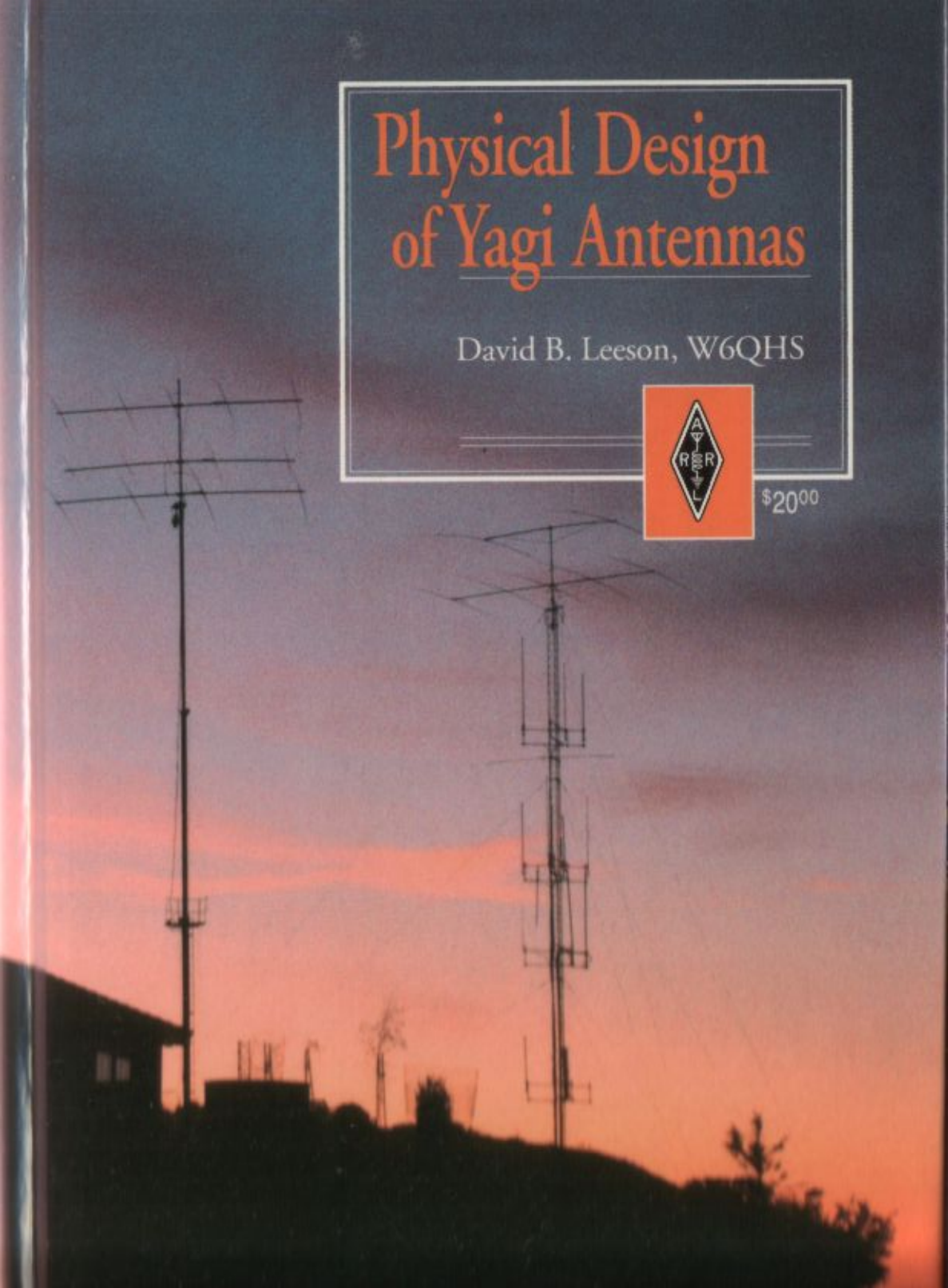
Physical Design of Yagi Antennas

David B. Leeson, W6QHS

ARRL

$20.00



"Winter winds reached speeds of 100 miles an hour. Outside on the 120-foot tower, the well-modified Cushcraft 40-2CD and Hy-Gain 205BA, 155BA and 105BA antennas never missed a beat. The strengthened TH7 tribander on the third tower also performed and rotated through the 48-hour combination of contest and winter storm."

Dr. David B. Leeson, W6QHS, provides the tools here to design and build robust Yagi antennas, using sound mechanical-engineering principles. Leeson shares in detail his procedures for determining mechanical stresses from wind and ice on elements, booms and masts. He includes abundant information on hardware and assembly techniques for survivable Yagi antenna systems. The electrical significance of the resulting physical design is also treated thoroughly.

You need no longer fear the consequences of wind or ice storms on your antennas. With this information, you can build or "beef up" existing Yagis so, like Leeson's, they will function in winds of 100 miles an hour and beyond.

Published by The American Radio Relay League



Contents

Diskette Availability

Abbreviations Used in This Book

About the Author

Introduction

Chapter 1 The Wind in the Yagis

1.1 Amateur Mechanical Engineering 1-1
1.2 The Hilltop Location 1-2
1.3 End of Innocence 1-2
1.4 Enter the Computer 1-3
1.5 Yagi-Uda 1-3
1.6 Physical Design of Yagi Antennas 1-4
1.7 Electrical Modeling of Yagi Antennas 1-7
1.8 Siting of Yagi Antennas over Sloping Foreground 1-10
1.9 Disclaimer 1-11

Chapter 2 Wind and Ice Loading

2.1 Wind Characteristics 2-1
2.2 Wind Speed Statistics 2-3
2.3 Structural Standards 2-6
2.4 Wind-Speed Factors 2-7
2.5 Wind Force on Cylinders 2-15
2.6 Ice Loading 2-21

Chapter 3 Strength of Structures and Materials

3.1 Force and Moments in Cantilever Structures 3-1
3.2 Section Modulus 3-3
3.3 Bending Stress 3-6
3.4 Yield Strength of Materials 3-7
3.5 Fasteners 3-8
3.6 Rivets and Hose Clamps 3-9
3.7 Aluminum Tubing 3-10
3.8 Aluminum Pipe as an Antenna Material 3-11
3.9 Electrolytic Potential and Corrosion 3-12
3.10 Summary of Analysis Procedure 3-13

Chapter 4 Yagi Element Design

4.1 Wind Force on Yagi Elements 4-1
4.2 Cylindrical Elements 4-3
4.3 Two-Section Tapered Elements 4-8
4.4 Multisection Tapered Elements 4-12
4.4.1 Constructing a Computer Spreadsheet 4-12
4.4.2 Some More Advanced Spreadsheet Techniques 4-14
4.4.3 Using the Spreadsheet 4-15
4.5 20-Meter Elements 4-20
4.6 15-Meter Elements 4-28
4.7 10-Meter Elements 4-32
4.8 Reinforcing Triband Antennas 4-33
4.9 Reinforcing Shortened 40-Meter Elements 4-36
4.10 Avoiding Fatigue Failures 4-38
4.11 Summary 4-40

Chapter 5 Attachment of Elements to a Boom

5.1 Mechanical Considerations 5-1
5.2 Calculation of Section Modulus and Maximum Moment 5-2
5.3 Effect of Wind Angle 5-4
5.4 Element Moments 5-6
5.5 The Hy-Gain Clamp 5-9
5.6 The Plate and U-Bolt Clamp 5-14
5.7 The Channel Clamp 5-18
5.8 Other Structural Fixtures 5-19
5.9 The Insulated Plate Clamp 5-20
5.10 Recommended Element Attachment 5-23

Chapter 6 Yagi Elements Arrayed on a Boom

6.1 Antenna Area and Wind Force 6-1
6.2 Boom Moment About the Mast 6-9
6.3 Torque Compensation 6-13
6.4 Boom Strength 6-16

Chapter 1

The Wind in the Yagis

Sec 1.1 Amateur Mechanical Engineering

If it didn't fall down, it wasn't big enough, is a commonly heard Yagi antenna comment. If this approach were applied to common civil structures such as buildings and bridges, this would be a far less safe world. Fortunately, it is possible to make Yagi antennas that can survive and be used and rotated, even in severe weather.

My hilltop site experiences storm winds of 120 miles per hour at least once every year. Learning how to make survivable antennas for this location led to the preparation of this book, the objective of which is to show how to model Yagi antenna mechanical characteristics to provide quantitative analysis and synthesis of survivable elements, booms, masts and rotators.

It is a further objective of this book to explore how to realize an accurate physical embodiment of a Yagi antenna from an arbitrary electrical design. The proliferation of computer-based analyses of Yagi antenna designs has resulted in a number of attractive electrical designs which can be realized as survivable physical antennas by the methods outlined in the following sections. The use of computers to calculate the performance characteristics of large Yagis has had a surprising local side effect to reduce QRM; all the big guns are seated in front of their computers searching for that extra half dB!

Along the way through this book, you will learn how to tell what to expect from a particular site and antenna, and how to improve mechanical strengths so the antenna, mast and rotator will survive the severe weather that seems to come with each year's DX contests. This is not a professional treatise on engineering analysis; it is limited to the level of sophistication sufficient to meet the needs of the radio amateur.

Sec 1.2 The Hilltop Location

It seems to be every radio amateur's dream to live on a mountaintop, and to build and enjoy Yagi antennas that dominate the high-frequency amateur bands. Even as a teenager, I could not help noticing that amateurs who lived on hilltops seemed to put out substantially better signals. My wife Barbara, KK6QM, is also an active amateur, and we were fortunate to be able to purchase a mountaintop on which to build the house and ham station of our dreams.

With a 360° clear view and 12° downslope in all directions from an altitude of 3400 ft, the results have been gratifying. Especially under marginal propagation conditions, the advantage of measurably increased low-angle radiation, even from relatively low antennas, makes a very big difference in DX and contest operation.

As shown in later sections, the advantages of a sloping foreground are sufficient to dwarf other antenna-gain improvement possibilities. With favorable siting, it is possible to obtain substantial advantages at low radiation angles compared to similar antennas on flat ground. Takeoff angle and slope are defined in Fig 1-1. The good results I have obtained with even relatively modest antenna sizes led to an interest in a quantitative understanding of the antenna possibilities for the hilltop location.

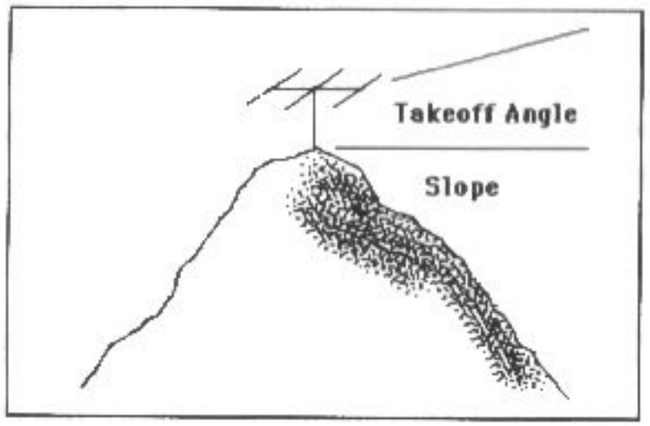


Fig 1-1—Mountaintop geometry.

Sec 1.3 End of Innocence

As is the case with most radio amateurs, for me it seemed only natural to consider the possibilities of improved antennas for even better results. In the past few years, the literature of Yagi electrical design has proliferated. This flowering owes much to the pioneering work of Dr James Lawson, W2PV.[1] One of the results of his work has been an

emphasis on high-frequency Yagis with boom lengths which are long enough to present serious mechanical issues.

Having lived in the mountains for some time, we knew our new location would be windy, but we had no real idea *how* windy until we watched one antenna after another being destroyed each winter season. With our gauge repeatedly registering winter wind speeds over 100 miles per hour, I realized we had a first-class problem on our hands. After a number of disappointing antenna and rotator failures, it was clearly time to abandon trial-and-error methods and apply engineering techniques to the mechanical analysis and design of Yagi antennas.

I could not help but notice a paucity of detailed material relating to the mechanical design of Yagi antennas. Two brief articles published in the early 1970s[2,3] and several that have subsequently been published[4-6] are initial steps toward filling this void for the radio amateur. This book is intended to clarify and expand these first steps to consider not only element design, but also boom, mast and rotator design.

Sec 1.4 Enter the Computer

As will be seen, the mechanical and electrical principals needed to analyze and design Yagi antennas are limited in number and complexity. However, the substantial quantity of arithmetic operations which must be repeated for these purposes would have been a deterrent to my interest if it were necessary to perform them all by hand calculator.

Fortunately, the last few years have seen the amazing development of personal computers and software that can make short (and painstakingly accurate) work of the computations required for the physical design of a Yagi antenna. Although there are certainly other choices, I have enjoyed the process of the physical design and of writing about it, using the Apple Macintosh desktop computer with the Microsoft Excel spreadsheet and Word applications, plus MacDraft mechanical drawing software.

Sec 1.5 Yagi-Uda

In his recently republished classic book on antenna design,[7] John Kraus, W8JK, relates the early history of the Yagi-Uda antenna. Shintaro Uda, then an assistant professor, published a series of articles

---

[1] References are listed at the end of each chapter.

beginning in 1926 describing novel array antennas with parasitic reflector and directors.[8] Hidetsugu Yagi, professor of engineering and recipient of a grant which supported the research done by Uda with Yagi's collaboration, toured the United States in 1928 to present talks on the antenna design in New York, Washington and Hartford.

In his 1928 article on the parasitic antenna,[9] Yagi noted that Uda had already published a number of papers on the antenna. Although Yagi apparently acknowledged that Uda's ingenuity was key to its development, the antenna has, since that time, been called a "Yagi." Although it seems impossible to reverse the practice of more than half a century, it is appropriate to note the role of Uda in the development of the antenna that is the subject of this book.

Sec 1.6 Physical Design of Yagi Antennas

The mechanical limitations of a given Yagi antenna are determined from the wind and ice environment to which it will be exposed, and from the strength of the materials and structure of the antenna elements, boom and mast. In order to determine the survival characteristics of Yagi antennas, in this book we will study the nature of wind and ice loading, the strength of structures and materials, and the implications of these factors for typical antennas. For a Yagi antenna in the high-frequency region, mechanical considerations generally dictate a tapered element.

We will begin by considering the characteristics of wind as they affect a mechanical structure such as an antenna. The simplifying models of wind-speed zones and of gust factor are introduced. Ice loading is modeled through the thickness of radial ice accumulation.

The forces on a cylindrical member such as an antenna element or boom are then modeled. Finally, the limitations of a stepped tubular structure in wind- and ice-loading environments are calculated directly in terms of wind-survival speed. Element designs may be analyzed or synthesized in terms of the length, diameter and thickness of a series of sections having strength and diameter that taper from the center to the tip.

This taper schedule can be determined from mechanical considerations for an element half whose length is approximately ¼ wavelength. This will be the initial step in the physical design of Yagi antennas. At the same time, the strength of various element attachment methods will be modeled. The possibility of simple improvements to

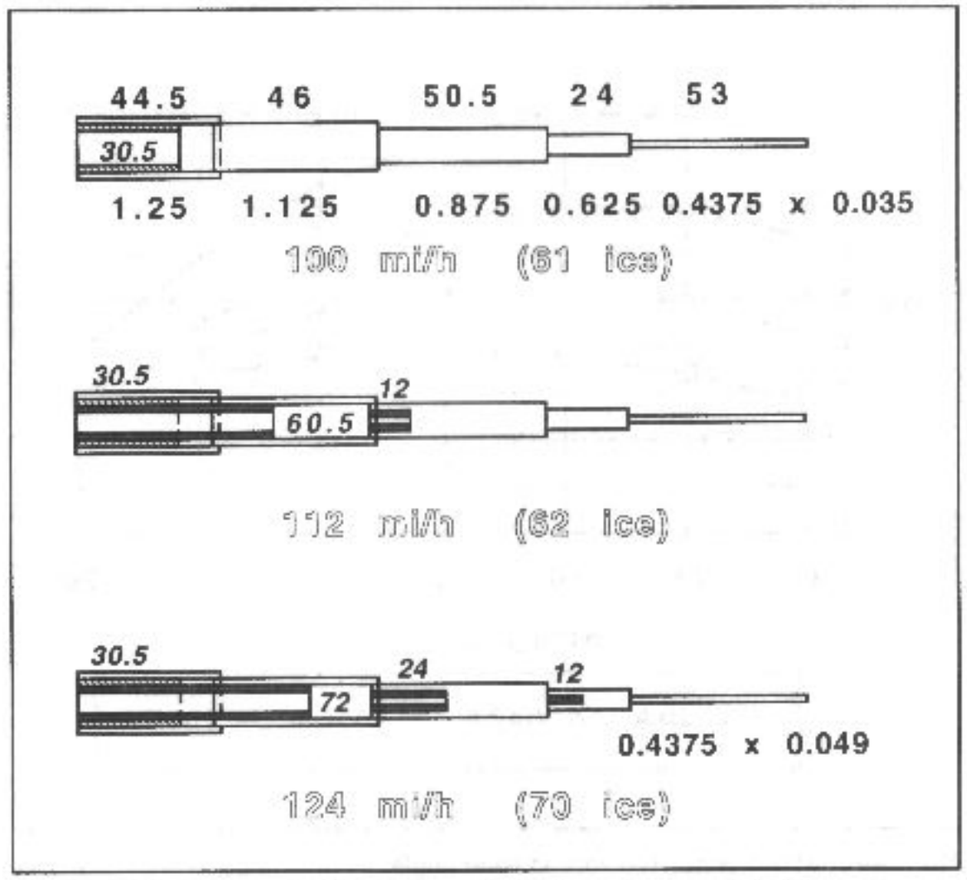


Fig 1-2—Reinforcement of Yagi elements. Values above and inside each element represent lengths in inches. Values below each element represent diameters in inches. Italic numbers indicate the lengths of telescoping reinforcements. Wall thicknesses are those appropriate for a telescoping fit.

the strength of existing antennas is also explored. An example of reinforcing a 20-meter element is shown in Fig 1-2.

We then turn to the issue of arraying the elements on the boom. Here we consider the force and torque resulting from wind pressure on the ensemble of elements and boom, from which boom strength, mast forces, and rotator torque are determined. The tendency of certain Yagis to rotate in the wind is quantified, and a method of compensation is introduced to reduce wind-induced rotating torque to zero (how many stuck or broken rotators have you experienced?).

Next, we consider the more complex problem of guyed elements or booms. The advantages of guyed construction for larger Yagis are quantified. Also, mast strength and antenna stacking are explored. The variation of antenna wind force with wind direction shown in Fig 1-3

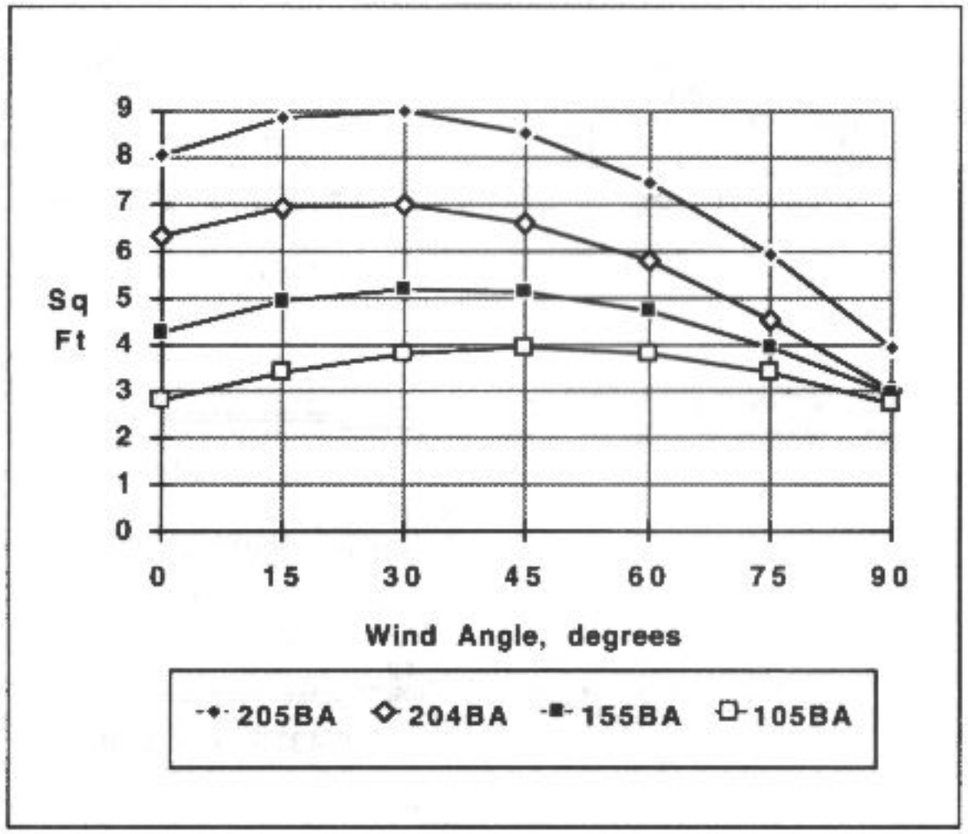


Fig 1-3—Effective wind area versus wind angle.

can be useful in promoting mast and tower survival in high winds. Comments on rotators and mast-torque isolation are included.

Next, we explore the physical realization of a given electrical design. Each dipole element of the antenna could, with some difficulty, be electrically measured and adjusted to yield the actual reactance at the center frequency. Even more extensive experimental design has been done on entire arrays, resulting in commercially available Yagi antennas having the advantage that the design process is, at the same time, an experimental confirmation of overall array performance.

Various electrical design procedures are discussed, and the results are applied to the physical realization or improvement of a number of existing Yagi designs. The ultimate objective is to be able to construct an antenna, given the underlying electrical design, with sufficient accuracy that no experimental adjustments are needed to obtain the expected performance. The complete range of steps is outlined in Table 1-1.

Table 1-1
Steps in Procedure for Physical Design of Yagi Antennas

Design Steps	*Source or Model*
Wind pressure and force	EIA RS-222 zones, local variations
Ice loading	Radial thickness 0.5 in.
Strength of cylindrical cantilever structures	Moments, section modulus, material
Mechanical design of tapered element, clamp	Sum of wind-force moments
Element location, mast torque, compensation	Sum of element and boom moments
Design of cantilever boom	Column strength
Boom and element guying	Beam-column calculations
Design of cantilever mast	Sum of wind-force moments, materials
Choice of rotator, torque isolator	Polar moment, data table
Yagi cylindrical-element electrical design	Cylindrical-element computer program
Dimensions for physical tapered elements	Taper algorithm
Element length correction for clamp, boom	Equivalent diameter
Siting and sloping foreground	Elevation model

Sec 1.7 Electrical Modeling of Yagi Antennas

Modeling is intended to avoid experimental difficulties through the development of approximations that are mathematically workable, yet sufficiently accurate to calculate the unknown impedances from the known dimensions and characteristics of dipole antennas. Modeling techniques are subject to experimental verification; measured performance is the final confirmation of the accuracy of a mathematical modeling process.

Electrical design of Yagi antennas is accomplished by modeling the self- and mutual impedances of an array of thin, cylindrical elements. With certain approximations such as sinusoidal variation of current and voltage along the dipole, Yagi design can be cast into analytical circuit matrix methods. The work of Lawson incorporates this

approach.[10] The antenna pattern and its key parameters, such as gain and front-to-back ratio, can be determined and plotted.

Once a satisfactory design composed of cylindrical elements is generated, the problem remains to find the equivalent tapered (survivable) element dimensions that provide the same self- and mutual impedance as the reference cylindrical elements. Lawson, in his classic work, derives through perturbation methods the equivalence between a given tapered element and the diameter and length of the cylindrical element having the same self-reactance. Lawson confirms that the effect of taper on resonant length is substantial, and his algorithm provides a means of calculating the required physical dimensions for a given electrical design.

In the case of broadband antennas and moderate tapers, it appears that this two-step process gives good insight into Yagi performance trade-offs, although perhaps not sufficiently so to allow exact physical realization without measurement and adjustment. Given an existing tapered antenna design, it is possible to use the same process in reverse to determine the equivalent cylindrical elements and calculate the performance of the antenna.

In the past decade, numerical methods have become powerful enough to permit the development of antenna-modeling programs based on fewer assumptions. Each element of the antenna is modeled as an interconnected group of short dipoles, as is the mutual impedance of the remaining elements. The self- and mutual impedances of short dipoles are quite accurately known. This process, known as the method of moments,[11] is used in the program NEC (for large computers) and MININEC (for minicomputers). These programs have been shown to yield results consistent with physical measurements. Versions of MININEC are available for use on personal computers.[12]

Although calculations based on the method of moments are substantially more accurate than analytic methods, the process is especially time consuming for synthesis. Because the numerical process confirms certain inaccuracies in Lawson's taper algorithm, I have developed an improved algorithm for converting tapered elements to their cylindrical equivalents. This new algorithm is derived from the work of Schelkunoff,[13] and is introduced in this book. Also, a more accurate model is developed for the equivalent diameter of irregular shapes such as element clamps.

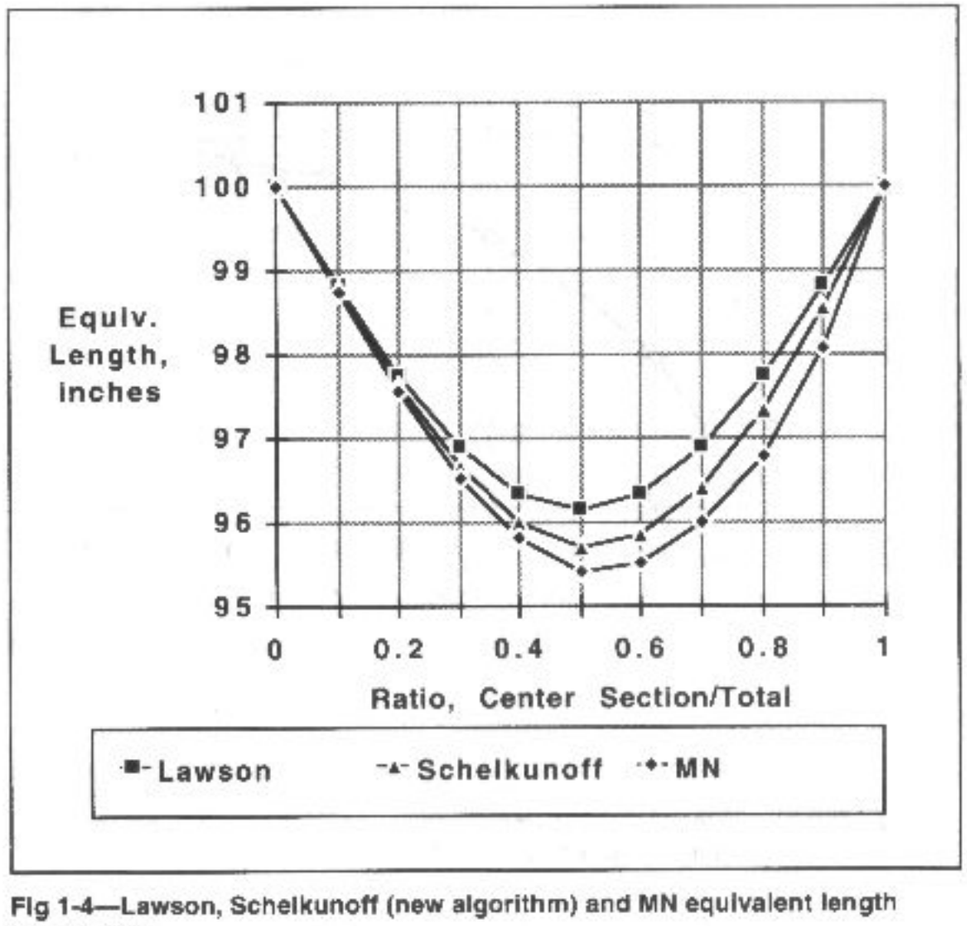


Fig 1-4—Lawson, Schelkunoff (new algorithm) and MN equivalent length comparison.

The results of MININEC have been applied to substantiate this improved process. Curves comparing the new algorithm, the Lawson method, and the method of moments calculations are plotted in Fig 1-4. The curves indicate the length of a cylindrical antenna equivalent to a dipole having two sections with a 2:1 ratio of diameter, shown as a function of the ratio of center-section length to the total length.

The new algorithm can be used to provide accurate Yagi modeling in connection with Yagi-design programs suitable for use on personal computers. Several such programs are available commercially, and are useful in determining the expected performance of existing antennas and exploring the potential for improved pattern, bandwidth or feedpoint impedance match. These programs have been used to assist with the electrical and mechanical design analysis of several popular commercial antennas and with the physical realization of several attractive published Yagi designs.

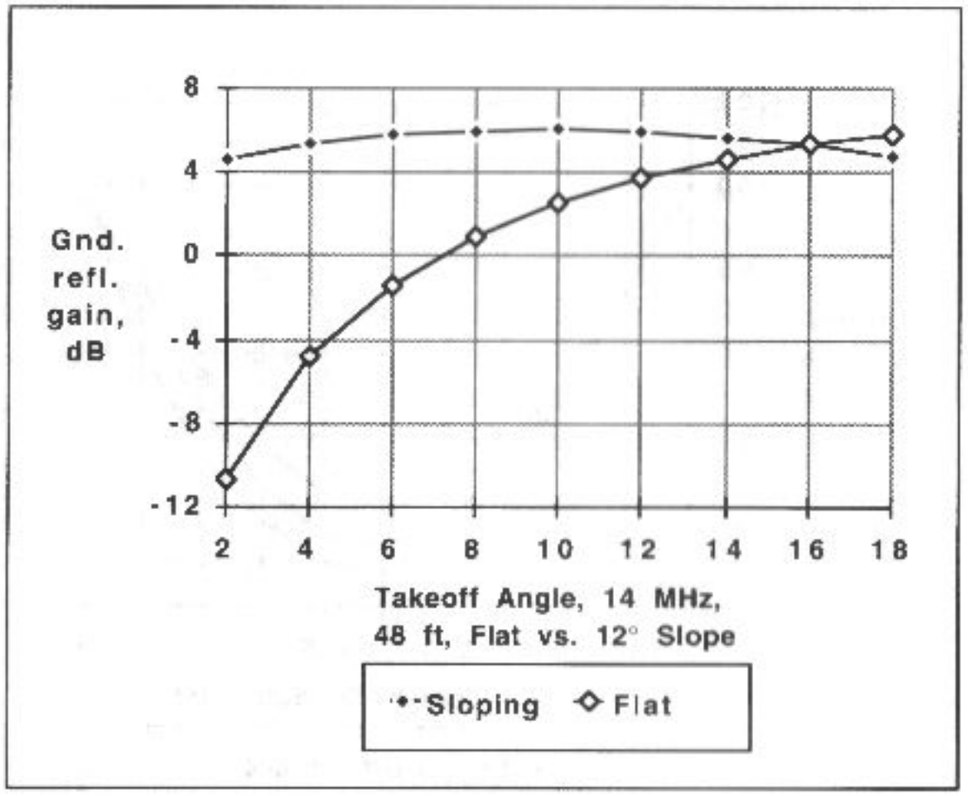


Fig 1-5—Ground-reflection gain comparison for 14-MHz Yagi at a height of 48 feet. For the sloping ground case, the slope is 12°.

Sec 1.8 Siting of Yagi Antennas over Sloping Foreground

In addition to greatly increasing my sensitivity to wind-loading issues, mountaintop life has satisfied my curiosity about the benefits of a sloping foreground. There is no question in my mind that the wind penalty is worth the tremendous low-angle radiation advantage obtained from the sloping foreground in a mountaintop location.[14-16] Our location looks just like the diagram in the Moxon (G6XN) HF antenna book,[17] and the results are so good that they dwarf the potential 0.5- to 1-dB gain improvements sought in individual Yagis.

My experience is that a slightly smaller antenna with a 6- to 12-dB low-angle advantage and no nulls in the useful elevation-angle range represents a good trade-off against a larger antenna over alluvial (horizontal) ground, since the latter must have essentially zero radiation at the horizon. This advantage can be seen in Fig 1-5. I also agree with Moxon and Beezley (K6STI)[18] that the front-to-back issue is better thought of as a statistical average over all undesired azimuth angles; high front-to-back ratio may even be undesirable for contest operation

in certain locations. On that basis, it is possible to find very satisfactory antennas that are not so big as a ¾-wavelength boom on 14 MHz.

In this book I discuss the reasons why substantial advantage is obtained by Yagi antennas above sloping ground, and why vertical stacking may not be appropriate to this siting. Some specific siting recommendations are derived from simple theoretical considerations, and the results of crude but convincing experimental confirmations are discussed.

Sec 1.9 Disclaimer

The concepts and techniques outlined in this book can provide an improved understanding of the physical design of Yagi antennas. However, I am not a professional mechanical engineer and this is not a professional mechanical engineering text. Additionally, I have no control over the reader's mechanical skills or workmanship. I regret the need in our litigous society for this disclaimer.

Although I have attempted to eliminate any errors of concept or calculation, the techniques and examples presented in this book are not guaranteed to produce unfailingly safe and survivable antenna designs. For professional assurance, the services of a licensed mechanical or civil engineer should be sought to certify the safety of any design or structure.

References

[1] J. L. Lawson, *Yagi Antenna Design* (Newington: American Radio Relay League, 1986).

[2] E. Fuller, "The Application of Stress Analysis to Antenna Systems," *Ham Radio*, Oct 1971, pp 23-31.

[3] J. Nagle, "How to Calculate Wind Loading on Towers and Antenna Structures," *Ham Radio*, Aug 1974, p 16-27.

[4] D. Weber, "Structural Evaluation of Yagi Elements," *Ham Radio*, Dec 1988, pp 29, 31-33, 35, 37, 38, 41, 42, 45, 46.

[5] D. Weber, "Vibration Induced Yagi Fatigue Failures," *Ham Radio*, Aug 1989, pp 9-12, 17, 18.

[6] D. Reynolds, "Wind Loading," *Radio Communication*, Apr/May 1988.

[7] J. D. Kraus, *Antennas*, 2nd ed. (New York: McGraw-Hill Book Co, 1988).

[8] S. Uda, "On the Wireless Beam of Short Electric Waves," *JIEE* (Japan), Mar 1926 through Jul 1929.

[9] H. Yagi, "Beam Transmission of Ultra Short Waves," *Proc IRE*, Jun 1928.

[10] See reference 1.

[11] See pp 359-408 of reference 7.

[12] B. Beezley, *MN* (computer program), 507½ Taylor St, Vista, CA 92084, 1989. Also R. W. Lewallen, *ELNEC*, PO Box 6658, Beaverton, OR 97007, 1990, and L. Gordon, *YAGIMAX*, PO Box 105, Hannibal, MO 63401, 1990.

[13] S. Schelkunoff, "Theory of Antennas of Arbitrary Size and Shape," *Proc IRE*, Sep 1941, p 493.

[14] P. D. Rockwell, "Station Design for DX, Part I—Antenna Topics and Siting," *QST*, Sep 1966, pp 50-54.

[15] S. Bally, K. Beeke, and R. Manton, "The Performance of H.F. Curtain Antennas with Irregular Foregrounds," *Proc Fourth International Conference on HF Radio Systems and Techniques*, IEE (UK), Apr 1988.

[16] M. Gibbings, "Moxon Slopes at VHF," *Radio Communication*, May 1988, p 342.

[17] L. A. Moxon, *HF Antennas for All Locations* (Potters Bar, Herts: RSGB, 1982), p 132.

[18] B. Beezley, *YO* (Yagi optimizer computer program), 507½ Taylor St, Vista, CA 92084, 1989.

Chapter 2

Wind and Ice Loading

Sec 2.1 Wind Characteristics

Strength analysis of tubular antenna structures requires an understanding of wind force and how it operates on a cylindrical element. Wind is a part of the experience for each of us, although not everyone may have experienced the awesome sound and force of winds over 100 miles per hour. Watching a Yagi antenna as it was being ripped apart in the wind was the beginning of my own curiosity to see what could be done to enjoy Amateur Radio in the winter.

Although the wind force on antennas would seem to be a well-studied problem, I found only one mechanical engineering reference, edited by C. Richards and published in England, that treated antennas as a separate specialty.[1] I obtained a copy at a university library, and was only the fourth person to check it out in 15 years!

Wind is the result of atmospheric pressure differences caused by weather systems. The gradient of atmospheric pressure (the difference in atmospheric pressure divided by the distance over which it operates) determines the average speed v_g of gradient wind flow aloft. Wind along the ground flows in a boundary layer that is retarded by surface roughness. Hence, there is an increase of wind velocity with height above ground. The rate of increase with height and the ratio of surface velocity to gradient velocity has been the subject of extensive measurement, and can be quantified by a simple mathematical model.

The wind gradient with height can be expressed as

$$v(z) = v(z_o)\left(\frac{z}{z_o}\right)^{1/\alpha} \qquad \text{(Eq 2-1)}$$

where

$v(z)$ = wind velocity at height z

$v(z_o)$ = wind velocity at the reference height z_o

6.5 Calculation of Boom Strength 6-18
6.6 Guyed Boom and Elements 6-28
6.7 Columns In Compression 6-32
6.8 Beam-Column Strength 6-33
6.9 Joining Boom Sections 6-41
6.10 Guyed Elements 6-45

Chapter 7 Masts and Rotators

7.1 Section Modulus and Material Strength 7-1
7.2 Steel Tubing and Pipe 7-2
7.3 Tubing Manufacturing Processes 7-3
7.4 Tubing Catalog Listings 7-5
7.5 Calculation of Mast Bending Moment 7-6
7.6 Stacking 7-10
7.7 Boom-to-Mast Attachment 7-21
7.8 Los Angeles County Specifications 7-23
7.9 Rotator Selection 7-24

Chapter 8 Self-Impedance of Yagi Elements

8.1 Reactance of a Dipole Near One-Half Wavelength 8-1
8.2 Equivalence Among Dipoles of Various Shapes . . 8-4
8.3 Effect of Element Shape 8-7
8.4 An Algorithm for Step-Tapered Radius 8-18
8.5 Comparison of the Algorithm with MININEC . . . 8-24

Chapter 9 Yagi Element Electrical Design Issues

9.1 Equivalent Diameter of Element Clamps 9-1
9.2 Algorithm Spreadsheets for Tapered Elements . . . 9-6
9.3 The Hy-Gain Insulated Clamp 9-8
9.4 Comments on Beta Matching 9-9
9.5 Element Reactance 9-16

Chapter 10 Siting and Installation of Yagi Antennas

10.1 Ground Reflection and Antenna Performance . . . 10-1
10.2 Avoiding Foreground Shadowing 10-6
10.3 Antenna Comparison Using Fading Statistics . . . 10-8
10.4 Experimental Verification of Taper Algorithm . . 10-10
10.5 Stacking Yagis—Equivalent to Foreground Slope? 10-14
10.6 Suggestions for Further Studies with MININEC . 10-15
10.7 A Broadband Matching Suggestion 10-16
10.8 Some Comments on Coaxial Cable 10-16
10.9 Where is the Power Measured? 10-18
10.10 Some Contest Tips 10-18
10.11 Some Suggestions for Antenna Manufacturers . . 10-19
10.12 Installation and Safety 10-20
10.13 Concrete Basics for Tower Installation 10-23

Chapter 11 Yagi Antenna Examples

11.1 Applying the Techniques of This Book 11-1
11.2 Calculating Expected Wind Conditions 11-2
11.3 Antenna Examples 11-4
11.3.1 Cushcraft 40-2CD 11-5
11.3.2 Hy-Gain TH5 11-11
11.3.3 Hy-Gain TH7 11-19
11.3.4 Hy-Gain 105BA 11-24
11.3.5 Hy-Gain 155BA 11-28
11.3.6 Hy-Gain 204BA 11-32
11.3.7 Hy-Gain 205BA 11-36
11.3.8 QHS N6ND510 11-41
11.3.9 QHS N6ND520 11-47
11.3.10 QHS N2FB420 11-55
11.4 Advanced Cylindrical Yagi Designs 11-59
11.5 Manufacturer's Mechanical Antenna Parts . . . 11-64
11.6 Mast Design Examples 11-64
11.7 Conclusions and Suggestions for the Future . . . 11-71

Index

The connotation $v(z_o)$ indicates v *as a function of* z_o. Similar connotations are used throughout this book.

Eq 2-1 is valid up to heights of approximately 1000 feet. The exponent $1/\alpha = 0.15$, ranging from $\alpha = 7$ to $\alpha = 2.5$, depending upon surface roughness and whether the equation is applied to mean or peak wind speeds.[2]

The height z_g at which the influence of ground friction has a negligible effect on the wind velocity (that is, the height at which the velocity is equal to the gradient velocity) varies with the degree of roughness.[3] The same exponential expression can be used to calculate the ratio of surface velocity v_s (at standard $z_o = 33$ ft) to gradient velocity v_g.

$$v_s = v(z_o) = v_g \left(\frac{z_o}{z_g}\right)^{1/\alpha} \qquad \text{(Eq 2-2)}$$

If we know α and z_g, we can solve for v_s/v_g from

$$\frac{v_s}{v_g} = \left(\frac{z_o}{z_g}\right)^{1/\alpha} \qquad \text{(Eq 2-3)}$$

Eq 2-3, with measured values of α and z_g, yields the results in Table 2-1.

We can see that there is considerable variation in surface wind velocity at standard height, depending upon surface roughness, and we also have Eq 2-1 to account for the effect of height above the surface. This concept permits comparison of anemometer records taken at locations differing in height and surface roughness. The relationship

Table 2-1
Wind Gradient Parameters

Surface Roughness	α	z_g	v_s/v_g
Flat open country	7	900 ft	0.62
Wooded areas, towns	3.5	1300	0.35
Large cities	2.5	1700	0.21

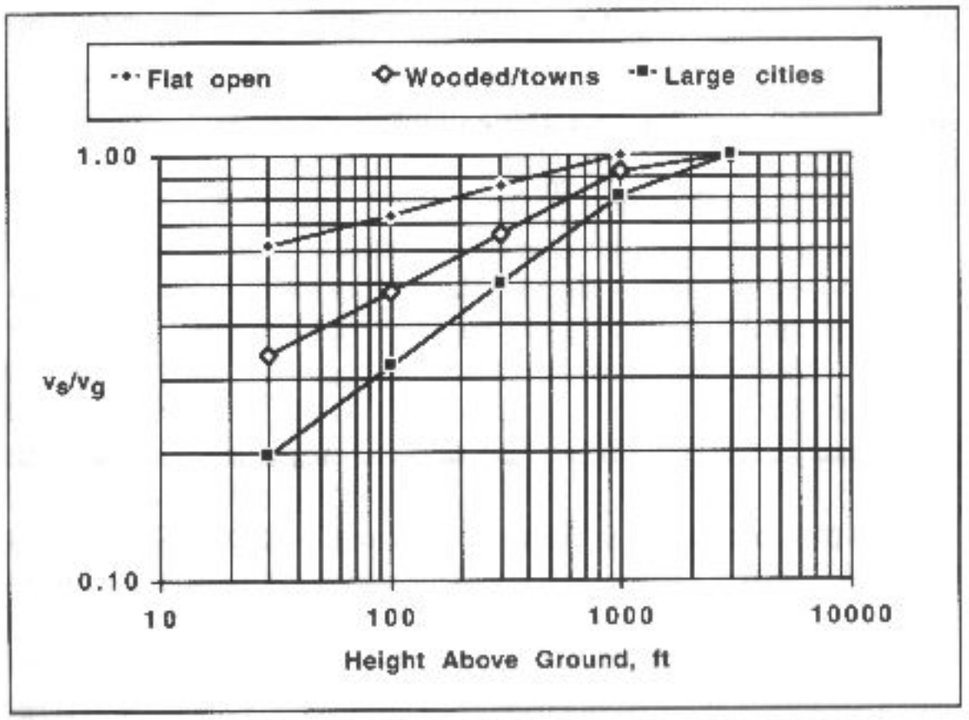


Fig 2-1—Ratio of surface wind to gradient wind as a function of height.

between wind velocity and height above ground for different surface roughness is shown in Fig 2-1.

Sec 2.2 Wind Speed Statistics

A structure will fail if the forces generated by the wind exceed, even momentarily, the structural strength. The dynamic amplification factor from the interaction of structural resonances with the intensity and frequency of wind fluctuations is beyond the scope of this book, as is the consideration of structural deflections on wind forces. In order to design safely against wind forces, we need to know the maximum gust affecting the entire structure that could be expected during the structure's service life. Because the velocity of wind is continually fluctuating, this is a question involving the statistics of wind velocity at the site over a period of years, as well as the statistical relationships between average and gust peak velocities.

During the period of a wind event such as a storm, the wind speed exhibits a statistical distribution such that the maximum speed over very short gust times (such as one second) is greater than the speed averaged over longer times (such as one minute or one hour). As an example,

Table 2-2
Wind Speeds over Various Averaging Times

1 Hour	*10 Min*	*1 Min*	*30 Sec*	*20 Sec*	*10 Sec*	*5 Sec*	*0.5 Sec*
40 mi/h	43	50	53	54	57	59	65
50	53	62	66	68	71	74	82
60	64	74	79	81	85	89	96
70	74	87	92	95	99	104	111

Table 2-2 shows measured values of maximum wind speed averaged over short periods of time t, for various values of mean hourly wind speed.[4]

By fitting a logarithmic curve to the data of Table 2-2, we can derive an approximate relationship between wind velocity and averaging time, which is given by

$$v(t) \approx v(t_o)\left[1 + k_o \log_{10}\left(\frac{t_o}{t}\right)\right] \qquad \text{(Eq 2-4)}$$

where

$v(t)$ = wind speed averaged over time t
$k_o = 0.158$ for $t_o = 3600$ seconds or 1 hour

When wind velocity is measured with a standard wind gauge such as a spinning-cup anemometer, the response time of the instrument is not necessarily short enough to read the peak gust. Instead, the meter reads a longer term average determined by its particular averaging time constant. For a given location or geographic area, statistics can be obtained or generated to give an idea of the recurrence of a given wind velocity.

In order to obtain standard measures of highest wind velocity, the instantaneous velocities are averaged over an interval that (1) is invariant with terrain variations, (2) is long compared to both structural natural frequencies and instrument response times, (3) is short compared to the duration of storm peaks, and (4) corresponds to a body of moving air sufficient to completely envelop a structure during a peak.

The US standard for basic wind velocity is the "fastest mile of wind," which is the extreme wind velocity averaged over a mile (that is, over the time required for the passage of one mile of wind). A secondary choice is the extreme wind velocity averaged over a minute, which, at 60 mi/h, is the same as the fastest mile of wind. The extreme mean hourly velocity is also considered significant, because the variation in the spectral density of wind speed has a relatively constant value for times of the order of one hour. UK standards define basic wind speed in terms of a 3-second gust, which is approximately 1.15 times the fastest-mile wind velocity. The mean minute wind velocity (and hence, the fastest-mile wind velocity) is approximately 1.25 times the mean hourly wind velocity.

The "basic wind speed" is the mean wind speed estimated to be exceeded on the average once in 50 years in a given geographic area at the standard height over open, level country with no obstructions. The basic wind speed can be determined for a given area from published charts and maps in various standards, and reflects the statistical variation over the years among many storms, rather than the gust statistics within a given storm.

Specific local wind statistics can be obtained from a number of sources including local airports, weather service offices and personal experience. Clues can be gained from observation of trees in the area; absence of forestation may be a clue to high winds. One excellent suggestion[5] is to ask the local power company engineering department what wind and ice statistics it uses to plan power-line installations near the site in question.

Structural failure is caused by the peak forces associated with peak wind speed, so the effect of gusting must be considered. Appropriate gust factors are used for structures which are small enough to be responsive to gusts of less than one-minute duration. One-second gusts, in a 60-mi/h basic minute wind, would have a downwind length of about 90 feet that would completely envelop a structure having a horizontal dimension of 15-30 feet. By comparison, a 10-second gust would envelop a structure having a horizontal dimension of 125 feet.

From the data of Table 2-2, the calculated ratio of one-second average gust speed to fastest-mile speed ranges from 1.25 to 1.18 over the range of fastest-mile speeds of 40 mi/h to 120 mi/h. It is accepted to use a 30% gust factor as a simple and conservative model for one-second peak wind (peak velocity equals 1.3 times the mean

velocity). Gusts are not always in the same direction as the general wind flow; vertical or transverse gusts are also considered to have a magnitude equal to 30% of the mean wind velocity. Note that tornadoes and tropical storms such as hurricanes are excluded from this description.

Sec 2.3 Structural Standards

Although the Uniform Building Code[6] (UBC) and the British Standards Institution Code of Basic Data for the Design of Buildings[7] treat wind survival of structures and towers, the accepted standard in the United States for antenna and tower engineering has been RS-222-C, published by the Electronic Industries Association (EIA).[8] Many tower companies include this document in their catalogs. The model presented in RS-222-C is based on 50-year mean recurrence intervals from the distribution of extreme winds in the United States.[9] It incorporates the 30% gust factor and includes tables that illustrate the increase of wind speed with height above ground, although for heights (300 ft) that are well above typical amateur installations. The model names three zones, as shown in Table 2-3.

