D E C I B E L

PART I

(By BORKING)

VU2 ???, de XXYZZ !!! Tnx for coming back to my CQ call. Well, you gave me a report 59 + 20 db. The other day my XYL was giving a similar report to someone else on the band. She drew my attention and at the end of the QSO I asked her what she meant by 20 db, she said simply decibel. Decibel means .... decibel ? That is the story now. All hams are not professionals. We come across house wife, merchant, pilot, etc, etc on the band. Why even a religious head like our respectful Swamiji. We cannot expect all of them to be mathematicians, engineers and so on. For those whose limitations are within a narrow band width an attempt is made from the basic fundamental of mathematics to explain this universal term "DECIBEL".

In arithmetic it is often useful to express number in their prime factors :-

$$\text{e.g. } 42 = 2 \times 21 = 2 \times 3 \times 7.$$

where 2, 3 & 7 are prime factors of 42, they cannot be broken down into simpler factors. In some numbers, prime factors are

$$\begin{aligned} \text{e.g. } 96 &= 2 \times 48 \\ &= 2 \times 2 \times 24 \\ &= 2 \times 2 \times 2 \times 12 \\ &= 2 \times 2 \times 2 \times 2 \times 6 \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

in short, this last expression can be simply written as

$$96 = 2^5 \times 3.$$

when the small "**index**" 5 simply "**indicate**" the number of limit the factor of 2, occurs. The expression 2^5 is called the "**fifth power of two** or **two to the fifth power**." Similarly, you may check for yourself for each, that

$$648 = 2^3 \times 3^4$$
$$\text{and } 1008 = 2^4 \times 3^2 \times 7.$$

(please note that 2^3 is called "two cubed and 3^2 is known as "three squared")

Power 10

As our number system is based on Ten, we frequently have to deal with power of Ten. For example, in expressing 300,000 in its convenient factor :-

$$\begin{aligned} 300{,}000 &= 3 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 3 \times 10^5 \end{aligned}$$

Now let us concentrate the application of powers of ten.

The Index Law

Before using indices and powers let us consider the rules to be followed in the use. i. e. how they behave when multiplied or divided, and so on.

Multiplication of Powers :-

e. g. To multiply 10^3 by 10^4
we could expand this statement as follows :-

$$\begin{aligned} 10^3 \times 10^4 &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^7 \end{aligned}$$

We started with 3 factors (each 10) multiplied together and then multiplied by 4 more factors, making now 7 factors in all. It should be noted that multiplying the power involves adding the indices. (3 + 4 = 7).

To sum up we could generalize the statement as

$$10^x \times 10^y = 10^{x+y}$$

When x and y indices.

Division of Powers:

In the same manner, if we divide 10^7 by 10^4 we clearly obtain 10^3

$$\text{i. e. } \frac{10^7}{10^4} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10}$$
$$= 10 \times 10 \times 10$$
$$= 10^3$$

Thus dividing one power by another power involves subtracting the indices. ($7 - 4 = 3$)

In general, $\frac{10^x}{10^y} = 10^{x-y}$ when x and y are indices.

Raising to a Power :-

Let us consider the following examples :-

$$8^2 = 8 \times 8 = 64$$

But $8 = 2 \times 2 \times 2$ and $64 = 8 \times 8$

$= 2^3$ $\qquad = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore (2^3)^2 = 2^6 = 64$ $\qquad = 2^6$

Thus squaring a power involves doubling the index.

Also $(2^2)^4 = 4 \times 4 \times 4 \times 4$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8$

and generally $(2^x)^y = 2^{xy}$.

Summing up :- Raising a power to a further power involves multiplying the index by that further power.

Taking the Square Root

As $8^2 = 64$, we can very well say that 8 is the square root of 64 or we can write $64 = 1$

But $\qquad 64 = 2^6$ a4d $8 = 2^3$

$\therefore 2^6 = 2^3$

So taking the square root of a power involves dividing the index by two. (Similarly, taking the cube root involves division by three).

