The Wave Antenna for Transmission and Reception

RONOLD W. P. KING, LIFE FELLOW, IEEE

Abstract—**The traveling-wave horizontal-wire antenna over the earth is analyzed in its original form with vertical ground connections (the Beverage antenna) and with these replaced by horizontal terminations. For transmission, the electromagnetic field of an antenna with optimum length is determined, both along the surface of the earth in air and in the earth, in an accurate but simple form that takes full account of the proximity of the earth (lake, sea) on the distribution of current. For reception, the induced currents in the antenna and in the load are determined for a field incident along the surface of the earth. The two types of termination are compared and their contributions shown to be small when the horizontal wire has a length near the optimum.**

I. INTRODUCTION

A USEFUL antenna for transmission between points on or near the surface of the earth is the wave antenna first described by Beverage *et al.* [1]. It consists of a long wire close to the surface of the earth (sea, lake, ice) and driven and terminated to carry a traveling wave of current. The Beverage antenna has been discussed primarily for reception [2]-[5]. In his book on traveling-wave antennas, Walter [6] assumes the current on the Beverage antenna to be $I_x(x) = I_x(0) \exp(ik_0 x)$, where k_0 is the *real wavenumber of air* and determines the far-field pattern of the wire alone. He states that "the presence of the ground complicates the problem" and that "the usual approach is to represent the ground as a complex dielectric and consider the total pattern of the antenna and its properly weighted image." This approach disregards the effect of the earth on the distribution of current, assumes that a valid image can be constructed to replace the earth, and provides no means for obtaining the field along the surface of the earth where a lateral wave dominates.

The current in and driving-point admittance of a horizontal-wire antenna over the earth are known [7]-[9]. When the wire is close to the earth, the wavenumber for the current is significantly different from the free-space wavenumber. Furthermore, a study of the electric field at the surface of the earth near the antenna indicates that the scattered field *cannot* be interpreted as due to the current in an image antenna at a specified distance from the boundary.

In the applications of the wave antenna, the field along the surface of the earth is of interest. This is the field involved in the transmission from a driven wave antenna to a second wave or other receiving antenna. It is the reflected field received by a Beverage receiving antenna from an obstacle near the horizon when this is illuminated by a radar beam. No accurate determination of this field appears to be available in the literature.

Manuscript received March 1, 1983; revised July 7, 1983. This work was supported in part by the Joint Services Electronics Program under Contract N00014-75-C-0648 and is based on research supported in part by the Office of Naval Research under Contract N00014-79-C-0419, both with Harvard University.

The author is with the Gordon McKay Laboratory, Harvard University, Cambridge, MA 02138.

II. DESCRIPTION OF THE WAVE ANTENNA AND ITS SURROUNDINGS

The conventional Beverage antenna shown in Fig. 1(a) consists of a long horizontal conductor that extends from $x = 0$ to $x = h$ in Region 2 at the constant height d above the x-axis and two vertical conductors in series with suitable impedances that assure a traveling wave along the horizontal wire. The vertical conductors enter the ground at $z = 0$ and continue down into Region 1 a distance l, where they end in radial ground networks. The radius of each conductor is a. An alternative form, shown in Fig. 1(b), consists of only a horizontal wire at the height d over the earth. It is driven at $x = 0$ at the distance l from the open end at $x = -l$, is loaded at $z = h$ by a series impedance Z_L, and ends at $z = h + l$.

The electrical properties of the two half-spaces, Region 2 (air) at $z < 0$ and Region 1 (earth, sea, lake) at $z > 0$, are characterized by the wavenumbers $k_2 = k_0 = \omega(\mu_0\epsilon_0)^{1/2}$ and $k_1 = \beta_1 + i\alpha_1 = \omega(\mu_0\tilde{\epsilon}_1)^{1/2}$, where $\tilde{\epsilon}_1 = \epsilon_1 + i\sigma_1/\omega$. Here $\epsilon_1 = \epsilon_0\epsilon_{1r}$ and σ_1 are the real effective permittivity and conductivity. Also useful are the wave impedances $\zeta_2 = \zeta_0 = (\mu_0/\epsilon_0)^{1/2} \sim 120\pi\ \Omega$ and $\zeta_1 = (\mu_0/\tilde{\epsilon}_1)^{1/2} = \omega\mu_0/k_1$. The time dependence is $e^{-i\omega t}$. The following conditions are imposed:

$$|k_1| \geqslant 3k_2, \qquad k_2 d \leqslant 0.2\pi, \qquad |k_1 l| \leqslant 0.2\pi. \tag{1}$$

A consequence of (1) is that the power radiated into Region 2 (air) is negligibly small compared to that transmitted into Region 1 (earth, sea, etc.). The horizontal wire is so closely coupled to the earth that it cannot properly be treated as a wire in air with only an added correction to take account of the earth. In effect, it is an *eccentrically insulated conductor lying on the earth*. The thickness of the air insulation is essentially the height d of the wire. When (1) is satisfied, the current in the wire is like that in a transmission line with the wavenumber $k_L = \beta_L + i\alpha_L$ and a characteristic impedance Z_c (that depend on the inductance, capacitance and resistance per unit length of the wire) and not like that of an antenna in air with the wavenumber k_2 of air. For an antenna in air, radiation is a property of the antenna as a whole; it cannot be assigned per unit length. For the antenna close to the earth, radiation is a property per unit length. The formula for k_L when $|k_1 d|$ is not too large is [7]:

$$k_L = k_2 \left\{ 1 + \frac{2}{\ln(2d/a)} \left[\frac{1}{(2k_1 d)^2} - \frac{K_1(2k_1 d)}{2k_1 d} + \frac{i\pi I_1(2k_1 d)}{4k_1 d} - i\left(\frac{2k_1 d}{3} + \frac{(2k_1 d)^3}{45} + \frac{(2k_1 d)^5}{1575} + \frac{(2k_1 d)^7}{99225} + \cdots \right) \right] \right\}^{1/2}. \tag{2}$$

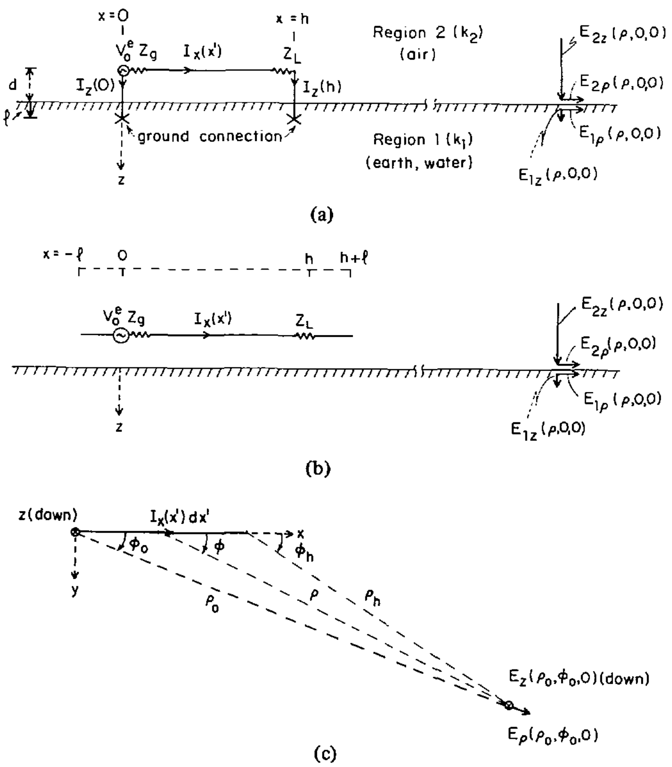

Fig. 1. Wave antennas. (a) Conventional Beverage antenna. (b) Horizontal-wire antenna. (c) Coordinates.

When $|k_1 d|$ is large [7]:

$$k_L \sim k_2 \left\{ 1 + \frac{2}{\ln(2d/a)} \left[\frac{1}{(2k_1 d)^2} - \frac{1}{2k_1 d} \left(\frac{\pi}{4k_1 d} \right)^{1/2} \cdot e^{-2k_1 d} \left(1 + \frac{3}{16k_1 d} - \frac{15}{128(2k_1 d)^3} + \cdots \right) + \frac{i}{2k_1 d} \left(1 - \frac{1}{(2k_1 d)^2} - \frac{3}{(2k_1 d)^4} - \cdots \right) \right] \right\}^{1/2}. \quad (3)$$

The formula for Z_c is

$$Z_c = (\zeta_2 k_L / 2\pi k_2) \ln(2d/a) = (60 k_L / k_2) \ln(2d/a)\ \Omega. \quad (4)$$

In (2) $K_1(x)$ and $I_1(x)$ are the modified Bessel functions. The current on the conductor has the general transmission-line form [9, p. 92, but for a monopole]:

$$I_x(x) = \frac{-iV_0^e}{Z_c} \frac{\sin[k_L(h-x) + i\theta_h]}{\cos(k_L h + i\theta_h)} \quad (5)$$

where $\theta_h = \coth^{-1}(Z_h/Z_c)$. When $Z_h = Z_c$, $\theta_h \to \infty$ and

$$I_x(x) = \frac{V_0^e}{Z_c} e^{ik_L x}. \quad (6)$$

Note that the wavenumber for the current is k_L, not k_2.