The wind pressure is defined in RS-222-C as the force per unit area on a flat surface, and may be calculated from

$$p = 40\left(\frac{v}{100}\right)^2 = 0.004v^2 \qquad \text{(Eq 2-5)}$$

where

p = peak pressure, lb/ft^2
v = mean wind speed, miles per hour (mi/h)

Note that pressure rises as the square of the wind velocity, so a 10% error in estimation of wind velocity results in a 20% error in the estimation of wind pressure.

Fig 2-2 shows the EIA wind zones for the 48 contiguous states. Most of the 48 states lie within Zone A, with Zone C locations primarily on the Atlantic coast of Florida and North Carolina. Zone B includes the Atlantic and Gulf coasts from Texas to Maine, plus areas near Lake Michigan, the northern Great Plains, the extreme Pacific Northwest coast and the California Gold Rush country. A detailed listing of zone boundaries by state and county is included in the EIA publication.

In 1986 the EIA published a revision of its RS-222-C, designated EIA-222-D.[10] This revision appears to conform RS-222-C with the

Table 2-3
Electronic Industries Association RS-222-C Wind Zones

EIA Zone	*Wind Pressure*	*Mean Velocity*	*Peak Velocity*
—	20 lb/ft^3	70.7 mi/h	92 mi/h
A	30	86.6	113
B	40	100.0	130
C	50	111.8	145
—	60	122.5	159

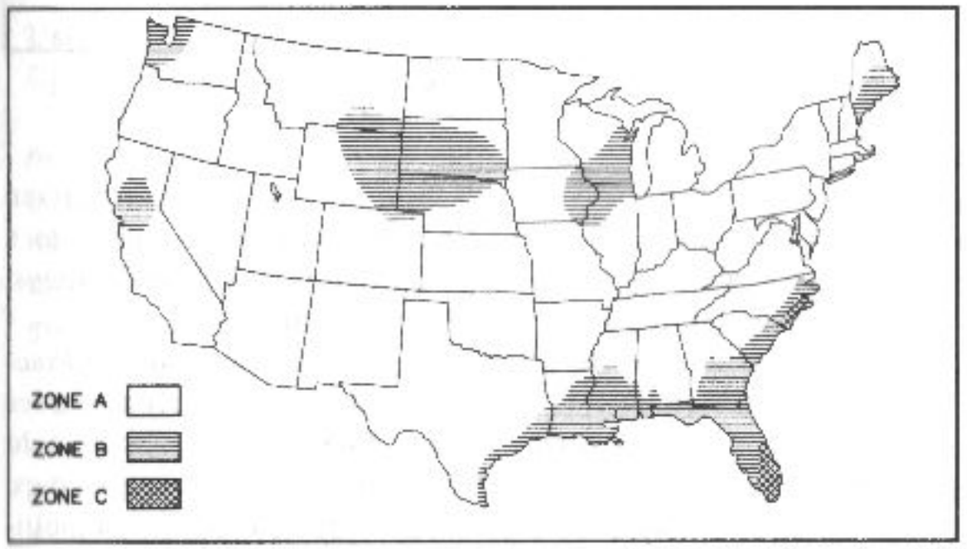


Fig 2-2—EIA RS-222-C wind zones. Also see Table 2-3.

UBC. The wind-zone map of RS-222-C is replaced by a similar basic wind-speed map[11] identical to that of the UBC, shown in Fig 2-3. The two EIA standards use somewhat different force constants to derive wind pressure on structures, but the differences in wind-speed maps tend to compensate to produce essentially the same pressure for a given locale.

Sec 2.4 Wind-Speed Factors

In order to apply the EIA model safely to amateur antenna practice, it is important to note that there are a number of situations

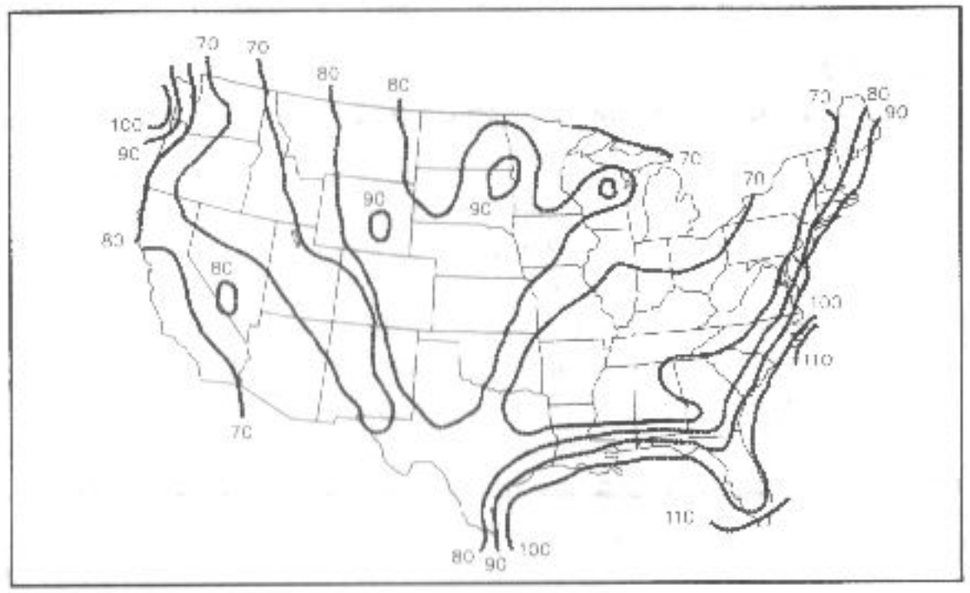


Fig 2-3—UBC and EIA-222-D wind-speed map.

where the wind substantially exceeds the mean wind speed for a given geographic zone. To apply general mean wind-speed estimates to a given site, we need to consider the local effects of hills and valleys, trees, buildings, and antenna height above ground.

The British Standards Institution's comprehensive Code of Practice (CP3)[12] for determination of wind loads treats separately the issues of basic wind speed, topography factor, ground roughness and height above ground, expected structure life, and wind-direction statistics. Each of these local variables is treated by a separate factor that multiplies the basic wind speed to estimate the value appropriate to a given site. Basic wind speed in the BSI Code of Practice is defined as the 3-second peak speed, and hence corresponds to approximately 1.15 times the EIA specification of mean wind speed.

S_1, the topography factor, takes account of large local variations in the ground surface, and deals with the effects of hills and the sheltering in valleys. The effects of ground roughness, small-scale obstructions such as trees and buildings, and height above the surface are combined in the terrain factor, S_2. Last, the statistical issue of the required structure life is treated in the safety factor, S_3.

In CP3, the local design wind speed v_s is calculated from

$$v_s = vS_1S_2S_3 \qquad \text{(Eq 2-6)}$$

where

v_s is obtained in mi/h

v = basic peak wind speed

The wind dynamic pressure q on a flat surface is calculated from

$$q = 0.00256v_s^2 \qquad \text{(Eq 2-7)}$$

The pressure on a structural element is given by

$$p = C_pq \qquad \text{(Eq 2-8)}$$

where

p is obtained in lb/in.^{2}

C_p = pressure coefficient determined by the structural shape

Using this definition of dynamic pressure, $C_p = 2$ for a flat plate, and $C_p = 1.2$ for a cylindrical shape.

The basic wind speed does not account for local topography features such as hills, valleys, cliffs, escarpments and ridges, which can significantly affect the local wind speeds. Near the summits of hills or the crests of cliffs and ridges, the wind is accelerated. In valleys or near the foot of cliffs, the wind may be decelerated. A local slope of 5% or greater is deemed to be significant in this respect.

A hill or ridge is defined as having a downwind slope greater than 5%, while a cliff or escarpment is defined as having a *downwind* slope of an essentially flat surface (less than 5%). The range for the topography wind factor is $1.0 \le S_1 \le 1.36$, which can make this factor at least as significant as variations in the basic wind speed itself.

Approximate equations for the value of the topography factor S_1 at the top of a ridge or cliff of slope Ψ are

$$S_1 = 1 + 1.2\Psi \text{ (for } 0.05 \le \Psi \le 0.3) \qquad \text{(Eq 2-9)}$$

$$S_1 = 1.36 \text{ (for } \Psi > 0.3) \qquad \text{(Eq 2-10)}$$

For the topographical feature, the slope Ψ is defined as the ratio of its height to the length of the upwind slope in the windward direction. The height is measured from the average foreground level, and the slope length is the horizontal distance from the foot of the feature to its peak. BSI CP3[13] contains a comprehensive appendix that treats locations on the side slope of hills and cliffs.

The terrain factor S_2 takes account of the combined effect of ground roughness, the variation of wind speed with height above ground, and the size of the building or component part under consideration. In conditions of strong wind, the wind speed generally increases with height above ground. The rate of increase depends on ground roughness and on whether short gusts or mean wind speeds are being considered. Smaller structures such as antennas are more affected by short gusts than are larger structures, for which a longer wind-averaging period is more appropriate. For HF Yagi antennas, the structural resonances may be sufficiently low that the structure does not fully respond to peak gusts of extremely short duration. Excellent reviews of mechanical resonance and vortex shedding frequencies for tubular structures are contained in two magazine articles.[14,15]

According to Sachs,[16] a mountain can be considered as an airfoil protruding into the otherwise uniform wind stream. As with an airplane wing, the wind accelerates over the top of the mountain in order to rejoin the stream on the downstream side. The degree of speed increase can be as much as 50%, or a 100% force increase above the nominal zone wind pressures. Because the increase in wind speed over a ridge is a result of the surface shape, the wind velocity above the surface eventually *decreases* with height to the undisturbed value. In my own case, I have determined through experience that I must design to 60 lb/ft^2, corresponding to a mean fastest-mile wind speed of 122.5 mi/h.

The formation of gusts is described as the peeling off of vortices from the foreground terrain. Statistics and spectral densities of the wind measurements described by Richards[17] do not suggest any better choice of gust factor than 30%. Because the gusts are of a vortex nature, the upward and transverse gust velocities can be estimated as just the 30% of the mean wind speed, with the peak in the line of the wind direction being 130% of the mean. On a steep up-slope, the mean wind has a considerable upward component that must be considered in boom design, for example. The reverse is true on the downslope side.

Sachs also presents measurements suggesting that an antenna mounted on or near a structure experiences wind forces that can be some 25% above the typical wind forces in the general area. Conversely, well forested or built-up areas tend to reduce surface winds through the effect of energy dissipation. The existence of surviving trees or structures in your area can help give an idea of your actual wind conditions.

Table 2-4
British Standards Institution CP3 Ground Roughness Categories

Category	*Description*	*Example*
1	Flat, open, level with no shelter	Airfields, grassland
2	Flat or undulating, with occasional small obstructions	Farmland
3	Numerous large obstructions, roof height 10 m	Forest, towns
4	High, closely spaced obstructions, roof height 25 m	City centers

Table 2-5
British Standards Institution CP3 Ground Roughness and Height Factor S_2

Height	*Ground Roughness Category*			
	1	2	3	4
<10 ft	0.83	0.72	0.64	0.56
15	0.88	0.79	0.70	0.60
30	1.00	0.93	0.78	0.67
70	1.06	1.03	0.95	0.79
130	1.12	1.10	1.05	0.97

The BSI Code divides ground roughness into four categories, and structure size into three classes. Antenna structures are included in Class A, which applies also to elements of buildings; Classes B and C apply to entire small and large buildings. Ground roughness classifications are shown in Table 2-4. Ground roughness has been shown to affect both surface velocity and the gradient of wind speed with height. The net result of these considerations is a table of values for the wind-speed factor S_2, derived from BSI CP3 and shown here in Table 2-5 for antenna structures.

The factor S_3 takes account of the degree of security required and the period of time the structure is required to be exposed to the wind. The factor is taken as $S_3 = 1.00$ except in cases of shorter exposure such

as temporary structures or in cases of structures where greater than normal safety is required.

The product of the wind-speed factors $S_1 \times S_2 \times S_3$ multiplies the basic wind speed to provide an estimate for a specific location. The extreme value of these factors can reflect a wind-speed increase of as much as a factor of 1.44 over the basic value for a particular area. The British Standards Institution code of practice gives a quantitative basis to account for site-specific wind variations.

The Uniform Building Code treats the same variables by means of factors multiplying the design *pressure*, rather than the wind speed, so the UBC factors are the square of the BSI factors. The UBC uses a coefficient C_e to combine the effects of height above ground, exposure, terrain roughness, obstructions, and a factor for gusts. Statistics of structure life and safety factor are included in a coefficient termed the importance factor, I.

Using the method of the UBC, the wind stagnation pressure at a standard height of 30 ft (≈ 10 m) for wind velocity v is calculated using essentially the same equation as in EIA-222-D and BSI CP3.

$$q_s = 0.00256v^2 \qquad \text{(Eq 2-11)}$$

The pressure on a structural element is given by

$$p = C_e C_q q_s I \qquad \text{(Eq 2-12)}$$

where

C_q = pressure coefficient for the structural shape (C_q = 2 for a flat plate, $C_q \approx 1.2$ for a cylinder)

q_s = wind stagnation pressure at the standard height of 30 ft

Under the UBC, exposure at a site is rated as Exposure B or Exposure C. Exposure C represents the most severe exposure and has terrain which is flat and generally open, extending at least one-half mile from the site. The less severe Exposure B terrain has buildings, forest or surface irregularities 20 ft or more in height covering at least 20% of the area, extending one mile or more from the site. The UBC does not incorporate a factor representing the effect of hills and valleys on the wind speed. The UBC wind pressure factor C_e for combined height, exposure, and gust factor is given in Table 2-6.

The UBC contains a wind map of the United States showing wind-velocity profiles in 10 mi/h increments. The data is generally

similar to the EIA RS-222-C map, but shows greater detail in coastal and high plains areas, as well as a considerable inland area having a maximum mean wind of 70 mi/h. The value of the importance factor is taken as I = 1.00 except for essential facilities which must be safe and usable for emergency purposes after a windstorm. In this case, I = 1.15; consideration should be given to using this higher wind pressure factor to plan amateur installations which must work during or after windstorms.

Using the wind-speed factors of the BSI and UBC methods, Table 2-7 and Fig 2-4 summarize the effects of local site conditions on the EIA RS-222-C 50-year maximum mean wind speeds at a height of 70 ft. The four columns of Table 2-7 correspond to BSI CP3 categories 1 through 4 of Table 2-5, and the hilltop category includes the slope factor.

One final point is that the density of air, and hence the wind force at a given speed, varies with absolute altitude. Per 1000 feet of altitude, the atmospheric density decreases approximately 3% and the air temperature decreases approximately 3.5°F. Although atmospheric variations are significant for aeronautical purposes, for most practical antenna situations the decrease in wind force with absolute altitude is not material.

The mean wind speeds determined for Table 2-7 and Fig 2-4 can be used to calculate the wind pressure from Eq 2-12, giving effect to gust factor and drag coefficient for a flat surface. These

Table 2-6
Uniform Building Code Height, Exposure and Gust Factor, C_e

Height	*Exposure*	
	B	C
0-20 ft	0.7	1.2
20-40	0.8	1.3
40-60	1.0	1.5
60-100	1.1	1.6
100-150	1.3	1.8

Table 2-7
Maximum 50-Year Mean Wind Speeds at a Height of 70 Feet

Zone	*Urban or Forested*	*Rolling Suburban*	*Flat Open Country*	*Bare Hilltop*
A	68 mi/h	82 mi/h	89 mi/h	125 mi/h
B	79	95	103	144
C	88	106	115	161

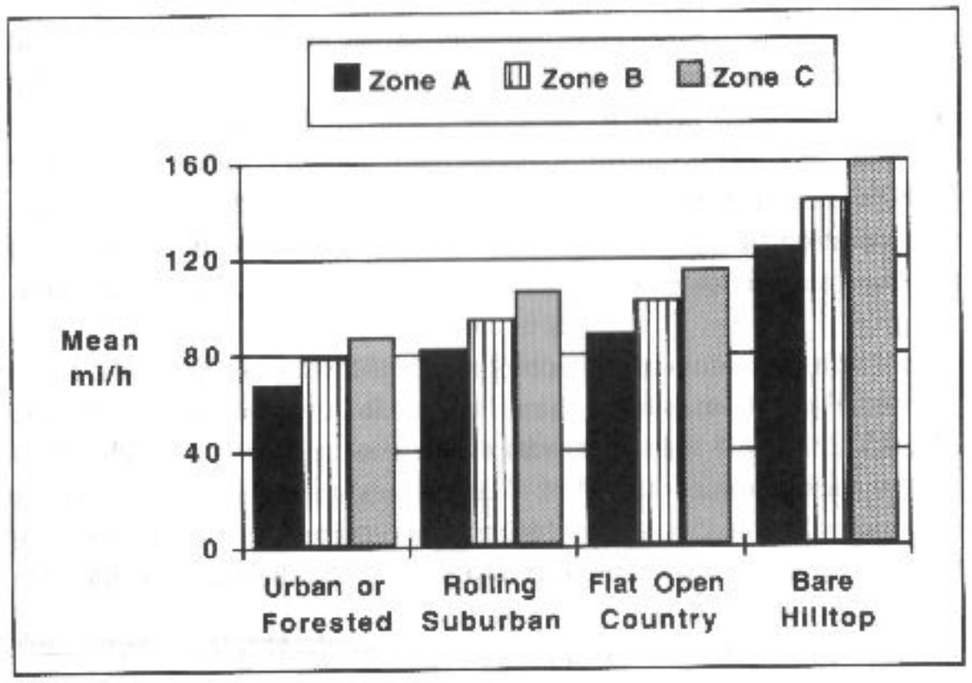


Fig 2-4—Mean wind speeds (RS-222-C) for 70-foot antenna height.

Table 2-8
Maximum 50-Year Wind Pressure at a Height of 70 Feet

Zone	*Urban or Forested*	*Rolling Suburban*	*Flat Open Country*	*Bare Hilltop*
A	19 lb/ft^2	27 lb/ft^2	32 lb/ft^2	62 lb/ft^2
B	25	36	42	83
C	31	45	53	104

are presented in Table 2-8. Note that a mountaintop site requires heroic design efforts, and that commercial designs providing survival at v_s = 70 mi/h (20 lb/ft^2) will be marginal for many sites while those providing survival at v_s = 100 mi/h (40 lb/ft^2) will serve a much wider range of sites.

To summarize, the complexities of wind-speed statistics and spectral densities can be simplified with acceptable safety to the statistics of the appropriate EIA RS-222-C zone, with the strong proviso that the

effects of nearby structures and wind obstructions, antenna height above ground, and site topography must be considered for a specific site.

Sec 2.5 Wind Force on Cylinders

Now that we have determined how to estimate mean or peak gust wind velocity at a given site, we turn to the question of how to calculate the wind force on a structural member. As with most aerodynamic problems, this can be approached with varying degrees of sophistication.

An understanding of the derivation of the force constant used in the various specifications will be helpful here. Dynamic pressure of a moving fluid, such as air, is defined from Bernoulli's equation

$$p_1 + 0.5\rho v_1^2 = p_2 + 0.5\rho v_2^2 \qquad \text{(Eq 2-13)}$$

where

p = air pressure
ρ = air density (assumed to be constant)
v = air velocity
Subscripts 1 and 2 denote these parameters at two different locations

Let us define position 1 as being far from the antenna and assume the wind is stopped completely at position 2 (the antenna position), so $v_2 = 0$. Then the pressure on the antenna, p_2, is greater than the static air pressure p_1 by $0.5\rho v_2^2$. This quantity is known as the dynamic pressure of the wind stream, and is denoted by q so that, for air of standard density

$$q = 0.5\rho v_1^2 = 0.00256v^2 \qquad \text{(Eq 2-14)}$$

In practice the wind does not come to a complete stop over the entire surface of the windward side of an obstruction, and there is also suction on the leeward side.[18] For a given shape, the total pressure is proportional to q. The proportionality constant C, generally called the shape factor, may be defined either as the ratio of force to the product of dynamic pressure multiplied by the normal (perpendicular) area, or as the ratio of actual pressure to dynamic pressure. Thus

$$C_f = \frac{\text{Force}}{Aq} \text{ or } C_p = \frac{\text{Pressure}}{q} \qquad \text{(Eq 2-15)}$$

In general, the relationship between pressure and velocity (p proportional to v^2) is expressed in the form

$$p = qC = p_o C v^2 \qquad \text{(Eq 2-16)}$$

This equation will be found in all of the structural standards, although the value of the constants depends on the choice of standard. The simplest picture is given by EIA RS-222-C, which states that the wind pressure on a flat plate is given by

$$p = 0.004v^2 \qquad \text{(Eq 2-17)}$$

where

p = pressure, lb/ft^2
v = site mean wind speed, mi/h

Eq 2-17 includes a gust factor and a force coefficient (drag factor) for flat surfaces. The force on a *flat* surface of a given projected area is the product of the pressure times the area, given by

$$F = pA \qquad \text{(Eq 2-18)}$$

where

F = force, lb
p = pressure, lb/ft^2
A = projected area, ft^2

In order to calculate the force on cylindrical elements, RS-222-C states that the pressure on cylindrical surfaces is 0.67 of that specified for flat surfaces. This means that the wind forces on cylindrical surfaces may be taken as approximately ⅔ those on flat surfaces, or that the equivalent projected area of a cylinder is ⅔ that of its actual value. Using the methodology of EIA RS-222-C, we calculate the force on a cylindrical element of projected area A square feet by

$$F = c_d pA \qquad \text{(Eq 2-19)}$$

where

F = force, lb
$c_d \approx 0.67$
$p = 0.004v^2$ (from Eq 2-17)
v = site mean wind speed, mi/h

If we use units of in.^{2}, lb/in.^{2}, and mi/h, the equation becomes

$$F = c_d pLD = 2.78 \times 10^{-5} c_d v^2 LD \qquad \text{(Eq 2-20)}$$

where

F = force, lb
$c_d = 0.67$ (drag coefficient for cylinders)
p = wind pressure, lb/in.^{2}
L = length of the element, inches
D = diameter of the element, inches
v = site mean wind velocity, mi/h

Using Eq 2-20, we can calculate the force from wind pressure on any part of the cylindrical elements of a Yagi antenna. We will use the same equation in our analysis of boom and mast wind forces. The drag coefficient is only an approximation, and depends upon structure dimensions and wind velocity. Also, we are assuming a rigid structure, ignoring any change of shape in response to its flexibility or sag. Nonetheless, this simple approach yields quite usable results for Yagi antenna installations.

It is important to note that this model is a considerable simplification of the aerodynamic force on cylindrical elements. The interaction of wind with structural elements does not scale just with structure size, but rather with a quantity called *Re*, the Reynolds number,[19,20] that relates dimension, velocity and viscosity. Also, the drag coefficient of a cylinder oblique (not perpendicular) to the wind is not the same as for the perpendicular case. However, the accuracy of the drag coefficient given here is adequate for the range of shapes and sizes found in amateur Yagi antennas and towers.

The wind force on any object is given by the relationship

$$F = c_d pA \qquad \text{(Eq 2-21)}$$

where

c_d = drag coefficient
p = wind pressure
c_d and p vary with wind speed v

For many shapes, the drag-force coefficient remains approximately constant over the entire range of wind speeds likely to be encountered. However, for objects of circular cross-section it varies considerably with the ratio of length to diameter, with turbulence caused by surface

roughness, and with *Re*. For most Yagi antennas, the ratio of length to diameter is large, so no correction need be made for finite lengths. The Reynolds number is given by

$$Re = \frac{Dv_s}{\upsilon} \qquad \text{(Eq 2-22)}$$

where

D = cylinder diameter, inches
v_s = wind speed, mi/h
υ = viscosity of air

At low Reynolds numbers the drag coefficient is constant. At some critical Reynolds number, there is a change in the wake developed behind the cylinder, and the force coefficient shows a sudden drop above this critical value.

The transition from subcritical to supercritical flow depends strongly upon whether the flow is smooth or turbulent. For laminar flow over a smooth cylinder, the transition takes place for a value of $Dv_s \approx$ 45 ft-mi/h = 540 in.-mi/h, assuming that the viscosity of air υ is reliably constant.

For turbulent flow over a rough surface, the transition is at a lower wind speed corresponding to $Dv_s \approx 5$ ft-mi/h = 60 in.-mi/h. For a smooth cylinder of D = 2 in., the transition from subcritical flow occurs at v_s = 540/2 = 270 mi/h! However, for a turbulent flow, which can be induced by a very nominal degree of surface roughness, the transition takes place at v_s = 60/2 = 30 mi/h. The application of aerodynamic devices such as surface roughness and vortex generators could prove fruitful in the reduction of antenna, mast and tower wind forces. It certainly works for dimpled golf balls! If we could count on turbulent flow for our crackle-finish antenna, we could use a lower value of drag coefficient. Conservative design, however, dictates a less aggressive choice.

Because of the variability of drag coefficient with dimension and wind speed, various building codes use differing values for wind pressure and force coefficients. Some codes differentiate between subcritical and supercritical flow in terms of cylinder diameter (for example, pressure coefficient C_p of BSI CP3 or C_q of the UBC). However, the approximation of a constant drag coefficient used in the EIA RS-222-C specification provides a satisfactory and conservative estimate of wind forces on Yagi elements and booms of normal size.

In the case of wind flow around a cylindrical element, the air flowing around each side of the cylinder rejoins the smooth flow a few cylinder diameters downstream. Thus, the forces on the elements and boom of a Yagi can be considered independently. The picture is not so simple for the inner ends of the elements when the boom is oblique to the wind. However, the calculation of the wind forces and moments of each element independently while using a constant drag coefficient seems quite acceptable, given the degree of approximation in our knowledge of the specific wind statistics themselves.

As defined in RS-222-C, the design pressure on a flat plate is given by $p = 0.004v^2$ lb/ft^2 (Eq 2-17). The velocity v is the zone mean wind speed averaged over one mile, and the expression incorporates a constant wind-speed gust factor of 1.3. A constant drag coefficient of $c_d \approx 0.67$ is used to relate cylindrical structural elements to a flat plate of equivalent area. The wind force on a cylindrical element of area A is thus

$$F = pA = 0.004v^2c_dA = 0.0027v^2A \qquad \text{(Eq 2-23)}$$

By comparison, the force is defined in EIA-222-D as the product of five parameters.

$$F = pA = 0.00256v^2K_zG_hC_fA \qquad \text{(Eq 2-24)}$$

where

$0.00256v^2 = q$ = dynamic pressure
v = fastest-mile wind speed
$K_z = (z/33)^{2/7}$ = an exposure coefficient, where z = height above ground to the midpoint of the section, feet
$G_h = 0.65 + 0.60/(h/33)^{1/7}$ = a gust-force factor, where h = total structure height above ground, feet
$C_f = 1.2$ = a force coefficient for thin cylinders (by comparison, $C_f = 2$ for flat structures)
A = area of cylinder = LD

Constraints:

$C_f = 1.2$ for $L/D > 7$
$1 \le K_z \le 2.58$
$1 \le G_h \le 1.25$

In the case of an antenna mounted atop a tower, z and h have the same value.

Table 2-9
Comparison of Pressure Calculations

Parameter	*EIA-222-D*	*RS-222-C*	*UBC*	*BSI CP3*
v	70 mi/h	70	70	80.6
q, p	12.54	19.6	13	16.6
G_h	1.25	1	1.3	1
C_f, c_d	1.2	0.67	1	1.2
p	18.8 lb/ft^2	13.1	16.3	19.9

For an example of h = z = 33 ft, EIA-222-D calculates the force on a flat plate to be

$$F = 0.00512v^2A \qquad \text{(Eq 2-25)}$$

and the force on a thin cylinder to be

$$F = 0.00384v^2A \qquad \text{(Eq 2-26)}$$

It is of interest to compare the results of EIA-222-D with those of RS-222-C and other standards. The UBC also uses the fastest-mile wind speed, while the BSI practice is based on a 3-second gust, which is approximately 15% greater than the equivalent fastest-mile speed. A 3-second gust speed of 80.6 mi/h is equivalent to the fastest-mile speed of 70 mi/h. Table 2-9 compares the pressure in lb/ft^2 on a thin cylindrical member as calculated for each of these standards at a height of 33 ft over flat, unobstructed ground.

An alternative comparison among the various standards results from running the calculations backwards, starting with the same pressure on a cylindrical element and calculating the wind speed that would cause that pressure using the method of each standard. The results of this comparison are shown in Table 2-10. Although RS-222-C would seem to be at a substantial variance with the other standards, the wind-speed maps on which EIA-222-D and the UBC are based show considerably lower wind speed (10-20%) in the same geographic areas. Thus, a map of *pressure*, rather than wind speed, would show reasonable agreement among the several standards.

Table 2-10
Comparison of Wind-Speed Calculations

Parameter	*EIA-222-D*	*RS-222-C*	*UBC*	*BSI CP3*
p	20 lb/ft^2	20	20	20
C_f, c_d	1.2	0.67	1	1.2
G_h	1.25	1	1.3	1
q, p	13.33	29.84	15.38	16.66
v	72 mi/h	86	78	70

NOTE: The wind speed for the BSI practice is given here in terms of fastest-mile speed, and corresponds to a 3-second gust of 81 mi/h.

The agreement among the various methods is comforting but not perfect, and suggests the use of the simplest model that reflects available basic wind-speed data, along with shape and exposure-correction factors. The spreadsheets presented in later chapters are based on the methodology of EIA RS-222-C, in which the flat-plate pressure and a drag coefficient are used to determine the forces on the antenna. The choice of a different standard results in a change in the force and drag constants (and also a different basic wind-speed map) that can be substituted into the calculations of antenna strength, if desired.

In any event, the determination of the relative strengths and survival speeds among competing antenna designs is consistent so long as the same force constants are used. A stronger design is stronger, no matter which specific constants are chosen. One can see that specific definitions of wind speed vary somewhat from standard to standard, and comparisons should be made with caution. Some manufacturers of towers and antennas have restated existing designs merely by noting that, under EIA-222-D, mean wind speeds of 70 mi/h and 90 mi/h provide essentially the same force as RS-222-C zones A (86.6 mi/h) and C (111.8 mi/h).

Sec 2.6 Ice Loading

In many parts of the world, buildup of radial ice on antenna elements is a serious consideration. A review of several radio-tower strength specifications[21] suggests that a radial ice thickness of ½ in. is

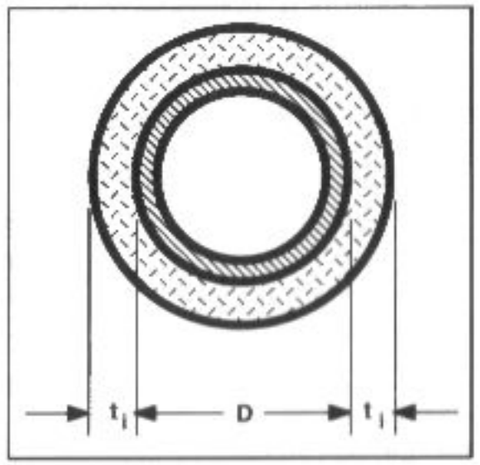


Fig 2-5—Tubular element with ice. D = tubing OD; t_i = ice thickness.

useful as a typical value for calculation. Under icing conditions, the effective diameter is the diameter of the tubing plus twice the ice thickness, as shown in Fig 2-5. This means that the element must carry not only the weight of the radial ice, but it also suffers the penalty of greatly increased wind area. Fortunately, it appears that for many locations, conditions favorable to ice buildup do not coincide with maximum wind, so design for high wind survival generally satisfies the requirements for icing survival.[22]

Later analysis of element structures includes specific treatment of wind survival with radial ice. The survival wind speed under heavy icing conditions is greatly reduced from that obtained in the absence of icing. The effect of wind force at high wind speeds is ultimately greater than the effect of the weight of the ice itself.

In order to design for ice loading, it is necessary to determine the expected ice thickness and wind-speed conditions for a given site. Once this is done, the analysis procedures for the condition of no ice loading can be applied. An alternative is to design for a very high wind speed, and then accept the resulting design as the best that can be done within reasonable weight and cost limitations.

Two interesting points in EIA-222-D are the specification of 0.5 in. of ice in the absence of other data,[23] and the statement that, because of low probability that an extreme ice load will occur simultaneously with an extreme wind load, the wind load can be reduced 25% (equivalent to 87% of basic wind speed) when considered to occur simultaneously with ice load.[24] Local experience over an adequate span of years can, of course, provide a more accurate picture for a particular site.

References

[1] P. Sachs, "Determination of Wind Forces Acting on Antenna Structures," Chapter 5 in C. Richards, *Mechanical Engineering in Radar and Communications* (London: D. Van Nostrand Reinhold, 1969), p 123.

[2] A. G. Davenport, "Rationale for Determining Design Wind Velocities," *Proc American Society of Civil Engineers, Journal of Structural Div*, May 1960, p 39.

[3] R. H. Sherlock, "Wind Forces on Structures: Nature of the Wind," *Proc American Society of Civil Engineers, Journal of the Structural Div*, Jul 1958, p 1708-1.

[4] See Table 5-8, p 152 of reference 1.

[5] Eric Scace (K3NA), private communication.

[6] "Wind Design," Section 2311, *Uniform Building Code* (Whittier, CA: International Conference of Building Officials, 1988), p 137.

[7] "Wind Loads," CP3: Chapter V, Part 2, *Code of Basic Data for the Design of Buildings* (London: British Standards Institution, 1972; amended 1988).

[8] Standard RS-222-C, *Structural Standards for Steel Antenna Towers and Antenna Supporting Structures* (Washington, DC: Electronic Industries Association, 1976.

[9] H. C. S. Thom, "Distribution of Extreme Winds in the United States," *Proc of the American Society of Civil Engineers, Journal of the Structural Div, Apr 1960, p 11.*

[10] Standard EIA-222-D, *Structural Standards for Steel Antenna Towers and Antenna Supporting Structures* (Washington, DC: Electronic Industries Association, 1986.

[11] H. C. S. Thom, "New Distribution of Extreme Winds in the United States," *Proc of American Society of Civil Engineers, Journal of Structural Div*, Jul 1968, p 1787.

[12] See D. Reynolds, "Wind Loading," Radio Communication, Apr 1988, p 252.

[13] See Appendix D, "Topography Factors," p 41 of reference 7.

[14] D. Weber, "Vibration Induced Yagi Failures," *Ham Radio*, Aug 1989, pp 9-12, 17, 18.

[15] A. Bolton, "Avoiding Wind Vibration Damage to Aerials," *Radio Communication*, Oct 1990, p 46.

[16] See p 152 of reference 1.

[17] See reference 1.

[18] G. B. Woodruff and J. J. Kozak, "Wind Forces on Structures: Fundamental Considerations," *Proc American Society of Civil Engineers, Journal of Structural Div*, Jul 1958, p 1709-1.

[19] A. Daugherty and A. Ingersol, *Fluid Dynamics* (New York: McGraw-Hill Book Co, 1954), pp 100 and 305.

[20] See Appendix G, "Wind Forces on Circular Sections," p 50 of reference 7.

[21] Unarco-Rohn Tower Catalog, 1989.

[22] See Appendix F, "Ice Formation on Structures," p 47 of reference 7.

[23] See Sec 2.1.2C, Appendix A, p 27 of reference 10.

[24] See Sec 2.3.15A, Appendix A, p 28 of reference 10.

Chapter 3

Strength of Structures and Materials

Sec 3.1 Force and Moments in Cantilever Structures

The science of the strength of structures and materials has been applied to human construction efforts since the first prehistoric occasion when a structure fell down. Galileo was the first to attempt to explain structural behavior on a rational basis. He studied tension and compression of beams used in the construction of hulls of ships. More recent examples of structural failure, particularly in bridges (static in 1897, dynamic in 1940) have further fostered the development of methodology to design structures and calculate their strengths.

Three separate considerations define the strength of a structure such as a Yagi antenna. They are (1) the way in which the structural elements are attached, (2) the cross-section shape of the structural elements, and (3) the strength of the materials.

Taking these in order, let's look at how the strength of, say, an aluminum element or a steel mast is defined.[1] The typical Yagi construction is intended to place electrical elements in position in the air, using support primarily from a single point of attachment of a boom to a rotating mast. This is done by holding the elements by their centers (each element half supported by its inner end), and by holding the boom by its center (each boom half supported by its inner end). This attachment is described as a cantilever attachment, and the principal force leading to wind failure is bending at the inner end of a section. The same issues apply to a mast retained at its lower end. Guyed elements and booms represent a more complex case of combined beams and columns that are treated in Chapter 6.

A physical force causing rotation or bending is called a torque or moment, and is measured in dimensions involving force and the distance

Diskette Availability

Chapter 4 of this book introduces the concept of using computer spreadsheets to design Yagi elements that will withstand high winds. Subsequent chapters extend this concept to the design of booms and masts, and to torque-balancing antenna systems. Equations required for the spreadsheet calculations are included in the text and tables.

The ARRL offers computer diskettes for the IBM PC/XT/AT and Macintosh computers as optional supplements to this book:

3½-in. PC, order no. 3827, $10
5¼-in. PC, order no. 3835, $10
3½-in. Macintosh, order no. 3843, $10

Each diskette contains spreadsheets to calculate (1) element areas from section dimensions, (2) element wind and ice survival, (3) boom survival and torque balance, (4) mast survival, (5) multisection mast survival, and (6) sloping foreground reflection gain. The spreadsheets are in Lotus 1-2-3 (WK1) format for IBM and compatible computers, and in Excel format for the Macintosh. The diskettes are copyrighted but not copy protected.

Spreadsheet programs are not included on these diskettes. To use these disk files, you must have your own spreadsheet program that is capable of using or translating from the WK1 (IBM PC) or Excel (Macintosh) format.

The spreadsheet files have been supplied by David Leeson, W6QHS, the author of this book, and the diskettes are made available as a convenience to the purchaser. The spreadsheets are rudimentary, but do assemble all the various equations and spreadsheet function commands required to perform the appropriate calculations.

over which it operates, such as foot-pounds, inch-pounds or meter-kilograms. Moment, force and distance are related by the equation

$$M = FL_m \qquad \text{(Eq 3-1)}$$

where

M = moment, inch-pounds (in.-lb)
F = force, lb
L_m = distance, in., over which it is applied

As a simple example, a 50-lb force applied 120 in. from the attachment of an element results in a bending moment or torque at the cantilever attachment of $M = 50 \times 120 = 6000$ in.-lb.

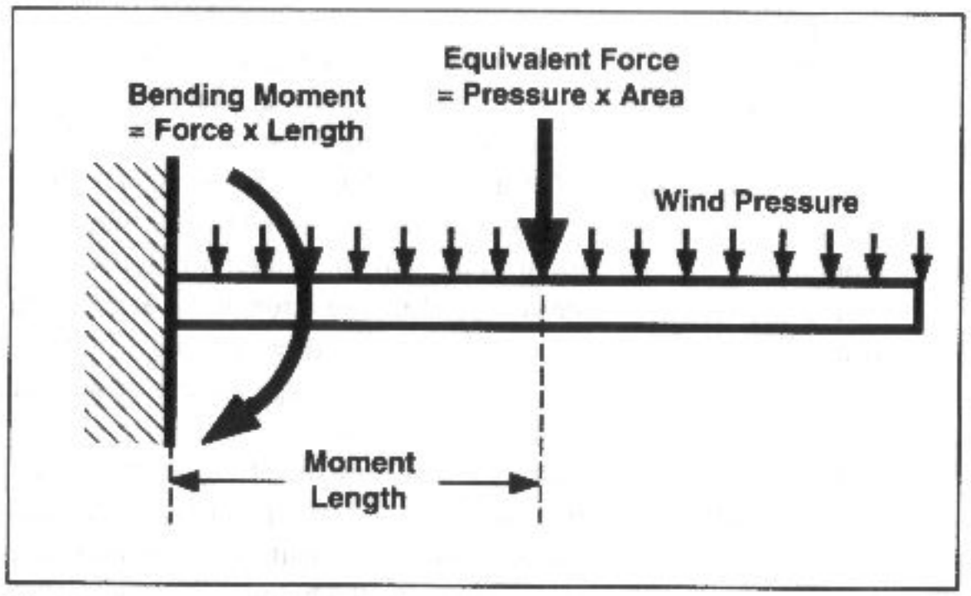


Fig 3-1—Moment on a cantilevered cylinder.

In the case of a force resulting from a uniform pressure such as wind or gravity, an equivalent force equal to the effective pressure multiplied by the affected area can be thought of as acting at a center of pressure or gravity, as shown in Fig 3-1. Such a force is expressed as

$$F = p_e A = p_e LD \qquad \text{(Eq 3-2)}$$

where

F = force, lb
p_e = effective pressure on the cylinder, lb (giving effect to drag coefficient)
A = area of the cylinder normal to the pressure, in.^{2}
L = length, in.
D = diameter of the cylinder, in.

In this case, the moment arm $L_m = L/2$, so the moment is given by

$$M = FL_m = p_e LDL_m = \frac{p_e L^2 D}{2} \qquad \text{(Eq 3-3)}$$

where L_m = moment arm, in.

As an example, for a cylinder of length L = 80 inches and diameter D = 0.75 inch, exposed to a mean wind speed of 100 mi/h, the bending moment is calculated as follows. Using Eq 2-20, the wind force is

$$F = 2.78 \times 10^{-5} c_d v^2 LD = 2.78 \times 10^{-5} \times 0.67 \times 100^2 \times 80 \times 0.75$$
$$= 11.18 \text{ lb}$$

This force is acting through a moment arm $L_m = L/2$. From Eq 3-3, the moment of force is

$$M = FL_m = 11.18 \times 40 = 447 \text{ in.-lb}$$

Sec 3.2 Section Modulus

The ability of a structural shape to withstand the application of a bending moment is determined by a property of the shape called its section modulus. The section modulus is determined only by the geometry of the cross-sectional dimensions of the structural shape, and is unique to any particular cross-section.

When a structural beam is bent, one surface is stressed in compression and the opposite is stressed in extension. The degree of stress varies linearly from one side of the beam to the other, and the neutral axis passes through the centroid of the cross-sectional area of the beam and is the location within the beam of zero stress under bending. The section modulus of a given cross-section is defined as the ratio of the rectangular moment of inertia I about the neutral axis of the cross-

section divided by the distance from the neutral axis to the most remote point on the cross-section in the direction of bending force.

The rectangular moment of inertia is calculated about the neutral axis by the integral over the area

$$I = \int y^2 dA \qquad \text{(Eq 3-4)}$$

where y = distance from the neutral axis

For a cylindrical shape, the rectangular moment of inertia is

$$I = \frac{\pi D^4}{64} \qquad \text{(Eq 3-5)}$$

where D = diameter of the cylinder, in., and the distance c from the neutral axis to the farthest point is the radius

$$c = \frac{D}{2} \qquad \text{(Eq 3-6)}$$

where c is in inches

Thus, the section modulus S of a cylindrical rod is

$$S = \frac{I}{c} = \frac{\pi D^3}{32} \qquad \text{(Eq 3-7)}$$

In the case of a compound shape such as a hollow cylinder, the moment of inertia of the cylinder is the moment of inertia of the outer diameter D minus the moment of inertia of the inner diameter d. In this case

$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} = \frac{\pi (D^4 - d^4)}{64} \qquad \text{(Eq 3-8)}$$

Because the distance c remains c = D/2, in this key example for tubular elements, the section modulus of a hollow cylindrical cross-section is given by

$$S = \frac{\pi (D^4 - d^4)}{32D} \qquad \text{(Eq 3-9)}$$

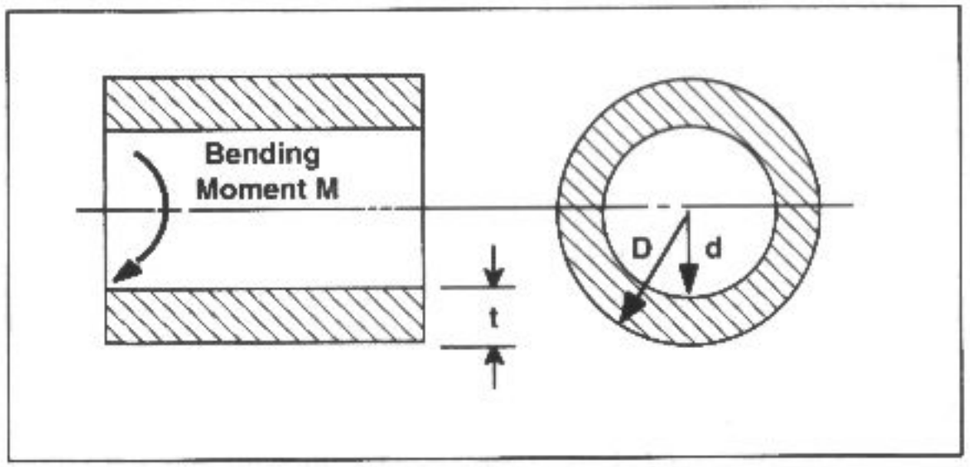


Fig 3-2—Dimensions for section modulus and moment.

where

S = section modulus, in.^{3}

D and d are the outer and inner diameters respectively, in inches, as shown in Fig 3-2

The section modulus can be restated in terms of diameter and wall thickness as

$$S = \frac{\pi [D^4 - (D - 2t)^4]}{32D} \qquad \text{(Eq 3-10)}$$

where

D = outside diameter, in.

t = wall thickness, in.

The section modulus uniquely defines the ability of a structural shape such as a tubular mast or element to survive bending. In the case of a shape which is not symmetrical, there is a section modulus defined for each axis.

Another quantity of interest in the analysis of element mounting brackets is the section modulus for each axis of a rectangular shape. This is given by

$$S_t = \frac{Wt^2}{6} \qquad \text{(Eq 3-11)}$$

and

$$S_W = \frac{W^2 t}{6} \qquad \text{(Eq 3-12)}$$

where

S_t = modulus for bending in the height direction of the rectangle
S_W = modulus for bending in the width direction
W = width of the rectangular shape, in.
t = thickness of the rectangular shape, in.

Sec 3.3 Bending Stress

When a structural shape is subject to bending, the material on one side of the shape is compressed and the material on the other side of the element is extended. The strength is defined by the yield strength, typically expressed as lb/in.^{2} (or psi), of the material in compression and tension. For an element in bending, the question of whether it will fail is determined by whether the maximum bending stress exceeds the yield strength of the material.

The maximum bending stress is related to the bending moment and the section modulus by

$$\sigma = \frac{M}{S} \qquad \text{(Eq 3-13)}$$

where

σ = bending stress, lb/in.^{2}
M = bending moment, in.-lb
S = section modulus of the structural element, in.^{3}

The element will not fail if σ is less than σ_{max}, where σ_{max} is the yield strength of the material in pounds per square inch.

Thus, we can say that the maximum moment M_{max} supportable by a structural element of section modulus S is

$$M_{max} = S\sigma_{max} \qquad \text{(Eq 3-14)}$$

Eq 3-14 is only approximate for materials that bend before failing, such as mild steel. This equation also does not consider stability issues such as buckling, and does not treat fatigue failure caused by repeated bending. On the other hand, it is conservative in that it does not give effect to relief of pressure from deflection. Despite these limitations, Eq 3-14 is a basic equation for structures, giving a good idea

of how strong a given antenna element, boom, or mast is. As long as σ, the bending stress, does not exceed the yield strength of the material σ_{max}, the structure in question will not fail. An additional safety factor is provided because the actual yield strength for a given material can substantially exceed the minimum specified.

If we calculate the bending stress $\sigma(v)$ corresponding to a wind of velocity v, we determine

$$\frac{v_{max}}{v} = \sqrt{\sigma_{max} / \sigma(v)} \qquad \text{(Eq 3-15)}$$

where v_{max} = maximum survivable wind velocity

Sec 3.4 Yield Strength of Materials

The yield strengths σ_{max} of aluminum and steel alloys commonly used in amateur antennas and masts are listed in Table 3-1.[2-4] Note that there are considerable differences among the various alloys and tempering; 6063-T5 is found in typical hardware-store aluminum tubing. 6063-T832 is an architectural grade used by many antenna manufacturers. It and 6061-T6 are worth the effort to obtain, as we shall see. The wide range of yield strengths makes it imperative that you know exactly what material you are using; unmarked or unknown material is useless in situations requiring strength for safety or survival.

The heat treatment of steel adds yet another dimension, and the specification of steel for masts and towers is a matter for professionals.[5] Without a certificate identifying the material and the subsequent heat treatment, a mast is an unknown quantity that could be a serious safety hazard.

The yield strength of a material determines the strength of an ideal structural shape. Irregularities in the structure, such as holes or sharp corners, have the effect of locally concentrating stress, which reduces the effective yield strength of the material. These "stress risers" can also generate minute cracks at points of high stress concentration. With repeated cycling of stress these cracks can spread to failure, a phenomenon called fatigue failure.

