Propagation coefficient of the Beverage aerial

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ABSTRACT

The propagation coefficient for the current flowing in a Beverage aerial driven by a generator is derived from the compensation theorem. Comparison is made with measurements and with other theoretical methods. It is concluded that the attenuation increases approximately as the square root of the frequency, reaching a limiting value at high frequencies. The velocity of propagation is less than the free-space velocity, but tends towards it at high frequencies. Expressions for the characteristic impedance of the aerial are also given.

LIST OF PRINCIPAL SYMBOLS

c = velocity of propagation in free space
$\mathbf{E}$ = electric-field strength
f = frequency, MHz
H = magnetic-field strength
h = height of conductor above ground
I = current flowing in conductor
K = contribution to the surface integral, per unit length along the direction of the conductor
K_r = $\epsilon_r - j1{\cdot}8 \times 10^4\sigma/f$, complex relative permittivity of ground
ℓ = distance between terminals A and B
v = velocity of propagation along the conductor
W = complex factor defined in Section 2.2
Z_{AB} = mutual impedance between terminals A and B
Z_C = characteristic impedance
α = attenuation constant
β = phase constant
γ = $\alpha + j\beta$ = propagation coefficient
ϵ_r = relative permittivity of ground
η = intrinsic impedance
λ = wavelength
σ = ground conductivity, S/m

Unprimed quantities (e.g. E_z) denote fields, currents etc. for the conductor when over perfectly conducting ground. Primed quantities (e.g. $\mathbf{E}'_z$) denote the corresponding quantities when the ground is imperfectly conducting. The subscript 0 (e.g. η_0) is used to denote the intrinsic impedance and propagation constant of free space. Corresponding quantities without subscripts refer to the lower medium.

1 INTRODUCTION

The Beverage aerial consists of a long wire supported on poles a few metres above the ground, terminated at its far end by a resistance. It was first used in the USA in 1922 for the reception of European v.l.f. transmitters, and aerials up to 12 km long were constructed.[1] Current applications include the reception of distant m.f. and h.f. transmitters. The use of the aerial for transmission has also been considered.

Whether the aerial is used for transmission or reception, a knowledge of the propagation coefficient for the current which would travel along it if it were driven is essential if its performance is to be assessed. The propagation coefficient is derived here by applying the compensation theorem,[2] and the result is compared with measurements and with other theoretical methods. Expressions for the characteristic impedance of the aerial are also derived.

2 THEORETICAL ANALYSIS

Fig. 1 shows a Beverage aerial which is driven at a pair of terminals A and has a second pair at B. The distance to the termination is assumed to be so large that no power is reflected from it when the current is attenuated by imperfectly conducting ground.

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If the ground were perfectly conducting and the aerial were correctly terminated, the mutual impedance Z_{AB} between terminals A and B would be

$$Z_{AB} = Z_C \exp(-\gamma_0 \ell) \quad (1)$$

where the symbols are defined above. The corresponding expression for imperfectly conducting ground is

$$Z'_{AB} = Z'_C \exp(-\gamma' \ell) \quad (2)$$

where Z'_C and γ' are the required values for the characteristic impedance and propagation coefficient of the aerial.

According to the compensation theorem,[2] if a change is made within a closed surface S which excludes two pairs of terminals A and B, the change in mutual impedance is given by

$$Z'_{AB} - Z_{AB} = \frac{1}{I(0)^2} \int_S (\mathbf{E}_A \times \mathbf{H}'_B - \mathbf{E}'_B \times \mathbf{H}_A) . d\mathbf{s} \quad (3)$$

where $\mathbf{E}_A$ and $\mathbf{H}_A$ are the electric and magnetic fields over S before the change is made when a current $I(0)$ is impressed at terminals A, $\mathbf{E}'_B$ and $\mathbf{H}'_B$ are the fields over S after the change when a current $I(0)$ is impressed at terminals B, and $d\mathbf{s}$ is an element of S regarded as a vector in the direction of the outward normal from the surface

In the problem considered here, S is the surface of the ground. Fig. 1 shows the co-ordinate system, the origin being situated below the terminals A. The surface S corresponds to the y = 0 plane.