The impedance of the antenna at the driving point $x = 0$ is $Z_{\text{in}} = Z + Z_g + Z_0$, where $Z = iZ_c \cot(k_L h + i\theta_h)$, Z_g is the impedance of the generator and Z_0 the impedance of the terminating section at $x = 0$. For traveling-wave operation, the effective impedance $Z_h = Z_L + Z_0$ of the terminating section at $x = h$ is made equal to the characteristic impedance Z_c so that $Z = Z_h = Z_c$. With the Beverage antenna, Z_0 is the impedance of each of the grounded vertical conductors. With the horizontal-wire antenna, Z_0 is the impedance of an open-ended section of the eccentrically insulated wire of length l; i.e., with $\theta_h \sim -i\pi$ for an ideal open end, $Z_0 = iZ_c \cot k_L l$.

III. THE ELECTRIC FIELD DUE TO DIPOLES WITH UNIT ELECTRIC MOMENT

The first step in determining the electromagnetic field generated by the currents in the several parts of the antennas shown in Fig. 1 is to express the field generated by an element of current $I_z(z')dz'$ in the vertical elements and by $I_x(x')dx'$ in the horizontal element. Evidently, there are two types of vertical elements, those on the boundary surface in Region 2 ($z \leqslant 0$) and those on the boundary surface in Region 1 ($z \geqslant 0$). The eccentrically insulated horizontal element is on the boundary entirely in Region 2.

Simple and accurate formulas for all components of the electromagnetic field in Region 1 and along the boundary in Region 2 are available when the infinitesimal vertical [10] or horizontal [11]-[13] electric dipoles are embedded in Region 1 ($z \geqslant +0$) or are on the surface in either Region 1 ($z = +0$) or Region 2 ($z = -0$). These are the fields of primary interest for transmission to a receiving antenna on the surface in Region 2 or below the surface in Region 1. They include transmission between points on or below the surface of the earth or ocean. To distinguish between the fields in Regions 1 and 2 due to vertical and horizontal elements in Regions 1 or 2, the following notation is used: $[E_{1\rho}(\rho, \phi, z)]_{2v}$. This gives the radial component (subscript ρ) of the electric field at the point (ρ, ϕ, z) in Region 1 (subscript 1) due to a vertical dipole in Region 2 (subscripts 2 and v on square bracket). Similarly, $[E_{2z}(\rho, \phi, 0)]_{1h}$ is the z-component of the electric field at the point $(\rho, \phi, 0)$, i.e., on the boundary surface, in Region 2 due to a horizontal electric dipole in Region 1. The formulas give the radial and vertical components of the electric field (E_ρ and E_z) and the transverse component of the magnetic field (B_ϕ) at distances from the source that are sufficiently great to make contributions from terms that involve $\exp(ik_1\rho) = \exp(-\alpha_1\rho)\exp(i\beta_1\rho)$ negligible. This is only a short distance (much less than a wavelength) when Region 1 is earth or water with practically available effective conductivity. To conserve space, the following notation is used

$$F(k_2\rho, k_1) = \frac{\omega\mu_0}{2\pi} [f_2(\rho)e^{ik_2\rho} + f_{21}(\rho)e^{ik_{21}\rho}] \quad (7)$$

$$G(k_2\rho, k_1) = \frac{\omega\mu_0}{2\pi} [g_2(\rho)e^{ik_2\rho} + f_{21}(\rho)e^{ik_{21}\rho}] \quad (8)$$

where

$$f_2(\rho) = \frac{ik_2}{\rho} - \frac{1}{\rho^2}; \quad g_2(\rho) = \frac{ik_2}{\rho} - \frac{1}{\rho^2} - \frac{i}{k_2\rho^3} \quad (9)$$

$$f_{21}(\rho) = -\frac{k_2^3}{k_1} \left(\frac{\pi}{k_2\rho} \right)^{1/2} \mathcal{F}(k_2\rho, k_1) \quad (10)$$

with

$$k_{21} = k_2(1 - k_2^2/2k_1^2) \quad (11)$$

and

$$\mathcal{F}(k_2\rho, k_1) = \tfrac{1}{2}(1+i) - C_2[k_2\rho(k_2^2/2k_1^2)] - iS_2[k_2\rho(k_2^2/2k_1^2)]. \quad (12)$$

TABLE I
ELECTROMAGNETIC FIELD OF AN INFINITESIMAL ELECTRIC DIPOLE ON OR BELOW THE SURFACE BETWEEN REGIONS 1 (EARTH, WATER) AND 2 (AIR); $I\Delta l = 1$ A·m

Component of the Field	Vertical Electric Dipole on Surface in Air	Horizontal Electric Dipole on Surface in Air or Water	Vertical Electric Dipole on Surface in Salt Water
$E_{2\rho}(\rho,\phi,0)$	$-k_1^{-1}F(k_2\rho,k_1)$	$-k_2k_1^{-2}G(k_2\rho,k_1)\cos\phi$	$-k_2^2k_1^{-3}F(k_2\rho,k_1)$
$E_{1\rho}(\rho,\phi,z)$	$-k_1^{-1}F(k_2\rho,k_1)\exp(ik_1z)$	$-k_2k_1^{-2}G(k_2\rho,k_1)\cos\phi\exp(ik_1z)$	$-k_2^2k_1^{-3}F(k_2\rho,k_1)\exp(ik_1z)$
$E_{2z}(\rho,\phi,0)$	$k_2^{-1}G(k_2\rho,k_1)$	$k_1^{-1}F(k_2\rho,k_1)\cos\phi$	$k_2k_1^{-2}G(k_2\rho,k_1)$
$E_{1z}(\rho,\phi,z)$	$k_2k_1^{-2}G(k_2\rho,k_1)\exp(ik_1z)$	$k_2^2k_1^{-3}F(k_2\rho,k_1)\cos\phi\exp(ik_1z)$	$k_2^3k_1^{-4}G(k_2\rho,k_1)\exp(ik_1z)$
$B_{2\phi}(\rho,\phi,0)$	$-\omega^{-1}F(k_2\rho,k_1)$	$-\omega^{-1}k_2k_1^{-1}G(k_2\rho,k_1)\cos\phi$	$-\omega^{-1}k_2^2k_1^{-2}F(k_2\rho,k_1)$
$B_{1\phi}(\rho,\phi,z)$	$-\omega^{-1}F(k_2\rho,k_1)\exp(ik_1z)$	$-\omega^{-1}k_2k_1^{-1}G(k_2\rho,k_1)\cos\phi\exp(ik_1z)$	$-\omega^{-1}k_2^2k_1^{-2}F(k_2\rho,k_1)\exp(ik_1z)$

$k_1^2 = \omega^2\mu_0(\epsilon_1 + i\sigma_1/\omega)$, $k_2^2 = \omega^2\mu_0\epsilon_0$; Conditions: $|k_1| \geq 3k_2$, $\rho \geq 5z$, $|k_1\rho| \geq 3$

Here $C_2(u)$ and $S_2(u)$ are Fresnel integrals defined by [14], [15]:

$$C_2(u) + iS_2(u) = \int_0^u (2\pi t)^{-1/2} e^{it}\,dt. \tag{13}$$

When $k_2\rho$ is sufficiently large, the Fresnel integrals are well represented by the leading terms in asymptotic formulas. With these,

$$F(k_2\rho, k_1) \sim G(k_2\rho, k_1) \sim -\frac{\omega\mu_0}{2\pi}\frac{k_1^2}{k_2^2}\frac{e^{ik_2\rho}}{\rho^2}; \qquad k_2\rho \geqslant 8|k_1^2|/k_2^2. \tag{14}$$

With this notation the expressions for the fields of infinitesimal vertical and horizontal electric dipoles with unit electric moments ($I\Delta l = 1$ A·m) located on the boundary between Regions 1 and 2 are summarized in Table I together with the limiting conditions. Formulas are given for vertical dipoles just above the boundary in Region 2 (air) and just below the boundary in Region 1 (earth, water, etc.). The formulas for the horizontal dipole are the same when the dipole is just above or just below the boundary. The formulas for the components of the electric and magnetic fields are for points on the surface in Region 2 ($z = -0$) and for points on and below the surface in Region 1 ($z \geqslant +0$). The rotationally symmetric field of the vertical dipole is smaller by the very small factor k_2^2/k_1^2 when the dipole is just below the surface in Region 1 than when it is just above the surface in Region 2. The directional field of the horizontal dipole is smaller by the factor k_2/k_1 than the field of the vertical dipole when this is in Region 2, and larger by the factor k_1/k_2 than the field of the vertical dipole when this is in region 1. It is, of course, possible to make the electric moment of a horizontal antenna sufficiently great so that its field is greater than that of a vertical antenna in Region 2 with a small electric moment. The fields in Table I are useful for the Beverage antenna which has both vertical and horizontal conductors in Regions 1 and 2.