After 100,000 load cycles, the effect of a small circular hole can be to reduce the effective material strength to as little as one-third of its normal value. For this reason, holes in structures should be located where the stress at peak load is safely below one-third of the material

Table 3-1
Yield Strength of Aluminum and Steel Alloys

Material	*Specification*	*Yield Strength*
Drawn aluminum tube	6063-T5	15,000 lb/in.^{2}
	6063-T832	35,000
	6061-T6	35,000
	6063-T835	40,000
	2024-T3	42,000
Aluminum pipe	6063-T6	25,000
	6061-T6	35,000
Extruded alum. tube	7075-T6	70,000
Aluminum sheet and plate	3003-H14	17,000
	5052-H32	22,000
	6061-T6	35,000
Structural steel	A36	33,000
Carbon steel, cold drawn	1016	50,000
	1022	58,000
	1027	70,000
	1041	87,000
	1144	90,000
Alloy steel	2330 cold drawn	119,000
	4130 cold worked	75,000
	4340 1550°F quench 1000°F temper	162,000
Stainless steel	AISI 405 cold worked	70,000
	AISI 440C heat-treated	275,000

yield strength. Specific examples of preferred locations for fastener holes are discussed in Sec 4.10.

Sec 3.5 Fasteners

While on the subject of the strength of materials, it is appropriate here to mention the subjects of graded bolts, rivets, hose clamps, and

corrosion from dissimilar metals. Ordinary hardware-store plated hex-head bolts with no markings on their head are not heat-treated, and are not recommended for antenna applications. In the US, stainless-steel bolts have two lines in a 120° pattern embossed on their hex head, and are recommended for all but the most critical applications. Heat-treated and plated grade 5 bolts are available from better hardware suppliers, and are identified by three lines in a star 120° pattern embossed on the head. I never use slotted-head screws on antennas, both because they are of uncertain material and because they are difficult to install without damaging the screw slots or injuring your hands, especially with a power screwdriver.

Based on my satisfactory experience in 20 years of car racing, I use grade 5 plated hardware for boom-to-mast attachment and other high-stress applications. I also drill and bolt through the booms for positive element location and rotation prevention. As matters of good practice, I assemble critical items with a torque wrench, lubricate all stainless fasteners before assembly to prevent galling, and never place a hollow tube in clamping compression without solid internal reinforcement to prevent crushing.

Aircraft bolts, grade 6, typically are designed to have only as much threaded area as a particular application requires. They are strong and durable, but not easy to find except at aircraft parts suppliers and commercial fastener companies. Last, specially heat treated grade 8 bolts, identified by a head pattern that looks like a centipede, are extremely strong but brittle, and are not recommended for antenna use except for tower base attachment as specifically recommended or supplied by the tower manufacturer.

Sec 3.6 Rivets and Hose Clamps

Pop rivets are available in several different compositions, including aluminum with an aluminum pin, aluminum with a steel pin, steel with a steel pin, and all-stainless steel. I recommend the all-aluminum or all-stainless pop rivets for such tasks as permanently fastening telescoping sections. I use a hand-held hydraulic riveter, and I recommend its use to avoid sore hands.

As a practical matter, I use all-stainless steel hose clamps for compressing the single end slot used to permit retention of adjustable telescoping elements. Note that there are two types available. One, called "stainless," incorporates a plated screw that rusts! The other,

called "all-stainless," is worth looking for. I get all the same hex size, so I can use the same nut driver or socket wrench everywhere. This tool, typically 5/16 in., can be conveniently mounted in one of the battery-powered electric drills now available. Also, it is worth finding the correct hose-clamp size to prevent extra strap from sticking out to catch on guy wires and other antennas.

Clamps should be positioned at the outer end of the slot for maximum closing force, and rotated so the draw screw mechanism is over the slot so as not to obstruct its closing. Excessive torque should be avoided to prevent stripping the clamp threads. There is available a simple T-handle torque wrench for hose clamps used in plumbing work; I recommend its use. A defective clamp should not be reused, since a clamp failure will result in an antenna failure. For that reason, I prefer redundant rivet fastening in contest antennas, and I avoid the "teeny-weenies" that are made for automotive fuel lines.

The smallest clamp with suitable strength is no. 6, which is useful for tubing sizes from 7/16 in. to 3/4 in. The no. 10 size is useful for 3/4 in. through 1 in. tubing, and the no. 12 or no. 16 sizes can be used up to 1 1/4 in. Manufacturers have found it acceptable to switch from proprietary clamps to all-stainless hose clamps with 5/16-in. hex head screws.[6] Some manufacturers prefer to use through-bolts at element-section joints, but fail to provide solid internal reinforcement to permit full tightening.

Sec 3.7 Aluminum Tubing

The range of available aluminum tubing suitable for Yagi construction is best seen in the catalogs of the major aluminum-tubing supply houses. Aluminum tubing is available in diameter sizes that increase in steps of 1/8 (0.125) inch. In most diameters a tube is stocked with a wall thickness of 0.058 inch, which provides a telescoping fit with 0.0045-inch clearance per side. I am told there is not such a readily available range of telescoping metric tubing sizes. Tubing in inch sizes generally comes in 12-foot lengths, although tubing and antenna kits to be shipped by United Parcel Service in the US are restricted to a maximum of 7 feet (84 inches).

The ready availability of drawn aluminum tubing makes building and modifying Yagi antennas a rewarding aspect of Amateur Radio. In most large cities in the US, the local telephone book has listings for "Aluminum Tubing" suppliers, many of whom can provide a catalog of

Table 3-2
Aluminum Pipe for Antenna Material

IPS No.	Schedule	OD, in.	ID, in.	Wall	Modulus, in.^{3}
3/4	40	1.050	0.824	0.113	0.071
	80	1.050	0.742	0.154	0.085
1	40	1.315	1.049	0.133	0.133
	80	1.315	0.957	0.179	0.161
1 1/4	5*	1.660	1.530	0.065	0.125
	10*	1.660	1.442	0.109	0.193
	40	1.660	1.380	0.140	0.235
	80	1.660	1.278	0.191	0.291
1 1/2	5*	1.900	1.770	0.065	0.166
	10*	1.900	1.682	0.109	0.260
	40	1.900	1.610	0.145	0.326
	80	1.900	1.500	0.200	0.412
2	5*	2.375	2.245	0.065	0.265
	10*	2.375	2.157	0.109	0.420
	40	2.375	2.067	0.154	0.561
	80	2.375	1.939	0.218	0.731
2 1/2	40	2.875	2.469	0.203	1.064
	80	2.875	2.323	0.276	1.339

*6063-T6 only (25000 lb/in.^{2}); others 6061-T6 (35000 lb/in.^{2}).

available aluminum tubing and pipe. The best catalogs list alloy and other physical parameters. Excellent tables of available sizes are listed in most amateur antenna publications, and so are not repeated here.

Sec 3.8 Aluminum Pipe as an Antenna Material

Aluminum pipe is approximately half the cost of aluminum tubing of the same general diameter and wall thickness. Note that pipe is sized by a nominal inside diameter ("iron pipe size," or IPS). As pipe is available in 6061-T6 alloy, it should be considered for element center sections and for boom sections. Pipe is not sized for telescoping, but telescoping is possible with suitable shimming or limited machining of the inner diameter. Table 3-2 shows a number of these possibilities.

As an example, a boom can be made without machining requirements with 2½ iron pipe size schedule 40 center section (2.875 in. OD, 2.469 ID), 2 IPS schedule 80 (2.375 in. OD, 1.939 in. ID), and 1½ IPS schedule 40 outer sections (1.900 in. OD, 1.610 in. ID). If required, the outer sections can be machined to take 1¼ IPS as an extension.

U bolts are made in sizes specifically for pipe outer diameters, so element and boom clamps can be readily fabricated in any of the styles shown in Chapter 5. If pipe is also used for the element center sections, the architectural clamps shown in that chapter can be applied directly. For element center sections, consider 1 IPS (1.315 in. OD, 1.049 in. ID) with shims to receive telescoped 1-in. tubing.

For any antenna applications of aluminum pipe, use only 6061-T6 alloy and avoid the much weaker 3003 and 5083 alloys that are suitable only for nonstructural piping. Again, it is imperative to know what you are using.

Sec 3.9 Electrolytic Potential and Corrosion

To minimize electrolytic corrosion, aluminum elements and booms should not be in moist contact with hardware that is at the opposite end of the corrosion-potential table. Table 3-3 summarizes metal electrolytic potentials.[7] This means that galvanized (zinc coated) hardware is preferred, as well as stainless steel, followed by cadmium and nickel-plated hardware. Brass (copper) hardware, thought to prevent corrosion because it does not rust, is in fact likely to result in accelerated corrosion of aluminum, being at the opposite end of the electrolytic potential table.

Table 3-3
Electrolytic Potential of Metals

Metal	*Potential, volts*
Aluminum	+1.67
Zinc	+0.76
Chromium	+0.71
Iron	+0.44
Cadmium	+0.40
Nickel	+0.25
Tin	+0.14
Lead	−0.13
Copper	−0.34
Silver	−0.80
Gold	−1.68

In moist contact with any of these elements in the presence of atmospheric oxygen, aluminum is the negative electrode of an electrical battery, and gradually loses material and strength. Similarly, the zinc of galvanized steel sacrifices itself before the steel corrodes, since zinc is more negative than iron.

Sec 3.10 Summary of Analysis Procedure

To analyze any Yagi structural element, we can proceed as follows.

1) Determine that the item is in cantilever attachment.
2) Determine the section modulus of the element.
3) Calculate the bending moment from wind force.
4) Determine the yield strength of the material.
5) Calculate the maximum survival wind speed and determine that it is greater than the highest to be expected over the antenna service life. If not, modify the design.

With the proviso that we need to consider each section of a tapered element separately, we can see that the basics are in place to understand and analyze the strength of a Yagi structure. The use of known, quality materials is mandatory if reliable results are to be obtained. Booms and elements may benefit from guying, the analysis of which requires a study of combined beams and columns in other than cantilever attachment. This is the subject of Chapter 6.

References

[1] E. P. Popov, *Mechanics of Materials* (Englewood Cliffs, NJ: Prentice-Hall, 1952).

[2] *Alcoa Structural Handbook* (Aluminum Corporation of America, 1956), p 52.

[3] H. Ryffel, *Machinery's Handbook* (Industrial Press Inc, 1988), p 512.

[4] C. Smith, *Engineer to Win* (Motorbooks International, 1984), p 66.

[5] See "Heat Treatment of Standard Steels," p 515 of reference 3.

[6] Telex Hy-Gain Product Update, "Tubing Clamp Addendum," part number 802739.

[7] P. Hawker, *Amateur Radio Techniques*, 7th ed. (London: Radio Society of Great Britain, 1980), p 335.

Abbreviations Used in This Book

A	wind zone designator, also area and signal amplitude
a	radius
ab, a × b, (a)(b)	product of a and b
α	radiation angle
B	wind zone, also buckling constant, electrical susceptance
β	wave number, also inductive transmission line impedance match
C	wind pressure factor (with subscript), also buckling constant
c	distance of farthest point from neutral axis, also velocity of light in vacuum, and, with subscript d, aerodynamic drag coefficient
c'	velocity of light in air
D	outer diameter, also buckling constant
d	inner diameter (also used as distance)
δ	fractional frequency offset
dB	decibels
E	elastic modulus
ε	dielectric constant
F	force
f	frequency
f(x)	function of x
ft	feet
G	electrical conductance, also gain and (with subscript) gust factor
h	height
I	moment of inertia
i	element number
IPS	iron pipe size, in pipe dimensions
in.	inches
K	constant or factor, also (with subscript) characteristic impedance
k	summation variable
L	length
λ	wavelength
ℓ	length (in wavelengths or inches)
lb	pounds
M	moment, also as M(β ℓ), dipole impedance function
MHz	megahertz
mi/h	miles per hour velocity
N	as N(β ℓ), dipole impedance function
n	element or boom section number
ω	angular frequency (*e.g.*, of rotation)
ν	viscosity
P	perimeter
p	pressure
ϕ	wind angle to antenna boom
ψ	slope
Q	moment of area about neutral axis, also electrical quality factor
q	dynamic pressure, also shear flow
R	electrical resistance
r	reflection coefficient, radius of gyration
Re	Reynolds number
ρ	density
S	section modulus, also (with subscript) wind speed factor
s	spacing
σ	bending stress
t	thickness (also time)
θ	wind angle to horizontal
u	dummy variable
V	distance rate of change of moment
v	velocity
W	width
w	weight
X	electrical reactance
x	distance along mechanical shape
Y	electrical admittance
y	distance across mechanical shape
Z	electrical impedance
z	vertical distance (also used as distance along mechanical shape)

1.25 × 0.058 × 48 in. diameter × wall thickness × length of tube, inches
20 ft × 2 × 0.25 in. length of tube (feet) × diameter × wall thickness (inches)

In drawings: All dimensions are in inches, feet and pounds. Italic type signifies dimensions of internal reinforcement tubing.

Chapter 4

Yagi Element Design

Sec 4.1 Wind Force on Yagi Elements

Now that we have all the basic concepts in hand, we can turn to a calculation of the forces and moments on practical Yagi elements. The calculation requires separate consideration of each section of a tapered element, and each section is affected by all the sections outboard of it. Accordingly, it is convenient to start from the outer end. We begin with a consideration of cylindrical elements and then return to the question of stepwise tapered elements.

Although the wind force is distributed, it is correct to assume that the total of the force on a cylindrical object of constant diameter has the same magnitude and effect as if it were concentrated at the center point of that section. The concept of center of force is related to the familiar concept of center of gravity.

The forces on a cylindrical aluminum element include horizontal and vertical wind force and the force from the weight of the element material. Both of these forces act at the center of the length of the cantilevered cylindrical element half. Under high wind conditions, the force in the direction of the wind is substantially larger than transverse gust or gravity forces. The analysis that follows is based on the approximation that the wind force determines the survivability of a cylindrical element, and that we can use a constant drag coefficient, ignoring the effect of Reynolds number variation. (The Reynolds number is discussed in Sec 2.5.)

From Eq 2-20, the wind force F for a cylindrical element is expressed as the product $F = c_d pA$, where $c_d \approx 0.67$, the drag coefficient for cylinders; p is the wind pressure, lb/in.^{2}, and A is the projected area of the element in square inches. Because $A = LD$ and $p = 2.78 \times 10^{-5} \times v^2$, we have

$$F = 2.78 \times 10^{-5} c_d v^2 LD \qquad \text{(Eq 4-1)}$$

where

L = element length, in.
D = element diam, in.
v = mean wind speed, mi/h

The dimensions are shown in Fig 4-1.

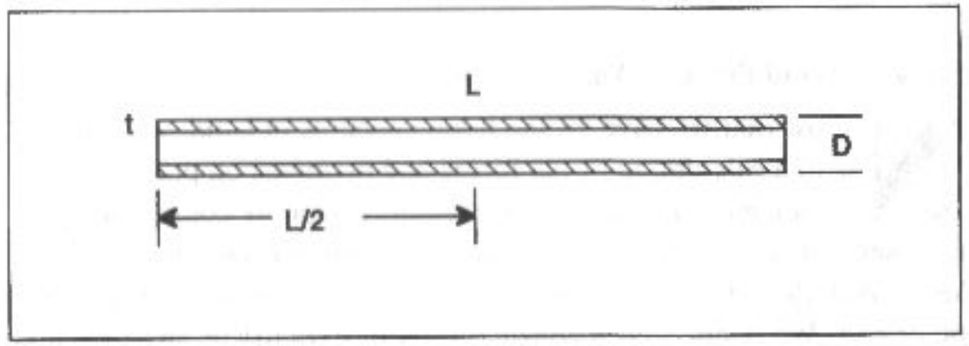


Fig 4-1—Dimensions for calculating the stress on a cylindrical element.

The moment of force, M, measured in in.-lb at the inner end of the element half is

$$M = \frac{FL}{2} = \frac{c_d p L^2 D}{2} = C_d p M_a \qquad \text{(Eq 4-2)}$$

where $M_a = \frac{AL}{2} = \frac{L^2 D}{2}$

M_a is defined as the moment of area, or $M_a = \int x dA$, measured in units of $in.^3$, of the cylinder. Moment of force is the moment of area multiplied by the pressure.

The stress, σ, is given by Eq 3-13 as $\sigma = M/S$ $lb/in.^2$. M is the bending moment, and S is the section modulus of the cylindrical element, given by Eq 3-10 as $S = \pi[D^4 - (D - 2t)^4]/(32D)$ $in.^3$, for element diameter D and wall thickness t.

Solving the stress equation above for the moment of force M, and setting the stress σ equal to the material yield strength σ_{max} (which we can find in Table 3-1), we have

$$M_{max} = S\sigma_{max} = c_d p_{max} M_a \qquad \text{(Eq 4-3)}$$

where M_a is the moment of area defined above. Eq 2-17 gives $p = 0.004v^2$ lb/ft^2. From this,

$$p_{max} = p_o v_{max}^2 \qquad \text{(Eq 4-4)}$$

where

p_{max} = maximum pressure, $lb/in.^2$
$p_o = 0.004/144 = 2.78 \times 10^{-5}$
v_{max} = maximum wind speed, mi/h

Substituting the right-hand term of Eq 4-4 into Eq 4-3 for p_{max}, taking the drag coefficient c_d as 0.67, and solving for v_{max}, we have

$$v_{max} = \sqrt{S\sigma_{max} / c_d p_o M_a} = 231.8\sqrt{S\sigma_{max} / M_a} \qquad \text{(Eq 4-5)}$$

If we know M_a, S and σ_{max} for a cylindrical element, we can calculate the survival wind speed v_{max} from Eq 4-5. For the single-section cylinder, M_a equals $L^2D/2$. Using this relationship, the expression for v_{max} becomes

$$v_{max} = \frac{327.8}{L}\sqrt{S\sigma_{max} / D} \qquad \text{(Eq 4-6)}$$

We can convert this expression to determine the longest survivable cantilevered length for a given v_{max}, which is

$$L_{max} = \frac{327.8}{v_{max}}\sqrt{S\sigma_{max} / D} \qquad \text{(Eq 4-7)}$$

Sec 4.2 Cylindrical Elements

Solid Rod

As an initial example, consider a 2-meter element half, made from solid rod of a commonly used aluminum alloy, 6061-T6, with the following properties:

Length	L = 20 inches
Diameter	D = ⅛ in. (0.125 in.)
Yield strength	σ_{max} = 35,000 $lb/in.^2$ (from Table 3-1)

For the rod we calculate the section modulus from Eq 3-7. This is $S = \pi D^3/32 = 1.9 \times 10^{-4}$ in.^{3}.

Calculating v_{max} by Eq 4-6, we determine that for this element the survival wind speed is

$$v_{max} = \frac{327.8}{20}\sqrt{1.9\times 10^{-4}\times 35000/\,0.125} = 120 \text{ mi/h.}$$

As a typical 2-meter element is at least as strong as this example, we can see that, absent icing, wind survival of the element itself is not a major issue for VHF Yagi antennas.

Hollow Tubing

Now consider a cylindrical element for application in the HF range, made of 6063-T832 and having the following properties.

Diameter	D = ½ in. (0.5 in.)
Wall thickness	t = 0.058 in.
Yield strength	σ_{max} = 35000 lb/in.^{2}
Density	ρ = 0.098 lb/in.^{3}

We wish to know the maximum section length for this element to survive in a wind of 100 mi/h, and also the weight of the section.

The inner diameter of the tubing is given by

$$d = D - 2t \qquad \text{(Eq 4-8)}$$

where

d = inner diam, in.
D = outer diam, in.
t = wall thickness, in.

For this example, d = 0.5 – 2(0.058) = 0.384 in.

From Eq 3-9, the section modulus is

$$S = \pi(0.5^4 - 0.384^4)/(32 \times 0.5) = 0.0080 \text{ in.}^3.$$

For v_{max} = 100 mi/h, we calculate the maximum survivable length from Eq 4-7 as

$$L_{max} = \frac{327.8}{100}\sqrt{0.0080 \times 35000/\,0.5} = 77.6 \text{ in.}$$

The weight of a solid rod is given by

$$w = 0.25\rho\pi LD^2 \qquad \text{(Eq 4-9)}$$

and for a hollow cylindrical tube by

$$w = 0.25\rho\pi L(D^2 - d^2) \qquad \text{(Eq 4-10)}$$

where

ρ = material density, lb/in.^{3}
L = length, in.
D = outer diam, in.
d = inner diam, in.

From Eq 4-10, the section weight $w = 0.25 \times 0.098\pi \times 77.6 \times (0.5^2 - 0.384^2) = 0.61$ lb.

Ice Loading

Next consider the addition of 0.5 in. of radial ice to the same element. This increases the diameter, but does not change the section modulus because the ice adds essentially no strength. We can calculate the maximum length with ice, L_{ice}, for this case as follows.

The outer diameter with ice is calculated as $D_{ice} = D + 2t_{ice} = 0.5 + 2(0.5) = 1.5$ in.

The maximum length for survival in a 100 mi/h wind, again using Eq 4-7, is

$$L_{ice} = \frac{327.8}{100}\sqrt{0.0080 \times 35000/\,1.5} = 44.8 \text{ in.}$$

The results of calculations of this type are presented in Figs 4-2 and 4-3 for several tubing sizes commonly used as element tips. These figures indicate the effect of diameter and survival wind speed on the maximum unsupported section length. Because a typical 14-MHz antenna requires element lengths of approximately 200 inches per half, we can see that these untapered examples are not useful for that application.

The equations for analyzing the single-section cylindrical element can be assembled into a computer spreadsheet, as illustrated in Table 4-1. The following example illustrates use of the spreadsheet for

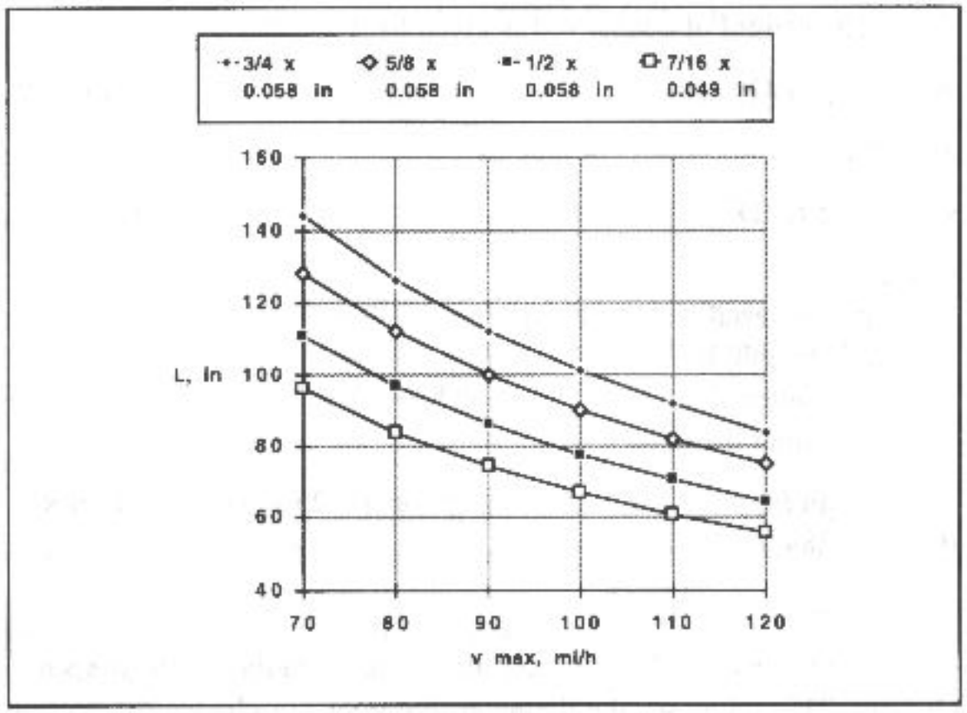


Fig 4-2—Maximum length versus wind speed for hollow cylindrical sections.

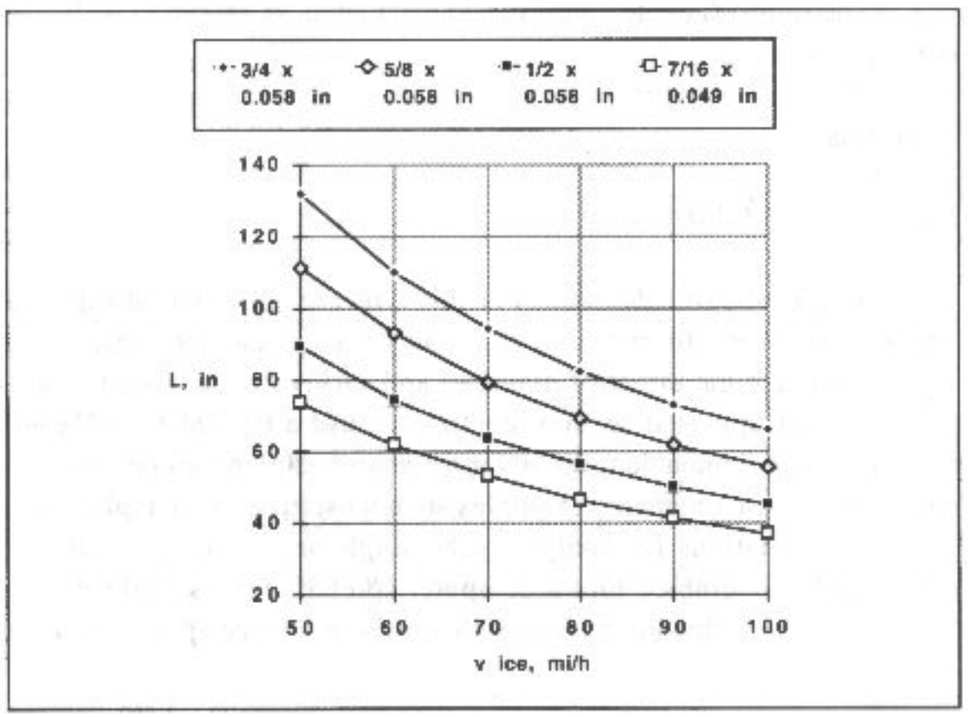


Fig 4-3—Maximum length versus wind speed with ½-in. ice load for hollow cylindrical sections.

Table 4-1
Spreadsheet Analysis of a Single-Section Cylindrical Element of Hollow Tubing

General entry parameters:	
Material yield strength	σ_{max}
Pressure coefficient	p_o
Drag coefficient	c_d
Radial ice thickness	t_i
Material density	ρ
Section entry parameters:	
Diameter	D
Wall thickness	t
Length	L
Calculated parameters:	
Section weight	$w = 0.25\rho\pi L[D^2 - (D - 2t)^2]$
Section wind area	$A = LD$
Section area moment	$M_a = AL/2$
Section modulus	$S = \pi[D^4 - (D - 2t)^4]/(32D)$
Survival speed	$v_{max} = \sqrt{S\sigma_{max} / (c_d p_o M_a)}$
Area with ice	$A' = L(D + 2t_i)$
Area moment with ice	$M_a' = A'L/2$
Section modulus	$S' = S$
Survival speed with ice loading	$v_{ice} = \sqrt{S\sigma_{max} / (c_d p_o M_a')}$

NOTE: The primed quantities refer to the calculation with radial ice. Note also that this spreadsheet may be used for solid rods by specifying the wall thickness t as half the section diameter D. (The "inner diameter" for a solid rod is zero.)

a Hy-Gain antenna tip section of 7/16-in. diam, 53-in. length, 0.035-in. wall thickness, and 6063-T832 aluminum alloy. Enter the following values into the spreadsheet.

Material yield strength $\sigma_{max} = 35000$ lb/in.^{2}
Constant p_o from Eq 4-4 $= 2.78 \times 10^{-5}$
Drag coefficient $c_d = 0.67$
Radial ice thickness $t_i = 0.5$ in.
Material density $\rho = 0.098$ lb/in.^{3}
Section diameter $D = 0.4375$ in.
Wall thickness $t = 0.035$ in.
Section length $L = 53$ in.

The results of spreadsheet calculations are

Section weight $w = 0.23$ lb
Section wind area $A = 23$ in.^{2}
Section moment of area $M_a = 614.5$ in.^{3}
Section modulus $S = 0.0041$ in.^{3}
Survival speed $v_{max} = 112$ mi/h
Area with ice $A' = 76$ in.^{2}
Moment of area with ice $M_a' = 2019$ in.^{3}
Section modulus $S' = S = 0.0041$ in.^{3}
Survival speed with ice loading $v_{ice} = 62$ mi/h

This type of analysis gives a clear idea of the limit on unsupported lengths of a cylinder, such as the tip section of a tapered Yagi element. The examples of flagpoles and other acrodynamic structures lead us to explore the requirement for tapered construction in Yagi elements.

Sec 4.3 Two-Section Tapered Elements

The analysis of multisection elements is much more complicated. In order to analyze a multisection tapered element, it is necessary to create a spreadsheet to keep track of the moment on each section of all the sections outboard of it. Although the analysis of the single cylindrical section would seem to indicate the desirability of the largest possible

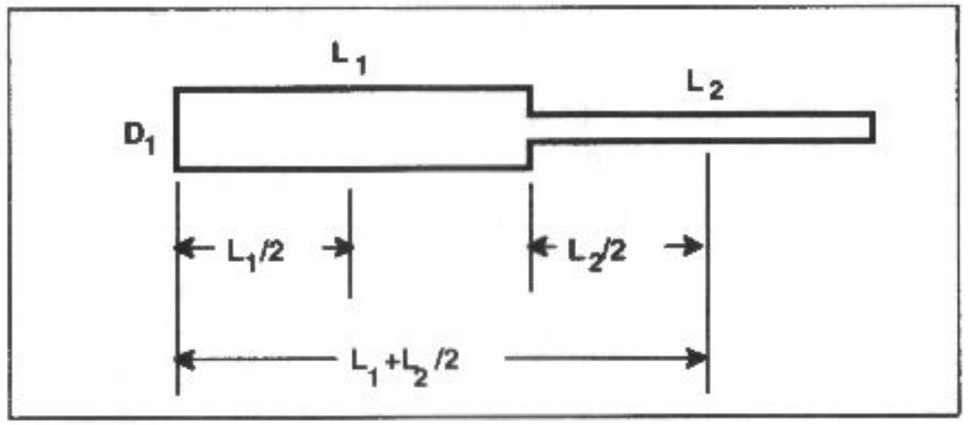


Fig 4-4—Dimensions for tapered-element moment calculation.

diameter, it will be seen that quite the opposite is true for the outer sections of tapered elements.

Consider now a tapered element consisting of two cylindrical sections of different diameters and lengths. To be consistent with Lawson,[1] number the sections and their dimensions from the inner end, and denote them in general by the subscript n. The sections and their dimensions are shown in Fig 4-4.

The moment of area at the inner end of the outer section is the same as the result M_a for the cylinder in Sec 4.2, and is given by

$$M_2 = \frac{L_2}{2} L_2 D_2 = \frac{L_2}{2} A_2 \qquad \text{(Eq 4-11)}$$

where $A_2 = L_2 D_2$ = the area of section 2 (see Fig 4-4)

The moment of area at the inner end of the inner section is given by the expression

$$M_1 = \left(L_1 + \frac{L_2}{2}\right) L_2 D_2 + \frac{L_1}{2} L_1 D_1 \qquad \text{(Eq 4-12)}$$

or, substituting $A = LD$ for sections 1 and 2,

$$M_1 = \left(L_1 + \frac{L_2}{2}\right) A_2 + \frac{L_1}{2} A_1 \qquad \text{(Eq 4-13)}$$

This equation can be rearranged to yield a general expression that lends itself to spreadsheet calculation of the tapered element parameters.

Table 4-2
Survival Wind Speed of 200-Inch-Length Two-Section Element Half

Tip Length	*Tip Diam*	*Center Diam*	*Center Wall*	*Area*	v_{max}	v_{ice}
200 in.	1.25 in.	1.25 in.	0.058 in.	250 in.^{2}	68 mi/h	51 mi/h
130	0.75	1.25	0.058	185	78	51
78	0.5	1.25	0.058	192	86	57
152	1.25	1.25	0.116	250	90	67
90	0.75	1.25	0.116	205	106	73
60	0.5	1.25	0.116	205	108	74

$$M_1 = M_2 + L_1\left(\frac{A_1}{2} + A_2\right) = M_2 + L_1\left[(A_1 + A_2) - \frac{A_1}{2}\right]$$

$$= M_2 + L_1\left(\sum_1^2 A - \frac{A_1}{2}\right) \qquad \text{(Eq 4-14)}$$

where $\sum_1^2 A$ is the sum of the areas of sections 1 and 2

To obtain moments of force from the moments of area, multiply by the effective pressure $c_d p$. See Fig 4-4. The quantity $c_d p A_2$ is the total force from L_2 at the tip of L_1. The resulting moment at the inner end of L_1 is $c_d p L_1 A_2$. The additional moment from the force on $L_1 A_1$ at the inner end of L_1 is $c_d p L_1 A_1/2$. The moment M_2 at the inner end of L_2 is $c_d p L_2 A_2/2$. As mentioned, Eq 4-14 is convenient for spreadsheet use, and the summation is over all sections from the tip to the section under consideration. The survival wind speed of a multisection element is the least (minimum) of the survival wind speeds of the individual sections.

No sooner do we begin calculation of the forces and moments of a practical tapered Yagi element than the complexity gets out of hand. Fortunately, these calculations are quite simple to perform using a spreadsheet application on a typical desktop computer.

Table 4-2 is the result of a series of calculations that show how the survival wind speed increases as we decrease the diameter of the tip

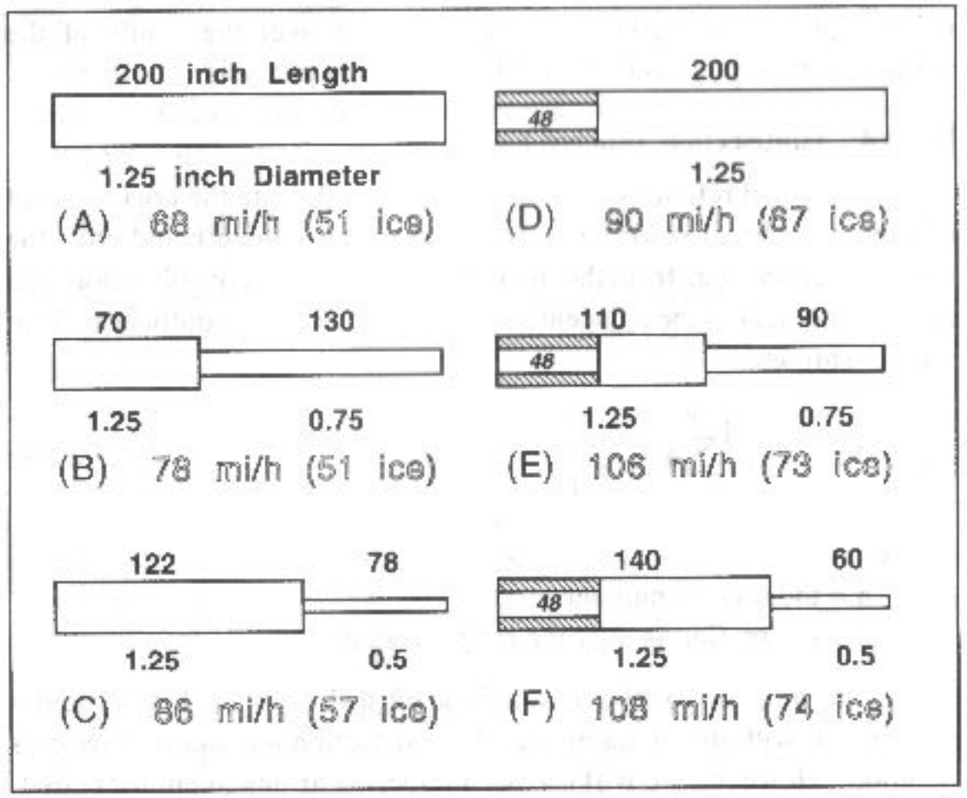


Fig 4-5—Development of two-section tapered elements. Elements at D, E and F are reinforced at the center section by an internal tube of 1.125-in. diameter and 0.058-in. wall thickness. For these, only the inner 48 in. (italic numbers) need to be reinforced to double wall thickness.

section while holding the total length constant. Fig 4-5 summarizes the same information in graphic form. Because the survival speed of the element is the minimum of the survival speeds of the individual sections, in these calculations the tip length is varied so the survival speed of all sections is approximately the same. *Decreasing* the tip diameter while holding the center section diameter constant results in successively stronger elements. The inclusion of center-section inner reinforcement with an additional telescoping 0.058-in.-wall tube results in a simple element that will survive EIA Zone B winds (see Table 2-3 and Fig 2-2).

This gives a general idea of the element design approach. A more gradual taper by means of additional sections offers more design flexibility, and permits the design of elements that can survive even greater wind speeds while minimizing overall weight and wind area. Even with

this simple configuration, the improvement over the limits of the untapered example shows the usefulness of the analysis.

Sec 4.4 Multisection Tapered Elements

A general relationship may be used to calculate the area moment of each cylindrical section in terms of the section length and area, the total of section areas from the tip to the section under consideration, and the area moment of the adjacent section closer to the tip (outboard). That relationship is

$$M_n = M_{n+1} + L_n\left(\sum_{n}^{tip} A - \frac{A_n}{2}\right) \qquad \text{(Eq 4-15)}$$

where

n = the section number

the summation is over the range from n to n_{tip}

The numbering of sections is a bit awkward in this particular expression, with the tip having the largest section number in Lawson's notation.[2] However, we will use his numbering arrangement for consistency.

4.4.1 Constructing a Computer Spreadsheet

Using the above equations, you can construct a useful computer spreadsheet of calculations to analyze or design an arbitrary tapered Yagi element. Table 4-3 summarizes the entries and equations used in the order they are required. All of the math functions needed to calculate the survival speed are available in spreadsheet programs such as Microsoft Excel for the Macintosh and Lotus 1-2-3 for the IBM PC. Because of possible differences in internal rounding, your results may differ very slightly from the examples given here.

Construct the spreadsheet with multiple columns, one column per element section. Enter the section numbers, diameters, wall thicknesses and lengths. The weight and area of each section are straightforward calculations, as are the total length, weight, and element area. The sums of the areas from the tip to the section under calculation are treated explicitly in their own row. This makes it easier to add or remove columns from the spreadsheet to accommodate elements of varying numbers of sections.

Table 4-3
Spreadsheet Calculation of Tapered-Element Section Survival Wind Speeds

General entry parameters:	
Material yield strength	σ_{max}
Pressure coefficient	p_o
Drag coefficient	c_d
Radial ice thickness	t_i
Material density	ρ
Section entry parameters:	
Section number	n
Diameter	D_n
Wall thickness	t_n
Length	L_n
Calculated section parameters:	
Section weight	$w_n = 0.25\rho\pi L_n[D_n^2 - (D_n - 2t_n)^2]$
Section wind area	$A_n = L_nD_n$
Total area	$(\Sigma A)_n = A_n + (\Sigma A)_{n+1}$
Area moment	$M_n = M_{n+1} + L_n[(\Sigma A)_n - A_n/2]$
Section modulus	$S_n = \pi[D_n^4 - (D_n - 2t_n)^4]/(32D_n)$
Survival wind speed, v_{max}	$v_n = \sqrt{S_n\sigma_{max} / (c_dp_oM_n)}$
Area with ice	$A_n' = L_n(D_n + 2t_i)$
Total area with ice	$(\Sigma A')_n = A_n' + (\Sigma A')_{n+1}$
Area moment with ice	$M_n' = M_{n+1}' + L_n[(\Sigma A')_n - A_n'/2]$
Section modulus	$S_n' = S_n$
Survival speed with ice	$v_n' = \sqrt{S_n'\sigma_{max} / (c_dp_oM_n')}$
Calculated half-element total parameters (make a separate column at right):	
Element length	$L = \Sigma L_n$
Element weight	$W = \Sigma w_n$
Element area	$A = \Sigma A_n$
Element survival wind speed	$v_{max} = MIN(v_n)$
Survival speed with ice	$v_{ice} = MIN(v_n')$

NOTE: $MIN(v_n)$ is a spreadsheet function to select the minimum from an array of values.

The equation for calculation of the moment for a given section involves the moment from the next column toward the tip, the length and area of the section under calculation, and the sum of section areas from the previous row.

The section modulus is calculated directly from the section diameter and wall thickness, and the survival speed v_{max} for each section is determined. The survival speed for the total element is the minimum of the section survival speeds. The same process is repeated with the addition of ice thickness to the diameter, in order to determine the survival wind speed with ice loading.

4.4.2 Some More Advanced Spreadsheet Techniques

In order to create a spreadsheet that accommodates a variable number of element sections, use the Excel spreadsheet function IF( ) to make cell values equal to 0 or blank for columns that are not used. As an example, for A_n' you can use

$=IF(A_n=0,0,L_n*(d_n+2*t_i))$ for Excel or

$@IF(A_n=0,0,L_n*(d_n+2*t_i))$ or

$@IF(A_n=0,"",L_n*(d_n+2*t_i))$ for Lotus 1-2-3.

For the section number, use

$=IF(A_n=0,0,(n-1)+1)$ or

$@IF(A_n=0,0,(n-1)+1)$.

Use the spreadsheet format commands to set the number format to 0;; in Excel or set the Global Format for Zero=Yes in Lotus 1-2-3, so that only positive nonzero entries are displayed. If you want to retain a zero in Lotus 1-2-3, such as in Section 0, enter '0 (quote zero).

As an alternative, you can define the cells as blank for the same logical argument using

$=IF(A_n=0," ",L_n*(d_n+2*t_i))$ for Excel (quote space quote) or

$@IF(A_n=0,"",L_n*(d_n+2*t_i))$ for Lotus 1-2-3 (quote quote).

As noted in Table 4-3, the Excel function $=MIN(v_n)$ (or the equivalent Lotus 1-2-3 function @MIN) indicates a spreadsheet function to select the minimum from an array of values, in this case the row containing v_n for each section. For the rows in which you search for a

minimum, such as the rows for v_{max} and v_{ice}, the MIN( ) function will show a value of zero if any cells are occupied by zero (or blank in Lotus 1-2-3). In this case, use the function IF( ) to make cells equal to " " in Excel or equal to 10E11 to get a cell full of ***** in Lotus 1-2-3 for columns that are not used, which will contain $A_n=0$. As an example, the spreadsheet formula for v_n is

$=IF(A_n=0," ",SQRT(S_n*\sigma_{max}/(c_d*p_o*M_n)))$ in Excel and

$@IF(A_n=0,10E11,@SQRT(S_n*\sigma_{max}/(c_d*p_o*M_n)))$ for Lotus 1-2-3.

This permits the $v_{max} = MIN(v_n)$ function to find the minimum value in active columns only, while ignoring the blank inactive columns (or those filled with asterisks from the cell-width overflow in Lotus 1-2-3, which assigns a value of 0 to a blank cell).

Once you learn how to use these logical functions, you can make spreadsheets in which the columns "light up" when you enter the diameter and length of each section. So that cell entries can be spread (copied) to the right or down as larger spreadsheets are constructed, use the S designation (for example, $BS5) to refer to the general-equation parameters such as yield strength and drag coefficient.

The objective of element design is to maximize the element survival speed while holding the overall length constant and minimizing element weight and wind area. By modifying the element section lengths, diameters and wall thicknesses, you can readily see how to optimize a given element for maximum survival wind speed consistent with constant total length, minimum weight, and minimum wind area. (Any change in taper distribution has second-order electrical consequences that are treated in Chapters 8 and 9.)

4.4.3 Using the Spreadsheet

To apply the spreadsheet, let's look at the mechanical design of a 14-MHz element presented in *The ARRL Antenna Book*.[3] The taper schedule for this element is shown in Fig 4-6A. Let us assume that the element is constructed from 6061-T6 or 6063-T832 alloy tubing, either of which has a yield strength $\sigma_{max} = 35000$ lb/in.^{2} and a material density $\rho = 0.098$ lb/in.^{3}. Let us also assume the ice thickness $t_{ice} = 0.5$ in.

Table 4-4 shows the results of the spreadsheet calculations. The wind and ice survival information for this element is summarized in graphic form in Fig 4-7. From these results, we can see that the

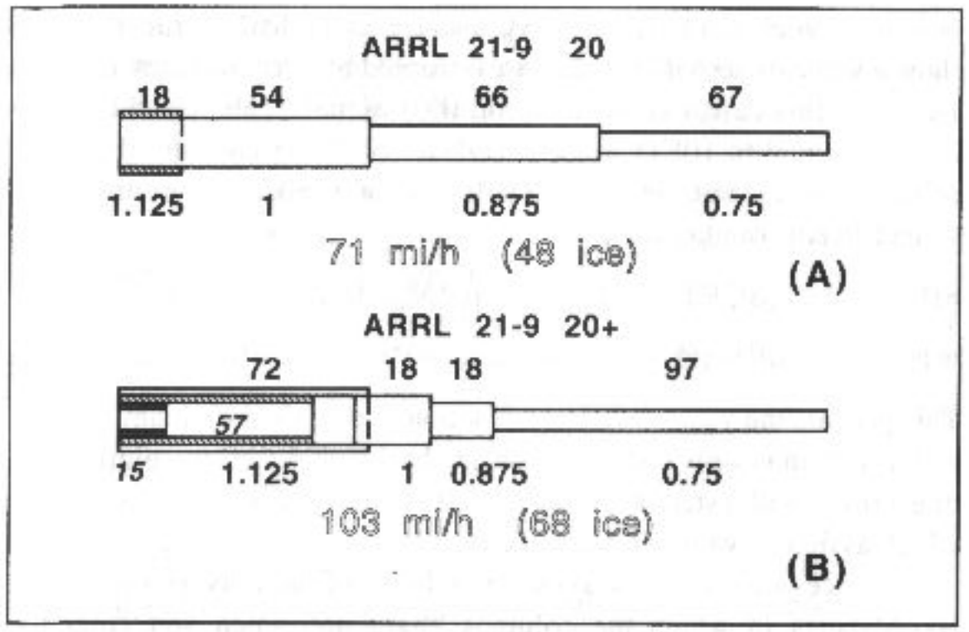


Fig 4-6—Development of *ARRL Antenna Book* 14-MHz improved element.[3] The drawing at A shows the original design; the modified design is shown at B. Italic numbers indicate the lengths of telescoping reinforcements.

Table 4-4
Spreadsheet Results for ARRL Antenna Book 14-MHz Element[3]

Section	*1*	*2*	*3*	*4*	*Total*
Diam, in.	1.125	1	0.875	0.75	
Thickness, in.	0.116	0.058	0.058	0.058	
Length, in.	18	54	66	67	205
Weight	0.65	0.91	0.96	0.83	3.35
Area, in.^{2}	20	54	58	50	182
ΣArea, in.^{2}	182	162	108	50	
Moment, in.^{3}	17294	14196	6906	1683	
Modulus, in.^{3}	0.0843	0.0382	0.0285	0.0203	
v_{max}, mi/h	96	71	88	150	71
$Area_{ice}$, in.^{2}	38	108	124	117	
$\Sigma Area_{ice}$, in.^{2}	387	349	241	117	
$Moment_{ice}$, in.^{3}	38306	31680	15750	3928	
v_{ice}, mi/h	64	48	58	99	48

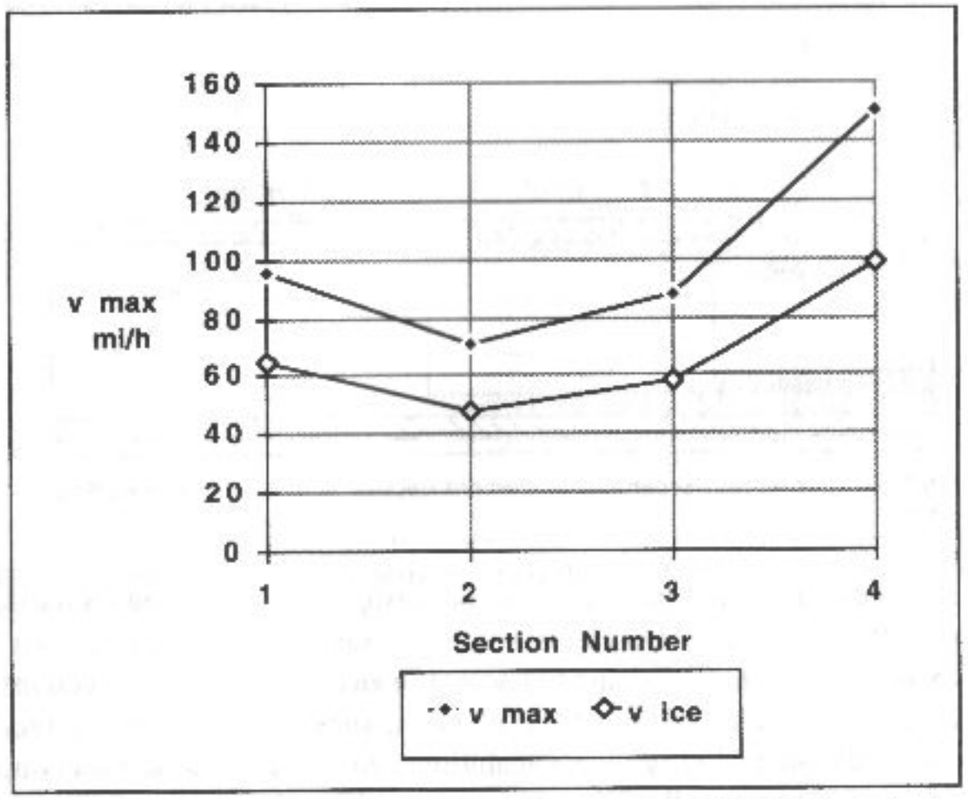


Fig 4-7—Section wind survival speeds for *Antenna Book* 14-MHz element of Fig 4-6A.