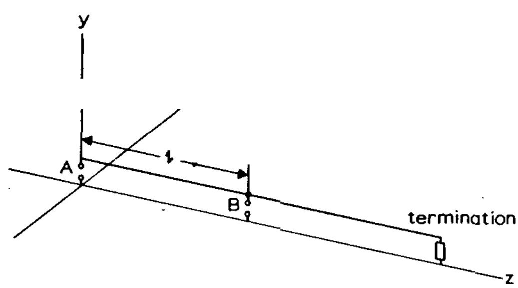

Fig. 1

Beverage aerial and co-ordinate system

Since the ground is initially perfectly conducting, the tangential component of $\mathbf{E}_A$ is zero and the vector product $\mathbf{E}_A \times \mathbf{H}'_B$ therefore makes no contribution to the surface integral. Over perfectly conducting ground, the only component of $\mathbf{H}_A$ which is present is H_x and it therefore follows that the only component of $\mathbf{E}'_B$ which needs to be considered is E'_z. Consequently, eqn. 3 simplifies to

$$Z_{AB} - Z'_{AB} = \frac{1}{I(0)^2} \int_S E'_z H_x \, dx \, dz \quad (4)$$

If the currents on the conductor are $I(z)$ and $I'(z)$ when the aerial is driven at terminals A and B, respectively, the contribution to the integral from a strip of ground of width δz extending to infinity on either side of the conductor is proportional to the product of the currents. The contribution from the strip may therefore be expressed in the form

$$\delta z \int^{\infty} E'_z H_x \, dx = K \, I(z) I'(z) \delta z \quad (5)$$

ductor and on the ground constants. Eqn. 5 enables eqn. 4 to be simplified to

$$Z_{AB} - Z'_{AB} = \frac{K}{I(0)^2} \int_0^{\infty} I(z) I'(z)\, dz \qquad (6)$$

When a current $I(0)$ is impressed at terminals A and the ground is perfectly conducting, the current distribution $I(z)$ is given by

$$I'(z) = I(0) \exp(-\gamma_0 z) \qquad (7)$$

When a current $I(0)$ is impressed at terminals B, it divides equally between the two branches of the aerial, and is given by

$$\left.\begin{aligned} I'(z) &= \frac{I(0)}{2}\left[-\exp\{\gamma'(z-\ell)\} + \exp\{-\gamma'(z+\ell)\}\right] \\ & \qquad\qquad 0 < z < \ell \\ &= \frac{I(0)}{2}\left[\exp\{-\gamma'(z-\ell)\} + \exp\{-\gamma'(z+\ell)\}\right] \\ & \qquad\qquad z > \ell \end{aligned}\right\} \qquad (8)$$

The second term on the right-hand side of eqn. 8 arises because the current is reflected at $z = 0$, where the terminals A are assumed to be open-circuited.

Substitution of eqns. 1, 2, 7 and 8 in eqn. 6, followed by integration between the appropriate limits, leads to

$$Z_c \exp(-\gamma_0 \ell) - Z'_c \exp(-\gamma' \ell) = K \frac{\gamma' \exp(-\gamma'\ell) - \gamma_0 \exp(-\gamma_0 \ell)}{\gamma'^2 - \gamma_0^2} \qquad (9)$$

Two special cases, $\ell = 0$ and $\ell = \infty$, may now be considered.

(a) As ℓ tends to infinity, the terms containing $\exp(-\gamma'\ell)$ tend to zero, and eqn. 9 is simplified to

$$Z_c = \frac{-K\gamma_0}{\gamma'^2 - \gamma_0^2} \qquad (10)$$

Eqn. 10 gives the following expression for the propagation coefficient:

$$\gamma' = \gamma_0 \left(1 - \frac{K}{\gamma_0 Z_c}\right)^{1/2} \qquad (11)$$

(b) When $\ell = 0$, terminals A and B coincide, and eqn. 9 is simplified to

$$Z_c - Z'_c = \frac{K}{\gamma' + \gamma_0} \qquad (12)$$

Elimination of γ' from eqns. 10 and 12 leads to the following expression for the characteristic impedance:

$$Z'_c = Z_c \left(1 - \frac{K}{\gamma_0 Z_c}\right)^{1/2} \qquad (13)$$

Eqns. 11 and 13 enable γ' and Z'_c to be determined if K is evaluated. From eqn. 5,

$$K = \frac{1}{I(z) I'(z)} \int_{-\infty}^{\infty} E'_z H_x\, dx \qquad (14)$$

Solutions of this integral are discussed in the following Section.