Note that the largest component of the electric field in Region 2 is E_{2z}, the largest in Region 1 is $E_{1\rho}$. The electromagnetic field of the horizontal dipole also has the components E_ϕ, B_ρ, and B_z; however, these are not significant in the operation of the Beverage and horizontal-wire antennas. They all have the factor $\sin\phi$ and are, therefore, zero in the direction $\phi = 0$ in which E_ρ, E_z, and B_ϕ have their maxima.

IV. THE ELECTRIC FIELD OF THE VERTICAL ELEMENTS OF THE BEVERAGE ANTENNA

The vertical parts of the Beverage antenna shown in Fig. 1(a) consist of two sections, one along the z-axis at $x = y = 0$, the other at $x = h$, $y = 0$. Each is composed of two parts: a length from $z = -d$ to 0 in Region 2 and a length from $z = 0$ to a radial ground system at $z = l$ in Region 1. Since it is assumed that d is electrically short in Region 2 so that $k_2d \leqslant 0.2\pi$ and l is electrically short in Region 1 so that $|k_1l| \leqslant 0.2\pi$, it follows that the currents in these lengths are sensibly constant. In both the lengths d and l, they are given by $I_z(x = 0) \sim -I_x(0)$ at $x = y = 0$ and by $I_z(x = h) \sim I_x(h)$ at $x = h, y = 0$. The associated electric moments are $-I_x(0)d$ and $I_x(h)d$ for the lengths in Region 2, $-I_x(0)l$ and $I_x(h)l$ for the lengths in Region 1.

The components of the electric field on the surface $z = 0$ in Region 2 due to vertical elements in Region 2 are taken from Table I multiplied by the respective electric moment. For the elements at $x = 0$ and $x = h$, they are

$$[E_{2z}(\rho_0, \phi_0, 0]_{2vc} = -k_2^{-1}d[I_x(0)G(k_2\rho_0, k_1) - I_x(h)G(k_2\rho_h, k_1)] \tag{15}$$

$$[E_{2\rho}(\rho_0, \phi_0, 0]_{2vc} = k_1^{-1}d[I_x(0)F(k_2\rho_0, k_1) - I_x(h)F(k_2\rho_h, k_1)] \tag{16}$$

where ρ_0 is the radial distance from the z-axis and $\rho_h = (\rho_0^2 + h^2 - 2h\rho_0\cos\phi_0)^{1/2}$. The components of the electric field in Region 2 due to the vertical elements in Region 1 are similarly obtained from Table I. They are

$$[E_{2z}(\rho_0, \phi_0, 0)]_{1vc} = -k_2k_1^{-2}l[I_x(0)G(k_2\rho_0, k_1) - I_x(h)G(k_2\rho_h, k_1)] \tag{17}$$

$$[E_{2\rho}(\rho_0, \phi_0, 0)]_{1vc} = k_2^2k_1^{-3}l[I_x(0)F(k_2\rho_0, k_1) - I_x(h)F(k_2\rho_h, k_1)] \tag{18}$$

where $|k_1l| \leqslant 0.2\pi$. It follows from (15) and (17) and from (16) and (18) that

$$\frac{[E_{2z}(\rho_0, \phi_0, 0)]_{1vc}}{[E_{2z}(\rho_0, \phi_0, 0)]_{2vc}} = \frac{[E_{2\rho}(\rho_0, \phi_0, 0)]_{1vc}}{[E_{2\rho}(\rho_0, \phi_0, 0)]_{2vc}} = \frac{k_2^2 l}{k_1^2 d} \sim \frac{k_2^3}{k_1^3}. \tag{19}$$

The last step follows since $k_2 d$ and $|k_1 l|$ are of comparable magnitudes ($k_2 d \leqslant 0.2\pi$, $|k_1 l| \leqslant 0.2\pi$). It may be concluded that the fields generated by the currents in the two vertical elements of length l in Region 1 are negligible compared to the fields generated by the currents in the vertical elements of length d in Region 2.

With $I_x(h) = I_x(0) \exp(ik_L h)$, $G(k_2\rho_h, k_1) \sim G(k_2\rho_0, k_1) \exp(-ik_2 h \cos\phi_0)$, and a similar expression for $F(k_2\rho_h, k_1)$, the field due to the vertical elements is

$$\begin{aligned}[E_{2z}(\rho_0, \phi_0, 0)]_{1vc} &+ [E_{2z}(\rho_0, \phi_0, 0)]_{2vc} \\ &\doteq [E_{2z}(\rho_0, \phi_0, 0)]_{2vc} = -k_2^{-1} d I_x(0) G(k_2\rho_0, k_1) \\ &\cdot [1 - e^{i(k_L - k_2\cos\phi_0)h}] \end{aligned} \tag{20}$$

$$\begin{aligned}[E_{2\rho}(\rho_0, \phi_0, 0)]_{1vc} &+ [E_{2\rho}(\rho_0, \phi_0, 0)]_{2vc} \\ &\doteq [E_{2\rho}(\rho_0, \phi_0, 0)]_{2vc} = k_1^{-1} d I_x(0) F(k_2\rho_0, k_1) \\ &\cdot [1 - e^{i(k_L - k_2\cos\phi_0)h}]. \end{aligned} \tag{21}$$

V. THE ELECTRIC FIELD OF THE HORIZONTAL ELEMENTS OF THE BEVERAGE AND HORIZONTAL-WIRE ANTENNAS

The horizontal conductor in Region 2 (air) at the electrically small height d over Region 1 (earth, water) has been analyzed as an eccentrically insulated antenna on the surface of Region 1. Its wavenumber k_L is given by (2), the characteristic impedance by (4), and the distribution of current by (5). When the terminating impedances $Z_L + Z_0 = Z_g + Z_0 = Z_c$, the current is a simple traveling wave given by $I_x(x) = I_x(0) \exp(ik_L x)$, where $I_x(0) = V_0^e/Z_c$.

The field generated by the current in the horizontal wire between $x = 0$ and $x = h$ is the superposition in proper phase and amplitude of the continuity of elements $I_x(x')dx'$ along the wire. If the radial distance from the origin at $x = y = z = 0$ to the point of observation at $(\rho_0, \phi_0, 0)$ is ρ_0, the radial distance from an element of current at x', $y = z = 0$, is $\rho_{x'} = (\rho_0^2 + x'^2 - 2\rho_0 x' \cos\phi_0)^{1/2}$. When $\rho_0^2 \gg h^2 \gg x'^2$, $\rho_{x'} \sim \rho_0 - x'\cos\phi_0$ in phases, $\rho_{x'} \sim \rho_0$ in amplitudes. It follows with (7) and (8) that

$$\begin{aligned} F(k_2\rho_{x'}, k_1) \sim \frac{\omega\mu_0}{2\pi} [f_2(\rho_0) e^{ik_2(\rho_0 - x'\cos\phi_0)} \\ + f_{21}(\rho_0) e^{ik_{21}(\rho_0 - x'\cos\phi_0)}] \end{aligned} \tag{22}$$

$$\begin{aligned} G(k_2\rho_{x'}, k_1) \sim \frac{\omega\mu_0}{2\pi} [g_2(\rho_0) e^{ik_2(\rho_0 - x'\cos\phi_0)} \\ + f_{21}(\rho_0) e^{ik_{21}(\rho_0 - x'\cos\phi_0)}]. \end{aligned} \tag{23}$$

The vertical electric field at $\rho_0^2 \gg h^2$ due to the elements of current $I_x(x')dx'$ in the horizontal wire ($0 \leqslant x' \leqslant h$) in both the Beverage and horizontal-wire antennas is

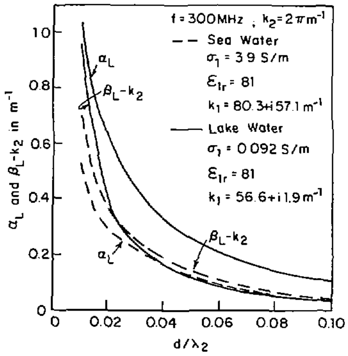

Fig. 2. Attenuation and phase constants of the current on the horizontal wire over sea and lake water.

$$\begin{aligned} [E_{2z}(\rho_0, \phi_0, 0)]_{hw} \\ = k_1^{-1} I_x(0) \cos\phi_0 \int_0^h F(k_2\rho_{x'}, k_1) e^{ik_L x'} dx'. \end{aligned} \tag{24}$$

