Approximate Methods for Calculating Z_0 and V.F. of "Wire Microstrip"

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The problem is to obtain the characteristic impedance (Z_0) and propagation velocity factor (V.F.) for a transmission line consisting of a round wire lying on the surface of a dielectric slab having a ground plane on the opposite surface. For example, this type of line was used for bias chokes in 24 GHz amplifiers by Hill [1]. An example will be used in the following where the wire diameter is 0.01 inch (#30 AWG) and the substrate is Rogers Duroid 5880, 0.025 inch thick.

Method 1 - Using MWI.EXE

MWI.EXE, free downloadable DOS software from Rogers Corp., calculates impedance and effective dielectric constant ($1/(\text{V.F.})^2$) for microstrip. It includes correction factors for the thickness of the microstrip conductor. So MWI can be used with a microstrip conductor 0.010 inch wide by 0.010 inch thick on a 0.025 inch substrate to approximate the 0.010 inch diameter wire. This provided values of Z_0 = 111 ohms and V.F. = 0.79 at 11 GHz. One might argue that a better approximation would result from using a square cross-section conductor with the same circumference as the round one, or 0.0079 inch on a side. This gives Z_0 = 121 ohms and V.F. = 0.79 at 11 GHz. The problem with this approach is that the original reference [2] for the formulas used in MWI does not provide ranges of validity for the conductor thickness or width-to-thickness ratio. So the accuracy is unknown.

Method 2 - Equivalent Capacitance

In the absence of dielectric the characteristic impedance of a transmission line consisting of a round wire over a ground plane is given by [3]

$Z_0 = 60 \cosh^{-1}(2h/d)$, or approximately by

$Z_0 \sim 138 \log_{10}(4h/d)$,

where d is the wire diameter and h is the distance from the centre of the wire to the ground plane. For a d = 0.010 inch diameter wire with its lower edge 0.025 inch from the ground plane, h = 0.030 inch and Z_0 = 149 ohms.

Now we can think of the 0.025 inch thick region between the wire and the ground plane as a capacitor, more or less. If this region is now filled with a material with a dielectric constant of 2.2 (for Duroid 5880) the capacitance of this region will be increased by a factor of 2.2. But in terms of capacitance this is the same as reducing the capacitor spacing by the same factor. So we should get an approximation of the correct impedance for the dielectric-loaded case by computing the impedance from the formula for the unloaded case, but with this region reduced to 0.025 x 1/2.2 = 0.0114 inch thick. This corresponds to h = 0.0164 inch and Z_0 = 111 ohms. Indeed, this is quite close to the value obtained with MWI.

The velocity factor for an unloaded line is just 1. In reality, the reduction of impedance from the unloaded case was actually done by dielectric loading, not thickness reduction. Therefore, since both Z_0 and V.F are proportional to the inverse of the capacitance per unit length, the same factor applies to the velocity factor as to the impedance. The velocity factor is therefore (111/149) x 1 = 0.745, which is not too far off the MWI result.

This method can also be used to examine the sensitivity to any air gap between the wire and the top surface of the substrate. As an example, let us introduce a 0.005 inch air gap. Now, the unloaded case has h = 0.035 inch and therefore Z_0 = 158 ohms.The unloaded equivalent to the loaded case has h = 0.0114 (equivalent dielectric thickness) + 0.005 (wire radius) + 0.005 (air gap) = 0.0214 inch, and therefore Z_0 = 128 ohms and velocity factor is 128/158 = 0.81. In bias choke applications the error in impedance (15% compared to no gap) is probably not significant but the error in velocity factor (9% compared to no gap) might be.

The accuracy of this method is also unknown, since the replacement of the dielectric layer by a thinner air layer is only an approximation and the transmission mode on this type of line is not truly TEM.

Summary

However, the reasonably close results from the two methods do create a degree of confidence that neither is badly in error. For a 0.010 inch diameter wire resting on a 0.025 inch Duroid 5880 substrate the predicted impedance is 111-121 ohms and the predicted velocity factor is 0.74-0.79. The results for velocity factor are perhaps not intuitive. I would have expected a significantly larger velocity factor than for microstrip line as not so much of the conductor is in contact with the dielectric. However, MWI provides a velocity factor of 0.77 for a 0.010 inch wide thin microstrip line on 0.025 inch Duroid 5880, about the same as predicted by these approximate techniques for the wire over the same substrate.

References

[1] Tom Hill, WA3RMX, "GaAsFET and HEMT Amplifiers for 24 GHz", Proceedings of Microwave Update '91, Arlington TX, 1991, pp.137-150 (reprinted in The ARRL UHF/Microwave Projects Manual, Volume 1, American Radio Relay League, Newington CT, 1996).

[2] E.Hammerstad and Oe.Jensen, "Accurate Models for Microstrip Computer-Aided Design", IEEE International Symposium on Microwave Theory and Techniques, Washington, 1980, pp.407-409.

[3] Reference Data for Radio Engineers, 5th Ed., Howard W. Sams& Co., Indianapolis, 1968, p. 22-22 (first equation derived from two-wire case).