2.1 Approximate solution

When the ground is perfectly conducting, the tangential magnetic field H_x is given by

$$H_x = \frac{I(z) h}{\pi(h^2 + x^2)} \qquad (15)$$

perfectly conducting ground is approximately equal to that over perfectly conducting ground, provided that the modulus of the complex relative permittivity K_r is greater than 10. This condition is satisfied by ground of good conductivity at all frequencies and by ground of poor conductivity (10^{-3} S/m) at frequencies below 2 MHz. It follows therefore that E'_z is given approximately by

$$E'_z = -Z_s H'_x = \frac{-Z_s I'(z) h}{\pi(h^2 + x^2)} \qquad (16)$$

where Z_s is the surface impedance of the ground. Z_s is equal to the intrinsic impedance of the lower medium, provided that H'_x does not vary over the surface to any great extent within a distance $1/|\gamma|$, where γ is the propagation coefficient of the lower medium.2 This condition is assumed to be satisfied for the approximate solution described in this Section.

Substitution of eqns. 15 and 16 in eqn. 14, with $Z_s = z$, leads to

$$K = -\int_{-\infty}^{\infty} \frac{\eta h^2 dx}{\pi^2 (h^2 + x^2)^2} - \frac{-\eta}{2\pi h} \qquad (17)$$

Substitution of this value of K in eqns. 11 and 13 then gives the following approximate solutions for γ' and Z'_c:

$$\gamma' = \gamma_0 \left(1 + \frac{\eta}{2\pi h Z_c \gamma_0}\right)^{1/2} \qquad (18)$$

$$Z'_c = Z_c \left(1 + \frac{\eta}{2\pi h Z_c \gamma_0}\right)^{1/2} \qquad (19)$$

2.2 More exact solution

In the approximate solution, it was assumed that $E'_z = -\eta H'_x$, and that H'_x is equal to the magnetic field over perfectly conducting ground. An expression for E'_z which does not require these approximations is derived in this Section. E'_z is given by Maxwell's equation $(\sigma + j\omega\epsilon) E = \text{curl } H$ as

$$(\sigma + j\omega\epsilon) E'_z = \frac{\partial H'_y}{\partial x} - \frac{\partial H'_x}{\partial y} \qquad (20)$$

If eqn. 20 is applied just below the surface of the ground, the value of E'_z at the surface is obtained.

For perfectly conducting ground, the magnetic fields above the surface are usually calculated by assuming that an image current $-I$ flows at a depth h below the surface. Here it is assumed that the magnetic fields above imperfectly conducting ground may be calculated in a similar manner, by assuming that the strength of the image current is ρI, where

$$\rho = \frac{1 - K_r^{1/2}}{1 + K_r^{1/2}} \qquad (21)$$

and K_r is the complex relative permittivity of the ground.* Although this procedure is somewhat empirical, it yields exact values for the fields when the ground is either perfectly conducting ($K_r = \infty$) or absent ($K_r = 1$); it would therefore be expected to be a reasonable approximation for intermediate values of K_r. It also yields exact values for the fields at a great height above a conductor of finite length, for all values of K_r.

The magnetic fields at the surface calculated in this way are

$$H'_x = \frac{K_r^{1/2} h I'}{\pi(1 + K_r^{1/2})(h^2 + x^2)} \qquad (22)$$

$$H'_y = \frac{x I'}{\pi(1 + K_r^{1/2})(h^2 + x^2)} \qquad (23)$$

The magnetic fields immediately below the surface are identical, since the boundary conditions require H to be continuous.