With (22), the integral to be evaluated is

$$\begin{aligned} \int_0^h e^{i(k_L - k_2\cos\phi_0)x'} dx' \\ = i[1 - e^{i(k_L - k_2\cos\phi_0)h}]/(k_L - k_2\cos\phi_0). \end{aligned} \tag{25}$$

There is a similar integral with k_{21} substituted for k_2 in (25). However, since $9k_2^2 \leqslant |k_1|^2$ it follows that, with $\phi_0 = 0$, $\exp(-ik_{21}h) = \exp[-ik_2 h(1 - k_2^2/2k_1^2)] \sim \exp(-ik_2 h)$ when $|k_2 h(k_2^2/2k_1^2)| \leqslant 0.2$, an inequality that it is usually desirable to satisfy. When it is not valid, the two terms can be treated separately without difficulty. When $\exp(-ik_{21}h) \sim \exp(-ik_2 h)$,

$$\begin{aligned} [E_{2z}(\rho_0, \phi_0, 0)]_{hw} \\ = I_x(0) \cos\phi_0 \frac{i[1 - e^{i(k_L - k_2\cos\phi_0)h}]}{k_1(k_L - k_2\cos\phi_0)} F(k_2\rho_0, k_1). \end{aligned} \tag{26}$$

Similarly,

$$\begin{aligned} [E_{2\rho}(\rho_0, \phi_0, 0)]_{hw} \\ = -I_x(0) \cos\phi_0 \frac{ik_2[1 - e^{i(k_L - k_2\cos\phi_0)h}]}{k_1^2(k_L - k_2\cos\phi_0)} G(k_2\rho_0, k_1). \end{aligned} \tag{27}$$

The quantities α_L and $(\beta_L - k_2)$ in the exponents with $\phi_0 = 0$ are shown graphically in Fig. 2 for sea and lake water at $f = 300$ MHz.

The horizontal-wire antenna has two horizontal terminations that extend a distance l beyond each end of the main section of length h. When $I_x(x') = I_x(0) \exp(ik_L x')$ is the current on the main section, the currents in the end sections are

$$I_x(x') = I_x(0) \frac{\sin k_L(l + x')}{\sin k_L l}, \qquad -l \leqslant x' \leqslant 0 \tag{28}$$

$$I_x(x') = I_x(0) e^{ik_L h} \frac{\sin k_L(h + l - x')}{\sin k_L l}, \qquad h \leqslant x' \leqslant h + l. \tag{29}$$

With these, the vertical component of the electric field due to each is

$$[E_{2z}(\rho_0, \phi_0, 0)]_{h0} = \frac{I_x(0)}{k_1 \sin k_L l} \cos \phi_0 \int_{-l}^{0} F(k_2 \rho_{x'}, k_1) \cdot \sin k_L(l + x')\, dx' \tag{30}$$

$$[E_{2z}(\rho_0, \phi_0, 0)]_{hh} = \frac{I_x(0) e^{ik_L h}}{k_1 \sin k_L l} \cos \phi_0 \int_{h}^{h+l} F(k_2 \rho_{x'}, k_1) \cdot \sin k_L(h + l - x')\, dx'. \tag{31}$$

With (22), a typical integral to be evaluated is

$$\int_{h}^{h+l} \sin k_L(h+l-x') e^{-ik_2 x' \cos\phi_0}\, dx' = e^{-ik_2 h \cos\phi_0} \frac{[C(k_2, k_L, \phi_0) + iS(k_2, k_L, \phi_0)]}{k_L^2 - k_2^2 \cos^2 \phi_0} \tag{32}$$

where

$$C(k_2, k_L, \phi_0) = k_L \; [\cos (k_2 l \cos \phi_0) - \cos k_L l] \tag{33}$$

$$S(k_2, k_L, \phi_0) = k_L \sin (k_2 l \cos \phi_0) - k_2 \cos \phi_0 \sin k_L l. \tag{34}$$

It follows that the fields due to the horizontal terminations are

$$[E_{2z}(\rho_0, \phi_0, 0)]_{h0} = \frac{I_x(0) F(k_2\rho_0, k_1) \cos \phi_0}{k_1(k_L^2 - k_2^2 \cos^2 \phi_0) \sin k_L l} \cdot [C(k_2, k_L, \phi_0) - iS(k_2, k_L, \phi_0)] \tag{35}$$

$$[E_{2z}(\rho_0, \phi_0, 0)]_{hh} = \frac{I_x(0) F(k_2\rho_0, k_1) \cos \phi_0 \, e^{i(k_L - k_2 \cos\phi_0)h}}{k_1(k_L^2 - k_2^2 \cos^2 \phi_0) \sin k_L l} \cdot [C(k_2, k_L, \phi_0) + iS(k_2, k_L, \phi_0)]. \tag{36}$$

The sum gives the field due to both terminations. It is

$$[E_{2z}(\rho_0, \phi_0, 0)]_{ht} = \frac{I_x(0) F(k_2\rho_0, k_1) \cos \phi_0}{k_1(k_L^2 - k_2^2 \cos^2 \phi_0) \sin k_L l} \cdot \{C(k_2, k_L, \phi_0)[1 + e^{i(k_L - k_2 \cos\phi_0)h}] - iS(k_2, k_L, \phi_0)[1 - e^{i(k_L - k_2 \cos\phi_0)h}]\}. \tag{37}$$

Similarly, the radial component of the electric field due to the currents in the terminations is

$$[E_{2\rho}(\rho_0, \phi_0, 0)]_{ht} = -\frac{k_2 G(k_2\rho_0, k_1)}{k_1 F(k_2\rho_0, k_1)} [E_{2z}(\rho_0, \phi_0, 0)]_{ht}. \tag{38}$$

The radial ground networks at the ends of the vertical ground connections of length l in the Beverage antenna are constructed of bare horizontal wires. The current on each radial wire has the approximate form $\sin k_1(S - s)$, where S is the radial length of the wire. Since $k_1 = \beta_1 + i\alpha_1$, where α_1 is significant, and the wires are bare, the current on each is quite rapidly attenuated. Consequently, the lengths S can be very small compared to the length h of the wire in air. Furthermore, with a radial distribution of wires, the fields of oppositely directed wires tend to cancel. It follows that the contributions to the field by the currents in the radially arranged horizontal ground wires are negligible.

VI. THE ELECTRIC FIELD OF THE BEVERAGE ANTENNA

The field of the Beverage antenna is generated by the currents in both the horizontal and vertical members in Region 2 (air). The contributions from the vertical and horizontal ground wires in Region 1 are negligible. The dominant components of primary interest are the vertical and radial components $E_{2z}(\rho_0, \phi_0, 0)$ and $E_{2\rho}(\rho_0, \phi_0, 0)$ in the air (Region 2) on the surface of the earth (Region 1) and the radial component $E_{1\rho}(\rho_0, \phi_0, z)$ in Region 1. These are obtained from (20) with (26) and (21) with (27). They are

$$E_{2z}(\rho_0, \phi_0, 0) = \frac{I_x(0)[1 - e^{i(k_L - k_2 \cos\phi_0)h}]}{k_1 k_2} \cdot \left[\frac{ik_2 F(k_2\rho_0, k_1) \cos \phi_0}{k_L - k_2 \cos \phi_0} - k_1 d G(k_2\rho_0, k_1)\right] \tag{39}$$

$$E_{2\rho}(\rho_0, \phi_0, 0) = \frac{-I_x(0)[1 - e^{i(k_L - k_2 \cos\phi_0)h}]}{k_1^2} \cdot \left[\frac{ik_2 G(k_2\rho_0, k_1) \cos \phi_0}{k_L - k_2 \cos \phi_0} - k_1 d F(k_2\rho_0, k_1)\right] \tag{40}$$

$$E_{1\rho}(\rho_0, \phi_0, z) = E_{2\rho}(\rho_0, \phi_0, 0) e^{ik_1 z}. \tag{41}$$

These expressions have the factor $\{1 - \exp [i(k_L - k_2 \cos\phi_0)h]\}$. For applications in communication and radar, it is desirable to select h, the length of the horizontal conductor and the distance between the two vertical terminations, to make the magnitude of this factor as large as possible in the direction $\phi_0 = 0$. The quantity to be maximized with respect to h is

$$|1 - e^{-\alpha_L h} e^{i(\beta_L - k_2)h}| = [1 - 2e^{-\alpha_L h} \cos (\beta_L - k_2)h + e^{-2\alpha_L h}]^{1/2}.$$

Differentiation with respect to h leads to the following maximizing equation:

$$\alpha_L \cos (\beta_L - k_2)h + (\beta_L - k_2) \sin (\beta_L - k_2)h = \alpha_L e^{-\alpha_L h}. \tag{42}$$