Negative indices :-

Now let us consider what it means to number when raised to a negative power.

for example $2^{-3} = ?$

Consider a typical example where 2^5 multiplied by 2^{-3}. Then if the ordinary laws of indices apply we should add 5 to -3. The sum total effect of "5 up" and "3 down" is clearly "2 up". Hence, if the laws of indices apply,

$$2^5 \times 2^{-3} = 2^2$$

i. e., $32 \times 2^{-3} = 4$

Thus 2^{-3} is the number which multiplies 32 to give the result 4.

Comparing with $32 \times \frac{1}{8} = 4$ we see that 2^{-3} has the same effect in multiplying as

$$\frac{1}{8} \text{ or } \frac{1}{2^3}$$

Then it is reasonable to use the notation 2^{-3} for the fraction $\frac{1}{2^3}$ (The "reciprocal" of 2). Similarly 10^{-4} can be interpreted as $\frac{1}{10^4}$ and so on

Application of indices in Technical Computation :-

In scientific and technological work we often have to handle very large or very small quantity

28 MHZ = 28,000,000 Hz

and 1 PF = .000,000,000,001 Farads. etc... and so many, Hi...28 MHZ could also be stated as 28×10^6 HZ and similarly 1PF = 1×10^{-12} Farads etc.

This index notation is particularly valuable when we have to substitute very large or very small values in a formula V = IR the units concerned are volts, amperes and ohms.

We often deal with tiny fractions of an ampere (milli and micro) and millions of ohms in telecommunication work. The use of indices helps to reduce the risk or errors in computation, for it renders unnecessary the continual writing and counting of rows of noughts.

I am sure the band condition is becoming bad now. QRM & QRN is becoming heavy. Let us call it a day. We will QSY to other band and try later. 73^3, 88^8, and 99^8.

DECIBEL

PART II

(By BORKING)

Thanks for answering back to my call. You have a lovely signal to day and happy to hear that you are also copying me very well at your end. Your simple dipole work well, Hi !! Ur Rx is very good. Let us go ahead now.

We were chatting the other day regarding the use of Powers of 10 when dealing with very large or very small numbers. When doing so it is a good rule to express all the numbers in "standard form", that is to move the decimal point so that there is one and only one **significant** digit to the left of it. For example : 3↓8960000 is more convenient to express as 3.896×10^7 (The decimal point shifted 7 places to the arrow position). And similarly, 0.000,000,00↓654 could also be expressed as 6.54×10^{-9} (The decimal point having been shifted 9 places in the reverse direction).

Summarizing all our QSO so for

A Process	B Example	C. Effect
MULTIPLICATION	$10^7 \times 10^4 = 10^{11}$	ADD INDICES
DIVISION —	$\frac{10^8}{10^3} = 10^5$	SUBTRACT INDICES
RAISING TO A POWER (e. g. cube —	$(10^2)^3 = 10^6$	MULTIPLY INDEX (by³)
TAKING THE (Square) ROOT	$\sqrt{10^6} = 10^3$	DIVIDE INDEX (by²)

Now let us reduce the loading further and try to reduce the labour of arithmetical calculations. They are the best basis of "**LOGARITHMIC**" tables and of the "**SLIDE RULE**".

Roger, happy that you had my QSP 100%. O. K. let us continue further now.

So far we have only met indices which are whole numbers. There are two special powers which require a little more explanation.

10^1 seems to be funny and unfamiliar as such. But if we remember that the index indicates how many times 10 is to be included as a factor, then we can find that it means simply 10 once only.

Now listen ? ... what about 10^0 ... Hi., thinking again of factors, this means "don't include this at all". In other words leave things as they are. You need'nt QSY up or down or change the direction to long path. Let us see how the simple speech compressor circuit works on your rig.

e. g., $10^3 \times 10^0$ Adding indices we have $10^{3+0} = 10^3$. No overloading Hi ... Hence multiplication by 10^0 leave the multiplication 10^3 unchanged, just as if it had been multiplied by **1**.

So whenever we come across 10^0 we write $10^0 = 1$.