Table 4-5
Improved 20-Meter Antenna Book Element Section Survival Wind Speed

Section	*1*	*2*	*3*	*4*	*5*	*6*	*Total*
Diam	1.125	1.125	1.125	1	0.875	0.75	
Thickness	0.174	0.116	0.058	0.058	0.058	0.058	
Length	15	42	15	18	18	97	205
Weight	0.76	1.51	0.29	0.30	0.26	1.20	4.33
Area	17	47	17	18	16	73	188
v_{max}	108	104	105	103	104	104	103
v_{ice}	73	70	69	68	68	68	68

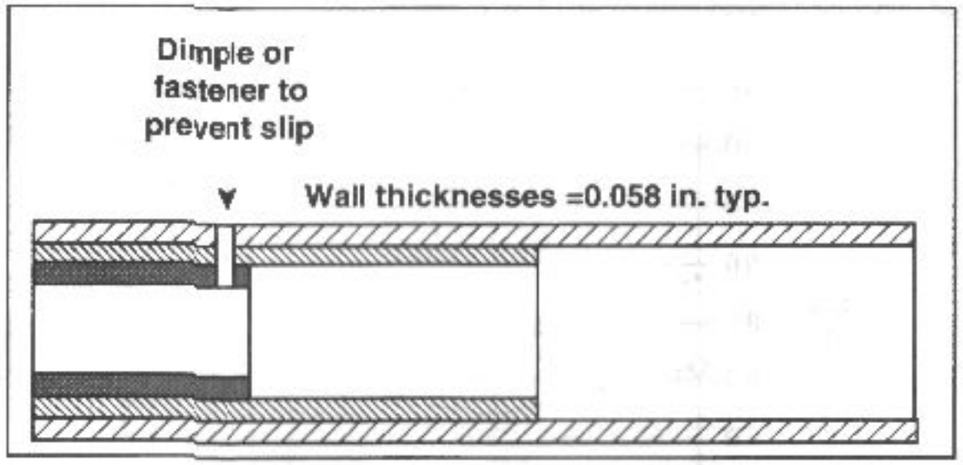


Fig 4-8—Internal reinforcement of element section with retention. See Sec 4.10.

maximum survival wind speed for this design is 71 mi/h, and 48 mi/h with 0.5 in. of ice. This element will not survive EIA Zone A wind conditions (see Table 2-3 and Fig 2-2). The analysis reveals that section 2 is the weakest portion of the element, substantially reducing the overall element wind survival capabilities. We can also see that section 4 could be substantially longer before it would be the limiting factor in element strength.

Using the same spreadsheet format with additional columns to account for internally reinforcing some sections, let's see what can be done to improve the strength of this element. We'll retain the same tubing sizes, and restrict ourselves to a maximum tubing length of 12 ft, a commonly available length. By adjusting the relative lengths of the sections, particularly lengthening the tip section, and by internally reinforcing the center section, we can realize substantial increases in v_{max} and v_{ice}.

Shown in Table 4-5 are the spreadsheet results, for brevity omitting the rows for area summations, moment, and section modulus. This element has a much more even distribution of section strength, and has greatly increased wind survival capacity. The center section has a total length of 72 in. per side, and is internally reinforced as shown in Figs 4-6B and 4-8. The graphic presentation of section survival wind speeds in Fig 4-9 shows the result of the improvement in strength through redistribution of section lengths.

The spreadsheet columns for the 1.125-in. diameter sections show that only 15 in. of the center-section tube can remain unreinforced.

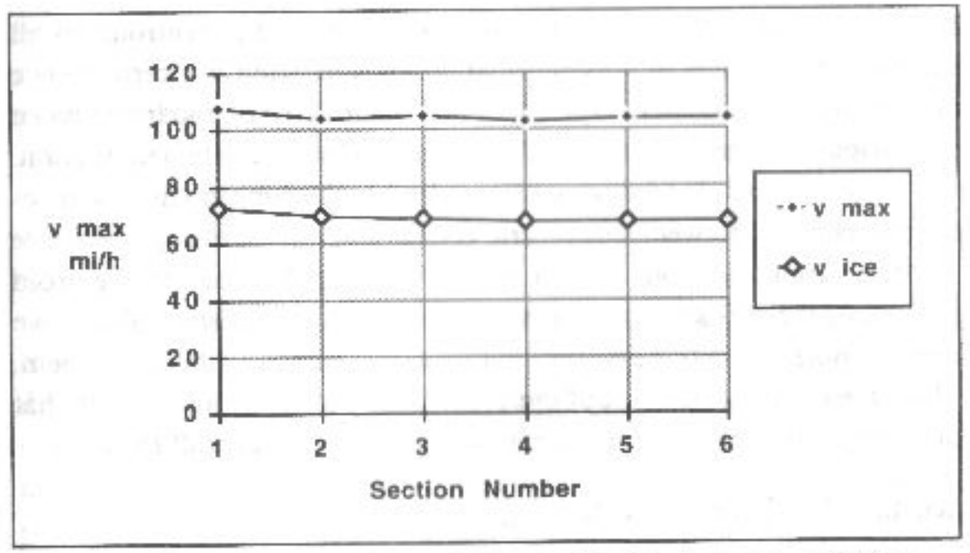


Fig 4-9—Section wind survival speeds for modified *Antenna Book* 14-MHz element of Fig 4-6B.

The 0.174-in. wall thickness of the inner section 1 indicates the need here for double reinforcement, with 15 in. of 0.875 × 0.058-in. tube telescoped into the 1 × 0.058-in. tube already reinforcing section 2 to a 0.116-in. wall thickness. The reinforcing tube inside section 2 is carried the full length of section 1, as well, for a total length in this example of 15 plus 42, or 57 in.

In many composite structures, such as riveted I beams, there exists the necessity of fasteners to carry the shear across the length of the boundaries between separate parts of the structure. The question of whether this fastening is necessary for the concentric telescoped cylindrical reinforcement is treated here.

The shear flow q, measured in lb/in., along a beam made of two separate elements[4] is

$$q = \frac{dF}{dx} = \frac{dM}{dx} \times \frac{1}{I} \int y dA = \frac{VQ}{I} \qquad \text{(Eq 4-16)}$$

where

V = rate of change of moment along the beam

I = rectangular moment of the entire composite beam

$Q = \int y dA$ is integrated over only one of the structural elements

The value of $Q = \int y dA$ is equal to the area of the beam element multiplied by the distance of its centroid from the neutral axis of the

composite beam. Because of cylindrical symmetry, the centroids of all the concentric tubes of a cylindrical composite beam are zero. Hence Q = 0 and q = 0, and there is no shear force across the boundary between the various concentric elements of a tubular reinforced element section.

This means that for composite cylindrical beams, no interconnection is required between concentric reinforcing tubes, except to assure that the various sections stay in place over time. Because the centroid of each of the tubes lies on the neutral axis (hence, no shear flow), we can combine their strengths without requiring fastening between them. Thus, the bending strength of the combined sections is identical to that of a single tube of the same overall diameter and total wall thickness.

Sec 4.5 20-Meter Elements

Using the same analysis method on other published and commercial 20-meter element designs, we can evaluate existing element designs and design improved ones. Figs 4-10 through 4-12 show the development of stronger elements based on existing popular designs. In each case, the use of 35000 lb/in.^{2} material such as 6061-T6 or 6063-T832 is assumed. For each design, a separate spreadsheet was used to vary element lengths and thicknesses to maximize strength while minimizing weight and wind area. Using the information of Sec 4.4, you can construct a similar spreadsheet for use in designing or analyzing other element structures.

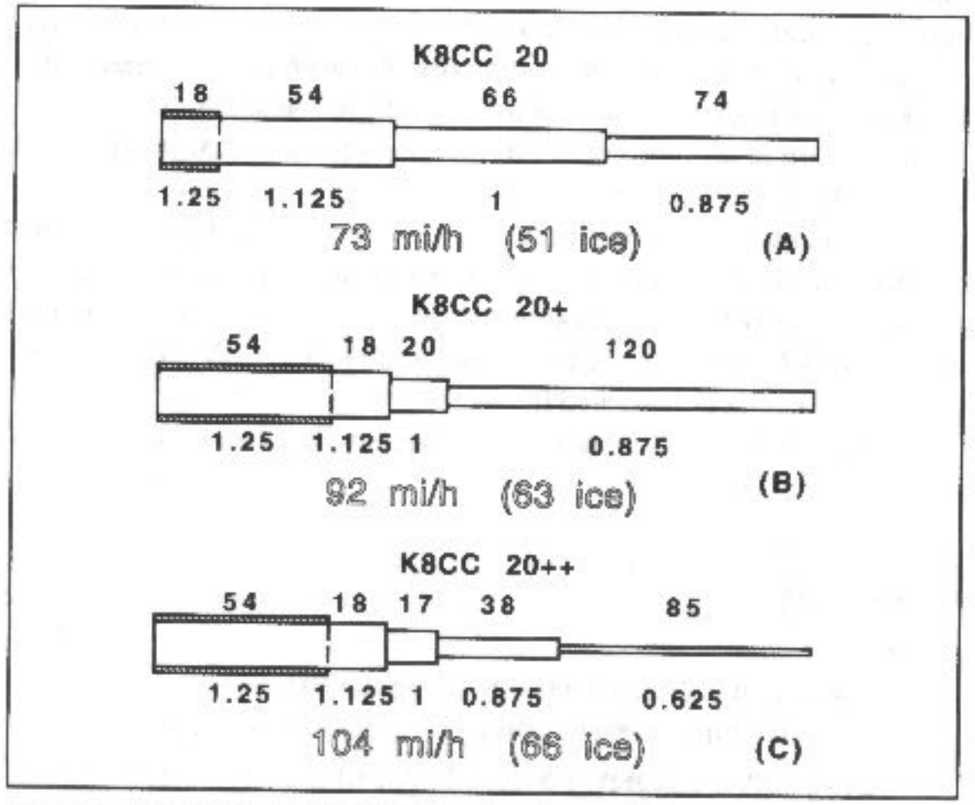


Fig 4-10—Development of K8CC 20-meter element.[5] The drawing at A shows the original design; modifications shown at B and C produce successively stronger elements.

First, let's look at a 1989 element design attributed to N6BV by K8CC.[5] This design, Fig 4-10A, has the same section lengths as the *Antenna Book* design previously presented, but with one size larger section diameters. Without adding any more tubing sizes or reinforcement, we can add considerable strength simply by redistributing the section lengths as shown in Fig 4-10B. If we add a smaller diameter tip section, we can realize further improvement, Fig 4-10C. The resulting element is suitable for EIA Zone B applications.

Another element design published by W2PV[6] is based on a commercially produced 20-meter Yagi. See Fig 4-11A. It incorporates a more gradual taper and has a smaller diameter tip section. Even without modification, this element will survive in Zone A winds. With the small changes in section lengths and diameters of Fig 4-11B, this element will survive Zone B winds.

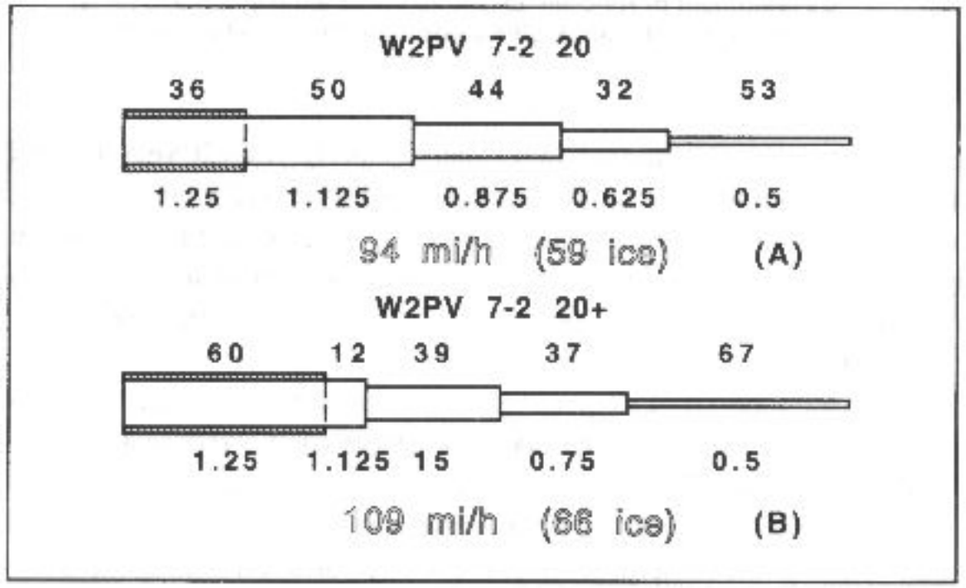


Fig 4-11—Development of W2PV 20-meter element.[6] At A, original design; at B, strengthened design.

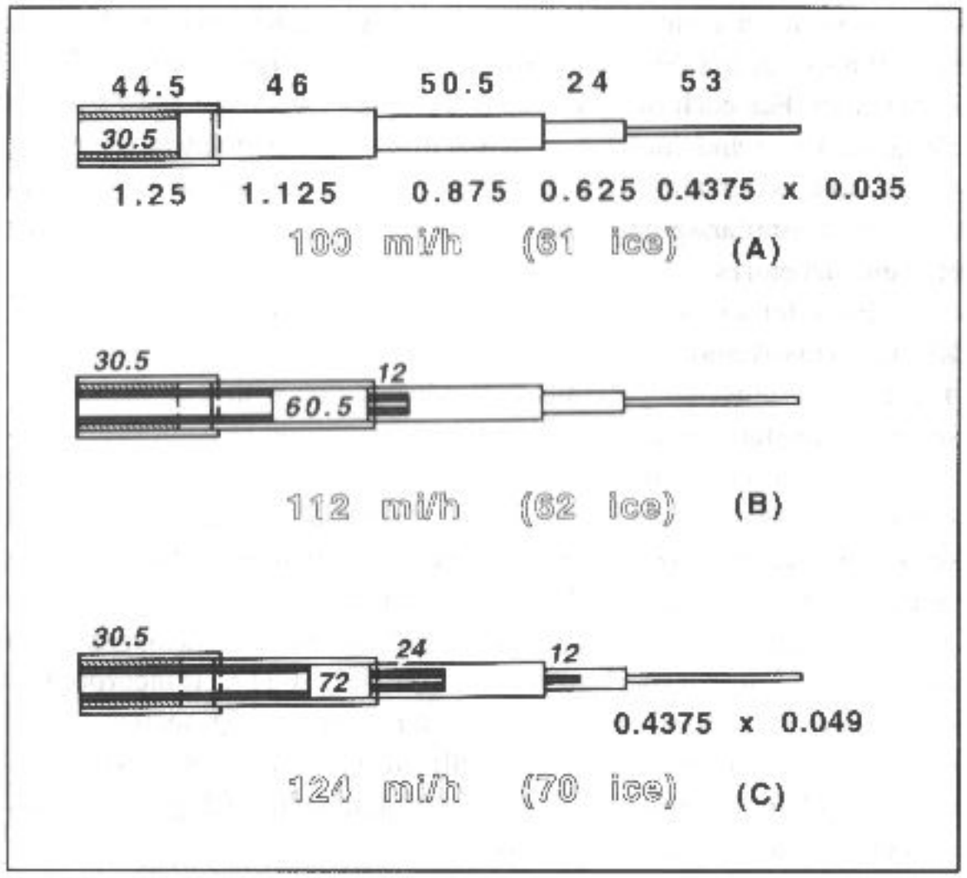


Fig 4-12—Development of Hy-Gain 204BA 20-meter element. The original design is shown at A, with successively stronger designs at B and C.

A popular commercial Yagi antenna, the Hy-Gain 204BA, is rated for 100 mi/h (Zone B) survival as delivered, Fig 4-12A. This element design shows the technique of rapid taper using a swaged joint between sections to minimize the diameter, and hence the wind area, of the tip section. These tip sections are 7/16-in. (0.4375-in.) diameter and 0.035-in. wall thickness.

In the analysis of a tapered element, the moment of each succeeding section from the tip includes the effect of the moments of the outboard sections. The best way to design elements from a mechanical viewpoint is to start with the tip, making it as long as is survivable, and then continue the same process inward. The variation among elements of a given Yagi to provide for reflector or director requirements is best accomplished by variations in the center section, keeping the outer sections constant once they have been designed for mechanical survival.

This approach minimizes weight, wind area and cost, by minimizing the requirements for reinforced large-diameter tubing. As it happens, the Hy-Gain elements are all designed this way, reflecting an engineering understanding of wind-stress issues. The development of this element is shown in Fig 4-12.

With simple modifications, the Hy-Gain 20-meter elements can be strengthened to survive Zone C winds of 111.8 mi/h and even higher speeds. For maximum strength, the tips are replaced with 6063-T832 tubing with 0.058-in. wall thickness. Alternatively, the originals can be internally reinforced, but the tubing size required is nonstandard and will require machining to fit. Tubing in this diameter is readily available in 0.049-in. wall thickness, which compromises the ice-loading capabilities to a slight degree (v_{ice} = 69 mi/h instead of 72 mi/h).

The 0.625-in.-diameter section is internally reinforced with 0.5 × 0.058-in. tubing, or alternatively with ¼ IPS aluminum pipe of 0.540-in. diameter. This section requires only a total wall thickness of 0.050 in. for v_{max} = 125 mi/h, so a number of lighter alternatives such as 0.035-in.-wall tubing may be used for reinforcement.

The 0.875-in.-diameter section is internally reinforced with 0.75 × 0.058-in. tubing. This section requires a total wall thickness of 0.083 in. to develop full strength, so lighter alternatives can be applied here as well.

It is attractive mechanically to provide reinforcement of the 1.25 × 0.058-in. inner section by a 1 × 0.058-in. tube centered within it as shown in Fig 4-12, extended to reinforce the second section as well. A question that arises in connection with strengthening this particular element is the strength in bending of two concentric tubes with space between them. As we have seen, no fastening is required so long as the tubes are concentric.

To determine bending strength, we need to know the section modulus of the combination. We know that, for a cylindrical structure of diameter D, the section modulus S is related to the rectangular moment of intertia I by the relationship

$$S = \frac{2I}{D} \qquad \text{(Eq 4-17)}$$

Therefore, if we know I, we can determine S. The moment of inertia of the two tubes taken together is just the sum of the moments of inertia of each of them, which we calculate as follows.

$$I_1 = \frac{\pi}{64}(D_1^4 - d_1^4) \qquad \text{(Eq 4-18)}$$

$$I_2 = \frac{\pi}{64}(D_2^4 - d_2^4) \qquad \text{(Eq 4-19)}$$

Combining Eqs 4-18 and 4-19, we have

$$I = I_1 + I_2 = \frac{\pi}{64}(D_1^4 - d_1^4 + D_2^4 - d_2^4) \qquad \text{(Eq 4-20)}$$

If we define the equivalent inner diameter d_e of the combination such that

$$I = \frac{\pi}{64}(D_1^4 - d_e^4) \qquad \text{(Eq 4-21)}$$

we can see by inspection that this equivalent diameter is

$$d_e^4 = d_1^4 + d_2^4 - D_2^4 \qquad \text{(Eq 4-22)}$$

and hence

$$d_e = (d_1^4 + d_2^4 - D_2^4)^{1/4} \qquad \text{(Eq 4-23)}$$

We can define an equivalent thickness for the calculation of I and S as

$$t_e = \frac{1}{2}(D_1 - d_e) \qquad \text{(Eq 4-24)}$$

By putting this concept to work, we find the equivalent wall thickness of the concentric combination of the 1.25-in. inner section and a 1-in. internal reinforcement member. For a wall thickness of 0.058 in., there is a space of 0.134 in. between the surfaces of the two tubes.

$D_1 = 1.25$ in.
$t_1 = 0.058$ in.
$d_1 = D_1 - 2t_1 = 1.134$ in.
$D_2 = 1$ in.
$t_2 = 0.058$ in.
$d_2 = D_2 - 2t_2 = 0.884$ in.
$d_e = (1.134^4 + 0.884^4 - 1^4)^{1/4} = 1.06$ in.
$t_e = 0.5(D_1 - d_e) = 0.095$ in.

We can use this 0.095-in. equivalent diameter in our spreadsheet calculations to determine the survival speed of the concentric combination.

This is a convenient way to reinforce the 204BA element inner section without requiring a change in electrical dimensions. For maximum strengthening, the inner sections require reinforcement with a 1 × 0.058-in. 6063-T832 or 6061-T6 tube, 72 in. long, inserted through the inner section and into the second section. In some versions, the existing reinforcing tubing is held in place by dimples in the element inner sections; these dimples must be filed smooth inside to insert the additional reinforcement tube required for higher strength. In order to support the reinforcement at the inner end of the first section, a short length of 1.125 × 0.058-in. tubing is required for those elements such as the directors that are not delivered with single reinforced first sections. For the less aggressive improvement to v_{max} = 112 mi/h, a reinforcement-tube wall thickness of 0.049 in. will suffice.

When I use these reinforcers on smaller elements, I use a dimple or pop rivet to hold them in place; these should be located as described in Sec 4.10. On sections where the material thickness is at least equal to three full threads, such as the inner end of this 14-MHz Yagi, the tubing can be drilled and tapped for a no. 10 stainless-steel machine screw, which also makes a secure point for connection to the driven element (at a possible risk of reduced element fatigue life).

Further details of this improvement for the 204BA Yagi, including the electrical designs of each of the modified elements, are presented in Chapter 11.

Given our ability to design high-strength elements, it is appropriate to ask what are the strongest elements that can be constructed with convenient tubing lengths. Fig 4-13 presents three similar element designs capable of withstanding Zone C wind forces with substantial safety margin. These elements represent some of the strongest 20-meter designs.

The elements are constructed with 6 ft (72 in.) center sections, which can be half of a continuous 12-ft length of tubing of 1.25-in. diam. With minor exceptions, the outboard sections are chosen to be lengths that are integral multiples of 6 in., including a minimum of 2 in. overlap at joints. The use of 18-in., 24-in. or 36-in. lengths cut from 12-ft standard delivery lengths helps keep scrap to a minimum. A standard plumber's tubing or pipe cutter makes short and accurate work of the

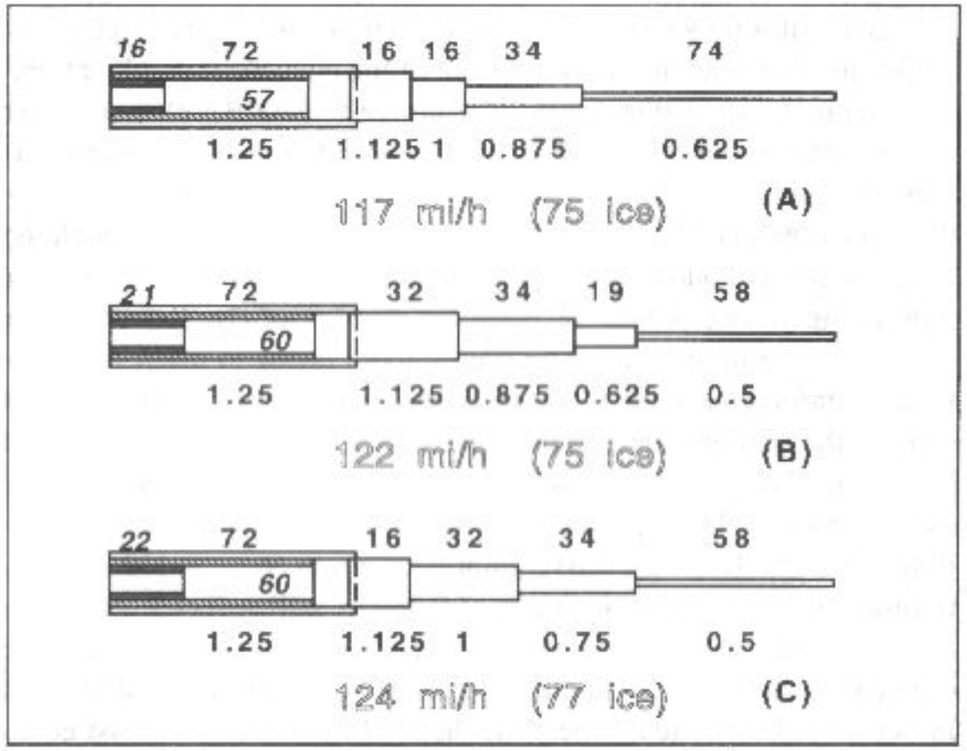


Fig 4-13—These 20-meter elements are designed for maximum strength.

cutting; deburring the inside of the cut with the blade on the side of the cutter is a necessity.

Each joint is slit lengthwise on one side with a hacksaw, and steps of two diameter sizes are made in a single connection as shown in Fig 4-14, using at least a 4-in.-long intermediate spacer of tubing slit at its outer end. This approach reduces the overall number of fasteners required, while achieving the desired advantages of diameter taper without the necessity of swaged joints. The hacksaw slits should be deburred inside and out with a round file and a flat file, and the two slits are aligned under the hose-clamp screw mechanism so the clamp can compress the slits without interference, Fig 4-14B.

The slits should be wide enough so they do not close completely when tightened on the inserted element section. This requires a slit width at least equal to $\pi\Delta D$, where ΔD is the difference in mating diameters of the telescoped tubing. For a 0.058-in. wall thickness tubing telescoped over tubing whose outer diameter is an exact multiple of $\frac{1}{16}$ in. (0.0625), the difference in diameter is $2(0.0625 - 0.058) = 0.009$ in., and the slit must be wider than $0.009\pi = 0.028$ in. Tighten the hose

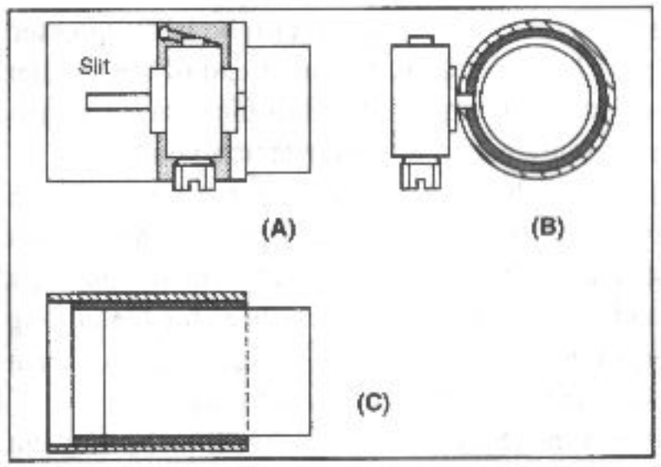


Fig 4-14—Three views of a two-step tubing joint. See text regarding width of slit, called out in A. During assembly, slits of the outer two diameters must be aligned, as shown at B, for proper compression.

clamp with a plumber's hose-clamp torque wrench, so the clamp threads are not damaged.

The degree of overlap of two concentric tubes in a joint is complex to calculate, but experience dictates a minimum of 2 in. for the slit joint between tubes with the above clearances. At least one commercially made antenna uses this overlap for the normal range of tubing sizes. If the slit is of adequate width and the clamp can be tightened

Table 4-6
Comparison of 20-Meter Element Designs

Reference, Fig No.	V_{max}	V_{ice}	*Weight*	*Area*
Ant. Book, 4-6A	71 mi/h	48 mi/h	3.35 lb	182 in.^{2}
K8CC, 4-10A	73	51	3.95	214
K8CC+, 4-10B	92	63	4.62	213
W2PV, 4-11A	94	59	3.79	186
204BA, 4-12A	100	61	3.45	190
Ant. Book+, 4-6B	103	68	4.33	188
K8CC++, 4-10C	104	66	4.23	191
W2PV+, 4-11B	109	66	4.30	189
204BA+, 4-12B	112	62	4.31	190
W6QHS-20, 4-13A	117	75	4.72	200
W6QHS-20+, 4-13B	122	75	4.80	197
204BA++, 4-12C	124	70	4.99	190
W6QHS-20++, 4-13C	124	77	4.78	195

properly around the periphery, the tip of the larger tube is compressed to fit the smaller tube exactly. In bending, the inner end of the smaller tube conforms to the inside of the larger tube until the bearing area is sufficient to reduce the pressure below the deformation point.

In order to preclude lengthwise slip in the joint, a fastener such as a rivet is much more secure than reliance on friction alone. A rivet provides the benefit of redundant fastening, as well. For the purist, a swaging tool can be made by substituting a flat wheel for the cutting wheel in a tubing cutter, so the outer tube can be swaged 0.0045 inch at a point behind the slit section to fit the inner tube exactly.

Table 4-6 summarizes the characteristics of the 20-meter element designs shown in the drawings of this section. Note the substantial variation in survival speeds and in weight and wind area for the various configurations. Two finer points are also worth mentioning for this and similar summary tables appearing later in this chapter: The element weight calculations ignore section overlap, and the Hy-Gain element calculations do not include the effect of the swage sections, which generally increase the survival speed slightly because of the reduced area.

Sec 4.6 15-Meter Elements

Improvements similar to those shown for 20-meter elements are possible for published 15-meter element designs. Although the shorter elements required for 15 meters are generally stronger than elements of

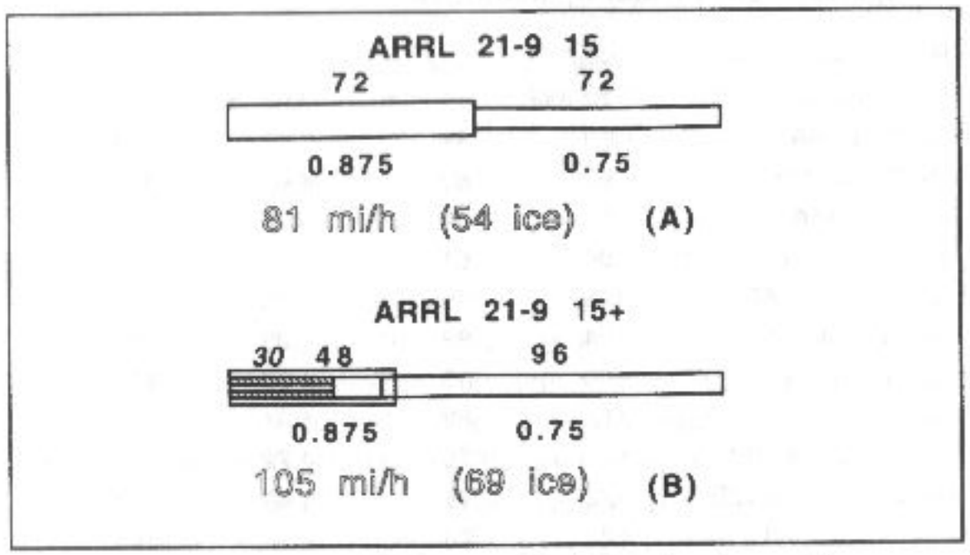


Fig 4-15—Development of *ARRL Antenna Book* 15-meter element.[7]

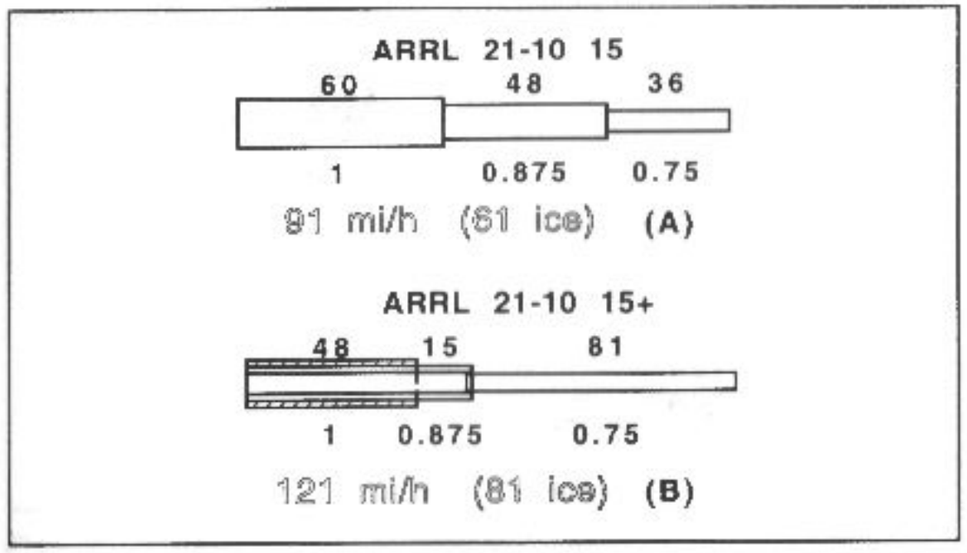


Fig 4-16—Development of more rugged *Antenna Book* 15-meter element.[8]

similar diameter for 20 meters, it is still necessary to redistribute the section lengths to achieve wind survival even for Zone A. Fig 4-15 shows the improvement possible with simple changes in the design published in *The ARRL Antenna Book*.[7]

The Antenna Book includes a more rugged element, which lends itself to strengthening for extreme environments. This element is shown in Fig 4-16.[8] Note the substantial tip-length increase needed to realize

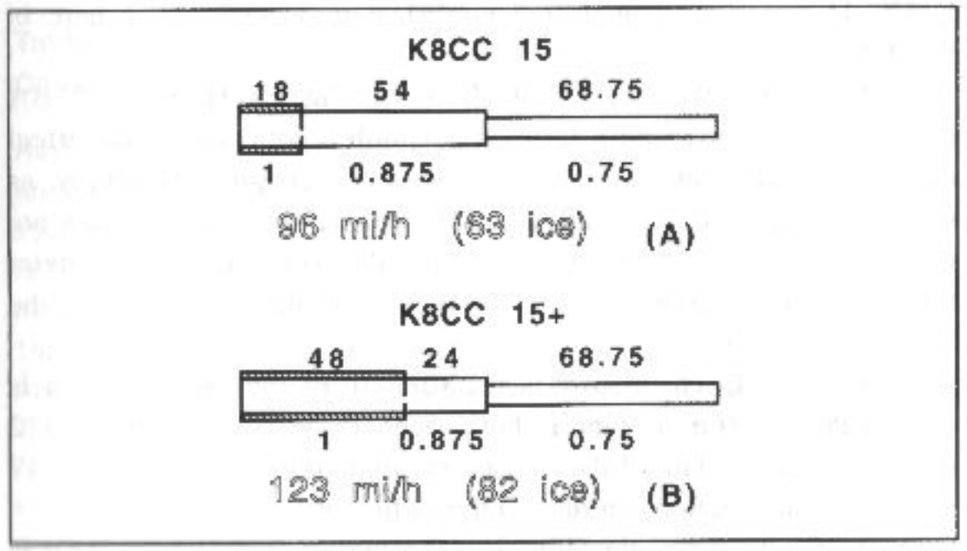


Fig 4-17—Development of the K8CC 15-meter element.[9]

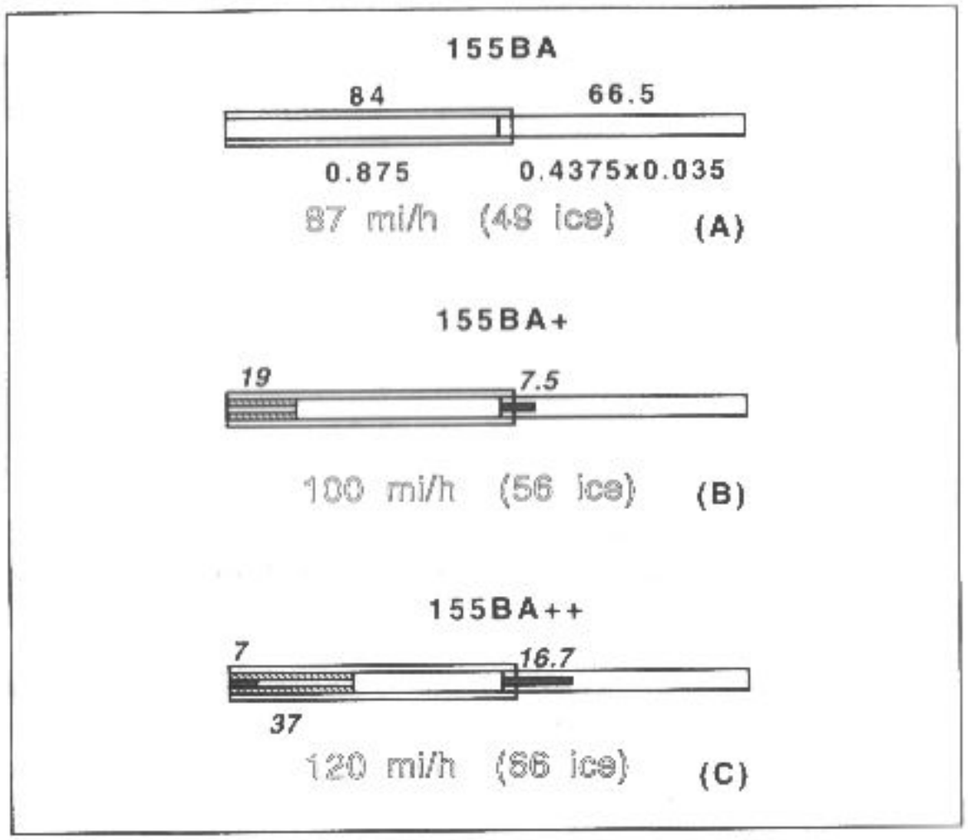


Fig 4-18—Development of Hy-Gain 155BA 15-meter element.

the improvement. Similar improvements are recommended for the K8CC 15-meter element design published in 1989,[9] shown here in Fig 4-17.

Because of the long, thin-wall tip sections, the Hy-Gain 155BA element is rated at 85 mi/h survival (87 mi/h if we consider the effect of the element clamp). This element can be strengthened simply, as shown in Fig 4-18. The reflector, driven and first director tips are reinforced with 0.375 × 0.065 × 18-in. tube with the outer diameter turned down to 0.358 in. to fit the 0.3675-in. inner diameter of the 0.4375 × 0.035-in. tips.

Also worth calculating is the strength of the swage, which is survivable in 120 mi/h wind in both cases. In order to survive in 122 mi/h wind, only 50 in. of the tip can remain unreinforced, and only 47 in. of the inner section can remain unreinforced.

As for 20-meter elements, it is instructive to determine what is the strongest 15-meter element that can be constructed from convenient

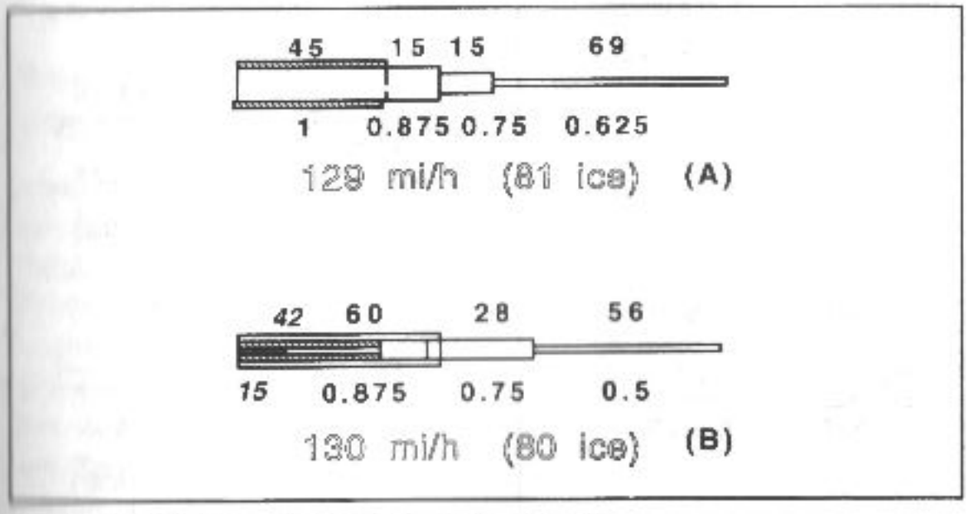


Fig 4-19—Maximum-strength 15-meter elements.

tubing diameters and lengths. Maximum-strength elements are shown in Fig 4-19. In the case of the first element, Fig 4-19A, the 60 × 0.875 in.-diameter tube is carried right through the 45 × 1-in. center-section tube. These element halves can, of course, be made from continuous center sections of 120 × 0.875-in. and 90 × 1-in. tubing, respectively.

For the 15-meter element designs considered in this section, Table 4-7 summarizes the element characteristics.

Table 4-7
Comparison of 15-Meter Element Designs

Reference, Fig No.	v_{max}	v_{ice}	*Weight*	*Area*
Ant. Book, 4-15A	81 mi/h	54 mi/h	1.94 lb	117 in.^{2}
155BA, 4-18A	87	49	1.37	103
Ant. Book rugged, 4-16A	91	61	2.15	129
K8CC, 4-17A	96	63	2.21	117
155BA+, 4-18B	100	56	1.68	103
Ant. Book+, 4-15B	105	69	2.26	114
155BA++, 4-18C	120	66	1.93	103
Ant. Book rugged+, 4-16B	121	81	2.74	122
K8CC+, 4-17B	123	82	2.72	121
W8QHB-15, 4-19A	129	81	2.52	113
W8QHB-15+, 4-19B	130	80	2.35	102

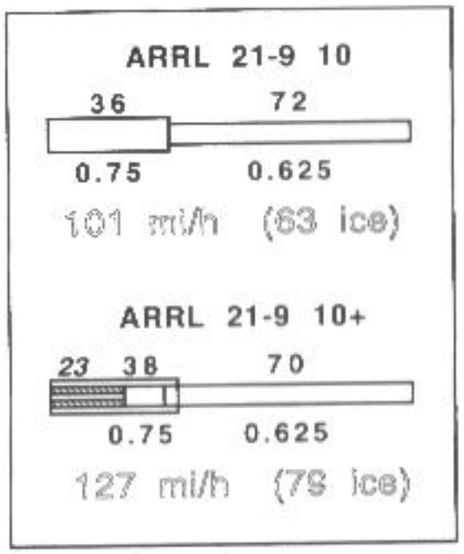


Fig 4-20—Development of *ARRL Antenna Book* 10-meter element.[10]

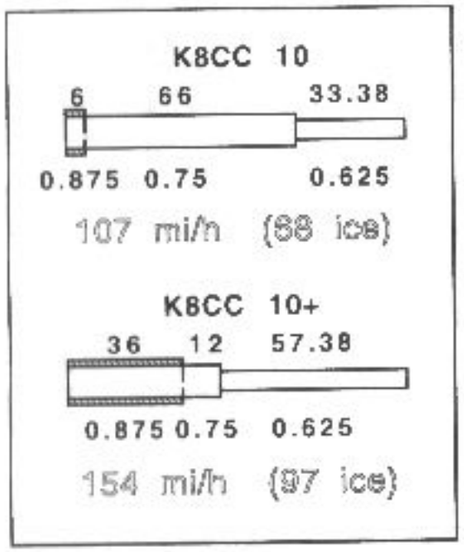


Fig 4-21—Development of K8CC 10-meter element.[11]

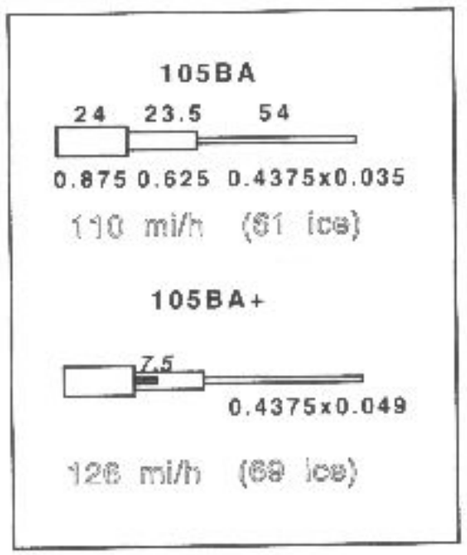


Fig 4-22—Development of Hy-Gain 105BA 10-meter element.

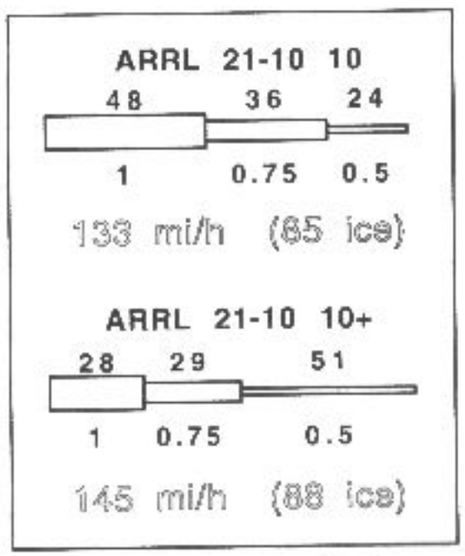


Fig 4-23—Development of *Antenna Book* more rugged 10-meter element.[12]

Sec 4.7 10-Meter Elements

The analysis and improvement of 10-meter elements is a more simple task, as the short overall lengths and smaller number of sections make for naturally stronger elements. Figs 4-20 and 4-21 reflect *The ARRL Antenna Book*[10] and K8CC[11] element designs, and steps for their improvement.

Table 4-8
Comparison of 10-Meter Element Designs

Reference, Fig No.	v_{max}	v_{ice}	*Weight*	*Area*
Ant. Book, 4-20 top	101 mi/h	63 mi/h	1.17 lb	72 in.^{2}
K8CC, 4-21 top	107	68	1.32	76
105BA, 4-22 top	110	61	0.76	60
105BA+, 4-22 bottom	126	69	0.91	60
Ant. Book+, 4-20 bottom	127	79	1.41	72
Ant. Book rugged, 4-23 top	133	85	1.44	87
Ant. Book rugged+, 4-23 bottom	145	88	1.23	75
K8CC+, 4-21 bottom	154	97	1.71	76

The commercial 105BA element is quite strong, but can also be improved simply as shown in Fig 4-22. Finally, the already strong *Antenna Book* heavy 10-meter element,[12] with minor modifications to the section length distribution of Fig 4-23, should survive in nearly any amateur application.

The comparisons among the various 10-meter element designs are summarized in Table 4-8.

Sec 4.8 Reinforcing Triband Antennas

Using the same techniques, we can determine how to strengthen the elements of triband antennas such as the Hy-Gain TH5, which has a 100 mi/h survival rating. Only the inner element sections of this antenna require strengthening to create a beam that can survive Zone C wind speeds with substantial safety margin. This is shown in Fig 4-24. To minimize scrap, reinforcement tubing lengths are chosen where possible to be multiples of 6 in., so they can be cut from tubing lengths that are multiples of 12 in. Alloys such as 6061-T6 and 6063-T832 with a 35000-lb/in.^{2} yield strength are assumed in the calculations.

The area of the traps is easily determined from their outer diameter, but their structural strength is an unknown. For simplicity, I assumed the traps to be mechanically equivalent to aluminum tubing of the same outer diameter and 0.058-in. wall thickness. Manufacturer's information would clearly permit a more accurate picture of element strength.