Since α_L and $(\beta_L - k_2)$ are generally of comparable size and less than one, the optimum value of h can be expected to be large enough so that $\alpha_L \exp(-\alpha_L h)$ is quite small. When this is true, the approximate condition for a maximum is

$$(\beta_L - k_2)\, h \sim \pi - \tan^{-1} [\alpha_L/(\beta_L - k_2)]. \tag{43}$$

Some solutions of (43) at $f = 300$ MHz are listed in Table II together with the associated small quantity $\alpha_L \exp(-\alpha_L h)$. With such lengths, the factor $\{1 - \exp(-\alpha_L h) \exp [i(\beta_L - k_2)h]\} \sim 1$. The quantity $|1 - \exp(-\alpha_L h) \exp [i(\beta_L - k_2)h]|$ is shown in Fig. 3 as a function of the length h for $d/\lambda_2 = 0.02$ for wires over both sea and lake water. It is seen that for lengths h less than half the maximizing value, the amplitude decreases rapidly. Since the exponential term in $|1 - \exp(-\alpha_L h) \exp [i(\beta_L - k_2 \cos \phi_0)h]|$ contributes negligibly to the amplitude when $\phi_0 = 0$ and h is at or near its maximizing value because

TABLE II
OPTIMUM LENGTHS FOR THE BEVERAGE ANTENNA OVER SALT AND LAKE WATER

d/λ_2	h (salt water)	$\alpha_L \exp(-\alpha_L h)$	h (lake water)	$\alpha_L \exp(-\alpha_L h)$
0.01	3.57 m	0.08	2.52 m	0.075
0.02	6.84	0.039	6.49	0.012
0.05	16.76	0.015	19.2	0.002
0.10	48.40	0.005	50.6	0.0005

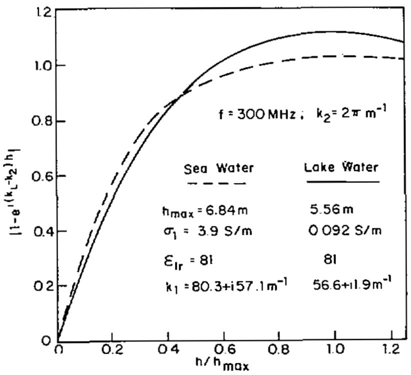

Fig. 3. The magnitude of the factor $[1 - e^{i(k_L-k_2)h}]$ as a function of the length h.

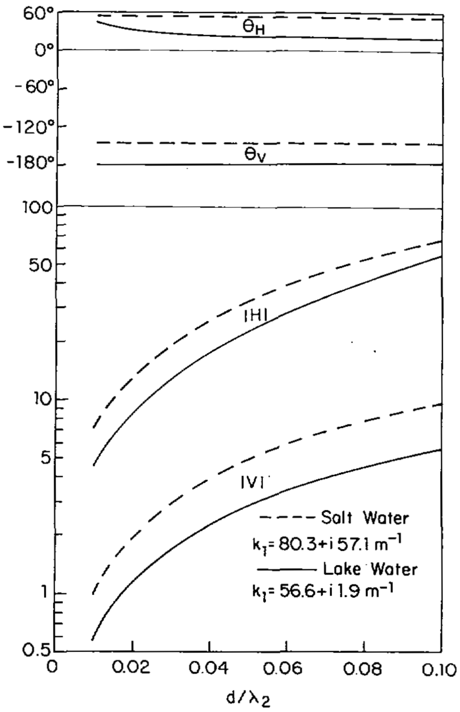

Fig. 4. Comparison of $H(0) = |H| \exp(i\theta_H) = ik_2(k_L - k_2)^{-1}$ and $V = |V| \exp(i\theta_V) = -k_1 d$ for sea water (σ_1 = 3.9 S/m, ϵ_{1r} = 81) and lake water (σ_1 = 0.092 S/m, ϵ_{1r} = 81) at f = 300 MHz.

$\exp(-\alpha_L h)$ is small, the same is true when $\phi_0 \neq 0$. This means that this term contributes negligibly to the directional properties of the Beverage antenna.

Since $G(k_2\rho_0, k_1)$ and $F(k_2\rho_0, k_1)$ in (39)-(41) differ only in the term $i/k_2\rho_0^3$, it follows that when $k_2\rho_0 > 1$, the quantities $H(\phi_0) \equiv ik_2 \cos\phi_0/(k_L - k_2\cos\phi_0)$ and $V \equiv -k_1 d$ are measures of the relative importance of the contributions to the field by the horizontal and vertical currents. With $k_L = \beta_L + i\alpha_L$, $k_1 = \beta_1 + i\alpha_1$, and k_2 real,

$$H(\phi_0) = \frac{k_2[\alpha_L + i(\beta_L - k_2\cos\phi_0)]\cos\phi_0}{(\beta_L - k_2\cos\phi_0)^2 + \alpha_L^2};$$

$$V = -(\beta_1 + i\alpha_1)d. \tag{44}$$

The direction of maximum for $H(\phi_0)$ is $\phi_0 = 0$ for which

$$H(0) = |H| e^{i\theta_H} = k_2[(\beta_L - k_2)^2 + \alpha_L^2]^{-1/2} \cdot e^{i\tan^{-1}(\beta_L - k_2)/\alpha_L}. \tag{45}$$

Also

$$V = |V| e^{i\theta_V} = (\beta_1^2 + \alpha_1^2)^{1/2} d e^{i[\tan^{-1}(\alpha_1/\beta_1) - 180^\circ]}. \tag{46}$$

Graphs of $|H|$ and θ_H, $|V|$ and θ_V as functions of d/λ_2 are in Fig. 4 for sea water with σ_1 = 3.9 S/m and ϵ_{1r} = 81 and for lake water with σ_1 = 0.092 S/m and ϵ_{1r} = 81. It is seen that $|H|$ is significantly greater than $|V|$ for all values of d/λ_2 for both lake and sea water. This is fortunate since the phase θ_V of V differs from the phase θ_H of H by roughly 180° so that $|H(0) + V| < |H(0)|$.

The magnitude of $H(\phi_0) + V$ as a function of ϕ_0 is shown in Fig. 5 specifically with d/λ_2 = 0.02 for both sea water and lake water. It is seen that there is a significant maximum at $\phi_0 = 0$ where $|H(\phi_0)| \gg |V|$. However, $H(\phi_0)$ decreases rapidly when $\phi_0 > 20°$ and for $\phi_0 > 45°$, $H(\phi_0)$ becomes negligible and V—which is independent of ϕ_0—dominates. Since no other factors in the formulas (39) and (41) for $E_{2z}(\rho_0, \phi_0, 0)$ and $E_{1\rho}(\rho_0, \phi_0, z)$ involve ϕ_0, the graphs in Fig. 5 represent the directional characteristics of the electric field.

VII. THE ELECTRIC FIELD OF THE HORIZONTAL-WIRE ANTENNA

The electric field of the horizontal-wire antenna includes contributions from the currents in the main section between $x = 0$ and $x = h$ with its traveling wave of current, $I_x(x') = I_x(0)\exp(ik_L x')$, and from the currents in the terminations with their standing waves of current given by (28) and (29). The combined field is obtained from (26) and (37). It is

$$E_{2z}(\rho_0, \phi_0, 0) = \frac{I_x(0)F(k_2\rho_0, k_1)\cos\phi_0}{k_1(k_L - k_2\cos\phi_0)} \Bigg[i\left(1 - \frac{S(k_2, k_L, \phi_0)}{(k_L + k_2\cos\phi_0)\sin k_L l}\right) \cdot \left(1 - e^{i(k_L - k_2\cos\phi_0)h}\right) + \frac{C(k_2, k_L, \phi_0)}{(k_L + k_2\cos\phi_0)\sin k_L l} \cdot \left(1 + e^{i(k_L - k_2\cos\phi_0)h}\right)\Bigg] \tag{47}$$

where $C(k_2, k_L, \phi_0)$ is given by (33) and $S(k_2, k_L, \phi_0)$ by (34). As with the Beverage antenna, the amplitude is maximized by selecting h as defined in (43). This is sufficiently great to make $\exp(-\alpha_L h)$ very small so that the exponential terms in (47) contribute negligibly. Hence

$$E_{2z}(\rho_0, \phi_0, 0) \sim \frac{I_x(0)F(k_2\rho_0, k_1)\cos\phi_0}{k_1(k_L - k_2\cos\phi_0)} \cdot \left[i + \frac{C(k_2, k_L, \phi_0) - iS(k_2, k_L, \phi_0)}{(k_L + k_2\cos\phi_0)\sin k_L l} \right]. \tag{48}$$