FRACTIONAL POWERS :

Consider as an index a simplest fraction e. g. one half. What do you mean by $10^{\frac{1}{2}}$?

If we apply the rule for multiplying them :
$(10^{\frac{1}{2}})^2 = 10^{\frac{1}{2}} \times 10^{\frac{1}{2}} = 10^{\frac{1}{2}+\frac{1}{2}} = 10^1$ or 10 itself.

So $10^{\frac{1}{2}}$ is the number which when multiplied by itself make 10 i. e., it is the "SQUARE ROOT" of 10. You can check this rule for raising to a power discussed the other day.

$(10^{\frac{1}{2}})^2 = 10^{\frac{1}{2} \times 2} = \mathbf{10}$

So again $10^{\frac{1}{2}}$ squared makes 10

Hence $10^{\frac{1}{2}} = \sqrt{10} = 3.16$

We could apply the procedure to any other fraction e. g. Say $10^{\frac{1}{4}}$ is the number which when multiplied by itself form four times ($10^{\frac{1}{4}} \times 10^{\frac{1}{4}} \times 10^{\frac{1}{4}} \times 10^{\frac{1}{4}}$) makes 10. Thus it is the fourth root of $10 = \sqrt[4]{10}$.

But also

$10^{\frac{1}{4}} \times 10^{\frac{1}{4}} = 10^{\frac{1}{4}+\frac{1}{4}} = 10^{\frac{1}{2}} \simeq \underline{3.16}$

$10^{\frac{1}{4}} \simeq \sqrt{3.16}$, i. e., $\underline{1.78}$

Similarly $10^{\frac{1}{8}} = \sqrt{10^{\frac{1}{4}}} \simeq \sqrt{1.78} \simeq \underline{1.33}$

From these three fractional power of 10, other numbers like $10^{\frac{3}{8}} = 10^{\frac{1}{4}+\frac{1}{8}} = 10^{\frac{1}{4}} \times 10^{\frac{1}{8}}$

$\simeq 1.78 \times 1.33$

$\simeq \underline{2.37}$

Let us now consider how this method can be used to calculate the value of powers of 10 for a whole series of indices lying between 0 and 1.

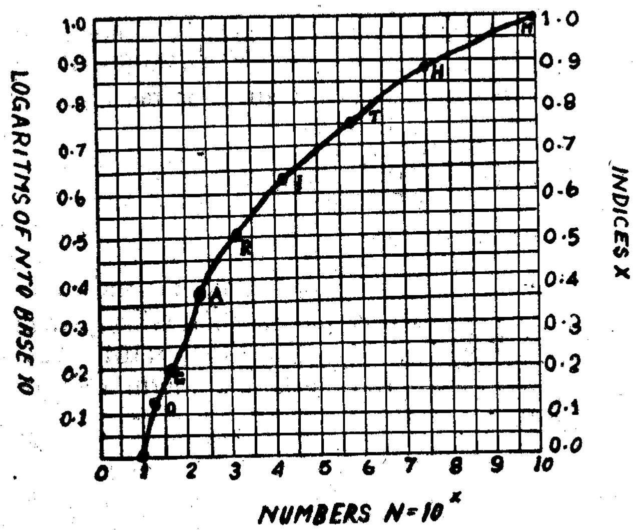
INDEX (x)	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
INDEX AS A DECIMAL	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
NUMBER (10^x)	1	1.33	1.78	2.37	3.16	4.22	5.62	7.50	10
REFERENCE (LOGARITHM)	L	O	G	A	R	I	T	H	M

Conversion of Powers of 10 Numbers and Viceversa

From the table we have made about a scale can be shown connecting the value of number 10^x with the value of the index x.

REFERENCE	L	O	G	A	R	I	T	H	M
INDEX x	0	0.125	0.25	0.375	0.5	0.625	0.75	0 875	1.0
NUMBER 10^x	1	1.33	1.78	2.37	3.16	4.22	5.62	7.50	10

But the graduations are not complete, and the lengths on the scale are not proportional to the numbers. In order to have a ready reference chart to read both the indices and the numbers on scale marked decimally, we can achieve this by setting out two scales with decimal divisions at right angles to one another on a piece of squared paper as shown below. (It is better to make a similar graph yourself to a longer scale).