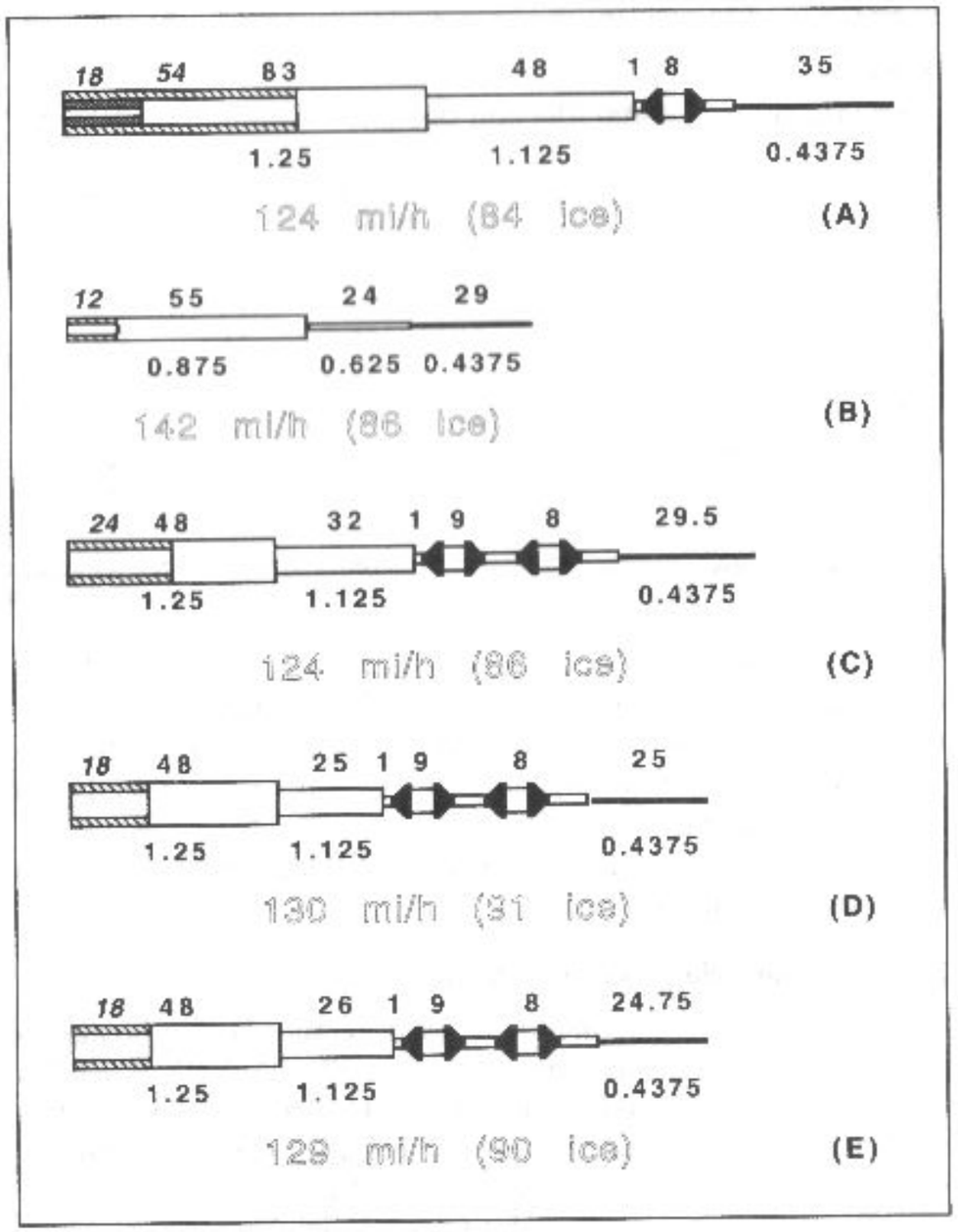


Fig 4-24—Hy-Gain TH5 element reinforcement.

However, I experienced no trap or tip failures of these particular elements at my very windy site, so the simplifying assumptions seem safe.

Note that the reinforcement tubing in the reflector, Fig 4-24A, consists of a 54-in. length of 1.125-in. tubing with an 18-in. length of 1-in. tubing inserted into the inner end. A fastener such as a dimple or pop rivet is necessary to keep the reinforcement tubing sections from

Table 4-9
Reinforced TH5

v_{max} = 124 mi/h, v_{ice} = 84 mi/h

Element	*Center Reinforcement*
Reflector	54 × 1.125 in., 18 × 1 in.
10-m reflector	12 × 0.75 in.
Rear driven	24 × 1.125 in.
Front driven	18 × 1.125 in.
Director	18 × 1.125 in.

Table 4-10
Reinforced TH7

v_{max} = 125 mi/h, v_{ice} = 76 mi/h

Element	*Center Reinforcement*	*Tip Replacement*
Reflector	54 × 1.125 in., 18 × 1 in.	
10-m reflector	18 × 0.75 in.	
Rear driven	30 × 1.125 in.	
Front driven	18 × 1.125 in.	
10-m director	—	Use 0.035-in. wall
15-m director	24 × 1.125 in., 12 × 0.5 in.	Use 0.049-in. wall
Director	18 × 1.125 in.	Use 0.049-in. wall

sliding out of position, as shown in Fig 4-8. Table 4-9 summarizes the reinforcement requirements for the TH5.

Exactly the same procedure can be applied to the Hy-Gain TH7. The results are summarized in Table 4-10. For this antenna, replacement of three director tip sections is required; this is accomplished on the 10-20 meter and 15-meter directors with $\frac{7}{16} \times 0.049$-in. tubing, and the removed $\frac{7}{16} \times 0.035$-in.-wall tubing is used to replace the thinner tip sections of the 10-meter director.

The element-to-boom clamps are adequate for all elements, with the possible exception of the multiband reflectors and the TH7 15-meter director. Parasitic-element clamps can be replaced by the larger diame-

ter clamps used for the driven elements, with a length of 1.5 × 0.125-in.-wall tubing (1.125 × 0.125-in. wall in the case of the 15-meter reflector) slit lengthwise and placed inside as a spacer. The electrical effect of this substitution is small, and the substantial section modulus improvement is helpful if there is any indication of wind gusting in the vertical plane, as is present over sloping ground.

Sec 4.9 Reinforcing Shortened 40-Meter Elements

One additional element issue is the survival wind speed of shortened 40-meter elements, such as those used in the Cushcraft 40-2CD, a popular 40-meter 2-element Yagi.[13] The possibilities for reinforcement are also of interest.

Figs 4-25 and 4-26 show the calculated survival wind speeds for these elements, and suggest some potential improvements. Fig 4-27 shows construction details. Without changing the electrical design the possibilities are somewhat limited, but substantial improvements are possible. Double and even triple internal reinforcing is required for this process, because the element diameters are small for the element lengths. The dimensions in italics in these figures reflect the dimension of the internal reinforcement in the smaller diameter section; to this

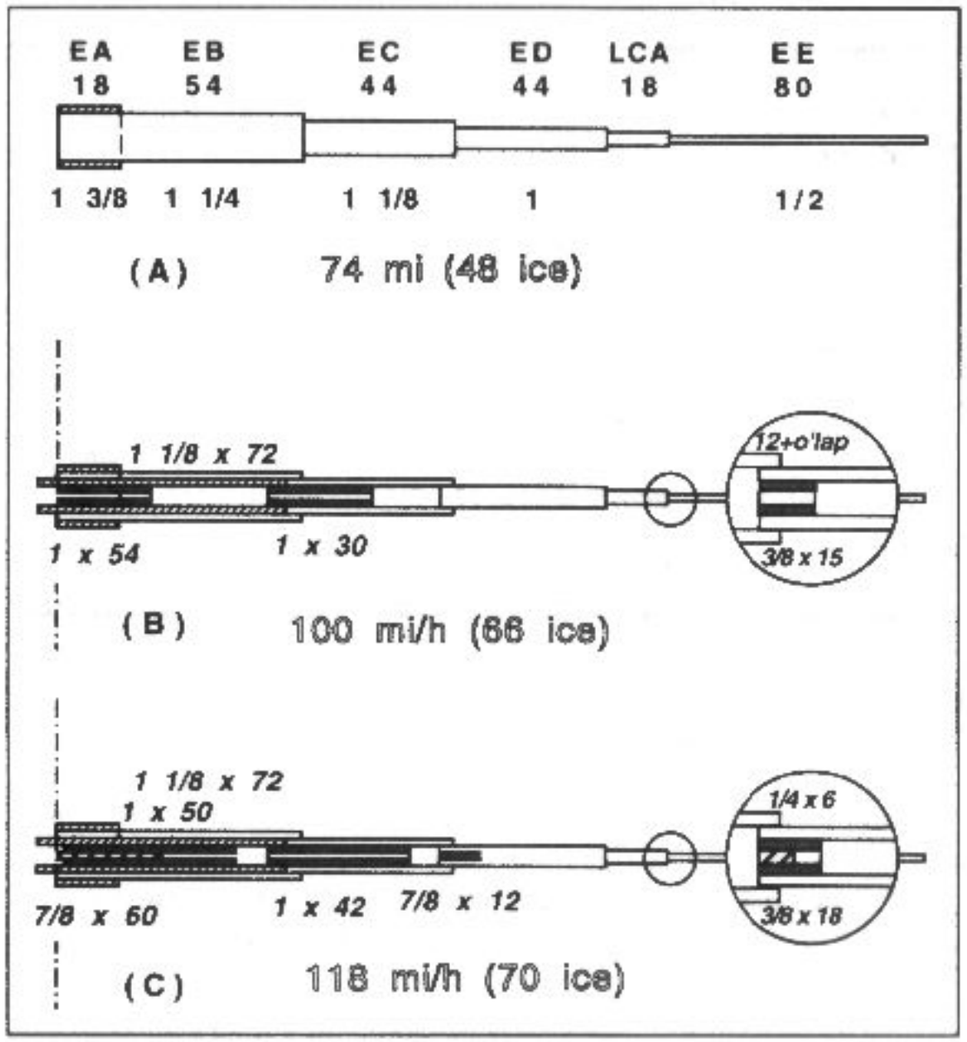


Fig 4-25—Reinforcement of Cushcraft 40-2CD reflector.

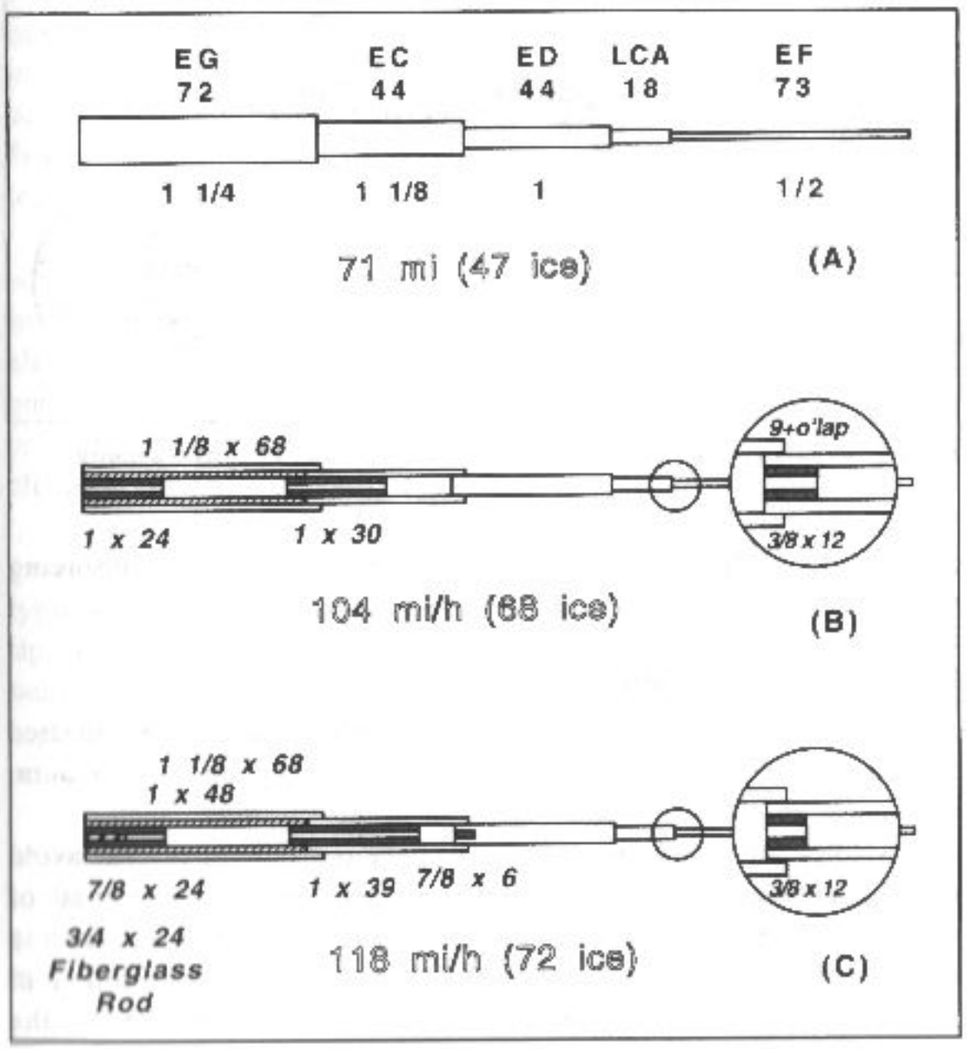


Fig 4-26—Reinforcement of Cushcraft 40-2CD driven element.

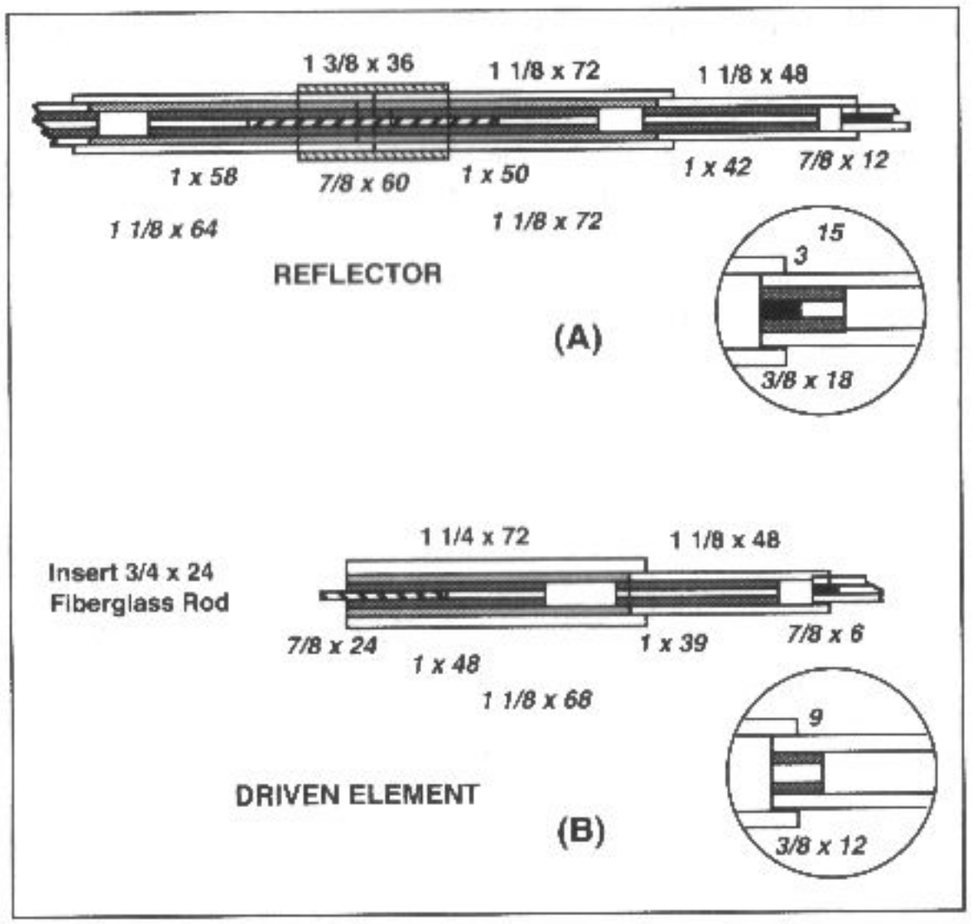


Fig 4-27—Construction details of reinforced Cushcraft 40-2CD elements.

must be added the section overlap. The dimensions of the reinforcing tubes themselves are also given in italic type.

Sec 4.10 Avoiding Fatigue Failures

Failure of elements caused by fatigue from vibration or repeated loading should also be considered. In the particular case of aluminum, fatigue failure can be the limiting factor in service life.

Holes are stress risers that can generate fatigue cracks; avoid them except where the stress is less than one-third the yield strength of the base material, as noted in Sec 3.4. To retain internal reinforcing sections, hammer a center punch or a ⅛-in. drift to make a dimple in the outer section that also dimples the inner section. Do not tear the surface; for multiple reinforcements, start with the inner two tubes.

The outer ends of element sections are lower stress locations, so it is acceptable to drill fastener holes through the overlap joints between sections. A hose clamp here can further reinforce the end of the joint against tearing. If a rivet is used to hold an internal reinforcing section, it should be located at the outer end of the reinforcement.

A hole for electrical connection at the high-stress inner end of the driven element might result in a reduction of element fatigue life. As wind stress is horizontal, the best location for a hole is on the underside of an element, drilled vertically. If the fatigue strength of the clamp itself is reliable, a clamp is a more conservative choice and avoids a hole.

These cautions also apply to holes in other high-stress locations (for example, boom-section inner ends or mast-section lower ends). The hundreds of thousands of load cycles found in aircraft and race cars do produce fatigue failures. Hy-Gain has used reinforcement retention screws at element inner ends, and I'm not aware of resulting fatigue failures. I've found typical Yagi failures to be bending or section-joint tear-out, except for element-tip fatigue from resonant vibration.

The wind velocity which excites resonances in the elements is well below the level of maximum wind speed, and is, in fact, typically in the region one might think of as just a breeze. The response of the element sections depends on their mechanical resonance and damping, and occurs over only a narrow range of wind speeds. However, if the critical speed happens to be present with high statistical probability, the elements can fail from fatigue.

A common failure is in the tip section. This process can be prevented by adding damping to the resonance of the tip section, typically by inserting a piece of waterproof rope inside the length of the tip, or by weighting the tip to reduce the mechanical resonant frequency below the vortex shedding frequency of the element shape.[14] Knot the rope or glue it in place with silicon caulk at the inner end so it can't slide out if the tip sags under heavy ice.

Other cures for vibration fatigue failures range from energy-absorbing rubber flappers clamped near the element tips[15] all the way to spiral-wound helical strakes consisting of dielectric tubing or rope, wound around the element and used as obstacles to vortex shedding. Recognizing the existence of this problem is half the battle. In my own experience the use of internal rope absorbers has proved adequate, but repeated failures may require stronger medicine.

Sec 4.11 Summary

The dimensions for the calculations in this chapter were taken from the manufacturer's manuals for the commercial antennas, and from ARRL publications for *The Antenna Book*, K8CC and W2PV elements. Several issues come forward in this effort. One is that the 15- and 20-meter published element designs can benefit from considerable mechanical modification in order to survive in Zone A. Another is that elements with minimal taper, such as in the 1988 15th edition of *The ARRL Antenna Book*, have substantially greater wind area and hence, more bending moment at the attachment point. Third, the commercially designed elements give a very good account of themselves because of judicious use of swage joints, allowing a greater step in diameter.

It is apparent that elements can be improved substantially without heroic measures. Analysis of the element-to-boom clamping method is handled through the same method as for the element tubing, except that the calculation of the section modulus of a complex shape is not quite so straightforward as for a cylinder. Element clamping structures must be strong enough to survive in the expected environment. The design and analysis of several element clamping means is the subject of Chapter 5.

Certain antennas such as the KLM KT-34 tribander employ elements with a substantial span in the horizontal plane. These elements are unable to bend horizontally in response to high winds without undue stress on the outboard joints. I was not able to apply this particular configuration to our windy location. It is possible that flexible cross-connections and supports that permit sliding will enhance the wind performance of antennas with this type of structure.

To summarize, it is possible to optimize element configuration to survive high winds. Reinforcement can often be accomplished internal to existing elements without electrical design changes, although the improved taper algorithm presented in Chapter 8 can be used to accommodate electrical changes and new designs. The use of heavy taper (small tip sections) is definitely helpful in minimizing undue stress on the inner sections, although this will result in a requirement for slightly increased physical length because of the electrical effect of taper. Certain commercial producers appear to have applied substantial mechanical engineering time to their designs, while other products and published designs can be improved in the mechanical area.

It is worth reiterating that there is substantial strength variation among aluminum alloys,[16] as indicated in Table 3-1, so one should be careful to verify what is being used. High-quality alloy tubing is normally marked all along its length, while hardware store tubing, which is typically 6063-T5, has only half the strength of the better choices, 6061-T6 or 6063-T832.

I regret not yet finding the time to analyze commercial or published designs for full-size 40- and 80-meter elements, but the spreadsheet concept presented here will allow you to do so. For 40- and 80-meter Yagis, the guying of elements is a consideration. Determining the strength of a guyed cylindrical element is not a cantilever problem, but is a beam-column strength problem that is considered in connection with boom guying in Chapter 6.

In addition to strengthening the elements as indicated in this chapter, I also drill the boom and use ¼-in. stainless through-bolts to hold the clamps and elements in position. The questions of boom strength, antenna wind torque balance, and rotator torque requirements are also examined in Chapter 6.

References

[1] J. L. Lawson, *Yagi Antenna Design* (Newington: American Radio Relay League, 1986).

[2] See reference 1.

[3] G. Hall, ed., *The ARRL Antenna Book*, 15th Ed. (Newington: ARRL, 1988), p 21-8, Fig 9. [Note: These designs are replaced with new, rugged versions in the 16th (1991) edition.]

[4] E. P. Popov, *Mechanics of Materials* (Englewood Cliffs, NJ: Prentice-Hall, 1956), p 132.

[5] D. Pruett, "Contest Aerials," *National Contest Journal*, Sep/Oct 1989, pp 23-26.

[6] See reference 1, p 7-9.

[7] See reference 3, p 21-8, Fig 9.

[8] See reference 3, p 21-9, Fig 10.

[9] See reference 5.

[10] See reference 3, p 21-8, Fig 9.

[11] See reference 5.

[12] See reference 3, p 21-9, Fig 10.

13 D. Leeson, "Strengthening the Cushcraft 40-2CD," *QST*, Nov 1991, pp 36-42.

14 D. Weber, "Vibration Induced Yagi Fatigue Failures," *Ham Radio*, Aug 1989, pp 9-12, 17, 18.

15 P. Hawker, "Elements Fatigued by Fluttering," *Radio Communication*, Nov 1989, p 36.

16 *Alcoa Structural Handbook* (Aluminum Corporation of America, 1956), pp 54-56.

Chapter 5

Attachment of Elements to a Boom

Sec 5.1 Mechanical Considerations

The attachment of elements to the boom involves both electrical and mechanical issues. In addition, the further electrical and mechanical complexities of insulated attachment, as for a driven element, must be considered. The electrical consequences of various mounting clamps are considered in Chapter 9.

We desire that the element clamp not fail from wind or rotational loading through any of the following mechanisms: (1) tearing at its edge from bending moment, (2) buckling under shear, or (3) collapsing at the bolt holes. We also desire that neither the element nor the boom buckle or collapse from the force of the mounting U bolts, for example, and that the U bolts not fail from the shear forces on them. Analysis of various clamping structures indicates that a clamp satisfying the requirement for adequate wind-force bending moment will generally meet the remaining criteria.

It should be noted that the problem of carrying forces to a thin boom from a thick clamp is not a straightforward one, and that simply placing thick U bolts around a thin boom section is not satisfactory. The forces must be carried gradually into a thin cylindrical section by means of a formed clamp that bears over a substantial area to keep the force per unit area down, or by means of a tapered wall thickness of the boom from its nominal value to a very heavy reinforcement (wall thickness at least equal to the U-bolt diameter or clamp-plate thickness) immediately under the U bolt.

In any event, it is poor practice to expect the clamp and element not to rotate if held by friction or even by a dimpling bolt; only a through-bolt in holes sized for a light drive fit will prevent element rotation over time. Although the strength of a very small-diameter bolt

About the Author

Dr. David B. Leeson, W6QHS, was first licensed as a radio amateur in 1952. With his wife Barbara, KK6QM, he shares an active interest in amateur radio from their California mountaintop home, and he is a director of the Northern California DX Foundation. He also drove to sports car racing national championships in 1978 and 1979.

Professionally, Dr. Leeson holds electrical engineering degrees from Caltech, MIT and Stanford (PhD), and is a Fellow of the Institute of Electrical and Electronic Engineers. He has served as a director of the Electronic Industries Association and the American Electronics Association. Since 1968, he has been the founding chairman of California Microwave, Inc.

Introduction

It was, as the story goes, a dark and stormy night. John, WØZZ, and Trey, WN4KKN, were CQing and dialing their way through the slow predawn hours of the ARRL DX contest. In the small hours, Kenji, KC6HNB, had worked out the 40-meter band of his JA compatriots and gone down from the wind and the rain of our mountaintop. K1EA's *CT* glowed brightly from the two computer screens of the W6QHS multi-single.

My alarm went off much too early, and I inched downstairs with half-opened eyes to make some coffee and take my turn at one of the rigs. Trey, ever bright and cheerful, commented, "Wow, the wind gauge never went under 50 the whole night! I think it peaked over 100, and the antennas worked and even rotated the whole time!" A gratified smile crept over my face as I listened to the soft hum of the amplifiers and computers against the muted howling of the wind in the Yagis.

Outside on the 120-ft tower, the well-modified Cushcraft 40-2CD and Hy-Gain 205BA antennas did their jobs right through the storm, while on the the Skyneedle the stack of Hy-Gain 204BA, 155BA and 105BA antennas never missed a beat. The strengthened TH7 tribander on the third tower also performed and rotated through the 48-hour combination of contest and winter storm. We did well for a first try as a team, winning multi-single for W6 with the highest top-ten score from the West Coast. Next year, with some really big antennas ...

This is the satisfying end of a long story that began with disappointment and heartache. After losing a series of towers and antennas to the 120-mi/h winter winds at our otherwise wonderful mountaintop site, I was convinced by my wife Barbara, KK6QM, that perhaps there ought to be an engineering solution to our problems. Susan and Dave Barton, KA6SEH and AF6S, founders of Personal Training Systems, ended our computer phobia and sealed our fate by teaching us what could be done with a Macintosh, then loaned us one of theirs until our own computer arrived.

This book, and the robust antennas we now enjoy, resulted. After I read the Yagi-element articles by Dick Weber, K5IU, of rotating tower fame, I saw that the techniques could be extended to computer spreadsheet practice. As I became more familiar with the computer, I prepared

itself might be adequate, at least ¼ in. diameter is required to prevent deformation of the holes in the boom. In order to carry the force away from the attachment point effectively, the inner boom reinforcement should be separately fastened to the outer boom section, typically with a small number of rivets or additional through-bolts.

Generally, the critical issue in a bolted hole is the bearing capability of the host material, and not the strength of the bolt. However, if the bolt hole is not a tight fit, the bolt strength is also substantially reduced. To bolt boom sections together, it is always good practice to use two bolts at right angles to each other, to reduce distortion and to provide redundancy. In the design of a plate clamp, bolt holes should be located so the hole *center* is a minimum of two hole diameters away from any plate edge; I prefer holes spaced three diameters from center to edge. Stainless-steel bolts are ideal for shear applications, but will gall if tightened in compression applications unless carefully lubricated. I use plated grade 5 steel bolts for boom-to-mast and mast-to-rotator clamps where the bolts are fully torqued.

The principal bending-moment consideration is the determination of clamp strength through calculation of the section modulus of the clamping structure in combination with the element center section. If the element is not continuous across the clamping structure (for example, the Hy-Gain attachment structure or a split driven element), the clamp itself must have adequate section modulus to carry the moment at the element center. In the case of an insulated mounting, the clamp must also be designed to constrain the insulator and element center-section ends.

On the other hand, if the element center section is carried across the clamping structure (as in the use of a plate with U bolts described by Lawson[1] and others), the requirements on the clamping structure are greatly reduced because the attachment structure only adds its strength to the already adequate element center section.

Sec 5.2 Calculation of Section Modulus and Maximum Moment

The calculation of the section modulus of a cylindrical section is straightforward. The same equation used in the determination of section modulus of an element section is used. To review, from Chapter 3 the section modulus of a cylindrical tube is given by Eq 3-7 as S = I/c. In the case of a compound shape such as a hollow cylinder, the moment of inertia of the cylinder is the moment of inertia of the outer diameter D

minus the moment of inertia of the inner diameter d. As given in Eq 3-8, $I = \pi D^4/64 - \pi d^4/64 = \pi(D^4 - d^4)/64$.

Because the distance to the point of maximum stress is c = D/2, the section modulus of a hollow cylindrical cross-section, as given by Eq 3-9, is $S = \pi(D^4 - d^4)/(32D)$, where S is the section modulus in $in.^3$, and D and d are the outer and inner diameters in inches.

The moments of inertia for the rectangular plate, another shape of interest, are

$$I_t = \frac{Wt^3}{12} \qquad \text{(Eq 5-1)}$$

and

$$I_W = \frac{W^3t}{12} \qquad \text{(Eq 5-2)}$$

where

I_t = moment of inertia in the height direction of the rectangle
I_W = moment of inertia in the width direction
t = thickness of the rectangular shape, in.
W = width of the rectangular shape, in.

As given by Eqs 3-11 and 3-12, the section modulus of a rectangular plate of thickness t and width W is $S_t = Wt^2/6$ and $S_W = W^2t/6$, where S_t is the section modulus in the thickness direction and S_W is the section modulus in the width direction. The calculation of the section modulus of more complex shapes is difficult, but the estimates derived from these two simple shapes serve to provide a conservative calculation of attachment clamp strength. As we have seen, the modulus of a complex shape such as a hollow tube is calculated by subtracting the moment of inertia of the inside shape from that of the outside shape.

The maximum moment supportable by any structural shape is given by Eq 3-14 as $M_{max} = S\sigma_{max}$, where M_{max} is the maximum moment and σ_{max} is the material yield strength. It should be noted that, while the yield strength of drawn tubing of the commonly used 6063-T832 aluminum alloy is 35000 $lb/in.^2$, the corresponding rating for flat plate is only 30000 $lb/in.^2$. The other commonly used alloy, 6061-T6, retains its 35000 $lb/in.^2$ rating in both forms. Stamped clamp shapes are typically made from more malleable alloys; the element clamps used on the Hy-Gain antennas are made with 3003-H14, a 17000 $lb/in.^2$ alloy.

In order to know the required strength of the element clamp, we need to know the bending moment at the survival speed of the element to be supported, and whether the clamp is required to support the entire bending moment of the element. If the element clamp supports the entire bending moment of the element, the maximum moment supportable by the clamp should be at least equal to the maximum moment supportable by the element at its survival wind speed v_{max}. The equations can be assembled into a simple spreadsheet for calculation, and the results for S_{clamp} and σ_{max} can be further integrated directly into the element spreadsheet derived in Chapter 4 to make use of the calculated value M_a, the area moment of the particular element.

Sec 5.3 Effect of Wind Angle

Many element clamp designs are quite strong in the horizontal direction, but may fail in the vertical direction. To consider the effect of wind at an angle to the horizontal, we can resolve a wind-force moment M into vertical and horizontal components, as shown in Fig 5-1. The horizontal component (width axis) of the moment is given by

$$M_W = M\cos\theta \qquad \text{(Eq 5-3)}$$

where

M = moment, in.-lb

θ = wind angle from the horizontal

The vertical component (thickness axis) of the moment is given by

$$M_t = M\sin\theta \qquad \text{(Eq 5-4)}$$

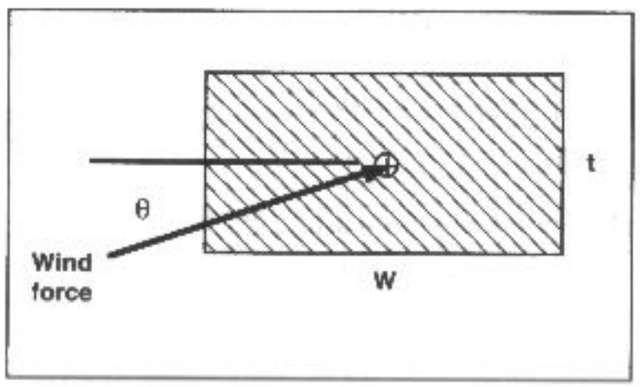


Fig 5-1—Wind force at an angle to a rectangular cross-section.

The stress from the horizontal component is given by

$$\sigma_W = \frac{M_W}{S_W} = \frac{M\cos\theta}{S_W} \qquad \text{(Eq 5-5)}$$

and the stress from the vertical component is given by

$$\sigma_t = \frac{M_t}{S_t} = \frac{M\sin\theta}{S_t} \qquad \text{(Eq 5-6)}$$

For a rectangular shape, the total stress is the sum of the two components

$$\sigma = \frac{M\cos\theta}{S_W} + \frac{M\sin\theta}{S_t} = M\left(\frac{\cos\theta}{S_W} + \frac{\sin\theta}{S_t}\right) \qquad \text{(Eq 5-7)}$$

Hence the equivalent section modulus S_θ is defined by

$$S_\theta = \frac{1}{\frac{\cos\theta}{S_W} + \frac{\sin\theta}{S_t}} \qquad \text{(Eq 5-8)}$$

and we can calculate M_{max} by the equation

$$M_{max} = S_\theta \sigma_{max} \qquad \text{(Eq 5-9)}$$

For clamps that are weaker in the vertical direction, relying on the assumption that the maximum wind is always horizontal leaves no safety margin against unexpected gusting conditions. For maximum safety factor, the clamps should be capable of withstanding the maximum expected wind speed in any direction, as gusts may take unexpected angles. Ideally, a clamp should have moment capability even in the vertical (clamp thickness) plane exceeding the greatest expected element moment.

It is appropriate to determine the survival capability of a clamp with a rectangular cross-section, even if it would not survive a vertical wind equal to the peak expected horizontal wind. To make this determination, we must choose an angle θ at which to calculate the clamp survival strength.

Over level ground, we could expect the maximum angle for full-velocity gusts to be in the range of $\theta = \arctan 0.3 \approx 17°$, as the gust factor is approximately 30%, or 0.3. If the antenna is sited over sloping foreground, the ground-slope angle must be added to this angle. As an

example, with a ground slope of 13° we might expect full-velocity wind gusts at angles up to $\theta = 17 + 13 = 30°$. Accordingly, the choice of θ is seen to be site dependent. It seems that the choice of $\theta = 15°$ over flat ground or $\theta = 30°$ over sloping ground will give a reasonable indication of the survival capability of clamps with a rectangular cross-section.

Sec 5.4 Element Moments

The element clamp must have the ability to withstand the moment of force of the element half it supports. In order to give an idea of the magnitude of this moment, Tables 5-1 through 5-4 summarize the wind area, the moments of force, and the area of the elements analyzed in Chapter 4.

The area is stated for the half element; the total area is, of course, double this number. The moment of force M is the moment of area M_a multiplied by the wind pressure at v_{max}, as given by

$$M = c_d p_o v_{max}^2 M_a \qquad \text{(Eq 5-10)}$$

It is significant to note that in Tables 5-1 through 5-4, the tabulated values were calculated to several decimal places but are rounded to whole numbers for presentation. Applying Eq 5-10 to the rounded values for v_{max} and M_a will yield somewhat different moments than those indicated.

Table 5-1
Comparison of 20-Meter Half Elements

Reference, Fig No.	*v_{max}*	*Area Moment M_a*	*Moment @ v_{max}*	*Area*
Ant. Book, 4-6A	71 mi/h	17294 in.^{3}	1630 in.-lb	182 in.^{2}
K8CC, 4-10A	73	21198	2086	214
K8CC+, 4-10B	92	20698	3220	213
W2PV, 4-11A	94	16315	2704	186
204BA, 4-12A	100	16581	3054	190
Ant. Book+, 4-6B	103	17319	3440	188
K8CC++, 4-10C	104	17062	3440	191
W2PV+, 4-11B	109	16383	3638	189
204BA+, 4-12B	112	16581	3874	190
W6QHS-20, 4-13A	117	17910	4527	200
W6QHS-20+, 4-13B	122	17153	4749	197
204BA++, 4-12C	124	16581	4695	190
W6QHS-20++, 4-13C	124	16809	4801	195

Table 5-2
Comparison of 15-Meter Half Elements

Reference, Fig No.	*v_{max}*	*Area Moment M_a*	*Moment @ v_{max}*	*Area*
Ant. Book, 4-15A	81 mi/h	8100 in.^{3}	999 in.-lb	117 in.^{2}
155BA, 4-18A	87	6498	906	103
Ant. Book rugged, 4-16A	91	8730	1338	129
K8CC, 4-17A	96	7773	1331	117
155BA+, 4-18B	100	6498	1202	103
Ant. Book+, 4-15B	105	7920	1616	114
155BA++, 4-18C	120	6498	1712	103
Ant. Book rugged+, 4-16B	121	8168	2241	122
K8CC+, 4-17B	123	7897	2241	121
W6QHS-15, 4-19A	129	7183	2241	113
W6QHS-15+, 4-19B	130	6377	1999	102

The optimum element design maximizes v_{max} while minimizing the area and the moment of area. If designed by shortening the center sections as required by electrical considerations, the driven and director elements will have the same v_{max} with smaller area and moments. Table 5-1 is a comparison of 20-meter element designs from Chapter 4, Table 4-6. These element lengths are representative of the longest element required for a Yagi antenna.

Note from Table 5-1 the substantial range of areas and moments, depending upon the degree of taper. The most efficient designs have areas in the range of 190 in.^{2} per side and moments of area in the range of 17000 in.^{3}. This results in moments of force of 3000 to 3400 in.-lb at wind speeds of 100 mi/h, 4100 in.-lb at 112 mi/h, and 4500 to 4800 in.-lb at wind speeds of 122 mi/h.

For the 15-meter designs of Table 5-2, it is possible to keep element-half areas in the range of 103 in.^{2}, and area moments in the range of 6500 in.^{3}. This results in element moments of force of 1200 in.-lb at 100 mi/h and 1800 to 2200 in.-lb at 122 mi/h.

Table 5-3
Comparison of 10-Meter Half Elements

Reference, Fig No.	V_{max}	*Area Moment* M_a	*Moment @* V_{max}	*Area*
Ant. Book, 4-20A	101 mi/h	3726 in.^{3}	709 in.-lb	72 in.^{2}
K8CC, 4-21A	107	3797	802	76
105BA, 4-22A	110	2538	575	60
105BA+, 4-22B	126	2538	737	60
Ant. Book+, 4-20B	127	3735	1120	72
Ant. Book rugged, 4-23A	133	4086	1338	87
Ant. Book rugged+, 4-23B	145	3420	1338	75
K8CC+, 4-21B	154	3695	1631	97

Table 5-4
Comparison of Triband and 40-Meter Half Elements

Reference, Fig No.	V_{max}	*Area Moment* M_a	*Moment @* V_{max}	*Area*
TH5	100 mi/h	16529 in.^{3}	3093 in.-lb	200 in.^{2}
TH7	100	16210	2978	199
TH5+, Table 4-9	124	16529	4735	200
TH7+, Table 4-10	125	16210	4752	199
40-2CD, Fig 4-25A	74	25180	2588	241
40-2CD+, Fig 4-25B	100	25180	4667	241
40-2CD++, Fig 4-25C	118	25180	6568	241

Survival speeds for the 10-meter elements summarized in Table 5-3 are naturally higher because of the shorter lengths and smaller diameters. The most efficient designs have areas in the range of 60 in.^{2}, and area moments in the range 2600 in.^{3}. This results in moments of force of 700 to 900 in.-lb for wind speeds in the 125- to 140-mi/h range.

It is interesting to note that the triband elements have areas and moments in the same range as 20-meter monoband elements. The representative shortened 40-meter elements of Table 5-4 have moments approaching 6600 in.-lb at 118 mi/h, and areas in the range of 240 in.^{2}.

With this data in mind, we can look at the survival capability of several representative element-to-boom clamps.

Sec 5.5 The Hy-Gain Clamp

The shape of the Hy-Gain clamp is that of a cylinder with two rectangular flanges. The mechanical configuration is shown in Fig 5-2. Note that t is defined here as the thickness of the material, which is half the overall thickness of the flanges.

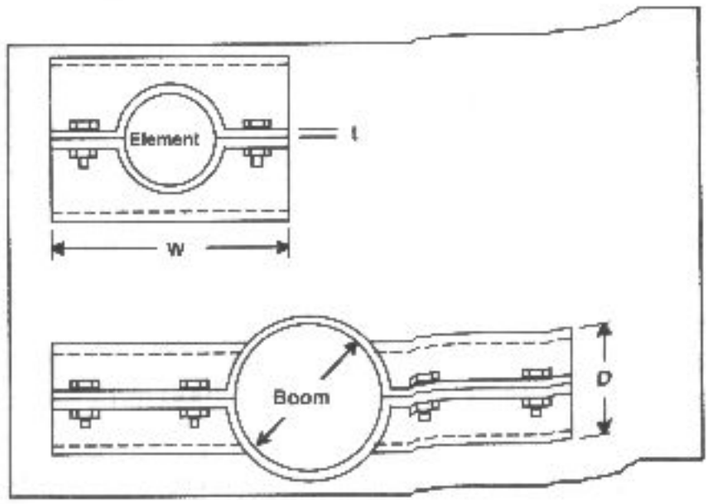


Fig 5-2—Hy-Gain element-clamp dimensions.

The moment of inertia of the cylindrical portion is

$$I = \frac{\pi [\, D^4 - (D - 2t)^4 \,]}{64} \qquad \text{(Eq 5-11)}$$

where

D = outer diameter of the cylindrical portion, in.
t = material thickness, in.

The moments of inertia of the two flanges, for the thickness and width directions, are

$$I_t = \frac{(W - D)\,(2t)^3}{12} \qquad \text{(Eq 5-12)}$$

and

$$I_W = \frac{(W^3 - D^3)\,(2t)}{12} \qquad \text{(Eq 5-13)}$$

The section modulus in each direction is the sum of the moments of inertia divided by the maximum radius. These are

$$S_t = \frac{I_t + I}{\frac{D}{2}} \qquad \text{(Eq 5-14)}$$

and

$$S_W = \frac{I_W + I}{\frac{W}{2}} \qquad \text{(Eq 5-15)}$$

The maximum moment in each axis is

$$M_t = S_t \sigma_{max} \qquad \text{(Eq 5-16)}$$

and

$$M_W = S_W \sigma_{max} \qquad \text{(Eq 5-17)}$$

The results of these section modulus and maximum moment calculations are summarized in Table 5-5 for σ_{max} = 17000 lb/in.^{2}, the yield strength of 3003-H14 alloy.

Note that the section modulus and maximum moment are considerably greater in the width direction, because of the extra strength of the flanges. If we model the clamp as a rectangle, the values of M_{max} as a function of θ for the clamps in Table 5-5 are shown in Fig 5-3.

Note that the range of moment capability is substantial, ranging in the example of the Hy-Gain 204 clamp from 2900 in.-lb to 8000 in.-lb, depending on the angle θ of the wind force. The values of M_{max} for θ = 15° and θ = 30° are tabulated as $M_{15°}$ and $M_{30°}$ in Table 5-5.

Now that we have these parameters, we can compare the mechanical capability of the mounting clamp to the moment of the element. For the elements treated in Chapter 4, we can find in Tables 5-1 through 5-4 the element moment of area or the bending moment at element survival speed. If we know the element moment of area M_a and the clamp maximum moment capability $M_{30°}$, we can determine the clamp survival speed from

$$v_{max} = \sqrt{S_{30°}\,\sigma_{max} / (c_d p_o M_a)} = 231.8\sqrt{M_{30°} / M_a} \qquad \text{(Eq 5-18)}$$

If we know the v_{max} we need, we determine

Table 5-5
Mechanical Parameters of Hy-Gain Element Clamps

	204 DR	*204, 105/155 DR*	*105/155*
Size	No. 14	No. 13	No. 4
Part Number	165920	165919	163764
L, in.	3.75	2.625	2.5
W, in.	3.25	3.25	2.5
D, in.	1.75	1.5	1.045
t, in.	0.125	0.125	0.085
S_W, in.^{3}	0.502	0.476	0.188
M_W, in.-lb	8534	8092	3195
S_t, in.^{3}	0.244	0.175	0.058
M_t, in.-lb	4148	2968	988
$M_{15°}$, in.-lb	5697	4841	1772
$M_{30°}$, in.-lb	4507	3630	1287

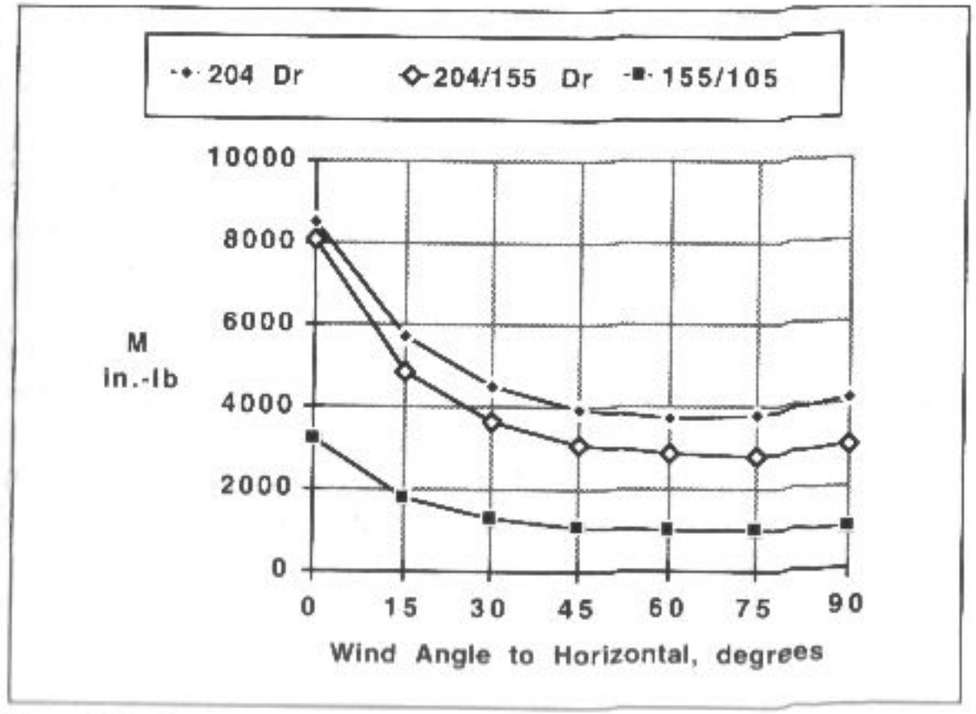


Fig 5-3—Maximum moment of Hy-Gain element clamps.

$$M_{30^\circ} = c_d p_o M_a v_{max}^2 = M_a \left(\frac{v_{max}}{231.8} \right)^2 \qquad \text{(Eq 5-19)}$$

Taking as an example the 204BA no. 13 clamp as used also on the 205BA, TH5, and TH7, we can see that the supportable moment M_{15° of 4841 in.-lb is roughly equal to the 4800 in.-lb bending moment of the longest element under horizontal wind conditions in the 122-mi/h range. Survival at these wind speeds depends upon the degree of vertical gusting, as the moment capability of the clamp at $\theta = 30^\circ$ is not adequate for the expected element moments at wind speeds above the manufacturer's design goal of 100 mi/h survival. These clamps have survived the highest mean wind speeds at my site, which I estimate to be between 112 and 122 mi/h. If we apply the equations above, for the 204BA++ we have $M_a = 16581$ in.^{3} from Table 5-1, and $M_{30^\circ} = 3630$ in.-lb from Table 5-5, so from Eq 5-18,

$$v_{max} = 231.8 \sqrt{3630/16581} = 108 \text{ mi/h}$$

If we want to know the M_{30° required for $v_{max} = 122.5$ mi/h, we have $M_a = 16581$ in.^{3} from Table 5-1 and we desire $v_{max} = 122.5$ mi/h, so from Eq 5-19 we determine

$$M_{30^\circ} = 16581 \left(\frac{122.5}{231.8} \right)^2 = 4631 \text{ in.-lb}$$

This suggests that, if we want to be absolutely secure, we should adapt the driven-element clamp for the reflector of these antennas. The driven-element clamp has a 1.5-in. inner diameter, so we must use a 3.75-in. length of 1.5 × 0.120-in.-wall tubing as a bushing to mount the 1.25-in. element center.

The calculated values for the Hy-Gain antennas used as examples are shown in Table 5-6. From this table we determine that the clamps are more than adequate if we do not have much vertical gusting. If instead we ask for the value of M_{30° required for $v_{max} = 122.5$ mi/h, we find that we need to use the heavier driven-element clamps, as summarized in Table 5-7.

We see that the 155BA and 105BA clamps are within their limits at their 100-mi/h rated wind speed, but are marginal at 122-mi/h wind speeds. I have not experienced bending of these clamps in the wind speeds experienced at my site. Note that the Hy-Gain elements are particularly well designed to minimize wind area and bending moment

Table 5-6
Wind Speed v_{max} of Hy-Gain Element Clamps

	204BA	*155BA*	*105BA*
M_{15° in.-lb	4841	1772	1772
M_{30° in.-lb	3630	1287	1287
M_a in.^{3}	16581	6498	2538
$v_{max,15^\circ}$ mi/h	125	121	194
$v_{max,30^\circ}$ mi/h	108	103	165

Table 5-7
Wind Speed v_{max} of Hy-Gain Driven-Element Clamps

	204DR	*155DR*
M_{30° req'd in.-lb	4631	1815
M_{15° in.-lb	5697	4841
M_{30° in.-lb	4507	3630
M_a in.^{3}	16581	6498
$v_{max,15^\circ}$ mi/h	136	200
$v_{max,30^\circ}$ mi/h	121	173

because of their taper and small-diameter tip sections. The element design is also efficient because the shorter driven and director elements are developed by shortening the center sections rather than the tip sections.