* *ρ is the Fresnel plane-wave reflection coefficient for normally incident waves*

The first term on the right-hand side of eqn. 20 may now be obtained by differentiating eqn. 23, and is given by

$$\frac{\partial H'_y}{\partial x} = \frac{I'(h^2 - x^2)}{\pi(1 + K_r^{1/2})(h^2 + x^2)^2} = \frac{H'_x(h^2 - x^2)}{K_r^{1/2} h (h^2 + x^2)} \tag{24}$$

The second term on the right-hand side of eqn. 20 may be obtained by assuming that the variation of H'_x below the surface is described by the equation

$$H'_x(y) = H'_x(0) \exp(\gamma_y y) \tag{25}$$

where γ_y is the y component of the propagation coefficient of the wave which propagates just below the surface.* In general, this wave may be resolved into an angular spectrum of plane waves. Here it will be assumed, for convenience, that a plane wave having no variation in the x direction predominates; this assumption is reasonable at high frequencies, but is liable to fail at low frequencies, where the conductor height is comparable with the penetration depth. For the dominant wave described below,

$$\gamma_y^2 + \gamma_z^2 = \gamma^2 \tag{26}$$

where γ_z is the z component of its propagation coefficient and γ is the propagation coefficient for plane waves in the lower medium.

Differentiation of eqn. 25 then gives

$$\frac{\partial H'_x}{\partial y} = \gamma_y H'_x \tag{27}$$

From eqns. 22, 26 and 27, it follows that

$$\frac{\partial H'_x}{\partial y} = \frac{-K_r^{1/2} h I'}{\pi(1 + K_r^{1/2})(h^2 + x^2)} \left(\gamma^2 - \gamma_z^2\right)^{1/2} \tag{28}$$

Now γ_z is equal to γ', which is to be determined, and $\sigma + j\omega\epsilon$ is equal to γ/η. Using these identities, it may be shown from eqn. 20, 24 and 28 that a more accurate expression for E'_z is

$$E'_z = \eta H'_x \left[\frac{h^2 - x^2}{K_r^{1/2}\gamma h\,(h^2 + x^2)} - \left\{1 - \left(\frac{\gamma'}{\gamma}\right)^2\right\}^{1/2}\right] \tag{29}$$

where H'_x is given by eqn. 22.

Substitution of eqn. 29 in eqn. 14, followed by integration, enables K to be expressed in the form

$$K = \frac{-\eta W}{2\pi h} \tag{30}$$

where

$$W = \frac{-K_r^{1/2}}{1 + K_r^{1/2}} \left[\frac{1}{2K_r^{1/2}\gamma h} - \left\{1 - \left(\frac{\gamma'}{\gamma}\right)^2\right\}^{1/2}\right] \tag{31}$$

Comparison of eqns. 17 and 30 shows that they are of the same form, and it therefore follows that the more exact solutions for γ' and Z'_0 are given by

$$\gamma' = \gamma_0 \left(1 + \frac{\eta W}{2\pi h Z_c \gamma_0}\right)^{1/2} \tag{32}$$

$$Z'_c = Z_c \left(1 + \frac{\eta W}{2\pi h Z_c \gamma_0}\right)^{1/2} \tag{33}$$

Since W contains γ', eqn. 32 leads to a quadratic equation in γ'^2. Although γ' may be found by solving this equation, it is more convenient to substitute the approximate solution for γ' in the expression for W and then to evaluate γ' and Z' directly from eqns. 32 and 33.

In principle, the value of γ' derived from eqn. 32 could then be substituted in eqn. 31 for a further iteration, but computation has shown that no worthwhile advantage is to be gained from this procedure.

3 APPLICATION OF THEORY

In applying eqns. 18 and 19, or eqns. 32 and 33, to practical problems, formulas for the following quantities are also required

(a) The characteristic impedance of the conductor over perfectly conducting ground

$$Z_c = 60 \log (h/a) \tag{34}$$

where a is the conductor radius.

(b) The intrinsic impedance of the ground

$$\eta = \eta_0 K_r^{-1/2} \tag{35}$$

where $\eta_0 = 120\pi$ and K_r is the complex relative permittivity, defined as

$$K_r = \epsilon_r - j\epsilon'_r \tag{36}$$

where $\epsilon'_r = 1.8 \times 10^4 \sigma/f$.

(c) The free-space propagation coefficient

$$\gamma_0 = j\beta_0 = j2\pi/\lambda_0 \tag{37}$$

where λ_0 is the free-space wavelength.

The more exact solution also requires the value of γ, the propagation coefficient in the lower medium, which is equal to $\gamma_0 K_r^{1/2}$.

In practice, the real part α' of the solution for γ' is small.

