequency, and shall be progressively overestimating the higher frequencies—so shall be always on the safe side.

To get the r.m.s. ripple voltage at the input to the smoother (V_i) we make the shunting assumption again and multiply I_i by the reactance of the reservoir capacitor C_r. Putting all this together:

$$V_i = I_i X_{cr} = \frac{I_i}{\omega C_r} = \frac{\sqrt{2} i_0}{\omega C_r} \quad \ldots \quad \ldots \quad (3)$$

What we want is V_0, the corresponding output ripple voltage. But since V_i/V_0 is what we have been calling a, we have

$$V_0 = \frac{V_i}{a}$$

$$\text{(substituting (3))} \quad = \frac{\sqrt{2} i_0}{a \omega C_r} \quad (4)$$

Taking the circuit of Fig. 7(a) and making the usual assumptions,

$$a = \omega^2 LC - 1$$

So (substituting in (4))

$$V_0 = \frac{\sqrt{2} i_0}{(\omega^2 LC - 1)\,\omega C_r} \quad \ldots \quad (5)$$

The same principle can easily be applied to any system for which the assumptions apply. If $\omega^2 LC$ is large enough for 1 to be neglected, the rule simplifies to:

Divide $\sqrt{2}$ times the output d.c. (in amps) by all the ωL's, ωC's, and R's used for smoothing, including the reservoir.

Applying this to Fig. 6(a) we have

$$V_0 = \frac{\sqrt{2} \times 0.1}{(2\pi \times 100)^3 \times 18 \times 8 \times 8 \times 10^{-12}} = 0.495\text{V}$$

This should, as we saw, be an overestimate (especially when rather a lot of current is being drawn in relation to the reservoir capacitance), and in fact the measured value was roughly 0.34V.

To get some practice in the use of all this groundwork you might care to design a smoother to give two or more outputs; say, 70 mA output roughly smoothed and a 10 mA output at a lower voltage, thoroughly smoothed.