By calculation, and from personal experience, the Hy-Gain 204BA no. 13 clamp will not provide an adequate safety factor in a Zone C environment if applied to 40-meter elements. The calculated values for the 40-meter element and clamp are given by $M_a = 25000$ in.^{3} from Table 5-4, and from Table 5-5, $M_{30^\circ} = 3630$ and $M_{15^\circ} = 4841$ in.-lb, so

$$v_{max,30^\circ} = 231.8 \sqrt{3630/25000} = 88 \text{ mi/h}$$

$$v_{max,15^\circ} = 231.8 \sqrt{4841/25000} = 102 \text{ mi/h}$$

Even with the less conservative selection of $\theta = 15°$, the parasitic-element clamps are marginal at the maximum wind-speed capabilities of the reinforced TH5, TH7 and 204BA reflectors. In each of these cases, consider using the larger driven-element clamp, with an appropriate short aluminum-tube bushing (spacer) to hold the element center section. A possible alternative for reflector clamps is to drill right across the boom end and continue the element internal reinforcement from one element half through to the other.

We can incorporate the clamp strength calculation directly into the element spreadsheet developed in Chapter 4. Following Lawson's lead, I denote the clamp as Section 0. By using the spreadsheet IF( ) function, the appropriate section modulus for a given clamp as listed in Tables 5-6 and 5-7 can be inserted into the modulus row of the spreadsheet, and the calculation of v_{max} will automatically incorporate the value of M_a for the element under consideration.

To prevent galling or tearing the aluminum clamp material, use small-diameter (SAE) stainless-steel flat washers under the clamping bolts and nuts. Lubricating the bolts and nuts before assembly with any of the popular spray lubricants will prevent seizing of the stainless steel.

Sec 5.6 The Plate and U-Bolt Clamp

Element mounting on a flat plate with U bolts around the element and the boom is a successful practice of long standing. An example is shown in Fig 5-4.

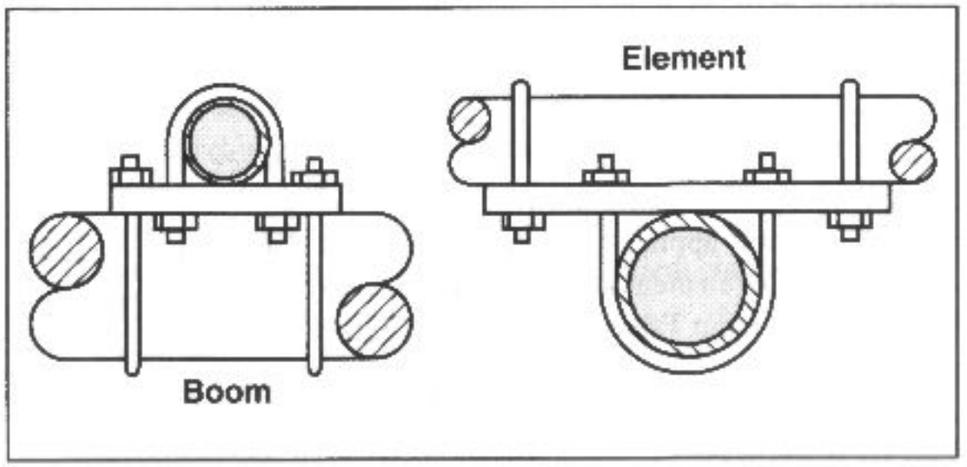


Fig 5-4—Plate element clamp.

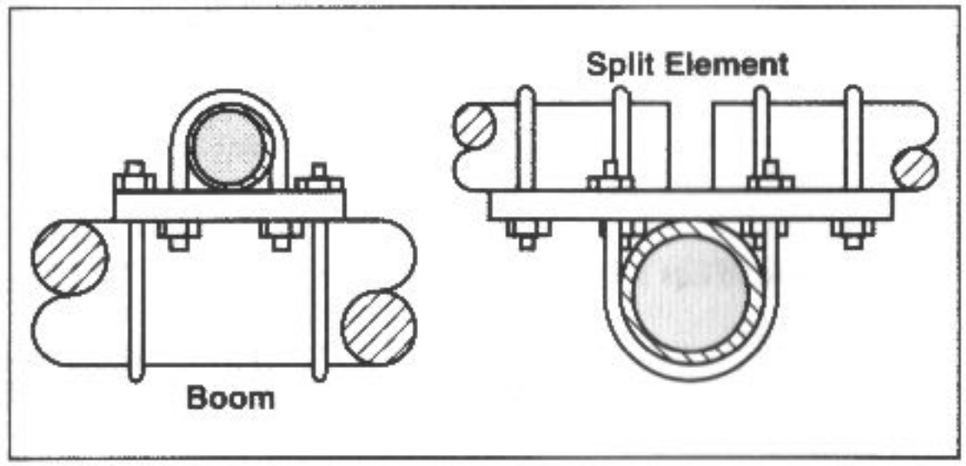


Fig 5-5—Split-element plate clamp. Insulating material, if used, is not shown.

For the case of a mounting plate as described by Orr[2] and also by Lawson,[3] the plate is not required to withstand the moment of the element, inasmuch as the element itself is carried through and is deemed to be strong enough to survive the environment for which it is designed. Even so, the plate needs to be sufficiently thick, wide and long to maintain the bearing stress on the plate holes, the shear stress on the U bolts, and the buckling stress on the plate. The details of the analysis of rectangular beams with moments at an angle to an axis of symmetry are found in structural engineering texts.[4] A guide to issues of buckling of plates is found in the Alcoa Handbook.[5] When in doubt, reinforce.

From the same analysis of wind angle as above, we are interested in the maximum moment $M_{30°}$ that can be supported by the plate at a wind angle $\theta = 30°$. In the case of a continuous element carried through the clamp, we arbitrarily choose $M_{30°} = M_{max}/3$. For the split-element plate clamp, shown for simplicity in Fig 5-5 without any insulation of the element halves, we choose $M_{30°} = M_{max}$.

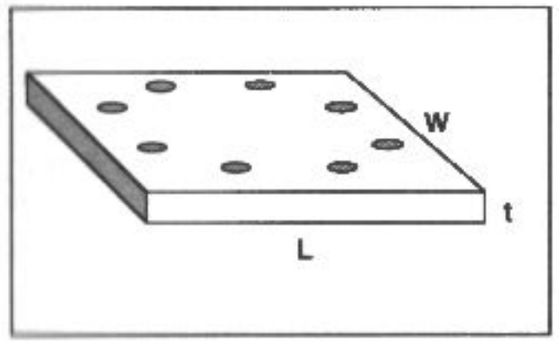


Fig 5-6—Clamp plate dimensions.

The moment capability of the rectangular plate shown in Fig 5-6 at $\theta = 30°$ can be calculated

from the section modulus in the thickness and width directions by the equation

$$M_{30^\circ} = \frac{\sigma_{max}}{\frac{\sin 30^\circ}{S_t} + \frac{\cos 30^\circ}{S_W}} \qquad \text{(Eq 5-20)}$$

Substituting from Eqs 3-11 and 3-12, we have

$$M_{30^\circ} = \frac{\sigma_{max}\, Wt}{\frac{3}{t} + \frac{5.2}{W}} \qquad \text{(Eq 5-21)}$$

With this information, we can examine existing element clamps and obtain some useful plate sizes for the elements analyzed in Chapter 4. See Table 5-8.

Limitations on a flat plate mount include the localized stress on the boom and element, which can be mitigated by reinforcing either the inside or the outside of both tubular members to prevent buckling. It is a good practice always to reinforce a tube in compression with an insert

Table 5-8
Mechanical Parameters of Plate Element Clamps

	20	*20+*	*15*	*15+*	*10*	*10+*
M_{max}, in.-lb	3500	4800	1400	2200	800	1400
Through element:						
W, in.	3	3	2	2	2	2
t, in.	0.25	0.25	0.25	0.25	0.25	0.25
M_{30°, in.-lb	1911	1911	1199	1199	1199	1199
Split element:						
W, in.	3.5	3.5	3	3.5	2	2.5
t, in.	0.3125	0.375	0.25	0.25	0.25	0.25
M_{30°, in.-lb	3453	4843	1911	2271	1199	1554
M_a, in.^{3}	17319	17910	7920	7897	3726	4086
$v_{max,30^\circ}$, mi/h	104	121	114	124	131	143

that fills the tube, so the tube itself is not crushed or deformed. In a split driven element, insulating reinforcement such as fiberglass rod can be carried across the gap to strengthen the element further.

Also of interest is the question of survival of clamps for the shortened 40-meter Yagi, the 40-2CD. Table 5-9 gives plate sizes that should be satisfactory for the element designs derived in Chapter 4. The values of $v_{max,30^\circ}$ do not include the strength of the driven-element insulator, and so are conservative.

For a clamp with a continuous element and two U bolts, the moment in the element section between the U bolts is constant at the level just outside the U bolts, and the force on each bolt equals the wind force on the element half. For a clamp holding each element half independently with two U bolts on each side, the force at each bolt is the element moment leveraged over the bolt spacing, and the moment decreases from the moment at the outer bolt to zero at the inner bolt. In this case, the force F in pounds on each U bolt is

Table 5-9
Mechanical Parameters of 40-Meter Element Clamps

	40-2CD	40-2CD+	40-2CD++
Through element:			
M_{max}, in.-lb	2585	4627	5879
W, in.	3.5	3.5	3.5
t, in.	0.25	0.25	0.25
M_{30°, in.-lb	2271	2271	2271
Split element:			
M_{max}, in.-lb	2251	4372	5506
W, in.	3.5	3.5	3.5
t, in.	0.25	0.375	0.4375
M_{30°, in.-lb	2271	4843	5769
$v_{max,30^\circ}$, mi/h	70	102	117

$$F = \frac{M}{s} \qquad \text{(Eq 5-22)}$$

where

M = moment, in.-lb

s = spacing between the two U bolts on the same element half

As an example, for s = 3 in. and M = 4800 in.-lb, the force F is 4800/3 = 1600 lb divided between the two legs of the U bolt. The force for each leg is then F/2 = 800 lb. The plate can withstand a maximum bearing force F_b determined by

$$F_b = P_b t d \qquad \text{(Eq 5-23)}$$

where

P_b = maximum bearing pressure of the material
($P_b \approx 56000$ lb/in.^{2} for aluminum)

t = plate thickness, in.

d = bolt diameter, in.

To continue the example above, consider a plate of t = 0.375 in. and U bolts of d = 0.3125 in. From Eq 5-23 the maximum bearing force per hole is $56000 \times 0.375 \times 0.3125 = 6562.5$ lb. This is safely above the 800 lb per bolt calculated above. Note, however, that the element and boom tubes must withstand the 1600-lb force over the concentrated area of the U bolt; substantial wall thickness is required here to preclude buckling. In practice, I prefer to fill the reinforced element tube entirely with a fiberglass or wood dowel at the clamping points.

I strongly recommend spending a few moments with a flat file to round the corners and edges of a mounting plate. This will prevent the otherwise inevitable cuts and bruises you'll receive while working with the antenna on the tower. Mounting plates for commercial VHF and UHF repeater antennas incorporate this feature.

Sec 5.7 The Channel Clamp

As can be seen from Table 5-9, the thickness of a plate clamp is a determining factor in its strength in the vertical direction. A structural shape that provides strength in both directions without the penalty of excessive weight is the aluminum channel. This is similar to the plate with bent edges shown by Orr.[6] An example is shown in Fig 5-7.

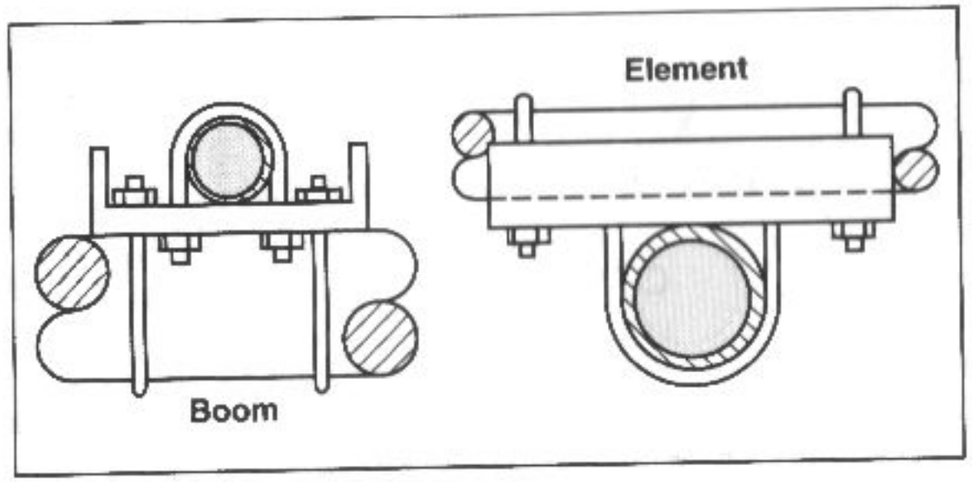


Fig 5-7—Channel element clamp.

Table 5-10
Mechanical Parameters of Channel Element Clamps

Sizes are American Standard Channel

	3-in.	*4-in.*
W, in.	3	4
t, in.	0.170	0.180
S_w, in.^{3}	1.10	1.92
M_w, in.-lb	38500	67200
S_t, in.^{3}	0.2	0.28
M_t, in.-lb	6000	8400
$M_{30°}$, in.-lb	10647	15648

The section modulus and other parameters are tabulated for commonly available structural shapes in the Alcoa Handbook.[7] Table 5-10 summarizes the strength parameters of 3-in. and 4-in. channel used as element mounting clamps. Note the quite substantial strength.

Sec 5.8 Other Structural Fixtures

A potentially useful fixture for element-to-boom clamping is made commercially for constructing architectural railings from standard pipe sizes.[8] This clamp, which is quite attractive in appearance, is

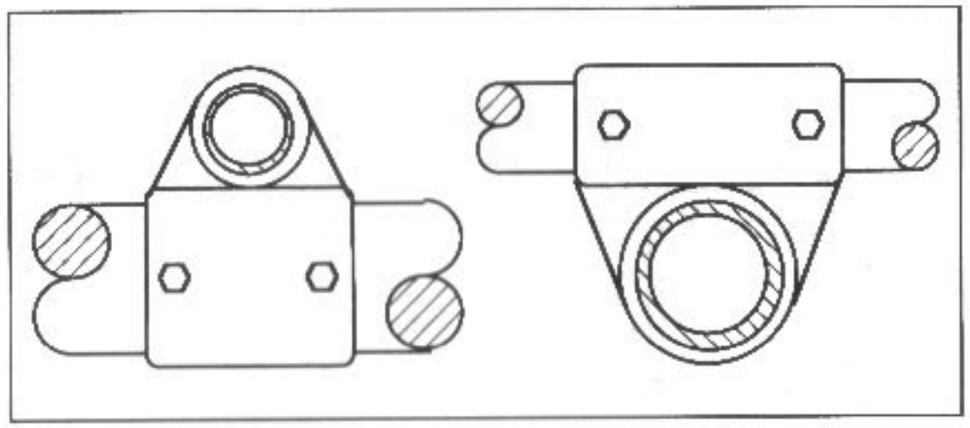


Fig 5-8—Pipe-railing element clamp. (See reference 8.)

similar to those used on commercial VHF Yagis. Shown in Fig 5-8, it can be used for tubing sizes by inserting a spacer made of standard-diameter aluminum pipe, with the appropriate inside diameter to take the element or boom tubing. (Recall that pipe dimensions are inside dimensions, while tubing is specified by outside dimensions.) This clamp is ideal for application with booms made from pipe.

Sec 5.9 The Insulated Plate Clamp

There are several methods of retaining an insulated split element. If the plate is strong enough, the mounting method shown in Fig 5-9 allows easy access to the very end of the element center section. This prevents the problem existing in enclosed insulated elements, where the

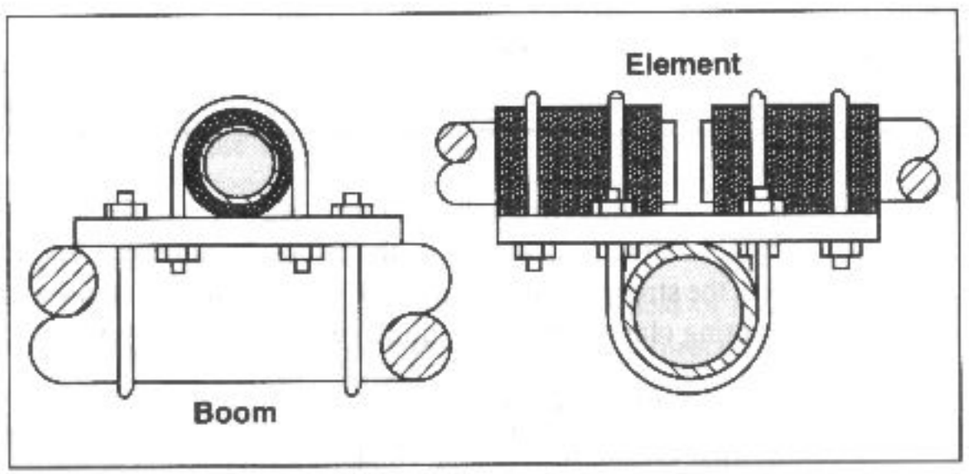


Fig 5-9—Split insulated element.

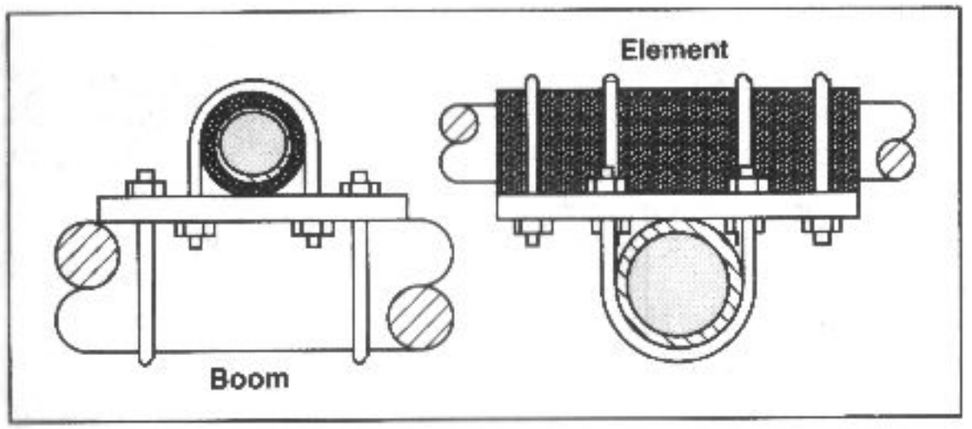


Fig 5-10—Full-tube insulated element.

necessary lead inductance to span from balun to element can be a substantial part of the antenna driven impedance.

An interesting insulated plate mount is used on the Cushcraft 40-2CD. This mount uses a thick-wall length of polycarbonate plastic pipe to contain the two driven-element halves, as shown in Fig 5-10. The mechanical strength of the driven-element mount is listed in Table 5-9 as 2271 in.-lb at $\theta = 30°$, compared to expected element moments in the range of 2600 to 5900 in.-lb (74 to 112 mi/h). Users who anticipate high wind or ice loading can increase the plate thickness to 0.375 in. or even 0.4375 in. to provide a vertical section modulus more nearly equal to that of the reinforced element centers. The horizontal modulus is, of course, much larger because of the width of the plate in that plane. The Cushcraft mount has provisions to access the elements through the insulating tube, but has the disadvantage that the connecting bolts clamp the insulator itself in compression. If the insulating material flows or deforms, mechanical contact is lost; it is better to clamp only to the element itself, and also to place a rod or dowel inside the element at the point of compression.

The strength of the 2-in.-diameter insulating junction of the driven element in Fig 5-10 is an unknown. The junction between the two element halves can be strengthened with fiberglass rod inside the aluminum tubing. A further recommendation is the use of additional internally threaded aluminum-rod spacers on the boom U bolts to contain the insulating pipe on each side, along with polycarbonate or metal caps to contain the inner ends of the element. This mount modification is shown in Fig 5-11.

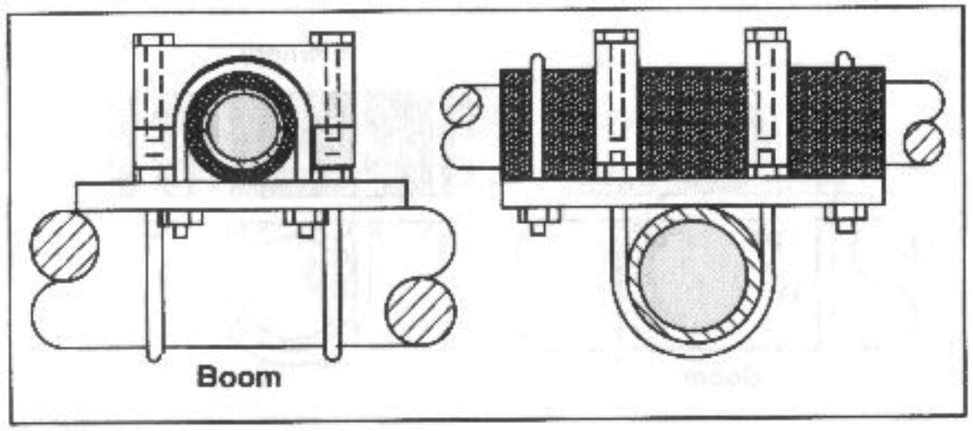


Fig 5-11—Strengthened mount for Cushcraft 40-2CD element.

Another potential approach to this general type of mount is to contain the element ends in polycarbonate or ABS insulating pipe, which in turn is contained within heavy-wall aluminum pipe U-bolted to a plate, as shown in Fig 5-12. The short sections of low-impedance transmission line thus formed are not part of the radiating structure, and their effect can be determined by analytical methods described in Sec 9.4 of Chapter 9. The suitability of a particular piece of plastic pipe for radio-frequency insulation can be evaluated quickly by placing a sample in a microwave oven; if it gets quite hot or melts, try another type or another manufacturer. Put the sample on a paper plate or paper towel and use a cup of water as a load for the magnetron.

If the element feed is not attached at the very inner end, a potential electrical problem with this type of mounting is the lead length needed

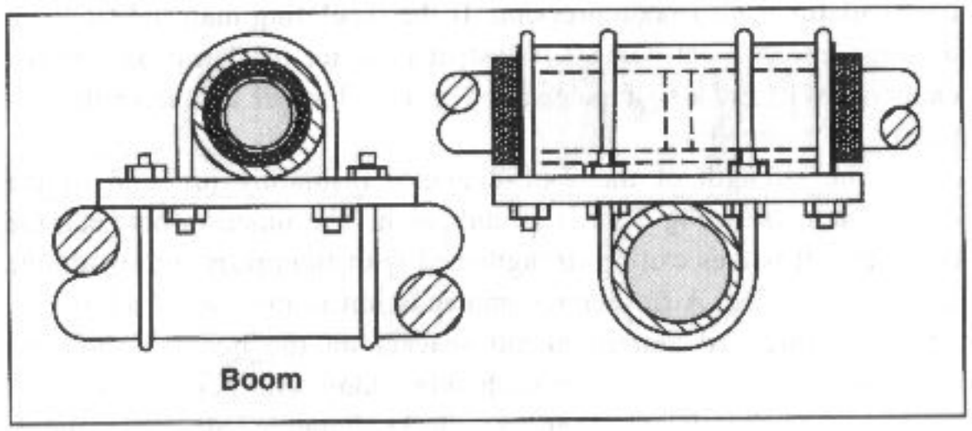


Fig 5-12—Pipe-mounted insulated element.

to reach the element beyond the insulating section. Long leads result in substantial series inductance in, for example, the Hy-Gain beta-match antenna center, which is examined in more detail in Chapter 9. The mounting method of Fig 5-12 is ideal for use with the pipe-railing clamps of Fig 5-8.

Sec 5.10 Recommended Element Attachment

The net objective of all these calculations is to be able to establish some concrete idea of which methods of element attachment are reliable under environmental stress, and how big the clamps need to be. From the calculations, it is apparent that the Hy-Gain element clamps are generally satisfactory for monoband antennas from 20 meters to 10 meters. The parasitic-element clamps are close to their strength limits for Zone C applications of triband antennas such as the TH5 and TH7, and are beyond their limits for Zone C application of shortened 40-meter elements. For these applications, you can use the larger driven-element clamp for reflector mounting.

From the calculations of mechanical strength, it is apparent that the plate sizes specified by W2PV[9] are somewhat larger than necessary. For continuous elements, the primary limitation on plate size is the bearing strength of the U-bolt holes and the buckling strength of the element and boom tubing. The use of aluminum channel is recommended for applications in which weight is a consideration, or in which extreme strength is required (for example, clamping 40- and 80-meter elements).

The plate clamps listed in Table 5-9 should suffice for 20- through 10-meter applications. A length of 4 in. per side for the larger clamps and 3 in. per side for the smaller ones provides adequate spacing for the U bolts around the element center section. The electrical characteristics of the clamps described here are treated in Chapter 9.

References

[1] J. L. Lawson, *Yagi Antenna Design* (Newington: American Radio Relay League, 1986).

[2] W. I. Orr, *Beam Antenna Handbook* (Wilton, CT: Radio Publications, 1955), pp 94-98.

[3] See reference 1.

[4] E. Popov, *Mechanics of Materials* (Englewood Cliffs, NJ: Prentice-Hall Inc, 1952), p 167.

[5] *Alcoa Structural Handbook* (Aluminum Corporation of America, 1956), pp 85-200.

[6] See reference 2.

[7] See reference 5, p 214.

[8] *1989 Catalog of Structural Fittings*, Hollaender Mfg Co, 10285 Wayne Ave, Cincinnati, OH 45215.

[9] See reference 1.

Chapter 6

Yagi Elements Arrayed on a Boom

Sec 6.1 Antenna Area and Wind Force

Now that we've looked at the elements themselves, it's time to see what happens when we array them on a boom. In order to do so, let's begin by looking at the effective area of the antenna. Next, it will be instructive to look at the rotational force (torque or moment) that the antenna places on the mast. Finally, we'll calculate the survival speed of the boom for unguyed and guyed configurations.

If the force from the wind area is too great, the mast or tower will suffer. If the mast torque is too great, the rotator will be unable to rotate the antenna in the wind or will be damaged. In this chapter, we'll see how to reduce this rotating torque to zero.

In order to calculate the wind force on part or all of a Yagi antenna, we need to know the effective wind area. The wind force acts only in the direction of the wind as defined in Fig 6-1; hence, the effective area of any part of the antenna is

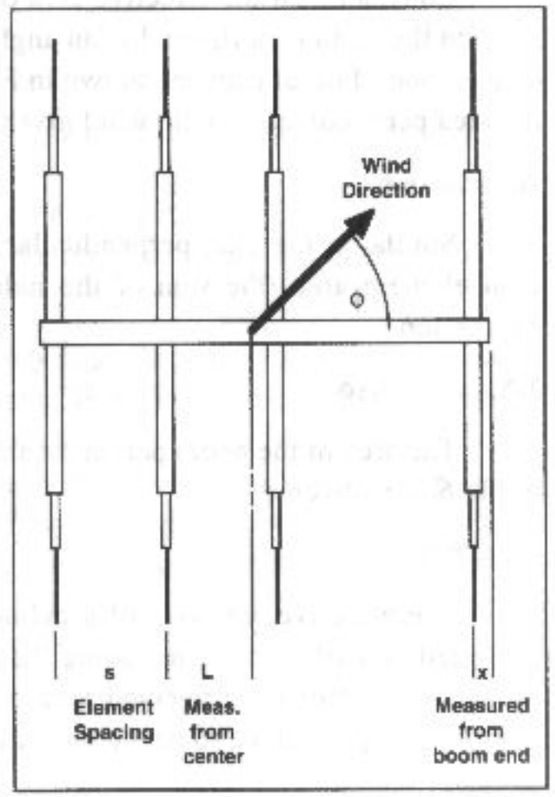


Fig 6-1—Wind angle ϕ and antenna dimensions defined.

a talk called "The Wind in the Yagis," which I gave at the Visalia International DX Convention in April 1989. I introduced the ideas of torque balance and rubber rotator isolators, and detailed the preliminary results of the spreadsheet analysis and the strengthening of several antennas I was using. The DX Convention was well attended, with DXers from Europe and all over the world. I was pleased to see several of the new ideas appear in subsequent computer programs and antenna talks.

Dave Siddall, K3ZJ, suggested that I expand the paper into a book, and mentioned the talk to Dave Sumner, K1ZZ, of ARRL HQ. Over dinner at the Sumner house with Paul Rinaldo, W4RI, the idea took shape for a comprehensive book on the mechanical and physical design of Yagi antennas.

The outstanding book, *Yagi Antenna Design*, by the late Jim Lawson, W2PV, and talks by Tom Schiller, N6BT, and Dean Straw, N6BV, alerted me to the new power of computer analysis. On-the-air discussions and visits over the years with Mike Schwab, OE6MBG, provided additional encouragement and an introduction to the use of the computer programs *YO* and *MN* by Brian Beezley, K6STI (which, along with *CT* contest software by Ken Wolff, K1EA, resulted in the broadening of our home computer lineup to include three PCs).

I was inspired to set my ideas in writing as a result of a memorable lunch that Mike Villard, W6QYT, arranged with Bill Orr, W6SAI, and Les Moxon, G6XN. As the concepts developed, I gave talks on various facets of the physical design of Yagis at the Dayton HamVention in 1990 and 1991, and at the Northwest DX Convention in 1990 in connection with the World Radiosport Team Competition (WRTC).

The objective of this book is to provide the tools to build robust physical Yagi antennas, without cut-and-try methods, from underlying cylindrical element electrical designs, by treating the following issues.

- Building standards and statistical models of wind forces on structures.
- Strength of structures, materials and fasteners.
- A mechanical strength spreadsheet developed to provide as direct output the wind survival speed for each section of a tapered element.
- Related strength spreadsheets for element attachment structures, guyed and unguyed booms, masts and impedance-matching networks.
- Introduction of torque-balancing elements and rubber rotator torque isolators.
- A new analysis of tapered element electrical design, leading to a new algorithm and spreadsheet to determine the electrically equivalent cylindrical element.
- Introduction of an accurate model for equivalent diameter of element clamps.
- Benefits of sites with sloping foreground.
- Illustrative physical design examples applying all of the techniques.

This book owes much to the efforts of Mark Wilson, AA2Z, and Chuck Hutchinson, K8CH, of ARRL HQ, and to the F. E. Terman engineering library of Stanford University. Review of draft manuscripts by Bill Myers, K1GQ, and Bill Shaheen, N1CQ, plus helpful comments and data from Roger Cox, WBØDGF, resulted in numerous improvements to the text. The patient and helpful editing of Jerry Hall, K1TD, is gratefully acknowledged.

This book, a weekend project that stretched to years, is dedicated to the memory of my late father, Herman L. Leeson, whose enthusiasm and patient sponsorship fostered my teenage introduction to Amateur Radio and a career in electronics.

Dave Leeson, W6QHS
Los Gatos, California
March 1992

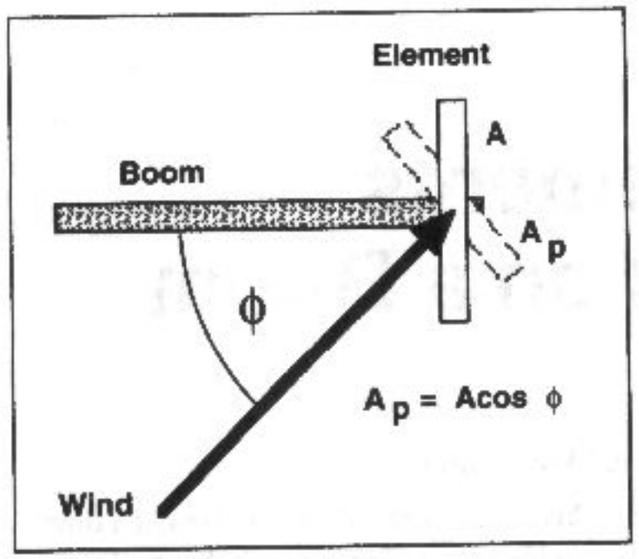


Fig 6-2—Element area perpendicular to wind direction.

the area of that part perpendicular to the wind. The wind force is the product of wind pressure and effective area, giving effect to wind angle ϕ and drag coefficient c_d.

Consider first the effective area of a cylindrical element of area A when the wind is horizontal at an angle ϕ to the lengthwise direction of the boom. For an element shown in Fig 6-2, the effective area A_p is the area perpendicular to the wind given by the expression

$$A_p = A\cos\phi \qquad \text{(Eq 6-1)}$$

Similarly, the total perpendicular element area is related to the total element area (the sum of the individual element areas) by the expression

$$\Sigma A_p = \Sigma A\cos\phi \qquad \text{(Eq 6-2)}$$

The area of the boom perpendicular to the wind direction, shown in Fig 6-3, is given by

$$A_p = A\sin\phi \qquad \text{(Eq 6-3)}$$

The effective area A_{eff} of a cylindrical element or boom is the projected area of a flat plate having the same wind force, and is given by the product of the projected area of the cylinder A and the drag coefficient c_d (assumed to be a constant).

$$A_{eff} = c_d A \qquad \text{(Eq 6-4)}$$

where $c_d \approx 0.67$

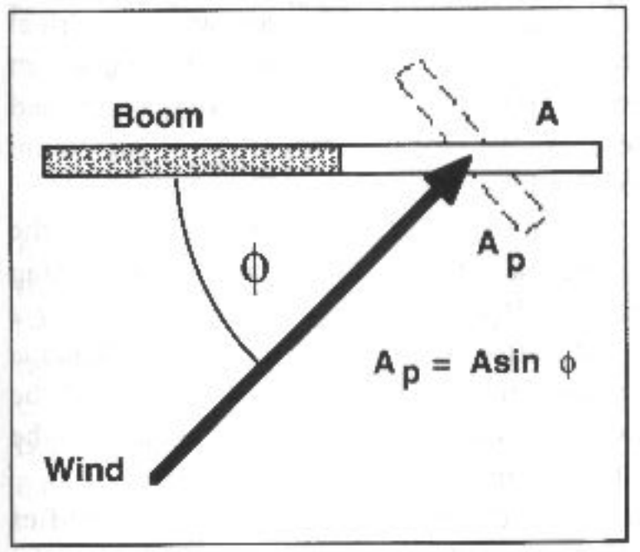


Fig 6-3—Boom area perpendicular to wind direction.

Combining these equations gives the effective wind area A_{eff} of a Yagi antenna as a function of wind direction ϕ, total element area A_{el} and boom area A_b, expressed as

$$A_{eff} = c_d(A_{el}\cos\phi + A_b\sin\phi) \qquad \text{(Eq 6-5)}$$

where

A_{el} = total element area (total of element section lengths times diameters)

A_b = boom area (length times diameter)

ϕ = angle of the wind to the boom as defined in Figs 6-1, 6-2 and 6-3

The variation of A_{eff} with ϕ depends on the relative magnitude of the element area and the boom area. For $\phi = 0°$, the effective area equals the element area (elements across the wind, boom along the wind). For $\phi = 90°$, the effective area equals the boom area (elements along the wind, boom across the wind). For wind angles between 0° and 90°, the effective area is greater than either the boom or element areas alone, and reaches a maximum given by

$$A_{eff} = c_d \sqrt{A_{el}^2 + A_b^2} \qquad \text{(Eq 6-6)}$$

at a wind angle given by

$$\phi = \arctan \frac{A_b}{A_{el}} \qquad \text{(Eq 6-7)}$$

To find the effective wind area of a Yagi antenna with cylindrical elements and boom, we calculate the area of each of the elements, sum them to find the total element area, calculate the area of the boom, and use Eq 6-6 for effective area. We calculate the angle of maximum effective area from the ratio of boom and element areas.

As with calculations for the elements and boom themselves, the wind force on the entire antenna is the product of antenna area, drag coefficient, and wind pressure. Because the air stream rejoins only a few diameters downstream, the effect of each of the cylindrical elements and the boom can be assumed to be independent of the rest of the structure. Accordingly, we sum the areas of the elements and of the boom without regard to shadowing.

For commercial antennas, the manufacturer generally specifies the effective maximum wind area of a given antenna, including the effect of drag coefficient of tubular elements and of worst-case wind direction, but excluding the effect of icing. To calculate the maximum wind force, we use the expression

$$F = pA_{eff} \qquad \text{(Eq 6-8)}$$

where

F = wind force

A_{eff} = effective maximum area

p = $0.004\,v_{max}^2$, the wind pressure specified for the maximum wind velocity expected

Note that convention is to include the drag coefficient in the effective area.

The expected worst-case wind- and ice-loading conditions can be estimated by the methods of Chapter 2, from querying the local power and telephone utilities, or from direct experience and measurement. Within reason and affordability, it is best to design as conservatively as possible. Also, as with other structural-strength calculations, the formal determination of strength limitations and safety factors is a task for a licensed structural or civil engineer. A possible source of assistance is the ARRL Volunteer Consulting Engineer program. Contact the Field Services Department at ARRL HQ for more information.

As an example, the Hy-Gain TH7 has a specified wind area of 9.2 ft^2. In nonicing conditions, the wind force in the worst-case wind

Table 6-1
205BA Element and Boom Areas

Note: This data takes swage sections into account.

Element	*Area*	*Spacing s*
Reflector	380 $in.^2$	
Driven	356	122.875 in.
Director	321	88.75
Director	310	83.00
Director	308	113.625
Element Total	1675 $in.^2$	
Boom	828	
Balun	15	
Boom total	843	

direction, for an EIA Zone B environment (p = 40 lb/ft^2) for which the antenna is rated by the manufacturer, is calculated by

Force = 40 $lb/ft^2 \times 9.2\ ft^2$ = 368 lb

It should be noted that icing increases the wind area of an antenna and mast to a very substantial degree. The effective area of an antenna and mast can be calculated with a known increase in element and boom diameter from radial ice loading.

Let's use the Hy-Gain 205BA as a more detailed example; the data in Table 6-1 is calculated from dimensions in the manual. The areas are actual projected areas, giving no effect to the use of drag coefficient. We can see that the total area of the elements, A_{el} = 1675 $in.^2$ = 11.6 ft^2 and the total area of the boom and balun, A_b = 843 $in.^2$ = 5.9 ft^2. The areas are calculated using the spreadsheet techniques introduced in Sec 6.5, and include the effect of the short swage sections.

Taking for this example the Hy-Gain 205BA under Zone C conditions (p = 50 lb/ft^2), we calculate the effective area and wind force as a function of the angle of the wind to the boom. We can then determine the maximum force and the wind angle at which it occurs, summarized in Table 6-2.

Table 6-2
Hy-Gain 205BA Effective Area, Wind Force

Wind Angle	*Effective Area*	*Wind Force*
0°	7.8 ft^2	390 lb
15	8.5	427
30	8.7	436
45	8.3	414
60	7.3	365
75	5.8	290
90	3.9	196

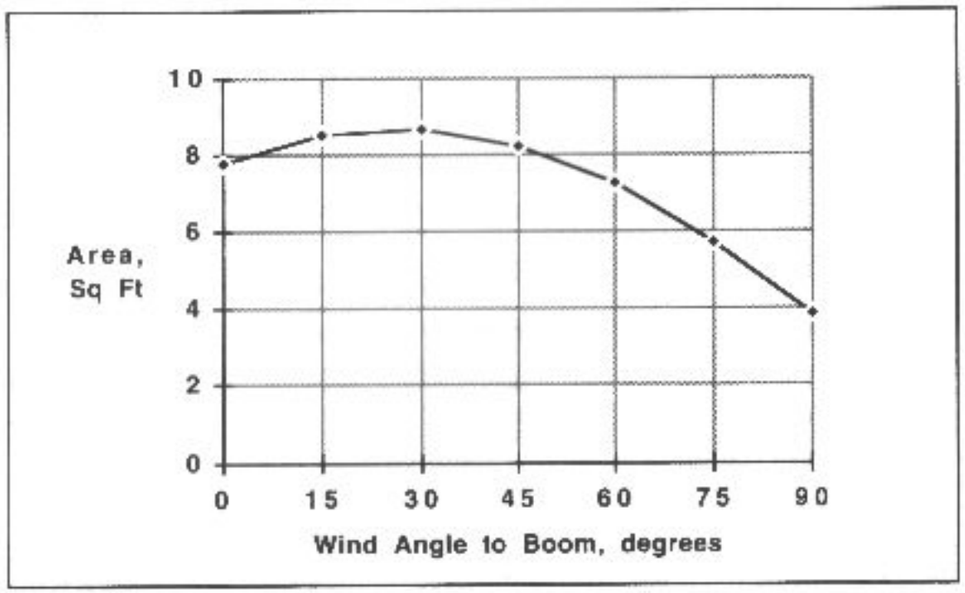


Fig 6-4—205BA effective area versus horizontal wind angle ϕ. (See Fig 6-1 for definition of ϕ.)

The total element effective area (7.8 ft^2) is the antenna effective area for $\phi = 0°$, and the effective boom area (3.9 ft^2) is the value for $\phi = 90°$. Note that the area and the resulting force change considerably with angle, peaking at about $\phi = 30°$ and dropping to a minimum at $\phi = 90°$. Using the equations above to calculate the maximum area and force, along with the associated wind angle, we find that

$c_d A_{el} = 0.67 \times 1675 = 1122$ in.$^2 = 7.8$ ft^2

$c_d A_b = 0.67 \times 843 = 565$ in.$^2 = 3.9$ ft^2

$A_{eff} = \sqrt{7.8^2 + 3.9^2} = 8.7$ ft^2

$F = 50 \times 8.7 = 435$ lb

$$\phi_{max} = \arctan\left(\frac{3.9}{7.8}\right) = 26.7°$$

The variation of this antenna's effective area with wind angle can be seen in Fig 6-4.

The above analysis assumes that the wind direction is in the plane of the elements and boom of the Yagi antenna. As discussed in Chapter 5, the plane of the wind can be at an angle θ to the horizontal. Over flat ground, the EIA model suggests that the maximum angle θ to the horizontal is $\theta \approx \arctan 0.3 = 17°$, because of the 30% gust factor. This is shown in vector form in Fig 6-5. Over sloping ground, we add the ground slope; if the ground slope is in the range of 0° to 12°, we should examine the total range of $0° \le \theta \le 30°$.

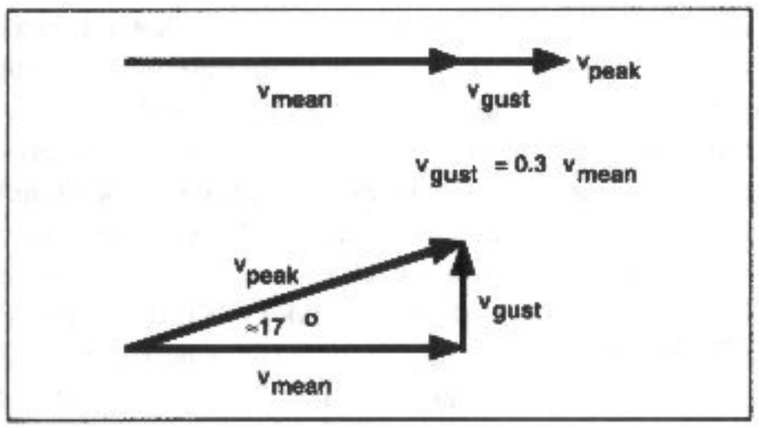


Fig 6-5—Mean and gust wind speeds.

For an antenna with the wind direction characterized by both the azimuthal angle ϕ and the angle to the antenna plane θ, the boom and element wind force do not go to zero when the wind is "end-on," as they do in the strictly horizontal case. The equation for the effective area in this case is

$$A_{eff} = c_d \cos\theta \{A_{el} \sin[\arccos(\cos\theta \sin\phi)] + A_b \sin[\arccos(\cos\theta \cos\phi)]\} \quad \text{(Eq 6-9)}$$

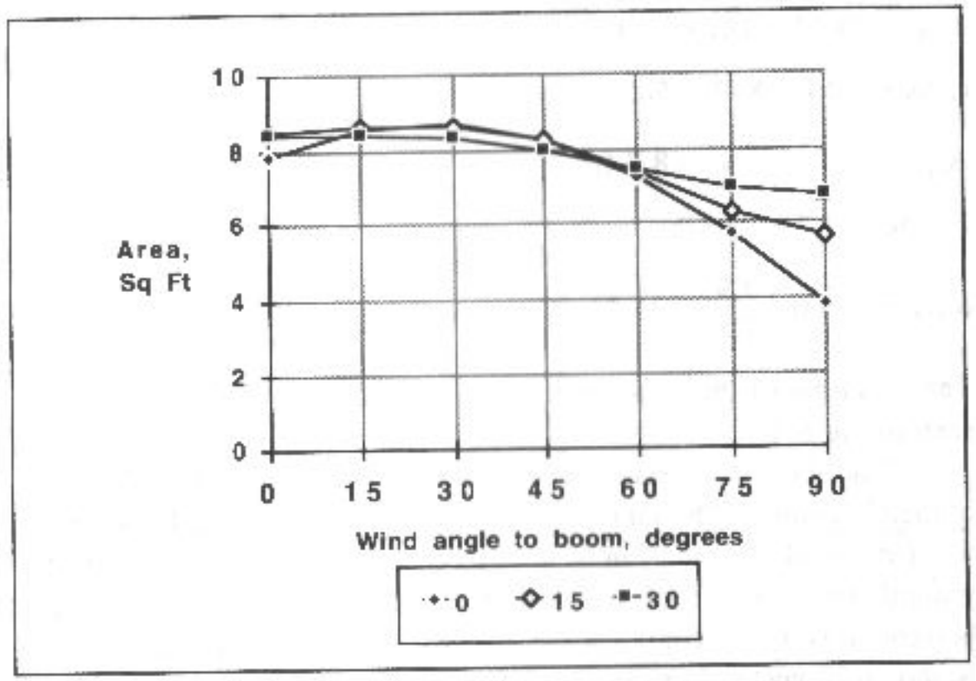


Fig 6-6—205BA effective area versus horizontal wind angle ϕ and vertical wind angle θ, represented by the three curves.

which, for $\theta = 0$, reduces to Eq 6-5. The effective horizontal wind area for the 205BA is shown in Fig 6-6 as a function of the azimuthal angle ϕ for values of θ, the angle to the horizontal, from $0° \le \theta \le 30°$.

The magnitude of the peak wind velocity varies with the angle from the horizontal. For $\theta = 0°$, the gust and mean winds are in the same direction, and the peak is 1.3 times the mean. For $\theta \neq 0°$, Table 6-3 summarizes the ratio of peak wind to the peak for $\theta = 0°$.

Table 6-3
Peak Wind Speed vs. Angle to Horizon

Angle to Horizontal, θ	*$V_{peak}/(1.3V_{mean})$*
0°	1.00
5	0.99
10	0.96
15	0.88

If we consider the further effect of sloping ground, the peak wind is at the same angle to the horizontal as the ground slope. Using this idea, we can determine the approximate effective horizontal wind area for a typical terrain situation.

The rather small effect of nonhorizontal wind can be accounted for by a modest safety factor, which greatly simplifies the calculation of wind forces and survival speeds. Thus, we can

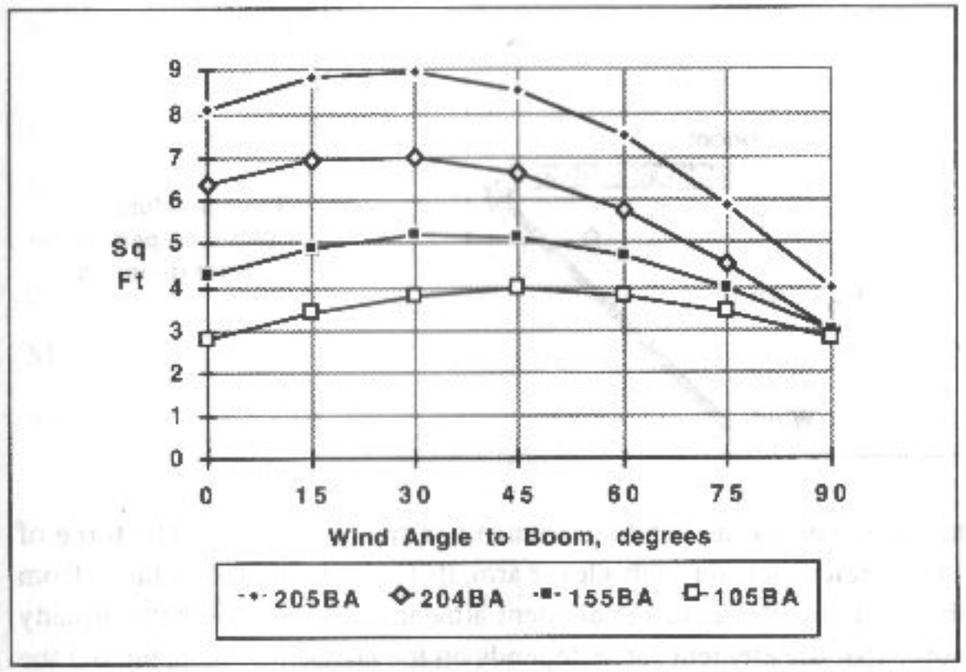


Fig 6-7—Effective wind area versus wind angle, ϕ.

evaluate antenna structures by means of horizontal calculations, with the understanding that there are occasions when the effects of nonhorizontal wind may represent a more rigorous survival limitation.