USE OF CONVERSION GRAPH

Now let us see how this graph we had prepared can solve your arithmetic problems. With this we are in a position to multiply number by adding indices and to divide by substracting indices. For example to multiply 2.35 by 3.75.

From the graph we can find as nearly as possible that

$$2.35 = 10^{0.372}$$
$$3.75 = 10^{0.575}$$

To multiply we add indices ;

$\therefore 2.35 \times 3.75 = 10^{0.372+0.575} = 10^{0.947}$

From the graph we find that the number corresponding to the index 0.947 is 8.8

$\therefore 2.35 \times 3.75 = 8.8$ approx. cross checking by ordinary arithmetic, we get 8.8125.

Similarly $\frac{3.75}{2.35} = \frac{10^{0.575}}{10^{0.372}} = 10^{0.575-0.372}$

$$= 10^{0.203} = 1.60$$

By ordinary long division we get 1.595 using the same principle we can determine the squares, square root etc.

The use of indices to substitute the simple processes of addition, subtraction, and halving for the more complicated process or multiplication, division and extracting the square root is so important that a special name has been given to the indices used in this way. They are called "LOGARITHMS". (In Greek " LGOS " means ratio and " RITHMOS " means - numbers as in "arithmetic"). Accordingly in our graph we have labelled a second verticle line to right "LOGARITHM of N to base 10". Thus our graph enables us to determine the "logarithm" of any number between 1 and 10 or knowing the logarithm, to find the corresponding number.

It should be noted that the verticle line on the left (for logarithm) can exact reproduction of the vertical on the right (for indices).

"So the logarithm to the base of 10 of any number of the index of that power of 10 which is equal to the number".

You may see that using the graph we cannot obtain very accurate results. Precalculated chart to a very high order accuracy are available in the form of standard table. Such a table for all numbers from 1.000 to 9.999, advancing 0.001 at a time, is known as "FOUR-FIGURE LOGARITHM" table. Thus this table covers the same range of numbers as our graph, but with far greater detail and precision. To summarise :- the tables enable us to express immediately any number between 1 and 10 as a power of 10.

DECIBEL

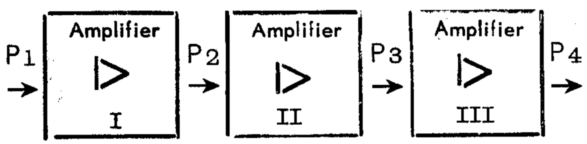
PART III **(By BORKING)**

Thank you once again for coming back to my call. Condition seems to be far better today. Not much of QRM. Having discussed before about "OM. LOGARITHM", you must have a clear picture of him by now. A simple, unassuming fellow. But a very useful guy indeed. In fact he is a storage circuit of a long bit of information. But he is so modest that he can put all those things in a nut shell like our little, but mighty ICs. You know how highly complicated, head reeling figures leading to several naughts could be expressed easily using the simple index law and its development to logarithm. The use of logarithm in mathematical expression has simplified calculations.

In radio & line communications, we are greatly concerned with the transmission of power from one point to another. The various media and circuits which constitutes the transmission system introduces losses or gain in the transmitted power. The overall performance of a system can be expressed by comparing the amount of power in the receiving signal (P_{out}) with the power transmitted (P_{in}). The overall power ratio M can then be expressed as follows :-

$$M = \frac{P_{out}}{P_{in}}$$

If M is greater than unity then there has been a gain of power in the system (amplification), but on the other hand if M is fractional then there has been a loss of power in the system (attenuation).



In the above figure, three amplifiers are shown connected in cascade or tandem. P_{input} in P_1 and P_{out} is P_4, while P_2 & P_3 are the signal powers at the two intermediate points. For this system the overall power ratio can be expressed by :-

$$M = \frac{P_2}{P_1} \times \frac{P_3}{P_2} \times \frac{P_4}{P_3} = \frac{P_4}{P_1}$$

From this it could be deduced that if the power ratio for each section of a system is known or if could be calculated, the overall performance of the system can then be obtained by multiplying the individual power ratios together. This looks very simple when it involves work with numbers that is not too large. However in practice while analysing a transmission system accurately a great deal of laborious multiplication may have to be carried out.