In order to compare the relative magnitudes of the fields due to

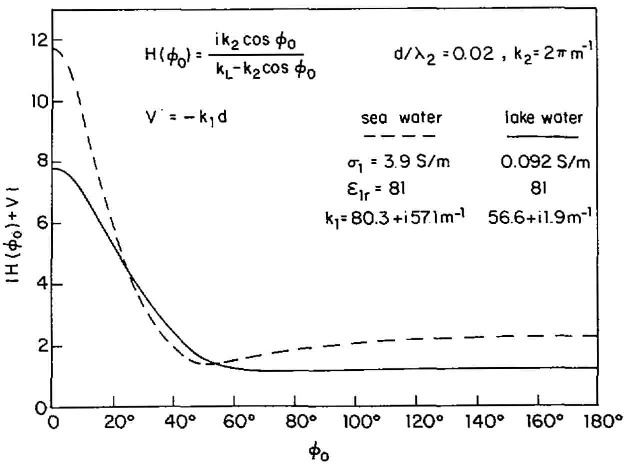

Fig. 5. The magnitude of the factor $H(\phi_0) + V$ as a function of the angle ϕ_0.

the currents in the long main section and in the much shorter terminations, let $\phi_0 = 0$ so that

$$C(k_2, k_L, 0) = k_L (\cos k_2 l - \cos k_L l) \tag{49}$$

$$S(k_2, k_L, 0) = k_L \sin k_2 l - k_2 \sin k_L l. \tag{50}$$

When $f = 300$ MHz, $d/\lambda_2 = 0.02$ and Region 1 is sea water, $k_L = \beta_L + i\alpha_L = 6.641 + i0.289$ m^{-1}. With $\beta_L l = \pi/2$, the square bracket in (48) with $\phi_0 = 0$ gives $1.014e^{i1.51}$ for the combined field, $1.0e^{i1.57}$ for the field of the main traveling-wave section alone, and $0.065e^{i0.18}$ for the field of the two horizontal terminations alone. Thus, the currents in the horizontal terminations increase the magnitude of the field of the current in the main section by about 1.5 percent. The corresponding magnitudes for the same antenna with vertical instead of horizontal terminations are 0.86 for the entire antenna, 1.0 for the horizontal part, and 0.14 for the vertical terminations. It is seen that the vertical terminations have a negative effect instead of the small positive effect of the horizontal ones.

VIII. SUMMARY: THE BEVERAGE AND HORIZONTAL-WIRE ANTENNAS FOR TRANSMISSION

The detailed analysis of both the Beverage and horizontal-wire wave antennas indicates that the contributions to the radiated field by the currents in the terminations are of minor significance when $|k_1 d|$ is small and the length h is adjusted for near optimum. The properties of both antennas are comparable and their field in Region 2 (air) is given by

$$E_{2z}(\rho_0, \phi_0, 0) = \frac{iI_x(0)[1 - e^{i(k_L - k_2\cos\phi_0)h}]}{(k_L - k_2 \cos\phi_0)} \cdot [E_{2z}(\rho_0, \phi_0, 0)]_h \tag{51a}$$

$$E_{2\rho}(\rho_0, \phi_0, 0) = \frac{iI_x(0)[1 - e^{i(k_L - k_2\cos\phi_0)h}]}{(k_L - k_2 \cos\phi_0)} \cdot [E_{2\rho}(\rho_0, \phi_0, 0)]_h \tag{51b}$$

$$B_{2\phi}(\rho_0, \phi_0, 0) = (k_1/\omega)E_{2\rho}(\rho_0, \phi_0, 0) \tag{51c}$$

where $[E_{2z}(\rho_0, \phi_0, 0)]_h$ and $[E_{2\rho}(\rho_0, \phi_0, 0)]_h$ are the components due to a horizontal electric dipole with unit moment. They are the same as in Table I with ρ changed to ρ_0 and ϕ to ϕ_0. The functions $F(k_2\rho_0, k_1)$ and $G(k_2\rho_0, k_1)$ in Table I are

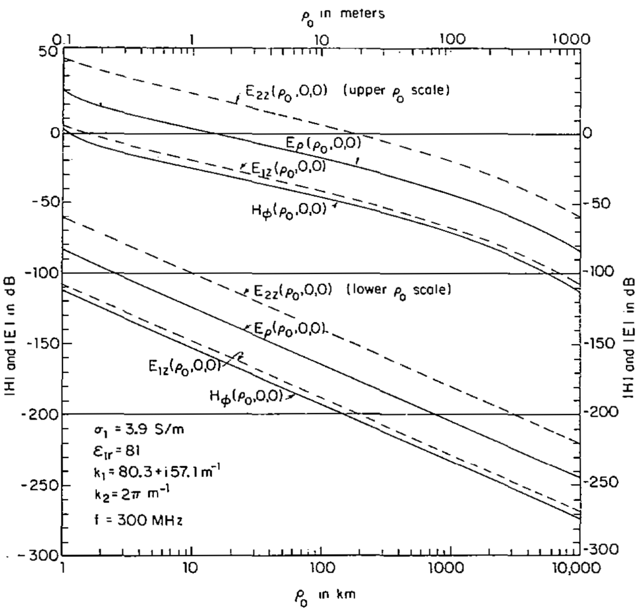

Fig. 6. The components of the lateral-wave electric and magnetic fields on the surface between air and sea water at f = 300 MHz due to a horizontal electric dipole with unit electric moment on the surface. ($|E_\rho|$ referred to 1 V/m at ρ_0 = 1.5 m; $|H_\phi|$ referred to 1 A/m at ρ_0 = 0.12 m; $|E_{1z}|$ referred to 1 V/m at ρ_0 = 0.15 m; $|E_{2z}|$ referred to 1 V/m at ρ_0 = 19 m.)

defined in (7) and (8) with (9)-(14) with ρ replaced by ρ_0 throughout.

It also follows that the field of the wave antennas in Region 1 (earth, sea) is

$$E_{1z}(\rho_0, \phi_0, z) = e^{ik_1 z}(k_2^2/k_1^2)E_{2z}(\rho_0, \phi_0, 0) \tag{52a}$$

$$E_{1\rho}(\rho_0, \phi_0, z) = e^{ik_1 z}E_{2\rho}(\rho_0, \phi_0, 0) \tag{52b}$$

$$B_{1\phi}(\rho_0, \phi_0, z) = (k_1/\omega)E_{1\rho}(\rho_0, \phi_0, z). \tag{52c}$$

These formulas are valid when the antennas are terminated so that the current in the long horizontal wire is a pure traveling wave with the form $I_x(x) = I_x(0) \exp(ik_L x)$.

Graphs of the magnitudes $[E_{2z}(\rho_0, 0, 0)]_{2h}$, $[E_{1z}(\rho_0, 0, 0)]_{2h}$, $[E_\rho(\rho_0, 0, 0)]_{2h}$ and $[H_\phi(\rho_0, 0, 0)]_{2h} = \mu_0^{-1}[B_\phi(\rho_0, 0, 0)]_{2h}$ are shown in Fig. 6 at the top with the upper scale in the range 0.1 m $\leqslant \rho_0 \leqslant$ 1000 m, and at the bottom with the lower scale in the range 1 km $\leqslant \rho_0 \leqslant$ 10 000 km. The graphs are for a unit horizontal electric dipole on the boundary between air ($k_2 = 2\pi$ m^{-1}) and sea water ($\sigma_1 = 3.9$ S/m, $\epsilon_{1r} = 81$, $k_1 = 80.3 + i57.1$m^{-1}) at $f = 300$ MHz. In the range $0.2 \leqslant \rho_0 \leqslant 20$ m, the fields decrease with radial distance approximately as $1/\rho_0$; in the range 200 m $\leqslant \rho_0 \leqslant \infty$, the fields decrease with radial distance as $1/\rho_0^2$.

IX. THE BEVERAGE AND HORIZONTAL-WIRE ANTENNAS FOR RECEPTION—THE CURRENT IN THE MAIN HORIZONTAL WIRE

The wave antennas shown in Fig. 1 are converted from transmission to reception by the substitution of a receiver with load Z_r for the generator with impedance Z_g at $x = 0$, $z = -d$, and the adjustment of $Z_0 + Z_r$ to Z_c. This is illustrated in Fig. 7 for the Beverage antenna where the receiver is represented by the impedance Z_r. In the following it will be assumed that both impedances Z_r and Z_L in conjunction with their grounding

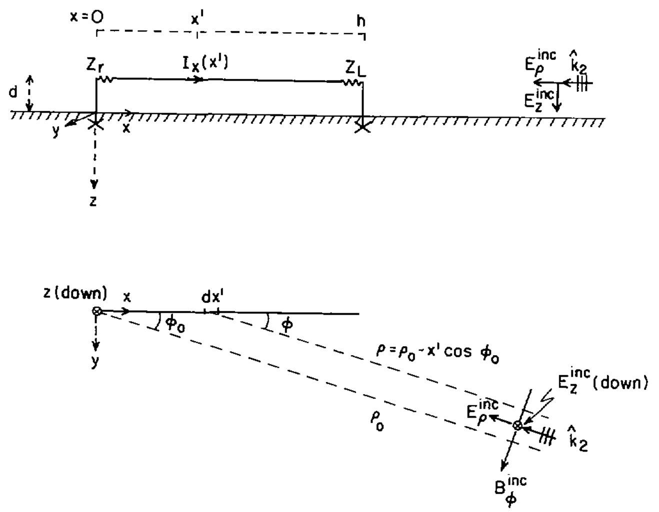

Fig. 7. Beverage receiving antenna in incident lateral-wave field.

impedances Z_0 are equal to the characteristic impedance Z_c of the eccentrically insulated horizontal wire. That is, $Z_r + Z_0 = Z_L + Z_0 = Z_c$.