For several antenna types, the variation of effective area with wind angle can be seen in Fig 6-7. This leads to two conclusions for these antennas. First, for antennas with the element area greater than the boom area, it is helpful to reduce the mast force in storm winds by parking them with the boom, rather than the elements, across the wind ($\phi = 90°$), although this advantage is reduced with sloping ground. Second, in cases where the element area dominates, it is mechanically helpful to minimize the overall antenna area by choosing the minimum number of elements that will provide the desired electrical performance on a given boom length. This is at odds with the electrical benefits of additional elements that may be necessary to meet simultaneous gain, pattern, VSWR and bandwidth conditions.

Sec 6.2 Boom Moment About the Mast

The wind forces on the elements and the boom acting through the lever arm of the boom length produce bending moments on the boom. These forces result in a rotating moment that the antenna places on the

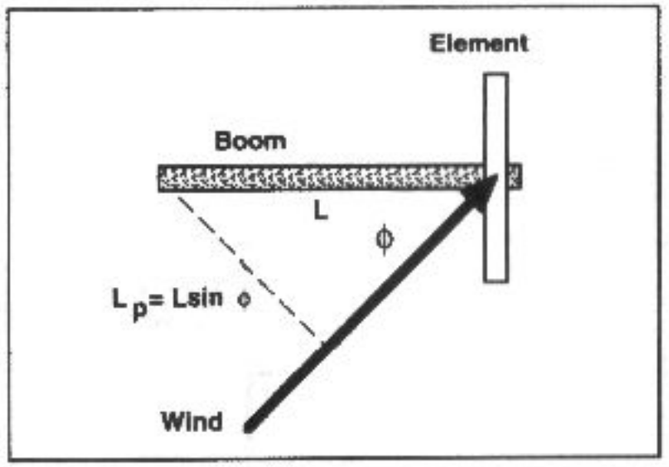


Fig 6-8—Moment length perpendicular to wind direction.

mast, as well as the bending moments in the boom itself. The force of each element acts through a lever arm, its length being the distance from the mast center line to the element attachment point. We have already noted that the element force depends on the area of the element and the angle the wind makes to the element, which is the complement of the wind angle to the boom.

The effective lever-arm length L_p through which the element or boom force operates is shown in Fig 6-8, and is given by the expression

$$L_p = L\sin\phi \qquad \text{(Eq 6-10)}$$

From Sec 6.1, we know that the force of each element is given by

$$F = c_d p A\cos\phi \qquad \text{(Eq 6-11)}$$

so the moment of force for each element is given by

$$M = L_p F = L\sin\phi \times c_d p A\cos\phi = c_d p A L\sin\phi\cos\phi \qquad \text{(Eq 6-12)}$$

For multiple elements, the total moment of force is seen to be

$$M_{el} = c_d p \sin\phi\cos\phi(\Sigma A_i L_i) \qquad \text{(Eq 6-13)}$$

where A_i and L_i are the area and distance from the mast (positive toward the director, negative toward the reflector) of element i.

We can define the location of the centroid of the ensemble of elements to be

$$L_c = \frac{\Sigma A_i L_i}{A_{el}} \qquad \text{(Eq 6-14)}$$

where $A_{el} = \Sigma A_i$, the total element projected area

We can restate the moment of force about the mast caused by the elements to be

$$M_{el} = c_d p L_c A_{el} \sin\phi\cos\phi \qquad \text{(Eq 6-15)}$$

Similarly, for the boom, the moment of force about the mast caused by the boom is

$$M_b = c_d p L_b A_b \sin^2\phi \qquad \text{(Eq 6-16)}$$

where

A_b = projected area (total length times diameter) of the boom
L_b = distance from the boom center of pressure (centroid of area) to the mast center line

The total moment of force about the mast is given by

$$M = c_d p \sin\phi(M_{el}\cos\phi + M_b\sin\phi) \qquad \text{(Eq 6-17)}$$

In many calculations, the moment of area M_a is useful because it is independent of wind speed. M_a is given by

$$M_a = \frac{M}{c_d p} \qquad \text{(Eq 6-18)}$$

The mast torque is calculated by totaling the moment of area contributions from the elements and from the boom, and then multiplying by the drag coefficient and wind pressure. The results of this calculation are summarized in Table 6-4 for Zone C wind pressure ($p = 50$ lb/ft^2).

The mast torque is zero at 0° because the elements are symmetrical about the boom, rises to a considerable peak near 45°, and then drops to a lower value at 90° caused entirely in this case by the moment of the balun. The nega-

Table 6-4
Hy-Gain 205BA Mast Rotating Torque

Wind Angle	*Mast Torque*
0°	0 in.-lb
15	–771
30	–1372
45	–1630
60	–1503
75	–1000
90	–264

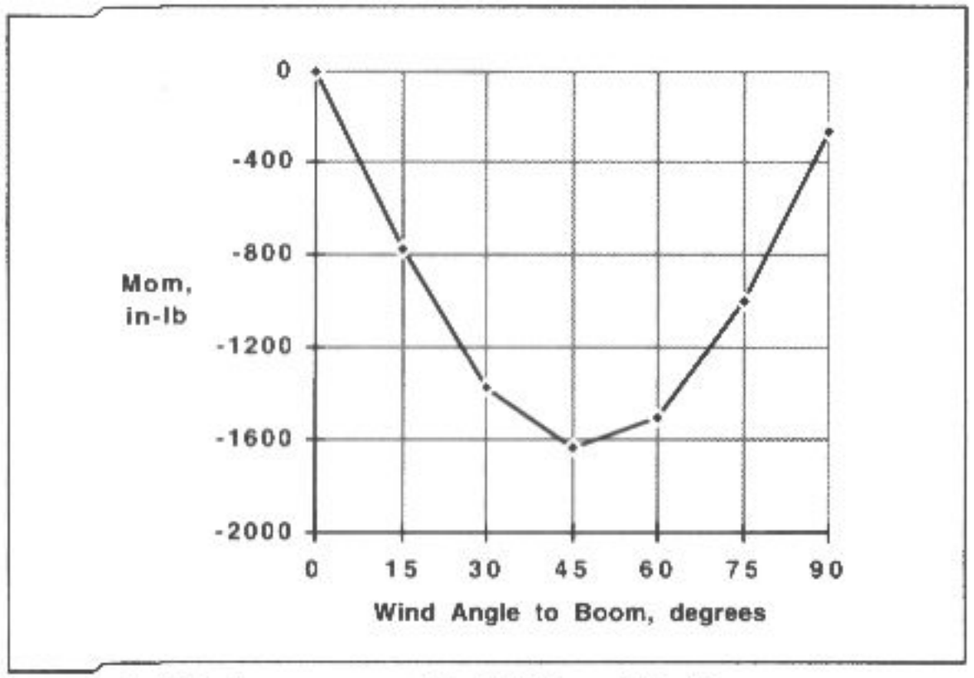


Fig 6-9—Rotating torque versus ϕ for 205BA, v = 112 mi/h.

tive sign of the torque means that this antenna will always tend to rotate away from the reflector end. The peak value of torque exceeds the rotating and braking ability of some commercial rotators, as we shall see in Chapter 7. The variation of rotating torque with wind angle can be seen clearly in Fig 6-9.

The angle for maximum moment is

$$\phi_{max} \approx 45° + 0.5\arctan\frac{M_b}{M_{el}} \qquad \text{(Eq 6-19)}$$

For the case of the 205BA data from Table 6-1, and using the method shown later for Table 6-9, we calculate

$M_{el} = A_{el}L_c = -12916$ in.^{3}

$M_b = A_bL_b = -1129$ in.^{3}

$pc_d = 50 \times 0.67$ lb/ft^2 $= 0.23$ lb/in.^{2}

$\phi_{max} = 45° + 0.5\arctan(1129/12916) = 47.5°$

$M_{max} \approx 0.233 \times 0.737[0.675 \times (-12916) + 0.737 \times (-1129)]$
$= -1630$ in.-lb

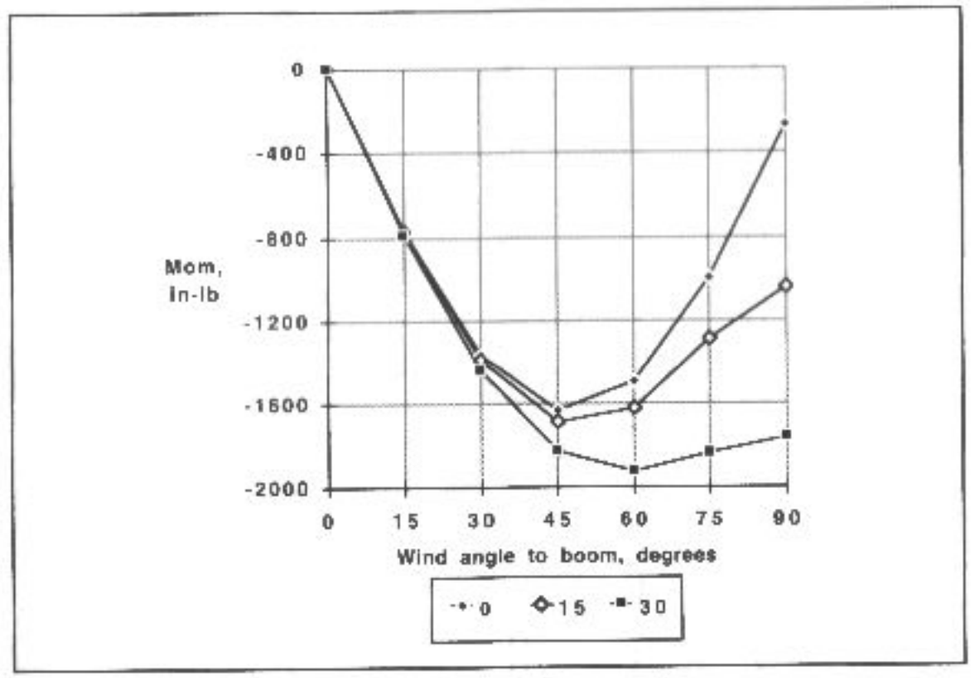


Fig 6-10—Mast moment versus ϕ and θ, v = 112 mi/h. The wind angle ϕ is shown along the horizontal axis; changes in θ are represented by the three curves.

Note: In this and some of the examples that follow, the values shown are rounded, while calculations were performed with several significant digits. The results appearing here therefore differ somewhat from the results obtained by using the rounded values shown.

A last question is the torque effect of nonhorizontal wind. Using the same nomenclature as for the example of effective area above, the equation for mast torque is

$$M = c_dp\sin\phi\{M_{el}\sin[\arccos(\cos\theta\sin\phi)] + M_b\sin[\arccos(\cos\theta\cos\phi)]\} \qquad \text{(Eq 6-20)}$$

which, for $\theta = 0$, reduces to Eq 6-17. Fig 6-10 shows the effect of wind at an angle θ from the horizon. Again, the increased mast moment near $\phi = 90°$ results because the elements are not end-on to a wind that is not in the plane of the elements and boom.

Sec 6.3 Torque Compensation

We can see that it is possible to make M equal zero for all angles if we separately make M_{el} and M_b equal zero. This means it is possible to compensate the moments of the elements and the boom to create a

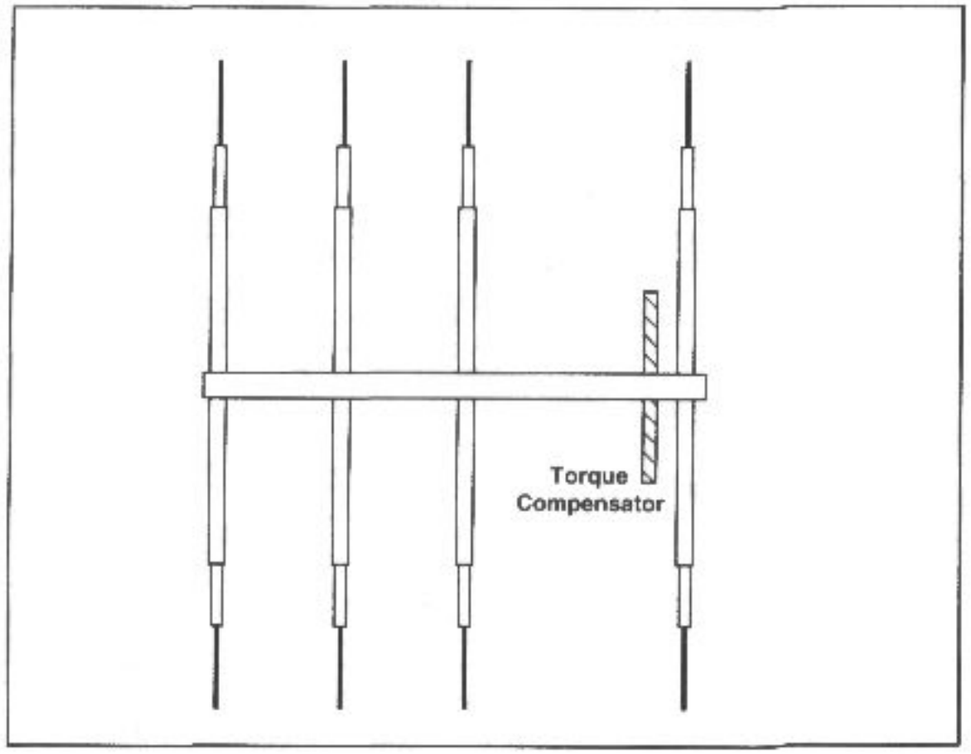


Fig 6-11—Element torque compensation can be provided by adding a small vestigial element.

condition of zero mast torque. Both the element-plane moments and the boom-plane moments must each separately be reduced to zero.

For the 205BA example, the boom-plane moment can be compensated by adding a small boom extension ΔL_b at the director end, or removing the same amount at the reflector end, to compensate the balun moment. The example here is for a tubular balun, so the drag coefficient $c_d = 0.67$ is the same for the balun and the boom tube. Ignoring the slight increase in the boom half-length, L_b equals 207 in. For the boom diameter $D_b = 2$ in., we calculate

$$M_b = -1129 \text{ in.}^3$$

$$\Delta L_b = M_b/D_b L_b = 1129/(2 \times 207) = 2.71 \text{ in.}$$

The boom-plane moment can also be balanced by a larger-diameter sleeve (ABS drain pipe works well) over the boom end opposite the balun. To be thorough, the additional area and moment of the coaxial cable to the driven element should be compensated.

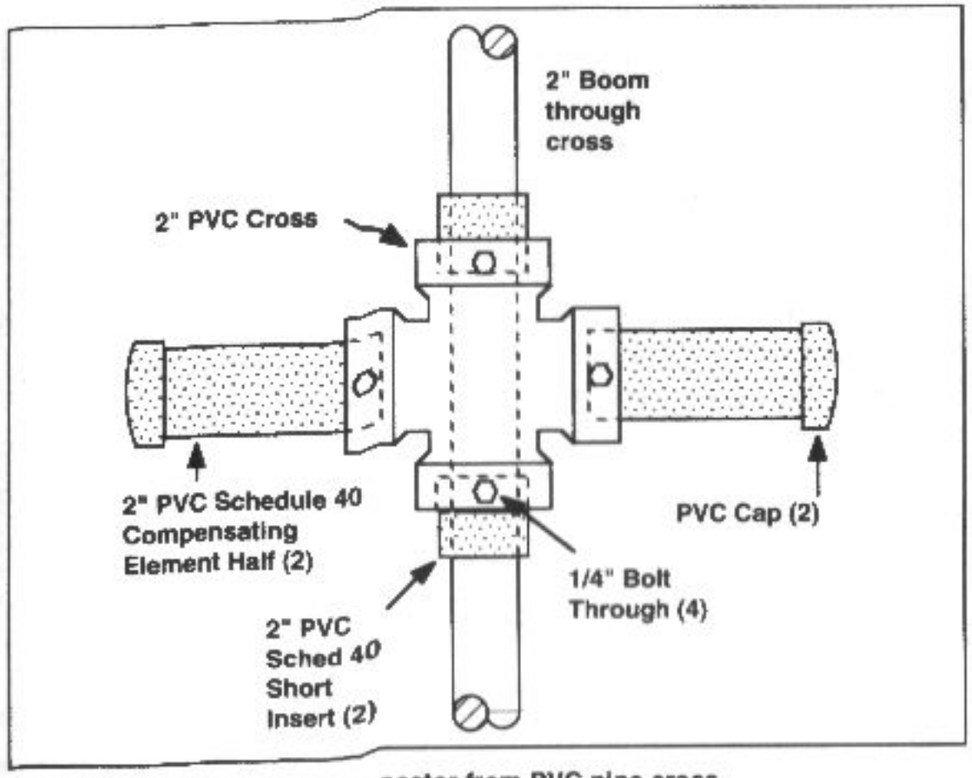


Fig 6-12—Wind torque compensator from PVC pipe cross.

The element-plane moment can be reduced to zero on some antennas by moving the entire ensemble of elements along the boom until the element moments total zero, making the distance from the mast to the centroid L_c equal zero. The boom as supplied by the manufacturer is not long enough to do this in the case of the 205BA, but the element moments can be balanced by the addition of a small vestigial element at the director end, as shown in Fig 6-11. In this case the addition of a 68-in.^{2} tubular element at a distance 12 in. closer to the mast than the director location will compensate the element-plane total moment to zero. This element can be made of plastic pipe or aluminum, and appropriate dimensions are 22.7 in. per side of 1.5-in.-diameter tubing. This element can be mounted with the same type of boom clamp as the elements.

Because of the potential for unwanted electrical interaction, compensating elements of any substantial size should be nonconducting. An example of a torque compensator made from PVC plastic pipe parts is shown in Fig 6-12. The side area of this compensator tends to balance the moment of the balun as well. In order to maintain symmetrical boom

dimensions and strength, a good location for the compensator is the same distance from the mast as the second element (typically the driven element) on the other end of the boom.

With successful compensation in both planes, the mast torque is zero for all wind angles, with only a slight increase in overall area because of the additions. I tried this on my own 205BA and could see the effect immediately; the tendency to wander was markedly reduced. Note that this compensation affects only the torque from linear wind, although from any direction. There are still vortex-induced rotating forces that are bigger in antennas with longer booms, and hence, greater polar moments of rotation result.

Some antennas, such as log periodics and antennas with asymmetrical booms or element placement, exhibit much higher mast torque than this example. On the other hand, antennas such as Hy-Gain's 105BA, 155BA and 204BA, are almost exactly torque balanced in both element and boom planes. In each case it is possible to adjust element locations slightly (but not spacings) for perfect balance, and to compensate for the balun moment by a slight addition at the director end or reduction at the reflector end of the boom.

As an aside, I always mount the beta-matching element and the balun on the side of the driven element toward the mast; this makes them easy to reach and also minimizes their moment around the mast point. It is possible to combine a beta and balun function in one low-profile unit by using semirigid coax for one side of the beta.[1] The center point at the boom is cold for RF, the cost is minimal, and you can't burn it out. Beta reactance is easily determined by equation, and easily measured by resonance with the equivalent capacitive reactance. I'm puzzled why the "beta-balun" is not offered with commercial antennas.

Sec 6.4 Boom Strength

The calculation of boom strength is similar to the calculation of element strength. For a cantilever attachment, the principal issue is bending caused by wind-loading torque. The boom problem is more complicated because the bending moment at any point along the boom must include the effect of the element areas. This means the calculation must be performed over the range of wind angles from $0° \le \phi \le 90°$.

The bending moment from the boom itself is determined by the same methods as used for cylindrical elements. We treat each half of the boom separately, as illustrated in Fig 6-13. For a constant-diameter

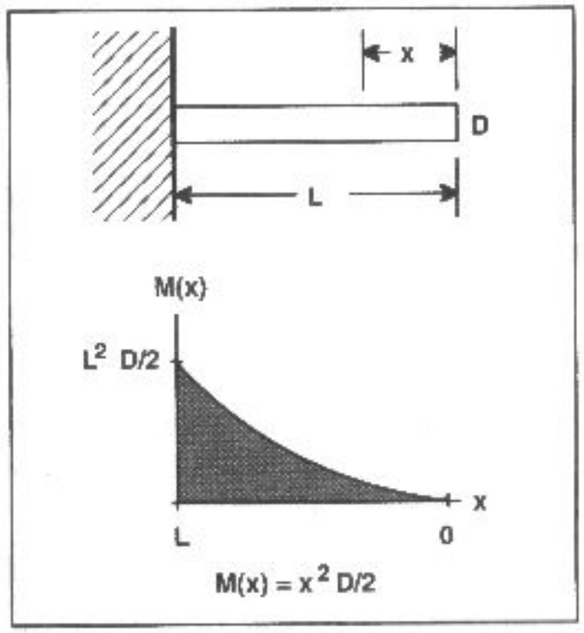


Fig 6-13—Moment of area for uniformly loaded cantilever beam.

boom, the area moment is calculated for $\phi = 90°$ as a function of x, the distance from the outer end, by the equation for a uniformly loaded cantilever beam

$$M_b = \frac{x^2 D}{2} \qquad \text{(Eq 6-21)}$$

which, for $\phi \neq 0°$ yields

$$M_b(\phi) = \frac{\sin^2\phi\, x^2 D}{2} \qquad \text{(Eq 6-22)}$$

To this we must add the bending moments caused by the area of the elements. For each element on a boom, we express the bending moment for $x \ge d_n$ as

$$M_n = (x - d_n)A_n \qquad \text{(Eq 6-23)}$$

where

x = distance from the boom end
d_n = location (measured from the boom end)
A_n = area of element n
$M_n = 0$ for $x \le d_n$

The total moment at any point x from the elements is obtained by summing over the elements on the boom half.

$$M_{el} = \Sigma(x - d_n)A_n \qquad \text{(Eq 6-24)}$$

where $x \geq d_n$

Combining, we determine that

$$M(\phi) = \sin^2\phi M_b + \sin\phi\cos\phi M_{el} \qquad \text{(Eq 6-25)}$$

At any point x, the angle ϕ of maximum moment is

$$\phi_{max} = 45 + 0.5\arctan\frac{M_b}{M_{el}} \qquad \text{(Eq 6-26)}$$

As with element analysis, we can make a spreadsheet to keep track of the arithmetic. The spreadsheet IF( ) function is a convenient way to account for the requirement that $M_n = 0$ for $x \leq d_n$.

Sec 6.5 Calculation of Boom Strength

We can now construct a set of spreadsheets to determine boom strength. First, we make tables of element dimensions using the equations of Table 6-5. We'll use a subscript i to identify the element number, starting with i = 1 for the reflector, and a subscript n to identify an element section, starting with n = 0 for the element clamp. For the Hy-Gain 205BA example, the results are shown in Table 6-6.

These dimensions are then used to calculate the element areas shown in Table 6-7. For simplicity, we ignore the minor effect of the swage dimensions, which amounts to an overestimation of area by approximately 1 in.^{2} per element.

Next, to determine the total moment of area of the elements and the centroid of the element moments, we construct a table using the element spacings, denoted s_i for the spacing between elements i – 1 and i, and element areas, denoted A_i. We define the station z_i as the distance of the element from the position of the reflector. If the element locations are specified, we enter the reflector location relative to the mast as L_1; if not, we can define the moment length L_i as the distance of the element from the center point of the element group (midway between the reflector and the farthest director). We take the element areas from Table 6-7.

The centroid location L_c is defined as the distance from the mast which, if multiplied by the total element area, provides the same moment of area, and hence the same rotating torque about the mast as that of the element ensemble. If the element ensemble is located so the

Table 6-5
Calculation of Element Area

Parameter	*Equation*
Element designation, number	Reflector, i = 1; Driven, i = 2, etc
Element section number	n
Element section diameter, in.	D_n
Element section length, In.	L_n
Element length, in.	ΣL_n
Element section area, in.^{2}	$A_n = L_nD_n$
Element area, in.^{2}	$A_i = \Sigma A_n$

Table 6-6
Element Section Dimensions

Element	*0*	*1*	*2*	*3*	*4*	*5*	*Total Length*
Diameter, in.	1.5	1.25	1.125	0.875	0.625	0.4375	
Reflector	2.625	41.875	46	50.5	24	53	218
Driven	3.75	31.25	46	50.5	24	50	205.5
Director	2.625	18.375	46	50.5	24	53	194.5
Director	2.625	14.125	46	50.5	24	53	190.25
Director	2.625	14.125	46	50.5	24	50	187.25

Table 6-7
Area of Element Sections

Element	*0*	*1*	*2*	*3*	*4*	*5*	*Total/ half, In.^{2}*	*Element area, In.^{2}*
Diameter, in.	1.5	1.25	1.125	0.875	0.625	0.4375		
Reflector	3.94	52.34	51.75	44.19	15.00	23.19	190.41	381
Driven	5.63	39.06	51.75	44.19	15.00	21.88	177.50	355
Director	3.94	22.97	51.75	44.19	15.00	23.19	161.03	322
Director	3.94	17.66	51.75	44.19	15.00	23.19	155.72	311
Director	3.94	17.66	51.75	44.19	15.00	21.88	154.41	309

Table 6-8
Calculation of Element Moments and Centroid

Parameter	*Equation*
Element spacing, in.	s_i ($s_1 = 0$)
Element group length, in.	$L_{el} = \Sigma s_i$
Element station relative to reflector, in.	$z_i = z_{i-1} + s_i$ ($z_1 = 0$)
Element location for moment calculation	$L_i = L_1 + z_i$, $L_1 = -\Sigma s_i/2 + L_{offset}$ or as specified
Area of element i, in.^{2}	A_i
Total area of elements, in.^{2}	$A_{el} = \Sigma A_i$
Moment of element i, in.^{3}	$M_i = A_i L_i$
Total of element moments, in.^{3}	ΣM_i
Location of element centroid	$L_c = \Sigma M_i / \Sigma A_i$

Table 6-9
Element Ensemble Positions, Moments and Centroid

Note: This data does not take swage sections into account.

Element	*i*	*Spacing*	*Station*	$L_{i,\ in.}$	*Ai, in.*^{2}	$A_{iLi,\ in.}^3$
Reflector	1		0	–204.13	381	–77733
Driven	2	122.875	122.875	–81.25	357	–28996
Director	3	88.75	211.625	7.5	322	2415
Director	4	83	294.625	90.5	311	28185
Director	5	113.625	408.25	204.125	309	63036
Total length, area, mom.		408.25			1680	–13093
Centroid location L_c = Σmoment/Σarea				–7.79 in.		

centroid is zero, the rotating torque from the elements is zero. The equations for calculation of element moments and centroid are summarized in Table 6-8. This results in the calculation for the 205BA shown in Table 6-9.

Now we construct a table for the boom-section dimensions, and calculate the area moments of the boom, section by section. As part of

Table 6-10
Calculation of Boom Area and Moments

Parameter	*Equation*
Boom section number	n
Boom section diameter, in.	D_n
Boom section length, in.	L_n
Boom length, in.	$L_b = 2\Sigma L_n$
Boom section wall thickness, in.	t_n
Boom section weight, lb	$w_n = 0.25\,\sigma\,\pi\,L_n\,[D_n^2 - (D_n - 2t_n)^2]$
Boom weight, lb	$w_b = 2\Sigma w_n$
Boom section area, in.^{2}	$A_n = L_n D_n$
Boom area, in.^{2}	$A_b = 2\Sigma A_n$
Sum of area plus outboard areas	$\sum_{n}^{tip} A_k,\ n \le k \le n_{tip}$
Moment of area, in.^{3}	$M_n = M_{n-1} + L_n\left[\sum_{n}^{tip} A_k - \frac{A_n}{2}\right]$
Section modulus, in.^{3}	$S_n = \pi[D_n^4 - (D_n - 2t_n)^4] / (32D_n)$

the calculation, we calculate the boom area and weight. We use the same method applied to multisection elements in Chapter 4. The equations for this construction are shown in Table 6-10. For the 205BA example, the boom-section dimensions, areas and moments are summarized in Table 6-11.

At this point, we have the total element area and the boom area, so we can determine the antenna effective area and the wind angle for the maximum wind force, as shown in Table 6-12. The effective area of the 205BA without a compensator is 8.7 ft^2 at an angle of 26.3° to the boom, as shown in the summary.

We now have both the element spacing and boom dimensions, so we can determine if there is sufficient boom length to move the element ensemble to a position that results in zero moment about the mast. This occurs when the centroid distance from the mast is reduced to zero by offsetting the element ensemble by a distance equal to the centroid

Table 6-11
205BA Boom Dimensions, Moments and Section Modulus

Section	*1*	*2*	*3*	*Total/half*	*Total*
Diameter, in.	2	2	2		
Length, in.	51	81	75	207	414
Thickness, in.	0.203	0.12	0.049		
Weight, lb	5.7	5.6	2.2	13.5	27.1
Area, $in.^2$	102	162	150	414	828
ΣArea, $in.^2$	414	312	150		
Moment, $in.^3$	42849	24336	5625		
Section modulus, $in.^3$	0.4685	0.2345	0.1430		

Table 6-12
205BA Effective Area

Quantity	$in.^2$	ft^2	*Equation*	*Area × c_d*
Boom area, A_b	828	5.8	$A_{eff} = c_d \sqrt{A_{el}^2 + A_b^2}$	3.9 ft^2
Element area, A_{el}	1680	11.7		7.8 ft^2
Total eff area, A_{eff}				8.7 ft^2
ϕ_{max}			$\phi_{max} = \arctan(A_b/A_{el})$	26.3°

distance from the middle of the element group. We need to allow for the end space required for the element clamping technique. If we offset the element locations, we need to add the amount of the offset, L_{offset}, to the location of the reflector in Table 6-8.

In the case of the 205BA, the element ensemble essentially fills the available boom length. If the boom were longer, the offset could be set equal to the centroid dimension. Although this eliminates the need for the compensating element, it results in a boom that is 2 × 7.79 = 15.58 inches longer. It is possible to use a combination of element relocation, element area variation, and compensation to achieve torque compensation.

The area of the compensating element required to reduce the element torque to zero is determined by the equations of Table 6-13.

Table 6-13
Compensator Area and Dimensions

Quantity	*Equation*	*205BA Value*
Centroid, offset	$L_c = \Sigma M_i / \Sigma A_i$	−7.79 in.
Compensator spacing re director	s_c	−12 in.
Compensator area moment	$M_c = -L_c \Sigma A_i$	13093 $in.^3$
Compensator area	$A_c = -L_c \Sigma A_i / (L_{dir} + s_c)$	68.15 $in.^2$
Compensator diameter	d_c	1.5 in.
Compensator length/half	$L = A_c/2d_c$	22.72 in.

Table 6-14
Calculation of Element Distances from Boom Ends

Parameter and Equation

Reflector distance from boom end:

$x_{1r} = (L_b - L_{el})/2 + L_{offset}$, or specified

Element distance from reflector end of boom:

$x_{ir} = x_{1r} + z_i$

Element distance from reflector end of boom:

$x_{id} = L_b - x_{ir}$

The equations are derived under the assumption that the compensation will be required at the director end. The area of the compensator can be added to the element area for the calculation of effective antenna area, and is used in the determination of boom strength. We can choose a point $s_c = -12$ in. (inboard) relative to the farthest director for the compensator location, and choose a diameter such as $d_c = 1.5$ in. that is similar in size to the elements, so that the Reynolds number *Re* will be in the same range, and so the compensator can be mounted using the same clamping method. For a given area, the larger the diameter of the compensator, the stronger it is because of both a larger section modulus and a shorter overall length.

One last calculation is necessary before determining boom strength. We need the element locations relative to the boom end, rather

Table 6-15
Element Locations on Boom

Element	*Station*	$x_{i\ refl}$	$x_{i\ dir}$
Reflector	0	2.88	411.13
Driven	122.875	125.75	288.25
Director	211.625	214.50	199.50
Director	294.625	297.50	116.50
Director	408.25	411.13	2.88

Table 6-16
Calculation of Boom Survival Speeds

Parameter	*Equation*
Boom section number	n
Boom station, in.	$x_n = \sum_{n}^{tip} L_n$
Boom moment, in.^{3}	$M_b(x_n) = M_n$
Moment of element i, in.^{3}	$M_i(x_n) = (x_n - x_{ir})A_i$ for $x_n \geq x_{ir}$
Total of element moments, in.^{3}	$M_{el}(x_n) = \Sigma M_i(x_n)$
Angle of maximum moment at x_n	$\phi_n = 45° + 0.5\arctan[M_b(x_n)/M_{el}(x_n)]$
Maximum moment	$M_n = M_b(x_n)\sin^2\phi_n + M_{el}(x_n)\sin\phi_n\cos\phi_n$
Survival wind speed, mi/h	$v_{max(n)} = 231.8\sqrt{S_n\sigma_{max}/M_n}$

than relative to the reflector or the mast. If the element locations are specified, we enter the reflector location relative to the reflector end of the boom as x_{1r}; if not, we can calculate the distance from the boom end to the reflector location x_{1r} as shown in Table 6-14. We will also use the element locations measured relative to the director end in the calculation of the strength of the director half of the boom. These locations for the 205BA are shown in Table 6-15 with no centroid offset.

Now we are ready to calculate the boom survival speed. We do this for both the reflector and director sides, working section by section.

Table 6-17
Boom Strength Calculation

Reflector side	*1*	*2*	*3*
Station	207	156	75
Boom moment	42849	24336	5625
Refl moment	77733	58312	27466
Driven moment	28996	10795	0
EI moment	106729	69107	27466
ϕ_{max}	56	55	51
M_{max}	78929	48802	16831
v_{max}	106	110	126

Director side	*1*	*2*	*3*
Station	207	156	75
Boom moment	42849	24336	5625
Dir1 moment	2415	0	0
Dir2 moment	28185	12302	0
Dir3 moment	63036	47287	22273
Comp moment	13093	9617	4097
EI moment	106729	69206	26370
ϕ_{max}	56	55	51
M_{max}	78929	48848	16294
v_{max}	106	110	128

The boom section numbers are designated by the subscript n as in Table 6-11, and the boom stations x_n are defined as the distance from the boom end to the boom-section joint at the inner end of section n. The boom moments are taken from Table 6-11, and the element moments are found in Table 6-9. The element numbers are designated as before by the subscript i. The equations for this calculation are summarized in Table 6-16.

The calculations for the director side are identical except the dimensions are those of the director-side element locations. Note that the calculation automatically ignores elements that belong on the other side of the boom center. The compensator is treated as an additional director. The boom-strength calculations for the 205BA, ignoring the boom swage joints, are summarized in Table 6-17 for $\sigma_{max} = 35000$ lb/in.^{2}.

The 205BA boom will survive a wind speed of 102 mi/h (95 mi/h for the inner swage). This seemingly overcomplicated set of calculations can be handled quite readily when reduced to a computer spreadsheet. Although the boom is carried directly through the boom-joint bracket in the 205BA, the strength of this bracket is an issue for several others of the Hy-Gain antenna family, such as the tribanders. Because the bracket is not symmetrical, the same considerations apply here as to the element-to-boom brackets described in Chapter 5.

The Hy-Gain boom-joint bracket is made from 5052-H32 alloy, having a yield strength σ_{max} = 22000 lb/in.^{2}. The average material thickness is t_{avg} = 0.112 in., the diameter of the cylindrical portion is D = 2.224 in., and the flange width is W = 4.275 in. with a lip of height h = 0.375 in. Using the rather complex technique of calculating the section modulus of a composite structure[2] to examine the boom construction of the popular Hy-Gain antennas, we find that the Hy-Gain boom center provides a section modulus of 0.4691 in.^{3}, which for σ_{max} = 22000 lb/in.^{2} provides M_{max} = 10320 in.-lb. The centroid of the bracket is offset by 0.5368 in. from the boom centerline, and the bracket strength is equivalent to a 2 × 0.111-in. 6061-T6 tube.

From this information and the boom moment determined from the boom spreadsheet, we can evaluate the bracket v_{max}. The flanges add materially to the strength of the cylindrical portion of the clamp by itself. The survival speeds shown in Table 6-18 represent the values that would result if the boom were constructed of separate halves joined only by the bracket. In the case of the 205BA, the boom is carried right through the clamp, so the calculation is not necessary. Several Hy-Gain antenna boom joints can be strengthened by the same technique, which is necessary if the bracket survival speed does not exceed the expected wind conditions. The calculation of v_{max} is for the conservative assumption of gusting at 30° from the horizontal. It can be seen that no reinforcing is required for the TH5 and 155BA, and that substantial improvement is possible for the TH7 and 204BA.

Table 6-18
Summary of Hy-Gain Boom-Center Reinforcement

Parameter	*TH5*	*TH7*	*105BA*	*155BA*	*204BA*
Boom moment, in.-lb	35332	53638	27040	36345	49314
Boom v_{max}, mi/h	129	129	100	157	135
Bracket v_{max}	125	102	143	124	106
Reinforced bracket v_{max}		119	156		124
Boom + tube v_{max}		129	124		135
Tube outer diameter, in.		1.568	1.875		1.568
Tube wall thickness, in.		0.287	0.058		0.162
Tube length, in.		12	60		12

The insert for these antennas must be drilled to match the bolt holes in the Hy-Gain bracket, and the insert for the 204BA must be machined from 1¾-in. tube down to an outside diameter of 1.568 in. to fit inside the existing 1¾ × 0.083-in. boom reinforcements. These dimensions were calculated by use of a spreadsheet based on the derivation given above.

An alternative approach that could be provided by the manufacturer or purchased as separate parts by the constructor uses the 1¾ × 0.083 × 71-in. boom insert, Hy-Gain part no. 171541, used on the 7-2 antenna. The insert is placed inside the nonreinforced 2 × 0.12 × 81-in. swaged boom sections (171540 used on the Hy-Gain 7-3, or 170424 used on the 205BA). The inserts may need to be drilled to match the boom-bracket bolt holes.

An interesting point is that the inside tube can be made from material with a lower yield strength, as the stress is proportional to the distance from the centroid. Beams made of different materials are analyzed[3] by transforming the combination to a section of a single material, with the transformation ratio governed by the elastic modulus E_1 and E_2, not the yield strength σ_1 and σ_2. Because almost all aluminum alloys have the same elastic modulus (E = 10.5 × 10^6 lb/in.^{2}), we can telescope different alloys and, subject to confirmation by careful analysis, use an inner reinforcement of lesser yield strength.

Using a stronger material for internal reinforcement provides no benefit unless the inner material takes essentially all the stress, because the failure of the material in bending will be at the maximum radius (that is, at the surface of the outer tube). The ratio of the allowable yield strengths is just the ratio of outer diameters of the telescoped tubes. For example, a 0.5-in. tube telescoped inside a 0.625-in. tube may have σ_1 = 0.5/0.625 × σ_2 = 0.8σ_2. Thus, for this case, a tube with σ_{max} = 35000 × 0.8 = 28000 can be used safely for reinforcement of a tube with σ_{max} = 35000.

Sec 6.6 Guyed Boom and Elements

If the strength of the boom as calculated above is not adequate to withstand the expected wind environment, we can improve matters by guying the boom. As we shall see, however, we need to be careful not to make things worse instead.

To determine the strength of a guyed boom, we must determine the effect of the guy on the cantilevered boom. The guy acts with force along its direction, which is the result of the force perpendicular to the boom necessary to satisfy the condition that the sum of all moments on the boom totals zero. (Otherwise, the boom would rotate.) This process would be simple were the boom pinned or pivoted at the center; under this assumption, although of opposite sign, the absolute value of the moment from the guying force perpendicular to the boom equals the moment at the center of the boom. Thus, for a boom of half-length L with guying at the tip, the perpendicular guy force in units of $in.^2$ is

$$F = \frac{-M(L)}{L} \qquad \text{(Eq 6-27)}$$

As before, to convert from units of area to units of force in lb, we multiply by the wind loading per area, $c_d p_o v_{max}^2$. The benefit of staying with units of area is that the calculations are independent of wind speed.

For a cylindrical boom of length L and diameter D with uniform loading, the moment of area at the center is

$$M(L) = \frac{L^2 D}{2} \qquad \text{(Eq 6-28)}$$

and the guy must provide a force normal to the boom equal to

$$F = \frac{-LD}{2} \qquad \text{(Eq 6-29)}$$

The total moment on the boom is the algebraic sum of the uniform loading moment and the moment from the guying force.

The moment from uniform loading is

$$M = \frac{x^2 D}{2} \qquad \text{(Eq 6-30)}$$

and the moment from the guying force is

$$xF = \frac{-xLD}{2} \qquad \text{(Eq 6-31)}$$

The total moment as a function of x, the distance from the boom tip, is

$$M_{tot} = \frac{x^2 D}{2} - \frac{xLD}{2} \qquad \text{(Eq 6-32)}$$

For this case, the maximum moment is in the middle of the boom, and is only one quarter of the maximum moment for the unguyed case. This summing of moments is illustrated in Fig 6-14.

There is a downside to guying. As we shall see, the guying action also results in a compressive force along the boom between the guying point and the point of boom attachment.

If we look at the case of a guy attached, not at the end (x = 0), but at an intermediate point L′ on the boom ($x_g = L - L'$), we have

$$F = \frac{-M(L)}{L'} = \frac{-L^2 D}{2L'} \qquad \text{(Eq 6-33)}$$

and the moment from the guying force is

$$(x - x_g)F = \frac{-(x - x_g)L^2 D}{2L'} \qquad \text{(Eq 6-34)}$$

where $x \geq x_g$

When $x < x_g$, the guying moment is M = 0. The total moment is then, for $x < x_g$

$$M_{tot} = \frac{x^2 D}{2} \qquad \text{(Eq 6-35)}$$

and for $x \geq x_g$

$$M_{tot} = \frac{x^2 D}{2} - \frac{(x - x_g)L^2 D}{2L'} \qquad \text{(Eq 6-36)}$$

This is more easily visualized with reference to Fig 6-15.

Because the boom is restrained rather than pivoted at the center, the calculation of the effect of guying is more complicated. This is termed a statically indeterminate case, because we cannot determine

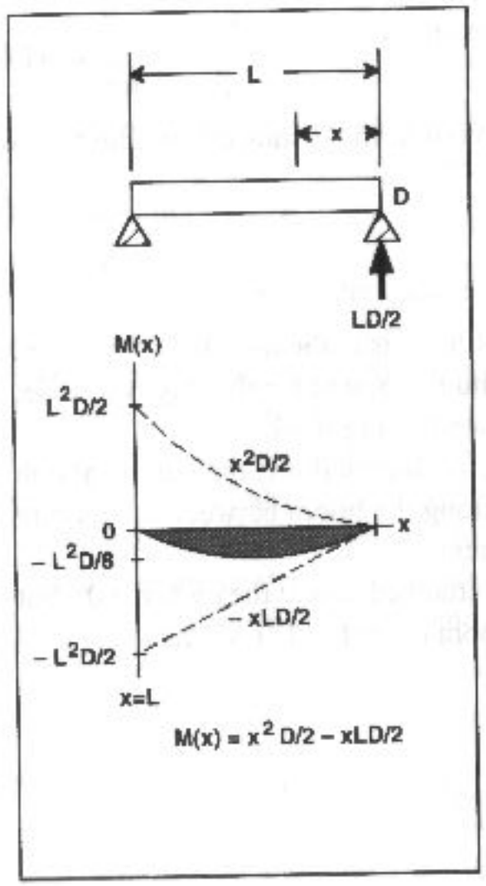


Fig 6-14—Moment for beam supported at both ends.

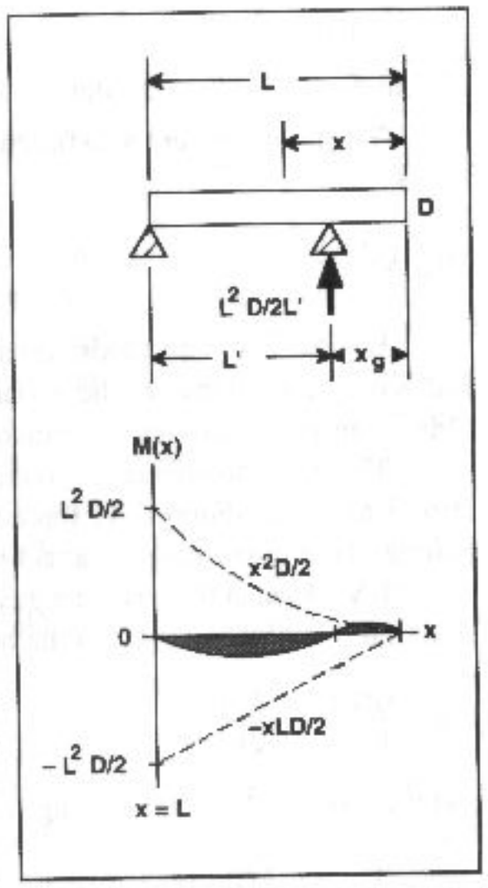


Fig 6-15—Moment for beam supported at end and intermediate point.

how much of the force is taken by the boom and how much by the guy without considering the dynamics of bending and spring constants.

If we consider the boom to have an effective spring constant of k at the point of guying, the force for a given displacement in the boom under a side force of F is given by $\Delta = F/k$. If the boom is guyed, the displacement for the guying system considered to have a spring constant k' is the same as the displacement of the boom under the same condition of sharing the side force. Typically, the guying system will have an effective spring constant k' that is an order of magnitude greater than that of the boom, k (that is, $k'/k = 10$). There will be additional guy force from the resting tension of the guys in the absence of wind stress.

The force, in units of in.^{2} if M(L) is in units of in.^{3}, at the guy perpendicular to the boom is

$$F(x_g) = \frac{k}{k'+k} \times \frac{M(L)}{L'} = \frac{M(L)}{L'(1+k'/k)} \qquad \text{(Eq 6-37)}$$

If we consider a boom made of n sections, the guy moment of area begins at L′, and as a function of section length L_n is given by

$$M_g(n) = L_n F(x_g) + M_g(n+1) \qquad \text{(Eq 6-38)}$$

The total moment of area at any point in the boom is the unguyed moment plus the guy moment

$$M_{tot}(n) = M_g(n) + M_n \text{ (unguyed)} \qquad \text{(Eq 6-39)}$$

Because the total moment may be larger at either the outer or inner end of a section, take the larger of the two

$$M_n = MAX(M_{tot}(n), M_{tot}(n+1)) \qquad \text{(Eq 6-40)}$$

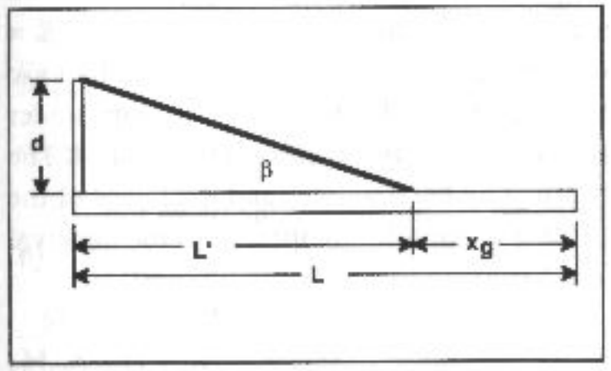


Fig 6-16—Guy-support geometry.

As illustrated in Fig 6-16, the angle of the guy is

$$\beta = \arctan \frac{d}{L'} \qquad \text{(Eq 6-41)}$$

where d = length of guy support or offset

The compression force, also in units of in.^{2}, from the guy tension is

$$F_c = \frac{F(x_g)}{\tan\beta} = \frac{F(x_g)L'}{d} \qquad \text{(Eq 6-42)}$$

The guy tension is

$$F_g = \frac{F(x_g)}{\sin\beta} \qquad \text{(Eq 6-43)}$$

and the support force is

$$F_s = F(x_g) \qquad \text{(Eq 6-44)}$$

To convert all these values to lb and in.-lb, we multiply by $c_d p_o v_{max}^2$.

Sec 6.7 Columns In Compression

The survival of a column in compression depends upon three parameters of the material, termed B, C and D, listed in tables of aluminum properties,[4] and upon a factor termed the slenderness ratio, KL′/r. In this factor, the choice of constant K depends upon the way in which the ends of the column are restrained, ranging from K = 0.5 for both ends completely fixed to K = 2.0 for one end fixed and one end completely free (cantilever compression member).[5] The choice of K = 0.7 corresponds to one end fixed and the other pinned, as is the case with guy restraint. The length L′ is the length of the column under compression, here the distance from the guy attachment to the mast. The radius of gyration r is determined by the diameter and thickness of the cylindrical boom member. For a column in compression, the survival compression pressure in lb/in.^{2} is given by

$$\sigma_{max\ col} = \frac{1.02\times 10^8}{(KL'/r)^2} \qquad \text{(Eq 6-45)}$$

where $KL'/r > C$ and

$$\sigma_{max\ col} = B - D\frac{KL'}{r} \qquad \text{(Eq 6-46)}$$

where $KL'/r < C$

Note that B is given in tables in units of kips (1000 in.-lb).