It is more convenient to calculate the attenuation per kilometre, since the latter quantity is usually measured. The attenuation per kilometre is equal to $8686\alpha'$ decibels.

it is also more convenient to express the imaginary part β' in terms of the velocity of propagation along the conductor, relative to the free-space velocity (v/c). This is given by

$$\frac{v}{c} = \frac{\beta'}{\beta_0} \tag{38}$$

and is known as the velocity ratio.

Fig. 2 shows how the velocity ratio and attenuation for the current flowing in a typical aerial varies with frequency, for a representative range of ground constants. Fig. 2a shows that the velocity ratio tends to the free-space velocity at high frequencies, and depends on the ground conductivity at low frequencies. Fig. 2b shows that the ground conductivity also controls the attenuation at low frequencies; but, at high frequencies, the attenuation tends to a limiting value which depends on the relative permittivity of the ground. Fig. 2b shows curves for three values of ϵ_r, for σ = 0·003 S/m; curves for other conductivities tend to the same limiting values. The curves of Figs. 2 and 3 were computed by the method described in Section 2.2.

Fig. 3 shows the variation of characteristic impedance with frequency. At high frequencies, it tends to the perfect-earth value, and at low frequencies it depends only on the ground conductivity.

It may be shown that the variations of propagation coefficient and characteristic impedance with frequency are exactly the same as those of an air-spaced transmission line of characteristic impedance Z_c into which a series impedance $Z = \eta W/2\pi h$ per unit length has been inserted. If Z is small compared with $\gamma_0 Z_c$, α' and β' are given approximately by

$$\alpha' \simeq \frac{Re(Z)}{2Z_c} \tag{39}$$

$$\beta' \simeq \beta_0 + \frac{Im(Z)}{2Z_c} \tag{40}$$

where Re(Z) and Im(Z) denote the real and imaginary parts of Z, respectively, and $j\beta_0$ is the free-space propagation coefficient. The characteristic impedance is given approximately by

$$Z'_c \simeq Z_c + \frac{Z}{2\gamma_0} \tag{41}$$

* *In eqn. 25. the exponent $\gamma_y y$ is positive because paver*

For the approximate solution, W = **1** and $Z = \eta/2\pi h$. At high frequencies, η, tends asymptotically to $\eta_0 \epsilon_r^{-1/2}$, and, consequently, γ and Z'_c also tend to asymptotic values. Eqns. **39-41** show why the attenuation tends to a value which depends on the relative permittivity, while the velocity ratio and characteristic impedance tend to the perfect-earth values.

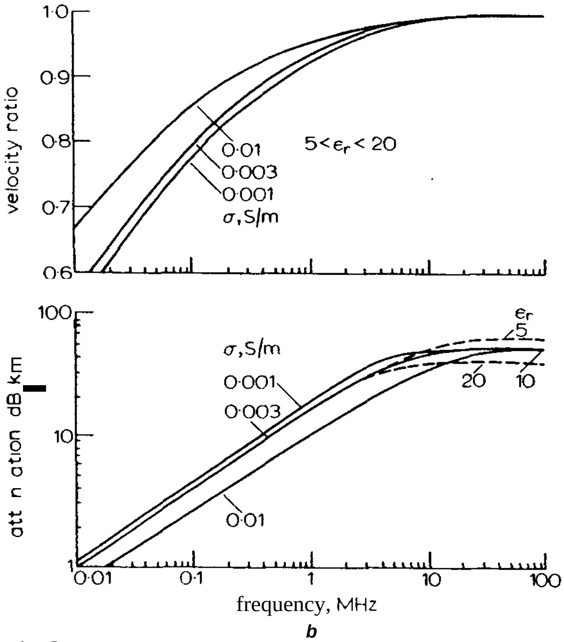

Fig. 2

Theoretical propagation coefficient

a Velocity ratio
b Attenuation
Conductor height = 3 m
Perfect-earth characteristic impedance = 426 Ω

At low frequencies, the real and imaginary parts of η are both approximately equal, and are proportional to $f^{1/2}$. At these frequencies, therefore, the attenuation increases as $f^{1/2}$ and the velocity of propagation deviates from the free-space velocity approximately as $f^{-1/2}$.