LOGARITHMIC TRANSMISSION UNIT :-

In order to satisfy the laborious procedure, we could conveniently use logarithm. From earlier discussions one could understand that the **"Logarithm of a given number is the power to which a base number must be raised to give that number".** (This is 10 in the case of Common Logarithm).

In other words, Logarithm are indices. The use of indices simplifies the Procedures of multiplication and division to one of addition and subtraction respectively. Thus a table of logarithm is a table of indices of 10.

For axample :-

$\log_{10} 100 = 2$ because $10^2 = 100$

$\log_{10} 1000 = 3$ because $10^3 = 1000$

$\log_{10} 2 = 0.3010$ because $10^{0.3010} = 2.$

Considering the example taken in the case of the three amplifiers in a system in which M is given by ;

$$M = \frac{P_2}{P_1} \times \frac{P_3}{P_2} \times \frac{P_4}{P_3} = \frac{P_4}{P_1}$$

Now consider taking logs throughout so that ;

$$\log M = \log_{10}\frac{P_2}{P_1} + \log_{10}\frac{P_3}{P_2} + \log_{10}\frac{P_4}{P_3} = \log_{10}\frac{P_4}{P_1} \qquad \text{..............(1).}$$

By definition the basic trasmission unit called the "BEL" is applied to the common logarithm of any ratio of two powers. In fact a more practical smaller unit which is ten times smaller is called the "DECIBEL" (dB).

$$10 \log_{10} M = 10 \log_{10} \frac{P_2}{P_1} + 10 \log_{10} \frac{P_3}{P_2} + 10 \log_{10} \frac{P_4}{P_3}$$

$$= 10 \log_{10} \frac{P_4}{P_1} = X \text{ dB}$$

As an example to this let us consider the following two cases to explain the Gain & Losses represented in decibels.

GAIN :-

If all the power ratios give a whole number, this represents a gain in the system. e. g.,

$10 \log_{10} M$ may be given by ;

$10 \log_{10} 2 + 10 \log_{10} 2 + 10 \log_{10} 2$

$= 3.01 \text{ dB} + 3.01 \text{ dB} + 3.01 \text{ dB} = 9.03 \text{ dB}$

This could be explained as that a gain of 9.03 dB in the system between points P_1 & P_4 or otherwise we could state that the level P_4 is 9.03 dB higher than the level P_1.

LOSS :-

If all the power ratios give fraction, this represents a steady loss of power throughout the system.

$10 \log_{10} M$ may be given by,

$10 \log_{10} 1/2 + 10 \log_{10} 1/2 + 10 \log_{10} 1/2$

$= 10\ (\bar{1}.699 + \bar{1}.699 + \bar{1}.699) \text{ dB}$

Which is the same as :-

$10\ (-3 + 2.097) \text{ dB}.$

$= 10\ (-0.903) \text{ dB}.$

$= -9.03 \text{ dB}.$

The negative sign indicates a loss. It should be noted that **you must not say that a loss of - 9.03 dB.** This is incorrect.

REFERENCE :-

In general the level of the chosen reference power is expressed as **"O" dBr".** Any power measured further along the system towards the receiving end is either represented by + X dBr if it is higher than the reference power or by — X dBr if it is lower than the reference power, while powers that are the same as the reference power are represented by O dBr.

It should be noted that in Line Communication practice, power levels are often referred to a standard reference power of 1 mW. A power of PmW is equal to 10 log P dBm., and a power level of 1 dBm has a value of 1.259 mW. A power of say 5 watts could be expressed as :-

$$10 \log \frac{5000}{1} \text{ dBm.}$$

$$= 10 \times 3.699$$

$$= 36.99 \text{ dBm.}$$

VOLTAGE and CURRENT RATIOS :-

Decibel notation can also be applied to express the ratios of Voltage, Current or Field Strength. The principle of deriving the value is the same as we discussed so far.