The current $I_x(0)$ in the load impedance can, of course, be obtained by an application of the reciprocal theorem to the transmitting antenna already analyzed. However, the distribution of current along the wire cannot be obtained in this manner. Actually, it is instructive to treat the wave antenna independently for reception.

An important initial question is the description of the incident electromagnetic field which induces currents in the receiving antenna. If the incident field is generated by an aircraft flying at some distance, the incident field is well approximated by a plane wave arriving along a path that subtends an angle ϕ with the x-axis along the horizontal wire and an angle θ with the vertical and polarized either in the plane of incidence or perpendicular to the plane of incidence. On the other hand, if the source of the incident field is a transmitting antenna—perhaps a Beverage antenna—on the surface of the earth or a missile near the horizon scattering a radar signal, the incident field is a lateral wave traveling along the surface. Whether the currents that generate the field are vertical or horizontal, the principal components of the lateral-wave electric field at moderate and large distances are E_z^{inc} and E_ρ^{inc} given by

$$E_{2z}^{\mathrm{inc}} = (A/k_2)G(k_2\rho, k_1), \quad E_{2\rho}^{\mathrm{inc}} = -(A/k_1)F(k_2\rho, k_1) \tag{53}$$

where F and G are defined in (7) and (8). Here ρ is the radial distance from a source on or very near the boundary surface between Regions 1 and 2 and A is an amplitude factor determined by the magnitude of the currents in the source.

If the incident field is referred to its value at $x = 0$, then

$$E_{2z}^{\mathrm{inc}}(x') = E_{2z}^{\mathrm{inc}}(0)e^{-ik_2x'\cos\phi},$$

$$E_{2x}^{\mathrm{inc}}(x') = -E_{2\rho}^{\mathrm{inc}}(0)\cos\phi\, e^{-ik_2x'\cos\phi}. \tag{54}$$

In these expressions it is assumed that the amplitude of the incident field is sensibly constant over the length h of the horizontal wire and that the phase varies progressively with the wavenumber k_2. This means that the distances ρ from points on the wire to the source are quite large compared to the length h: $\rho \gg h$.

The current in the horizontal wire shown in Fig. 7 is induced entirely by $E_{2x}^{\mathrm{inc}}(x)$. It can be determined by treating the incident field as a continuous distribution of generators along the wire. The current due to a generator with the electromotive force (emf) $E_{2x}^{\mathrm{inc}}(x')dx'$ at x' is

$$dI_x(x) = Z_c^{-1}E_{2x}^{\mathrm{inc}}(x')\,dx'e^{ik_Lx}e^{-ik_Lx'}, \quad x' \leqslant x \leqslant h \tag{55}$$

$$dI_x(x) = Z_c^{-1}E_{2x}^{\mathrm{inc}}(x')dx'e^{-ik_Lx}e^{ik_Lx'}, \quad 0 \leqslant x \leqslant x'. \tag{56}$$

The current due to the entire incident field $E_{2x}^{\mathrm{inc}}(x') = -E_{2\rho}^{\mathrm{inc}}(0)\cos\phi\exp(-ik_2x'\cos\phi)$ is

$$I_x(x, \phi) = -\frac{E_{2\rho}^{\mathrm{inc}}(0)\cos\phi}{Z_c}\left\{e^{ik_Lx}\int_0^x e^{-ik_Lx'} \cdot e^{-ik_2x'\cos\phi}\,dx' + e^{-ik_Lx}\int_x^h e^{ik_Lx'} \cdot e^{-ik_2x'\cos\phi}\,dx'\right\}. \tag{57}$$

This integrates into

$$I_x(x, \phi) = -\frac{iE_{2\rho}^{\mathrm{inc}}(0)\cos\phi}{Z_c(k_L^2 - k_2^2\cos^2\phi)}\{2k_Le^{-ik_2x\cos\phi} - (k_L - k_2\cos\phi)e^{ik_Lx} - (k_L + k_2\cos\phi) \cdot e^{i[k_L(h-x) - k_2h\cos\phi]}\}. \tag{58}$$

The values at $\phi = 0$ for maximum received signal and in the terminations with $x = 0$ and $x = h$ are readily obtained.

Equation (58) can be written in the following manner:

$$I_x(x, \phi) = \frac{E_{2\rho}^{\mathrm{inc}}(0)\cos\phi}{Z_c}[Ae^{ia}e^{-ik_2x\cos\phi} + Be^{ib}e^{ik_Lx} + Ce^{ic}e^{ik_L(h-x)}] \tag{59}$$

where the real coefficients A, B, C and the arguments a, b, c are readily obtained from (58). $E_{2\rho}^{\mathrm{inc}}(0)$ is given by (54) with (53). Note that the three traveling waves in (59) have quite different amplitudes and phases. One of these, represented by $A\exp(ia)\exp(-ik_2x)$ with $\phi = 0$, travels with the real wavenumber k_2 of the incident electric field so that its amplitude remains constant as it progresses from $x = h$ to $x = 0$; the phase is linear and advances by 2π in each wavelength in air ($\lambda_2 = 2\pi/k_2$). A second wave, represented by $B\exp(ib)\exp(ik_Lx)$, travels with the wavenumber $k_L = \beta_L + i\alpha_L$ in the opposite direction, i.e., from $x = 0$ to $x = h$. Its phase constant β_L is greater than k_2 and the wave is exponentially attenuated with the factor $\exp(-\alpha_Lx)$. The third wave, given by $C\exp(ic)\exp[ik_L(h - x)]$, travels from $x = h$ to $x = 0$ like the first wave but with the wavenumber k_L instead of k_2. It is exponentially attenuated with the factor $\exp[-\alpha_L(h - x)]$ as $(h - x)$ increases from $x = h$ to $x = 0$. The resultant current, represented by $D(x, 0)\exp[id(x, 0)]$, is the superposition of the three current waves.

The three current waves and the total current are shown quantitatively in Fig. 8 for a wire with the optimum length $h = 6.84$ m at a height $d/\lambda_2 = 0.02$ over sea water when $f = 300$ MHz. The first wave has the large constant amplitude A and the linear phase $(a - k_2x)$; the second wave has a very small amplitude $B\exp(-\alpha_Lx)$ that decreases from $x = 0$ to $x = h$ and a linear phase $(b + \beta_Lx)$; the third wave begins with a large amplitude $C\exp[-\alpha_L(h - x)]$ at $x = h$ that decreases expo-

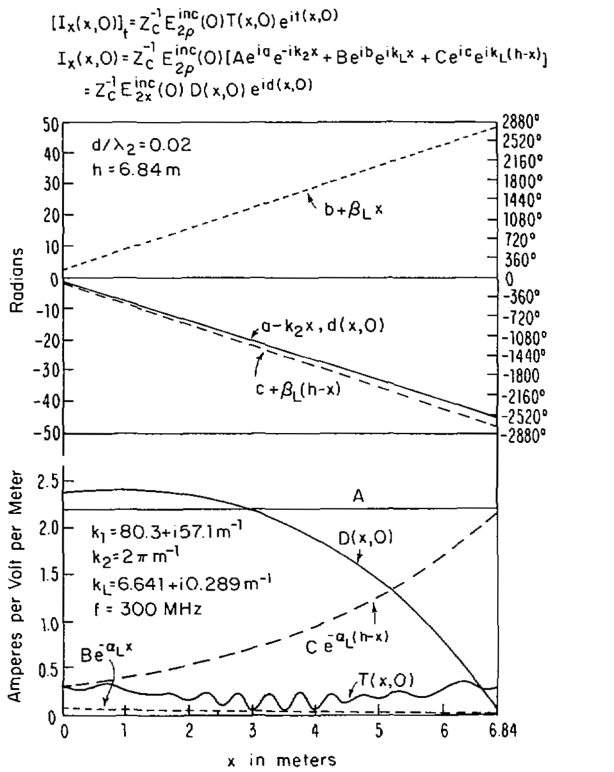

Fig. 8. Components of current on Beverage receiving antenna with optimum length over sea water. $[I_x(x, 0)]_t$ is due to voltages induced in terminations; $I_x(x, 0)$ is due to voltages induced in main horizontal section.

nentially toward $x = 0$. The linear phase $[c + \beta_L(h - x)]$ begins at $x = h$ almost 180° out of phase with the phase $(a - k_2x)$ of the first wave but, since $\beta_L > k_2$, this phase difference diminishes to virtually zero at $x = 0$. The current associated with this wave almost cancels the current of the first wave near $x = h$ where both are nearly equal in amplitude and opposite in phase. The rapid decrease of the third wave as $(h - x)$ increases and the simultaneous decrease in the phase difference allow the resultant current $D(x, 0)$ to grow rapidly from $x = h$ to $x = 0$. The second wave is generally unimportant because its amplitude is so small. However, very near $x = 0$ its contribution is to reduce the amplitude of the resultant current. If the length of the wire were increased beyond the optimum value, this decrease would become significant. As it is, the current in the load at $x = 0$ is at a maximum.