The trends described above apply also to the more exact solution. The principal differences between the two solutions arise at high frequencies, where the factor W differs appreciably from unity, but the only significant effect at high frequencies is an appreciable reduction in the attenuation.

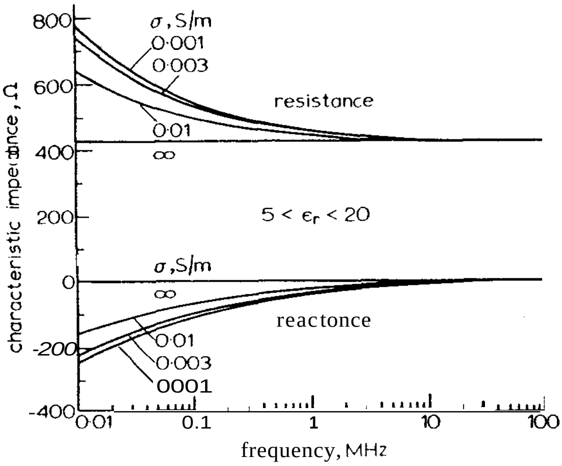

Fig. 3

Theoretical characteristic impedance

Conductor height = 3 m
Perfect-earth characteristic impedance = 426 Ω

Eqns. **39** and **40** also show that increasing the height of the conductor reduces both the attenuation and the velocity deviation; the effect is augmented by the small increase in Z_c. Raising the height of **a** Beverage aerial is not necessarily advantageous, however, as greater voltages are induced in the vertical end wires. The use of two or more conductors in parallel offers no advantage, since Z_c is reduced and the attenuation is thereby increased.

4 COMPARISON WITH MEASUREMENTS

The impedance/frequency characteristic of an 880 m Beverage aerial was measured with the aerial open-circuited at its far end. The aerial consisted of a single conductor of bare 6 s.w.g. (**4.88** mm-diameter) copper wire erected at a height of **3·0** m above ground.

The velocity ratio was calculated by comparing the measured electrical length with the electrical length which the aerial would have had if v were equal to c. The attenuation was derived from the relative amplitudes of the forward and reflected waves, the reflection coefficient at the far end being assumed to be unity.

In Fig. **4**, the measured values are compared with computed values, by the method described in Section 2. 2, for two ground conductivities; the conductivity below the aerial is believed to lie between these values. The measured velocity ratios agree well with theory, but the measured attenuations are somewhat lower than the theory predicts.

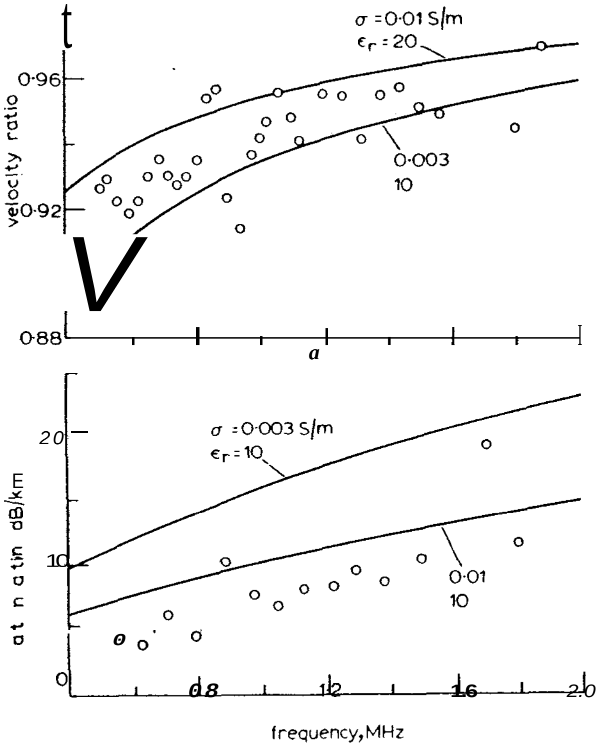

Fig. 4

Comparison of theoretical and measured propagation coefficients

a Velocity ratio *b Attenuation*
Conductor height = 3 m
Perfect-earth characteristic impedance = 426 Ω