Let us consider two resistors R_1 & R_2 having the same value R ohms. Assume that the voltage developed across one resistor due to a current I_1 amperes flowing in it is V_1 volts, and that the voltage developed across the other resistor due to a current I_2 amperes flowing in it is V_2 volts. Then the power, P_1, dissipated in the first resistor R_1 is given by ;

$$P_1 = \frac{V_1^2}{R_1} = I_1^2 R_1$$

And the power P_2, dissipated in the second resistor R_2 is given by ;

$$P_2 = \frac{V_2^2}{R_2} = I_2^2 R_2$$

Therefore the relation between P_1 & P_2 can be expressed in dB and is obtained from ;

$$10 \log_{10} \frac{P_2}{P_1} \text{ (If } P_2 \text{ is greater than } P_1)$$

$$= 10 \log_{10} \frac{V_2^2}{R_2} \Big/ \frac{V_1^2}{R_1} \text{ or } 10 \log_{10} \frac{I_2^2 R_2}{I_1^2 R_1}$$

Now since R_1 & R_2 are equal we have ;

$$= 10 \log_{10} \frac{V_2^2}{V_1^2} \text{ or } 10 \log_{10} \frac{I_2^2}{I_1^2}$$

$$= 20 \log_{10} \frac{V_2}{V_1} \text{ or } 20 \log_{10} \frac{I_2}{I_1}$$

Similarly the field strength of radio waves is usually expressed in terms of voltage developed across a unit length of aerial situated at a point in the path of the wave. However, it will be much more convenient to measure the current flowing in the aerial. Therefore, if the same type of aerial is used to measure the field strength at two different points A & B in the path of the radio wave the "path attenuation" between the two points can be determined in dB from expression :-

$$\text{Path Attenuation} = 20 \log_{10} \frac{\text{Aerial current at A}}{\text{Aerial current at B}}$$

A few more examples on the calculations of gain or loss in a system expressed in decibel will make the reader feel more confident and clear on this topic.

Example 1.

A transistor amplifier stage is found to have a voltage gain of 35. Express this in dB.

The gain = 20 log 35.
= 20 × 1.544.
= 31 dB apprx.

Example 2.

The field strength of an M. F. transmitter at night fluctuates between levels of 180 and 20 μv/m. Express this fluctuations in dB.

$$\text{The variation} = 20 \log_{10} \frac{180}{20}$$

= 20 log 9

= **19 dB.**

From the table given below you could find yourself how large figures raising to several degits could be easily expressed using decibel notation.

Decibel	Power. $10 \text{ Log } \frac{P_1}{P_2}$	Current or Voltage $20 \text{ Log } \frac{V \text{ or } I}{V \text{ or } I}$
1.	1.259	1.122
2.	1.585	1.359
3.	1.995	1.413
4.	2.512	1.585
5.	3.162	1.778.
6.	3.981	1.995
7.	5.012	2 239
8.	6.310	2.512
9.	7.943	2.818
10.	10	3.162
20.	100	10
30.	1000	31.62
40.	10000	100
50.	100000	316.3

Finally summing up all our talks so far, we could conclude that the practical value of decibel arises from the logarithmic nature. This permits the enormous ranges of powers involved in communication work to be expressed in terms of decibels without running into inconveniently large numbers. The logarithmic character of a decibel also make it possible to express the ratio of input to output powers in complicated circuits as the some of the ratios, expressed in decibels, of the input and output powers of different parts of the circuit. Further, the decibel is very convenient for expressing sound intensities since the effect that sound waves have on the ear is roughly proportional to the logarithm of the intensity. Thus the term "DECIBEL", find a very useful yard stick in our comparison.

I hope my signal is still holding 5/9 +20 dB... Hi.

Thanks very much for listening. 73^s, 88^s & 99^9 ... CHEEERRU.

Your QSL'S Via P. B. 6538 Bombay - 400 026.