X. THE BEVERAGE AND HORIZONTAL-WIRE ANTENNAS FOR RECEPTION–THE CURRENT DUE TO THE TERMINATIONS, IN THE LOAD, AND IN THE ENTIRE ANTENNA

The vertical conductors at $x = 0$ and $x = h$ that connect the horizontal wire to ground in the Beverage antenna are excited by the large z-component E_{2z}^{inc} of the incident field. The electrically short conductor at $x = h$ maintains the voltage $E_{2z}^{inc}(h)d$ across the end of the eccentrically insulated horizontal wire. With E_{2z} positive down, this is in the positive direction for the current. Similarly, the vertical conductor at $x = 0$ maintains the voltage $E_{2z}^{inc}(0)d$ but, with E_{2z} down, this is in the negative direction. Since the horizontal wire is terminated in Z_c at each end, the currents along the wire are

$$[I_x(x)]_h = \frac{E_{2z}^{inc}(h)d}{Z_c} e^{ik_L(h-x)} \tag{60}$$

$$[I_x(x)]_0 = -\frac{E_{2z}^{inc}(0)d}{Z_c} e^{ik_Lx}. \tag{61}$$

However, $E_{2z}^{inc}(h) = E_{2z}^{inc}(0) \exp(-ik_2h \cos\phi)$, so that

$$[I_x(x, \phi)]_h = \frac{E_{2z}^{inc}(0)d}{Z_c} e^{i[k_L(h-x) - k_2h\cos\phi]} \tag{62}$$

and the sum of the two currents is

$$[I_x(x, \phi)]_t = -\frac{E_{2z}^{inc}(0)d}{Z_c} \{e^{ik_Lx} - e^{i[k_L(h-x) - k_2h\cos\phi]}\}. \tag{63}$$

This is the superposition of two attenuated traveling waves moving in opposite directions. At large distances from the source,

$$E_{2z}^{inc}(0) = -\frac{k_1}{k_2}\frac{G(k_2\rho, k_1)}{F(k_2\rho, k_1)} E_{2\rho}^{inc}(0) \sim -\frac{k_1}{k_2} E_{2\rho}^{inc}(0). \tag{64}$$

Hence

$$[I_x(x, \phi)]_t \sim \frac{k_1}{k_2}\frac{E_{2\rho}^{inc}(0)d}{Z_c} \{e^{ik_Lx} - e^{i[k_L(h-x) - k_2h\cos\phi]}\}$$

$$= \frac{E_{2\rho}^{inc}(0)}{Z_c} T(x, \phi) e^{it(x,\phi)}. \tag{65}$$

The magnitude of this current with $\phi = 0$ is shown as $T(x, 0)$ in Fig. 8. The oppositely directed traveling waves excited by the voltages maintained across the ends of the vertical conductors at $x = 0$ and $x = h$ set up a standing wave. However, the magnitude of the current due to the voltage induced on the vertical elements is quite small compared with that of the currents induced directly in the horizontal wire. Specifically in the load, with $\phi = 0$, the sum of the currents is

$$I_x(0, 0) = -\frac{E_{2\rho}^{inc}(0)}{Z_ck_2}\left[\frac{ik_2}{(k_L - k_2)} - k_1d\right][1 - e^{i(k_L - k_2)h}]$$

$$= -\frac{E_{2\rho}^{inc}(0)}{Z_ck_2} [H(0) + V][1 - e^{i(k_L - k_2)h}] \tag{66}$$

where $H(0)$ and V are defined in (45) and (46). This current in the load could have been obtained directly from the analysis of the transmitting antenna by means of the reciprocal theorem.

The contribution to the current in the traveling-wave section between $x = 0$ and $x = h$ of a horizontal-wire receiving antenna due to the presence of the horizontal terminations could be evaluated without difficulty. However, it follows from the transmitting case that their contributions are small enough to be negligible so that it may be assumed that the entire current at any point is given by (58) or (59) and the current in the load by

$$I_x(0, 0) = -\frac{iE_{2\rho}^{inc}(0)}{Z_c(k_L - k_2)} [1 - e^{i(k_L - k_2)h}]. \tag{67}$$

XI. CONCLUSION

The transmitting and receiving qualities of the Beverage and horizontal-wire antennas over a conducting or dielectric earth have been determined analytically specifically for transmission and reception of waves traveling along the surface of the earth. Full account is taken of the effect of the earth on the currents in the antenna and its terminations and on the transmitted and incident fields. These latter are necessarily surface or lateral waves which have been accurately represented by newly available formulas.

NOMENCLATURE

$[\cdot]_h$	Field due to horizontal *unit* dipole on boundary in Region 1 or 2.
$[\cdot]_{1v}, [\cdot]_{2v}$	Field due to vertical *unit* dipole in Region 1, 2.
$[\cdot]_{1h}, [\cdot]_{2h}$	Field due to horizontal *unit* dipole in Region 1, 2.
$[\cdot]_{1vc}, [\cdot]_{2vc}$	Field due to vertical conductor at $x = 0$ or $x = h$ in Region 1, 2.
$[\cdot]_{hw}$	Field due to horizontal wire from $x = 0$ to $x = h$.
$[\cdot]_{h0}, [\cdot]_{hh}$	Field due to horizontal terminations of length l at $x = 0, x = h$.
$[\cdot]_{ht}$	Field due to both horizontal terminations.

ACKNOWLEDGMENT

The author is grateful to Ms. Margaret Owens for correcting the manuscript.

REFERENCES

[1] H. H. Beverage, C. W. Rice, and E. W. Kellog, "The wave antenna," *Trans. AIEE*, vol. 42, p. 215, 1923.
[2] H. Busch, "Theories der Beverage antenne," *Jahrb. Telegr. Telef.*, vol. 21, pp. 290–312, 1923.
[3] F. Ollendorf, *Die Grundlagen der Hochfrequenztechnik*. New York: Springer, 1925, pp. 576–583.
[4] J. Aharoni, *Antennae*. Oxford at the Clarendon Press, 1946, pp. 223–229.
[5] J. D. Kraus, *Antennas*. New York: McGraw-Hill, 1950, pp. 412–413.
[6] C. H. Walter, *Traveling-Wave Antennas*. New York, Dover, 1965, pp. 315–318.
[7] R. W. P. King, T. T. Wu, and L. C. Shen, "Horizontal-wire antenna over a conducting or dielectric half-space: Current and admittance," *Radio Sci.*, vol. 9, pp. 701–709, July 1974.
[8] R. M. Sorbello, R. W. P. King, K.-M. Lee, L. C. Shen, and T. T. Wu, "The horizontal-wire antenna over a dissipative half-space: Generalized formula and measurements," *IEEE Trans. Antennas Propagat.*, vol. AP-25, pp. 850–854, Nov. 1977.
[9] R. W. P. King and G. S. Smith, *Antennas in Matter*. Cambridge, MA: M.I.T. Press, 1981, pp. 89–109.
[10] R. W. P. King, "New formulas for the electromagnetic field of a vertical electric dipole in a dielectric or conducting half-space near its horizontal interface," *J. Appl. Phys.*, vol. 53, pp. 8476–8482, Dec. 1982.
[11] T. T. Wu and R. W. P. King, "Lateral waves: A new formula and interference patterns," *Radio Sci.*, vol. 17, pp. 521–531, May–June 1982.
[12] ——, "Lateral waves: New formulas for $E_{1\phi}$ and E_{1z}," *Radio Sci.*, vol. 17, pp. 532–538, May–June 1982.
[13] R. W. P. King and T. T. Wu, "Lateral waves: Formulas for the magnetic field," *J. Appl. Phys.*, vol. 54, pp. 507–514, Feb. 1983.
[14] E. Jahnke and F. Emde, *Tables of Functions*. New York: Dover, 1945, p. 36.
[15] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965, p. 300, eqs. (7.3.3) and (7.3.4).
[16] J. S. Belrose, J. Litva, G. E. Moss, and E. E. Stevens, "Beverage antennas for amateur communications," *Q.S.T.*, pp. 22–27, Jan. 1983.

Ronold W. P. King (A'30–SM'43–F'53–LF'71), for a photograph and biography please see pages 519 and 520 of the May 1981 issue of this TRANSACTIONS.