Impedance/frequency measurements were also made on a **420** m aerial which was otherwise identical but was terminated in a ***500* Ω** impedance. The measured impedance was approximately equal **to** the characteristic impedance of the aerial; more precise values of Z'_c were derived from the geometric means of measurements made at pairs of frequencies chosen so that the electrical lengths of the aerial differed by $\lambda/4$. Fig. **5** shows **a** comparison of values **cal**culated in this way with values computed by the more exact method; the agreement with theory is seen to be reasonable. The characteristic impedance was also calculated from the measurements on the open-circuited ***880*** m Beverage aerial, and similar results were obtained, but the scatter of the calculated values was somewhat greater because of the greater variation in the measured impedances, caused by the aerial being open-circuited.

A comparison has **also** been made with velocity ratios and attenuations measured at very low frequencies by Beverage et al.[1] One of their aerials consisted of a single 10 a.w.g. (2·59 mm-diameter) bare copper conductor erected at a height of 8 m over ground whose conductivity was said to be 5×10^{-3} S/m. The measured velocity ratios and attenuations are compared with theoretical values, calculated by the method described in Section 2.2, in Table 1.

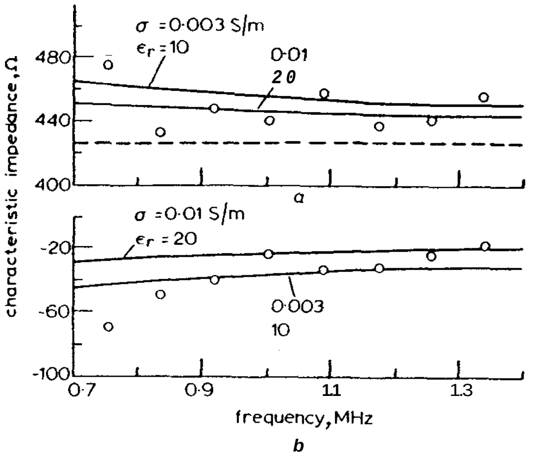

Fig. 5

Comparison of theoretical and measured characteristic impedance

a Resistance
b Reactance
—— theoretical
---- perfect-earth characteristic impedance
O measured values
Conductor height = 3 m

TABLE 1

COMPARISON OF THEORETICAL AND MEASURED PROPAGATION COEFFICIENTS

Frequency, kHz	12	20	30
Theoretical velocity ratio	0.823	0·857	0.880
Measured velocity ratio	0·827	0.840	0·853
Theoretical attenuation, dB/km	0.40	0.54	0.68
Measured attenuation, dB/km	0·20	0·27	0.34

Table 1 shows that the theoretical velocity ratios agree reasonably well with measurements, but that the theoretical attenuations are twice as large as the measured values. This ratio is believed to be lortuitous, since closer agreement between measured and computed attenuations is obtained at higher frequencies.

Beverage also describes measurements made on 2-wire and 4-wire aerials; these show lower velocity ratios and higher attenuations than those measured with single-wire aerials. This result is consistent with theory.

5 COMPARISON WITH OTHER THEORIES

It is of interest to compare the present theory with other methods which have been used to solve the same problem. Of particular interest is the method used by Carson,[3] which is restricted to low frequencies because displacement currents in the ground are neglected.* Carson's method was later extended to higher frequencies by Wise.[5]

Carson expresses the field above the ground in the form of an angular spectrum of plane waves, and equates this to the sum of the fields due to the current in the conductor and the conduction current in the ground. This leads to an expression for the series impedance, arising from the imperfect conductivity, in the form of a definite integral. Carson's paper contains curves and formulas which enable the integral to be evaluated.

Carson's formulas and curves are expressed in terms of a parameter r, which may be shown to be equal to $2\beta_0 h(\epsilon_r')^{1/2}$, where ϵ_r' is the imaginary part of the complex relative permittivity of the ground, defined in eqn. 36. Thus r varies as $f^{-1/2}$ and is small at low frequencies. An interesting feature of Carson's theory is that the real part of the integral tends to a limiting value when r is small. A consequence is that the series resistance is then directly proportional to frequency and that the attenuation increases in the same way. This result conflicts with that derived from the compensation theorem, which yields attenuations proportional to $f^{1/2}$ at low frequencies and is not confirmed by Beverage's measurements, which also vary as $f^{1/2}$ (as can be seen from Table 1) even though r is small. Attenuations measured by Jenssen[6] at medium frequencies also varied less rapidly than Carson's theory predicts. Variation as $f^{1/2}$ is to be expected intuitively, because both the complex relative permittivity of the ground and the penetration depth vary inversely as $f^{1/2}$.

Wise extended Carson's theory to include the effect of displacement currents, and his theory is not therefore subject to an upper frequency limit. Wise replaced Carson's parameter r by a complex parameter which can be shown to be equal to

$$2\beta_0 h\{j(K_r - 1)\}^{1/2}$$

At low frequencies, $K_r \gg 1$ and $jK_r \simeq \epsilon_r'$, and the two parameters have similar values; the theories then give identical results. At high frequencies, $K_r \simeq \epsilon_r$, and it may be shown that the series impedance tends asymptotically to $\eta/2\pi h$, where η is the intrinsic impedance of the ground. This result is identical with that given at all frequencies by the compensation theorem, using the approximate solution described in Section 2.1.

Reference must also be made to Kikuchi,[7] who also extended Carson's theory to higher frequencies; he does not appear to have been aware of the paper by Wise. Kikuchi's paper contains a curve showing the theoretical attenuation of a conductor 7·5 m above ground of good conductivity (10^{-2} S/m) at frequencies between 1 and 10^4 MHz. The attenuation increases, following Carson's theory, up to 10 MHz and then falls, reaching a minimum value at about 100 MHz. At even higher frequencies, it assumes the attenuation appropriate to an axial cylindrical surface wave[8] supported by a conductor of finite resistance in free space. This result is hardly surprising, because Kikuchi assumes the relative permittivity of the ground to be unity, and, consequently, the ground approximates to free space at frequencies above 1000 MHz. Unfortunately, Kikuchi's paper does not appear to contain sufficient information to enable attenuations to be calculated for more practical cases.†

6 CONCLUSIONS

Use of the compensation theorem has shown that the velocity of propagation of the current flowing on a Beverage aerial when driven approaches the free-space velocity at high frequencies. At low frequencies, the velocity is less than the free-space velocity, the reduction in velocity depending on the ground conductivity and the height of the conductor. These factors also govern the rate of attenuation at low frequencies, where the theory indicates that the attenuation is proportional to the square root of the frequency. At high frequencies, the attenuation increases to a limiting value which depends on the relative permittivity of the ground.

Increasing the height of the conductor reduces the attenuation and makes the velocity of propagation more nearly equal to the free-space velocity. In a long Beverage receiving aerial, this would result in an improvement in sensitivity and directivity, provided that the greater voltages induced in the vertical end wires can be cancelled. The use of multiple wires reduces the velocity and increases the attenuation, and therefore degrades the performance of a Beverage aerial.

The theory shows that the characteristic impedance of the aerial is capacitive and is greater than the characteristic impedance of an identical aerial erected over perfectly conducting ground. At high frequencies, however, the characteristic impedance tends towards the perfect-earth value.

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* *A similar solution was published independently by Pollaczek*[4]

† *It is perhaps worth noting that the more exact solution described in Section 2·2 also shows a reduction in attenuation at very high frequencies if ϵ_r is specified as unity, although the result obtained differs from Kikuchi's*

Theoretical propagation coefficients show reasonably good agreement with measurements. At medium frequencies, the theoretical velocity of propagation agrees well with measured values, but the attenuation is slightly overestimated. At very low frequencies, the measured velocity still shows good agreement with theory, but the theoretical attenuation is too great by a factor of about two. Comparison between theory and measurement at high frequencies has yet to be made.

The theory based on the compensation theorem is relatively easy to apply, and appears to give results at medium and high frequencies which are sufficiently accurate for practical purposes. At high frequencies, the theory is consistent with the more complicated Carson-Wise theory, but the two theories disagree at very low frequencies. Here the Carson-Wise theory predicts attenuations proportional to f, where f is the frequency, while the present theory suggests that they vary as $f^{1/2}$. Although the approximations contained in the theory described in this paper are responsible for overestimating the attenuation at very low frequencies, the rate at which the attenuation varies with frequency is less liable to error and is, moreover, consistent with measurements.

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