We can approximate $r = \sqrt{I/A}$, the radius of gyration of the column, by the equation

$$r = \sqrt{2}\,\frac{D}{4}\left(1 - \frac{t}{d}\right) \qquad \text{(Eq 6-47)}$$

The maximum force, lb, a column will support is

$$F_{max} = A\sigma_{max\ col} \qquad \text{(Eq 6-48)}$$

where A = the smallest cross-section area of the column

For a cylinder, the cross-section area A in.^{2} is given by

$$A = 0.25\pi[D^2 - (D - 2t)^2] \qquad \text{(Eq 6-49)}$$

Sec 6.8 Beam-Column Strength

If a column is subject also to side force, it is termed a beam-column. The solution for maximum survival compression force and bending moment is a nonlinear problem. The relation between maximum simultaneous force and bending moment in a beam-column[6] is given by

$$\frac{M}{M_{max}} = \left(1 - \frac{F_c}{F_{max}}\right)^2 \qquad \text{(Eq 6-50)}$$

where $M_{max} = S\sigma_{max}$, the maximum bending moment

For small values of F_c/F_{max}, we can approximate this expression by

$$\frac{M}{M_{max}} \approx \frac{1 - kF_c}{F_{max}} \qquad \text{(Eq 6-51)}$$

From Fig 6-17 we can see that a reasonable approximation for $0 < F_c/F_{max} < 0.6$ is given by k = 1.67, so that a conservative linear relationship between bending and compression in a beam-column is given by

$$\frac{M}{M_{max}} + \frac{1.67F_c}{F_{max}} = 1 \qquad \text{(Eq 6-52)}$$

where F is in lb and M is in in.-lb

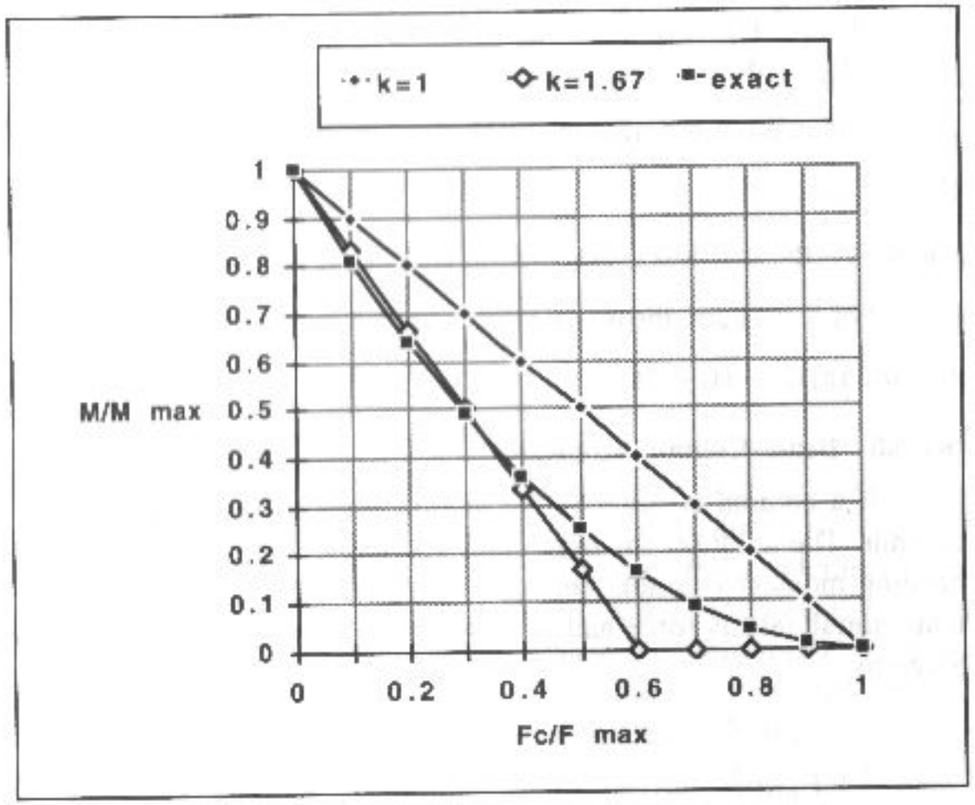


Fig 6-17—Bending versus compression limits in a beam-column.

Recalling that $M(\text{in.-lb}) = M(\text{in.}^3)c_d p_o v_{max}^2$ and $F(\text{lb}) = F(\text{in.}^2)c_d p_o v_{max}^2$, we can substitute and solve for the survival speed v_{max} of each section n by

$$v_{max}(n) = \frac{231.8}{\sqrt{\frac{1.67F_c}{A_n\sigma_{max\,col}} + \frac{M_n}{S_n\sigma_{max}}}} \qquad \text{(Eq 6-53)}$$

where F_c is in in.^2 and M_n in in.^3

To formulate a spreadsheet for the computer, we can assemble the equations for the new set of calculations as shown in Table 6-19. The compression buckling constants and yield strength are chosen for 6061-T6 and 6063-T832 alloy. This spreadsheet is a continuation of the spreadsheet for the unguyed case, and carries over a number of the parameters derived in Table 6-17.

Several of the parameters in the table are the same for all boom sections. It is convenient to list them separately as in Table 6-20. For

Table 6-19
Calculation of Guyed Boom Survival

Parameter	*Equation*
From previous tables	$n, D_n, t_n, M_n, S_n, \sigma_{max}$
Guyed length L′, in.	$L' = L - x_g$
Boom/guy spring constant ratio	$k'/k = 4$
Slenderness ratio constant	$K = 0.7$
Compression buckling constants	$B = 38300, C = 63, D = 0.202$
Guy moment at section 1, in.^3	$M_{g1} = -M_1(\text{unguyed})/(1 + k/k')$
Guy moment at section n, in.^3	$M_{gn} = M_{gn-1} - M_{g1}L_n/L'$
Total moment, in.^3	$M_n = M_{gn} + M_n(\text{unguyed})$
Greater of inner or outer end moment	$M_{n,max} = MAX(M_n, M_{n+1})$
Radius of gyration, in.	$r_n = \frac{\sqrt{2}}{4} D_n \left(1 - \frac{t_n}{D_n}\right)$
Column buckling pressure, in.^2	
For $KL'/r_n > C$:	$\sigma_{max\,col} = 1.02 \times 10^8/(KL'/r_n)^2$
For $KL'/r_n < C$:	$\sigma_{max\,col} = B - DKL'/r_n$
Cross section (bearing) area, in.^2	$A_n = 0.25\pi [D_n^2 - (D_n - 2t_n)^2]$
Survival wind speed, mi/h	
If guyed:	$v_{max}(n) = \frac{231.8}{\sqrt{1.67F_c/(A_n\sigma_{max\,col}) + M_n/S_n\sigma_{max}}}$
If unguyed:	$v_{max}(n) = 231.8\sqrt{S_n\sigma_{max}/M_n}$

this antenna, I have selected $L' = 207 - 75 = 132$ inches, and $k'/k = 4$. The results of the boom-strength calculation are not sensitive to the relative value of the spring constants, so long as $k' \gg k$.

With these parameters we can quickly derive the values for the boom sections, yielding finally the survival wind speed for the sections of the guyed boom. The survival speed for the entire boom is, of course, the smallest of the values for v_{max}. We can see from Table 6-21 the substantial improvement in survival capability over the unguyed case.

Table 6-20
Parameters for Guyed Boom Strength

Parameter	*Equation*	*Value*
Guyed length	$L' = L_1 + L_2$	132 in.
Ratio of spring constant	k'/k	4
Slenderness ratio constant	K	0.7
Compression buckling constants		
B		38300
C		63
D		0.202

Table 6-21
Guyed Boom Strength Calculation

(Continuation of Table 6-17 calculations)

Reflector side	*1*	*2*	*3*
Guy moment	–63143	–38747	
Total moment	15786	10054	16831
Max(n,n+1)	15786	16831	16831
r	0.69	0.70	
KL'/r	134	133	
$\sigma_{max\ col}$	5674	5796	
Bearing area	1.15	0.71	0.30
$V_{max\ guyed}$	181	144	126
Director side	*1*	*2*	*3*
Guy moment	–63143	–38747	
Total moment	15786	10101	16294
Max(n,n+1)	15786	16294	16294
r	0.69	0.70	
KL'/r	134	133	
$\sigma_{max\ col}$	5674	5796	
Bearing area	1.15	0.71	0.30
$V_{max\ guyed}$	181	145	128

Table 6-22
Guy Support Strength

Parameter	*Equation*	*Value @ $V_{max\ guyed}$*
Guy angle	$\beta = \arctan(d/L')$	10°
Length	$L_s = d/\cos 30°$	27.7 in.
Diameter	D_s	0.75 in.
Wall thickness	t_s	0.058 in.
Radius of gyration	$r_s = \frac{\sqrt{2}}{4} D_s \left(1 - \frac{t_s}{D_s}\right)$	0.26 in.
$\sigma_{max\ col}$	$\sigma_{max\ col} = 1.02 \times 10^8/(KL_s/r)^2$	18340 lb/in.^{2}
Bearing area	$A_s = 0.25\pi [D_s^2 - (D_s - 2t_s)^2]$	0.13 in.^{2}
F_{max}	$F = A_s \sigma_{max\ col}$	2384 lb
Side force at guy, in.^{2}	$F(x_g) = c_d p_o v_{max}^2 M(L)[L'(1 + k'/k)]$	–157 lb
Support force	$F_s = -2F(x_g)$	314 lb
Guy force	$F_g = F(x_g)/\sin\beta$	879 lb

In fact, this design has survived a number of mountain winters with no boom failure.

As a final step, we calculate the column strength of the boom supports, assuming we have selected a length that corresponds to a horizontal distance d = 24 in., and assuming a support angle of 30° to the horizontal. This is summarized in Table 6-22.

We can see that the support is not highly stressed, and that the guy force can be handled comfortably by a modest cable or rope capable of handling 879 lb force with a reasonable safety factor. The new Phillystran[7] nonmetallic guy cable HPTG 1, which can be clamped using standard wire rope clamps and does not require potted end fittings, is a useful product for this application. Be sure to follow instructions for the number and orientation of the clamps (the U bolts go around the short "dead" end as on wire rope) and for capping the exposed ends. The general layout of boom guying is shown in Fig 6-18.

If the boom needs guying, it is wind, and not gravity, that represents the most severe stress. Thus, it makes sense to use three boom

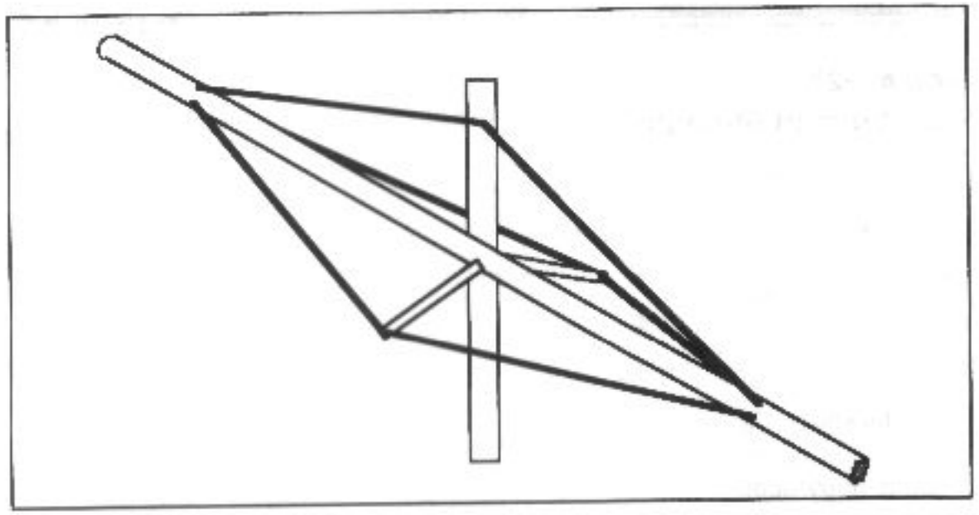


Fig 6-18—Triple boom guying.

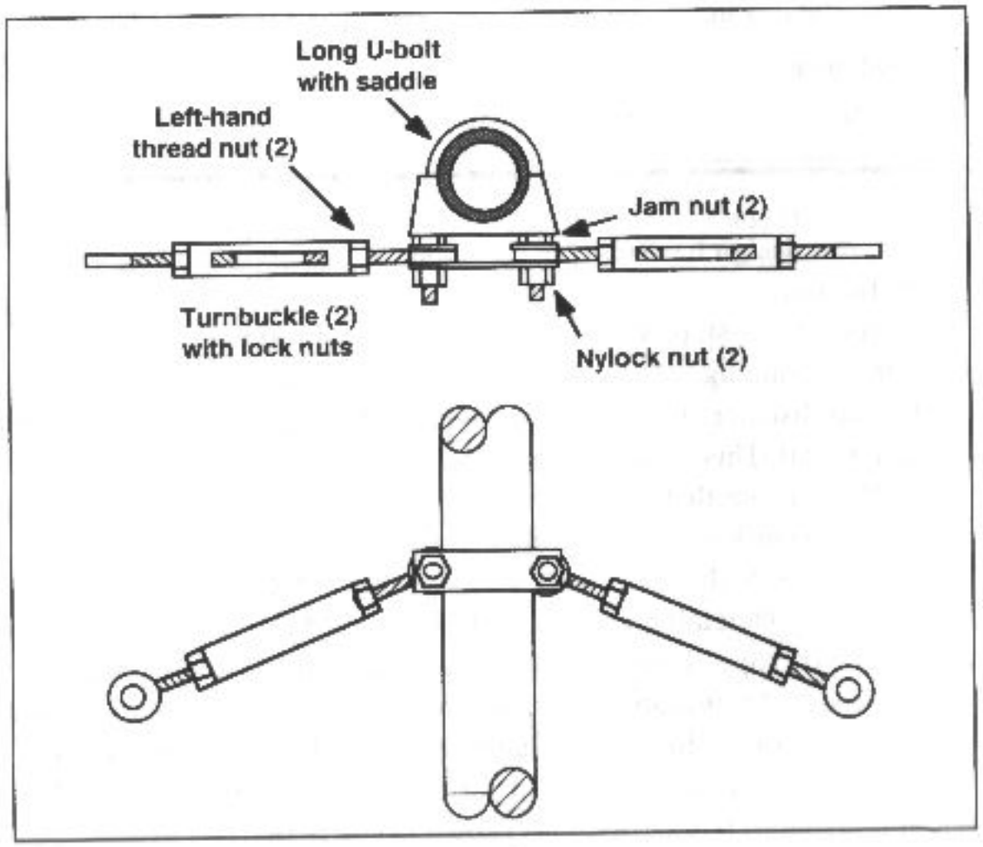


Fig 6-19—Mounting turnbuckles to a mast.

guys if any are used at all. The guys must be attached to the mast for maximum strength. Two ways of attaching the boom guys to the mast above the boom are shown in Figs 6-19 and 6-20. The height of the U bolt above the boom should be the same as the length of the side-guy supports calculated above.

I have found that the turnbuckle can be secured by using two nuts (one left-hand thread) on the turnbuckle eye threads. When the adjustment is satisfactory, the turnbuckle is held and the two nuts tightened against the body with a suitable end wrench. Stainless or plated nuts are well worth the search. Aluminum aircraft jam or check nuts are available in both left- and right-hand thread, but I have not found left-hand-thread plated nuts. Elegant but expensive streamlined stainless turnbuckles with spring safety stops are available from aircraft and marine supply houses.

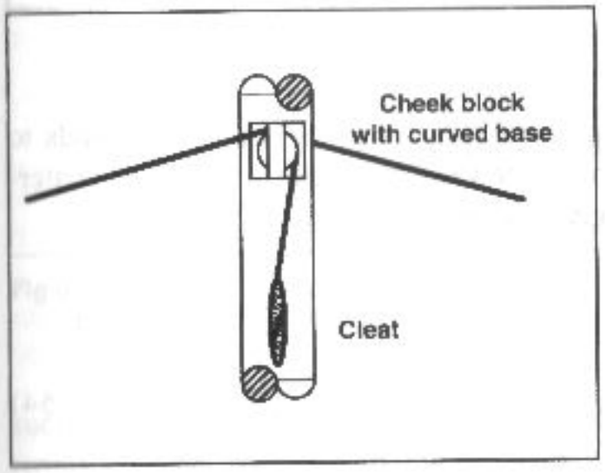


Fig 6-20—Guy mount with marine pulley and cleat.

An alternative means of attaching boom guys to the mast is the use of marine pulleys in a form known as a cheek block, Fig 6-20. The guy can be marine line (*rope* for landlubbers), and is secured in an aluminum or plated cleat that is bolted to the mast or to a plate mounted with a U bolt. This arrangement allows tensioning of the boom guys without the usual tower-top acrobatics.

As we have seen, the supports for side guys do not bear any major stress. Side-guy supports can be made from ready-made aluminum pipe 45° sweeps, threaded couplings and threaded pipe sections, as in Fig 6-21. The through-bolt threads are covered with short sections of aluminum tubing, and the guy ropes are tied to the support outside the bolts.

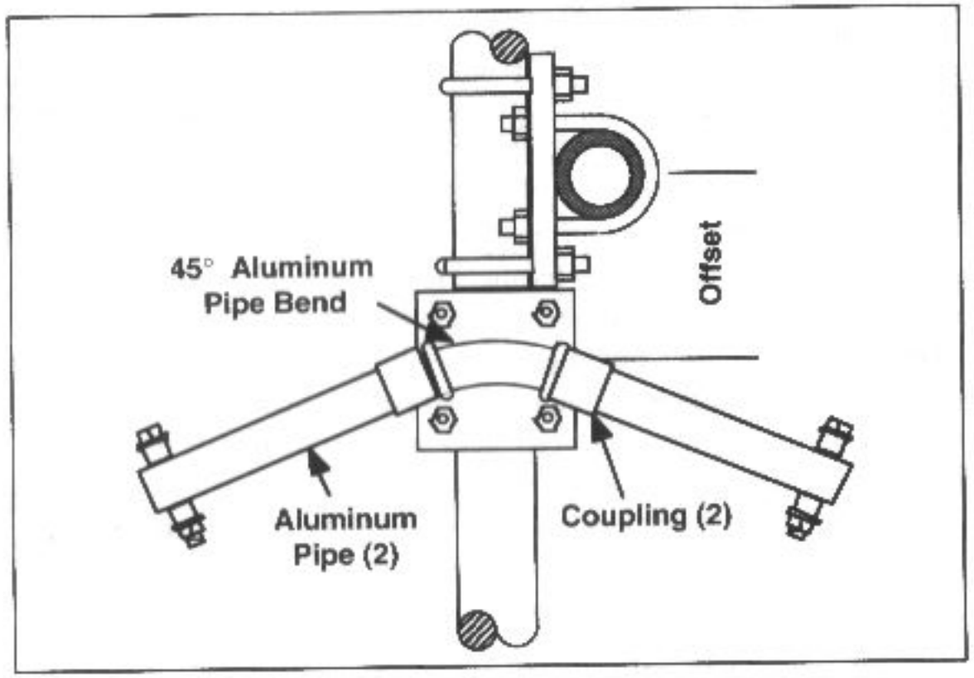


Fig 6-21—Side-guy support.

The offset below the boom centerline allows the use of 45° bends to achieve the 60° guy locations. The vertical offset from support centerline to boom centerline is given by the equation

$$\Delta h = d\cos 30°(\tan 30° - \tan 22.5°) + \left(\frac{W}{2\sin 22.5°}\right)\left(\frac{1}{\cos 22.5°} - 1\right)$$

(Eq 6-54)

where

d = length of guy support or offset
W = width of pipe bend in Fig 6-21

The first term corrects for the difference between 60° and 45° geometry, and the second accounts for the fact that the bend radius falls short of the projected point of intersection of the guy-support tubes. For the case of d = 24 in. and W = 10 in., the total offset $\Delta h \approx 4.5$ in. This is a convenient amount to avoid the boom-to-mast plate, and 45° bends, couplings and aluminum pipe are available from electrical supply houses. The total length of the guy support pipe on each side, including the length of half the pipe bend, is $L = d/\cos 22.5° = 24/0.92 \approx 26$ in.

Sec 6.9 Joining Boom Sections

Commercially available Yagi antenna kits are often derelict in providing adequate fastening between sections of telescoping or swaged boom tubing. Some manufacturers use hose clamps as with elements, and several provide a single ¼-in. bolt to join sections. The bearing capability of a hole in thin tubing can be calculated, but the use of more than one stainless bolt with reliable Nylock nuts provides a safety factor and redundancy against the substantial twisting stresses that arise in high wind conditions. In the case of added guying, the boom compression forces carried across the joint may be substantially higher than the manufacturer had in mind.

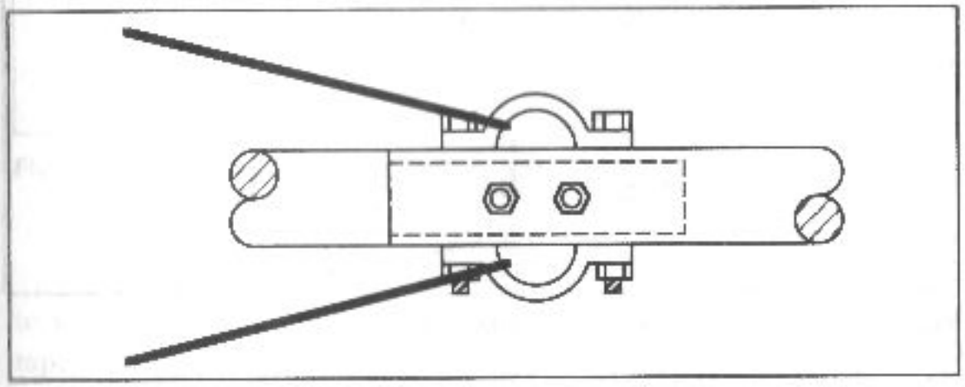


Fig 6-22—Bolted boom joint with guying eyes.

The use at a joint of a substantial number of bolts with nut plates, such as Pem-nuts, inside the smaller tube permits gripping the two tubes firmly together and is a reliable but inconvenient approach. Instead, I use four through-bolts, in pairs at right angles to minimize distortion of the boom tubes, as shown in Fig 6-22. If I am using boom guying, this is a convenient point to mount marine stainless eye straps to receive the guys, which can be wire rope or marine line. The bolts should be spaced to fit these loops, or at least several shank diameters apart, to prevent tearing between the holes. Use stainless-steel washers under all bolt heads and nuts to reinforce the boom material. Aircraft lock nuts should be used here. Internal cross tubes drilled to fit the bolt shanks will reinforce the boom from the inside and permit complete tightening of the bolts. It is a good idea to use an insulated-thimble guy termination to prevent static from metal-to-metal movement.

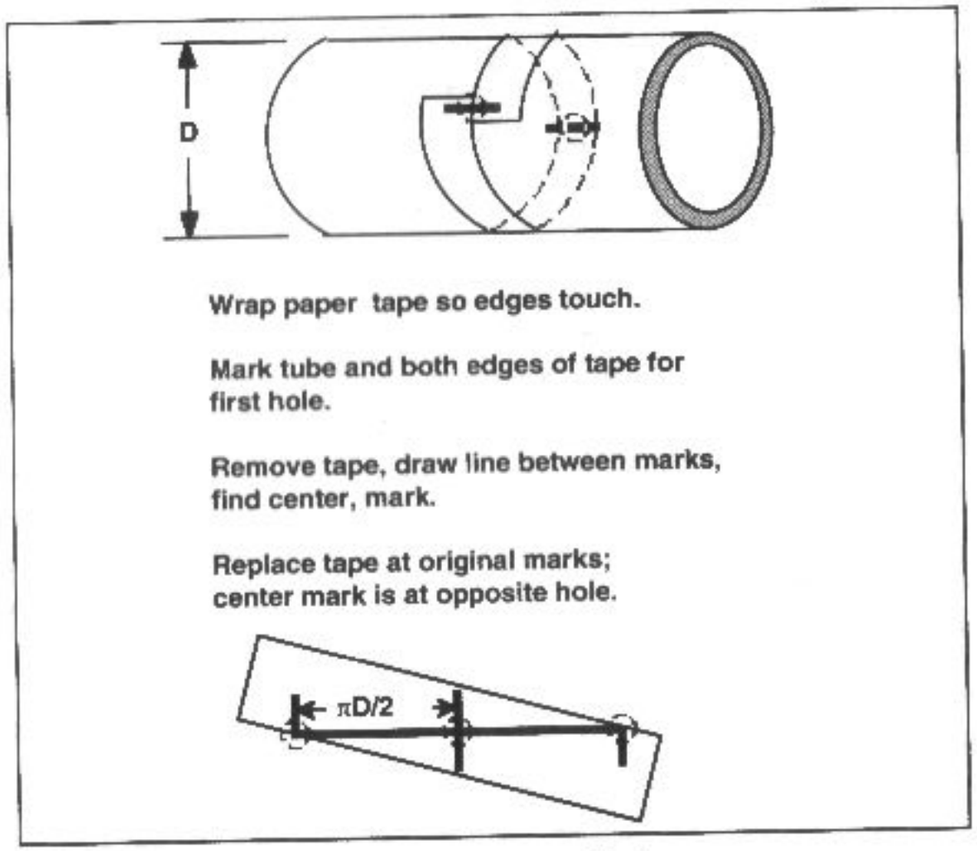


Fig 6-23—How to drill a straight hole in a round tube.

A problem that arises is the need to drill accurately through the boom tube for fastening one section to another. In the spirit of *Zen and the Art of Motorcycle Maintenance*,[8] it offends my sense of symmetry to drill a hole that is not precisely centered on both sides. This can be done easily with professional shop equipment, such as a drill press or mill, but it can also be accomplished with the aid of a clever measuring scheme using paper masking tape, a pencil, and a tape measure. I am sorry that I cannot recall a proper reference for this technique, but it did not originate with me.

As shown in Fig 6-23, wrap the tape in a spiral around the tube so the opposite edges of the tape touch at the point you will center-punch for the first hole. Mark both edges of the tape and the boom at this point. Now remove the tape and measure and mark the center point on a line between the two edge marks. Then replace the tape on the tube, matching up the original marks. The center point is directly opposite the first hole; center punch here and drill both holes from the outside.

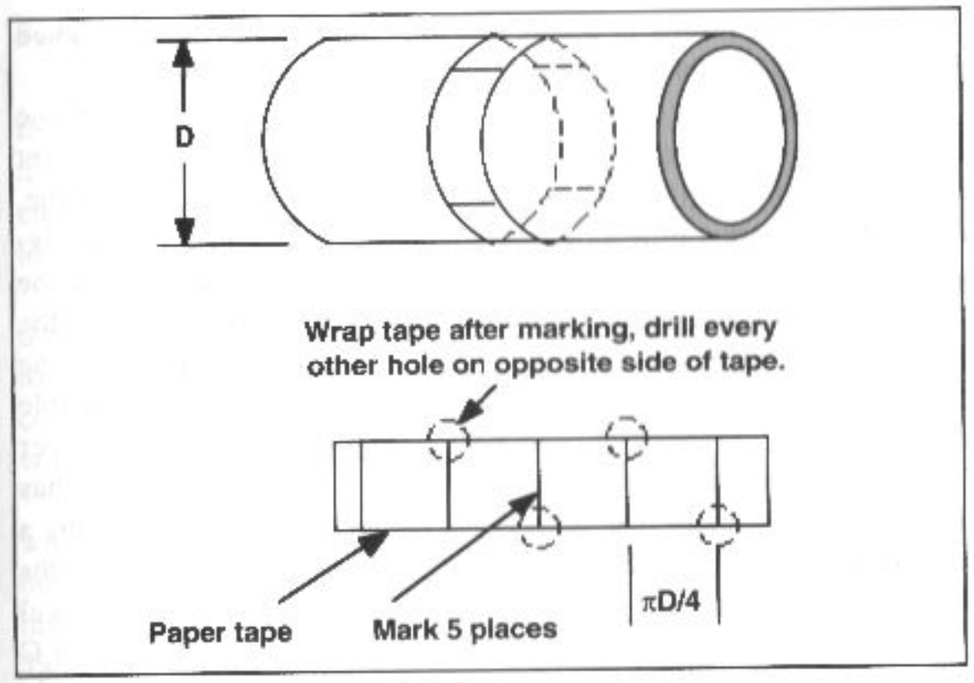


Fig 6-24—How to drill right-angle holes in a round tube.

Another helpful method is to mark paper tape at intervals equal to ¼ circumference, or πD/4, as shown in Fig 6-24. I use 1½-in.-wide tape with the lines laid out with a carpenter's square so I can use either edge of the tape for marking. Wrap the tape around the boom so that it makes a continuous ring, thus ensuring straight sides. Use alternating sides of the tape to make a set of holes at right angles to each other directly through the center-line of the boom. I use a fixture made from a muffler clamp and a short length of angle aluminum to mount a level on the boom so all the holes are in the same planes. This is also useful to get all the elements mounted in the same plane and at right angles to the mast.

It is common practice to drill a hole 1/64 in. larger than the bolt diameter (for example, a 17/64-in. drill for a ¼-in. bolt), but a closer tolerance hole gives less opportunity for relative motion that will eventually chew up the thinner wall tubing. Although bolt strength is rarely an issue, an oversize hole also reduces the effective bolt shear strength. The practice of sizing the hole with a tapered drift can be helpful here, as it is in tower-leg assembly. A light-drive fit is defined in aircraft practice as having a clearance of approximately 0.001 in. per inch of

diameter, which results in a hole into which the bolt will move when pressed by the handle of a hammer under the weight of one's body.

Best tolerances result from assembling the boom joint with one bolt, then drilling and reaming the remaining holes. Mark the boom joint halves, ideally with a letter punch set, so assembly is always the same. Although they are a little extra work to find, aircraft bolts (AN designated) are designed to bear on the smooth shank rather than on the threads in this type of application. If the bolt grip length exceeds the compressed diameter of the boom, washers are needed to prevent the nuts from bottoming before they grip. Fine-thread aircraft bolts suitable for this application are designated AN3 through AN20, with the number indicating the diameter in sixteenths of an inch. Thus, an AN4 bolt has a ¼-in. body diameter. The body diameter designator is followed by a dash number indicating the total length (not the grip length) in eighths of an inch. The standard bolt is cadmium-plated heat-hardened nickel steel (2330); corrosion resistant material is designated by the letter C before the dash number. Standard aircraft usage requires a through hole in the shank or head for safety wiring. The letter A after the dash number indicates an undrilled shank. Hence, a ¼ × 2⅞-in. stainless AN bolt, which has a 1 15/16-in. grip length suitable for a 2-in. boom, would be designated AN4-C23A.

The AN73 through AN81 (MS20073) bolt series is similar to the AN3 through AN20, but includes coarse-thread bolts for those who prefer them. The appropriate fine-thread lock nut is designated AN365-428A, denoting an elastic stop nut of ¼ in. (4/16-in.) diameter, 28 threads per inch, with a nonmetallic insert. At least one full thread of the bolt should show outside the lock nut, and the proper torque for this particular size is 8 ft-lb (96 in.-lb).

For light duty such as boom joints, one can shorten the threaded portion of standard stainless bolts and use washers under the nut to provide a thread-free grip range. Always be sure that the nut cannot bottom on the last thread, and lubricate to avoid galling and seizing. For permanent connection, rivets expand to fill the hole so there is no side motion. For those interested in knowing more about fasteners and construction techniques in general, the aircraft[9] and race-car[10] literatures are highly recommended. Note particularly the range of threaded eyes and clip-lock turnbuckles (MS21251) available from aircraft practice.[11]

Sec 6.10 Guyed Elements

In Chapter 4 we restricted our analysis to unguyed elements. In the case of 40-meter elements, even shortened ones, heroic measures are required to design an element capable of surviving a Zone C environment. With the tools we have developed for boom calculations, let's look at what can be done with guyed elements.

The Cushcraft 40-2CD, Fig 4-25A, will serve as an example for these calculations. Because there is no transverse structure analogous to the elements on a boom, the guyed element calculations require no consideration of wind angle. The maximum moment is simply for the wind normal to the element.

To proceed, we begin the calculations for an unguyed element with two major objectives—to calculate the survival speed v_{max} of the outer sections, and to find the moment M(L) at the attachment point for the unguyed element. We construct a spreadsheet identical to that of Table 4-3, down to the row dealing with the calculation of v_{max} for the case of no ice and no guying. By inspection, we can find the outboard sections for which our survival objective is met. We will attach the guys at the inner end of the most inward section that meets our wind criteria. In the case of the 40-2CD antenna, this means we will be guying the three inner sections. The sum of their lengths is $L' = 116$ in., the guying attachment distance.

Now we can choose a support dimension $d = 32$ in. so that $\arctan(d/L') = 15°$, and we can expect that $k'/k \approx 10$. Using M(L) from the unguyed element calculations, we have as in Eq 6-37

$$F(x_g) = \frac{M(L)}{L'(1 + k'/k)} \qquad \text{(Eq 6-55)}$$

where $F(x_g)$ is expressed as in.^{2}

Beginning at the point of guy attachment x_g, the moment, measured as in.^{3}, from the guy force at the inner ends of the section to which it is attached is

$$M_g(n) = L_n F(x_g) \qquad \text{(Eq 6-56)}$$

and the guy force moment at the inner ends of the more centerward sections is

$$M_g(n) = L_n F(x_g) + M_g(n+1) \qquad \text{(Eq 6-57)}$$

The total moment at the inner end of each guyed section n is given by totaling the guy force moment and the moment from the wind on the unguyed element, or

$$M(n) = M_g(n)\, M_{gn} + M_n(\text{unguyed}) \qquad \text{(Eq 6-58)}$$

We know that the maximum bending moment expressed as in.-lb for each section is $M_{max} = S_n\sigma_{max}$. Now we must explore the compressive force caused by guying and the maximum column strength of the inner, guyed sections.

The compressive force in the element guyed sections is given by

$$F_c = \frac{F(x_g)L'}{d} \qquad \text{(Eq 6-59)}$$

As before, in Eq 6-47, we calculate the radius of gyration to be

$$r = \sqrt{2}\,\frac{D_n}{4}\left(1 - \frac{t_n}{D_n}\right) \qquad \text{(Eq 6-60)}$$

and taking the smallest value among the sections for safety, we calculate the column buckling pressure

$$\sigma_{max\ col} = \frac{1.02 \times 10^8}{(KL'/r)^2} \qquad \text{(Eq 6-61)}$$

Table 6-23
Parameters for Guyed Element Strength

Parameter	*Equation*	*Value*
Guyed length	$L' = L_3 + L_2 + L_1$	116 in.
Support distance	$d = L'/4$	36 in.
Ratio of spring constant	k'/k	10
Normal guy force	$F(x_g) = -M(L)/[L'(1 + k'/k)]$	–200 in.^{2}
Compression force	$F_c = F(x_g)L'/d$	743 in.^{2}
Radius of gyration	$r = \sqrt{2}\,\frac{D}{4}\left(1 - \frac{t}{D}\right)$	0.39 in.
$\sigma_{max\ col}$	$\sigma_{max\ col} = 1.02 \times 10^8/(KL'/r)^2$	2323 lb/in.^{2}
σ_{max}		35000 lb/in.^{2}

Table 6-24
40-2CD Guyed Element Spreadsheet

Section name	*EA*	*EB*	*EC*	*ED*		*CA*	*EF*			
Section	1	2	3	4	5	6	7	8	9	Total
Diameter, in.	1.375	1.25	1.125	1	1	0.875	0.5	0.5	0.5	
Length, in.	18	54	44	12	32	18.5	3	13	64	258.5
Thickness, in.	0.058	0.058	0.116	0.116	0.058	0.058	0.174	0.116	0.058	
Weight, lb	0.42	1.15	1.59	0.38	0.54	0.27	0.05	0.18	0.51	5.08
Area, in.^{2}	25	68	50	12	32	16	2	7	32	241.9
ΣArea, in.^{2}	242	217	150	100	88	56	40	39	32	
Moment, in.^{3}	25465	21333	11427	5930	4800	2490	1600	1482	1024	
Modulus, in.^{3}	0.076	0.062	0.084	0.064	0.038	0.029	0.012	0.011	0.008	
v_{-ax}, mi/h	75	74	118	142	122	147	120	120	121	74
Guy moment	–23150	–19558	–8781							
Tot mom., in.^{3}	2315	1775	2646	5930	4800	2490	1600	1482	1024	
Greater moment	2315	2646	5930							
A, in.^{2}	0.24	0.22	0.37							
$v_{max\ guyed}$	132	121	125	142	122	147	120	120	121	120

and the bearing area

$$A_n = 0.25\pi\,[D_n^2 - (D_n - 2t_n)^2] \qquad \text{(Eq 6-62)}$$

The maximum compressive force is then $F_{max} = A_n\sigma_{max\ col}$, and we can use the same approximations as in the case of the boom calculations to determine the survival speed of each of the guyed sections, using the equation

$$v_{max}(n) = \frac{1}{\sqrt{\dfrac{1.67F_c}{A_n\sigma_{max\ col}} + \dfrac{M_n}{S_n\sigma_{max}}}} \qquad \text{(Eq 6-63)}$$

In tabular form, the parameters necessary for calculation of the guyed element section strength are shown in Table 6-23.

Table 6-25
Guy Support Strength

Parameter	*Equation*	*Value*
Length	$L_s = d/\cos 30°$	41.5 in.
Diameter	D_s	0.75 in.
Wall thickness	t_s	0.058 in.
Radius of gyration	$r = \sqrt{2}\,\frac{D_s}{4}\left(1 - \frac{t_s}{D_s}\right)$	0.26 in.
$\sigma_{max\ col}$	$\sigma_{max\ col} = 1.02 \times 10^8/(KL_s/r)^2$	10861 lb/in.^{2}
Bearing area	$A_s = 0.25\,\pi\,[D_s^2 - (D_s - 2t_s)^2]$	0.13 in.^{2}
F_{max}	$F = A_s\sigma_{max\ col}$	1369 lb
Support force	$F_s = 2F(x_g)$	106 lb
Guy force	$F_g = F(x_g)/\sin\beta$	204 lb

Using these parameters, we can construct the complete spreadsheet for the element sections. They are designated by the manufacturer's letter code. For this configuration, the element will survive at $v_{max} = 120$ mi/h, with minimal wall strengthening. This is, of course, a substantial improvement over the stock element, although there is a question regarding the unknown strength of the loading-coil section. With guying, the standard element-mounting clamps will suffice. The spreadsheet equations are summarized in Table 6-23 and the results in Table 6-24.

Of course, an element in compression should have more than just a hose clamp to hold the section joints. Some form of multiple pinning, either by rivet or by machine screw, is mandatory to prevent the compression force from collapsing the element.

The final step is to confirm the strength of the guy cable and support, as in Table 6-25. The guys are typically tightened to some 10% of the expected force, so this must be added to the calculated force from the wind. A substantial safety factor can reasonably be accommodated by even the lightest guy cables. The use of nonconducting cable, such as fiberglass guy cable or Kevlar sailboat line, is recommended. If rope

is used, be sure it does not stretch when wet, because bad weather is the time we need the guying.

References

[1] W. I. Orr and S. D. Cowan, *Beam Antenna Handbook* (Wilton, CT: Radio Publications, 1983), p 170.

[2] E. P. Popov, *Mechanics of Materials* (Englewood Cliffs, NJ: Prentice-Hall, 1952), p 102.

[3] See reference 2, p 113.

[4] *Alcoa Structural Handbook* (Aluminum Corporation of America, 1956), p 56.

[5] See reference 4, p 104.

[6] See reference 4, p 121.

[7] *Phillystran Data Sheet*, Apr 1990, United Ropeworks (USA) Inc, 151 Commerce Dr, Montgomeryville, PA 18936-9628.

[8] R. Pirsig, *Zen and the Art of Motorcycle Maintenance* (New York: William Morrow, 1974).

[9] L. Reithmaier, *Standard Aircraft Handbook* (Blue Ridge Summit, PA: Aero Publishers, Div of TAB Books, 1986).

[10] C. Smith, *Prepare to Win* (Blue Ridge Summit, PA: Aero Publishers, Div of TAB Books, 1975).

[11] *Catalog of AN/MS/NAS Fasteners*, Coast Fabrication, 17712 Metzler Ln, Huntington Beach, CA 92647.

Chapter 7

Masts and Rotators

Sec 7.1 Section Modulus and Material Strength

We come now to the questions of antenna wind force, mast strength and stacking.[1] We can determine the force from each antenna, to which we must add the wind force distributed over the mast itself. With these parameters and the locations of the antennas we can calculate the moment at the mast base, typically the point of maximum stress.

Because the mast is tubular, we can use the concepts applied to the elements and boom to determine its section modulus, and can compare the maximum stress to the yield strength of the mast material. For survival in high wind, the stress from wind force on a mast carrying one or more Yagi antennas must not exceed the maximum stress capability of the mast at the location of maximum moment.

We have seen that the stress capability of a cylindrical structural element such as the mast is determined by the section modulus of the mast geometry and the yield strength of the mast material. We'll take these parameters one at a time. Although the mast is more properly a beam-column because of the compression force of the antenna weights, we will ignore the vertical dead load and make the simplifying assumption that the bending stress dominates.

First, the section modulus of a tubular mast is determined by the same equation used for elements. The section modulus S for tubing of diameter D and wall thickness t is

$$S = \frac{\pi(D^4 - d^4)}{32D} \qquad \text{(Eq 7-1)}$$

where

D = outer diameter
d = inner diameter = D – 2t

The maximum moment M_{max} for a mast of section modulus S and yield strength σ_{max} is

$$M_{max} = S\sigma_{max} \qquad \text{(Eq 7-2)}$$

The yield strength σ_{max} of the mast material is a property of both the alloy chosen and the mast fabrication process. Typically, a steel material is chosen for reasons of strength, cost, and the fact that steel is ductile enough to fail more safely by bending rather than by snapping off. Higher strength steel alloys are generally more brittle. Aluminum masts can be comparable in strength to steel only if the material is a very expensive alloy; if the material is unknown or is of lesser strength, the use of aluminum for mast applications is too risky and is not recommended. Also, aluminum provides substantially reduced fatigue life.

For a comparison of yield strengths of materials that could be used for masts, refer to Table 7-1. Note that the alloys and heat treatment available in bar and sheet form are not necessarily available in tubing. Heat treatment is a major variable in steel. Check it out—it makes a big difference!

Note the wide range of yield strengths, which is accompanied by a commensurate range of costs. If you do not know for sure what material a mast is made of, you are taking a big risk. Ask suppliers for specific alloy information. Ideally the mast material should be marked or accompanied by a supplier's certificate. Some of the higher strength steels are not available in a galvanized finish, and that creates corrosion problems. Because it is not rated for mechanical strength, the use of galvanized water pipe for an antenna mast is definitely not recommended unless the bending moment at maximum wind is essentially zero (the antenna is located immediately at the junction of the mast and the tower top).

Sec 7.2 Steel Tubing and Pipe

Antenna masts are generally made of steel tubing or pipe that is purchased by the fabricator and subsequently galvanized. Tubular products are made in a variety of ways, using a range of steels. Fabrication categories of carbon steel tubing are electric resistance welded (ERW), cold-drawn butt welded (CDBW), seamless hot finish (HF), cold drawn (CD), and drawn over mandrel (DOM). Steel alloys are listed by the four-digit numbers representing the amount of carbon and other elements in the steel composition. Tubing is available in both low carbon (1010, 1018), high carbon (1020, 1026), and high strength (4130) alloys. Structural tubing and pipe are considered separate categories, which may conform to ASTM[2] specifications such as A 120 and A 500.

Table 7-1
Yield Strength of Tubing and Pipe

Description	*Material*	*Yield Strength, lb/in.²*
Pipe	A 120	Not rated
Pipe	A 53, A 106	30000
CREW	1010	32000
HREW	1010	35000
HF	1018	35000
STRUCT	A 500 Grade A	39000
STRUCT	A 500 Grade B	46000
CDBW	1010	50000
STRUCT	High Strength	55000
CD	1018	60000
CD	1026	65000
DOM	1020	70000
CD Alloy	4130	70000

Tubing strength is affected by both the selection of steel alloy and the fabrication process. Mast strength is determined by yield strength rather than tensile strength ratings of tubing materials. The yield strengths resulting from the various steels and fabrication processes are summarized in Table 7-1. The substantial variation is apparent; a lack of appreciation of the implications here could result in serious consequences. Certain of the abbreviations are explained further in Sec 7.3. Some masts offered by tower suppliers are said to provide a certified yield strength of 90000 lb/in.^{2} in 1027 DOM.

Sec 7.3 Tubing Manufacturing Processes

In order to match cost to application, a number of tubing manufacturing processes have been developed. Many applications would not be cost effective with the more expensive alloys and processes. The processes are listed here in order of the strength of the resulting tubing.

Electric resistance welded (ERW) tubing is available in 1010 and 1020, and is manufactured by forming flat rolled steel into a tubular shape and welding the edges. Yield strength for 1010 cold rolled and 1010 hot rolled are shown in Table 7-1; these types of tubing are designated by the abbreviations CREW (cold rolled electric welded) and HREW (hot rolled electric welded). Yield strength of corresponding 1020 CREW is 38000 lb/in.^{2} and HREW is 36000 lb/in.^{2}.

Structural-steel tubing is formed and electric resistance welded from flat rolled steel. Standard structural-steel tubing is listed as STRUCT. Structural tubing is also available in a high-strength form.

Cold-drawn butt welded (CDBW) tubing is cold drawn to size from hot rolled, continuous welded material. This tubing is typically available in 1010, although it is also available in 1018 and 1020 for higher strength.

Carbon steel seamless mechanical tubing is forged, and is commonly available in 1018. Cold drawing works the steel material and results in substantially stronger tubing. Seamless tubing is generally indicated in catalog listings by the four-digit alloy designation, along with the description cold drawn (CD) or hot finished (HF). Cold-drawn tubing is available in 1020, 1026 or 1027 steel with higher carbon content to provide higher yield strengths in the range of 65000 lb/in.^{2} to 70000 lb/in.^{2}.

Drawn-over-mandrel (DOM) tubing, also known as welded and drawn (WD), is formed from 1020 or 1026 strip and electric resistance welded, then subsequently cold drawn over a mandrel, working the metal and the weld to produce a closer tolerance, stronger tube. DOM tubing in 1020 provides a yield strength of 70000 lb/in.^{2} up to 2.75 in. diameter, and 60000 lb/in.^{2} for larger diameters.

Alloy steel seamless (SS) mechanical tubing is cold drawn by the seamless process, and is available in several chromium-molybdenum (chrome-moly) alloys such as 4130. For years this tubing has been the standard of the aircraft industry, where low price is not a typical feature. These alloys are intended to be heat treated and hardened to high strengths by the purchaser, but unless otherwise specified, the tubing as delivered provides yield strengths in the range of 70000 lb/in.^{2}. Aircraft alloy structural (AAS) tubing is a 4130 product that is specified for aircraft structural applications. It is available in heat-treat condition N, providing a yield strength of 70000 lb/in.^{2}, and HT-125, providing a yield strength of 100,000 lb/in.^{2}.

Pipe is a tubular product intended primarily for conveying fluids, but it is also used in structural applications. As distinguished from tubing, pipe is commonly produced in greater quantities, in relatively fewer standard sizes, and to less exacting specifications. Carbon-steel pipe is made using both seamless and welded processes. Pipe is produced to conform to specifications ASTM A 120, A 53 and A 106. No physical properties are specified for A 120 pipe; A 53 and A 106 boiler pipe both provide yield strengths as listed in Table 7-1.

Sec 7.4 Tubing Catalog Listings

Although we would expect to be able to purchase a finished mast from a tower supplier, it is interesting to see what the manufacturer sees when he consults a catalog listing of available steel tubing. Armed with this information, we can better evaluate the resulting mast product, and can even undertake to specify a particular process and alloy to obtain a required strength. Typical steel tubing catalog listings[3] are shown in Table 7-2. Price comparisons among the various categories are recommended.

Steel tubing with 0.120-in. wall thickness is intended for telescoping applications, and can be welded into reinforced mast structures with tapering wall thickness and strength. This process, analogous to the strength tapering of antenna elements described in Chapter 4, can result in a lighter and less costly mast for a given strength. After telescoping, the tubing is welded at the bottom and through several holes in the side, then galvanized overall. Because hot galvanizing involves a high temperature, we need to be certain that the steel strength is not adversely affected.

Detailed information on alloys and heat treating is beyond the scope of this book. These issues, which underlie the unique usefulness of steel, are the subject of extensive literature.[4,5] Generally, heat treating is the domain of the subsequent user or fabricator of a steel tubing product, not the original supplier or distributor.

Of course, the catalog range of diameters and wall thicknesses is much broader. Galvanized 2-in. masts sold for Amateur Radio service range from HREW 1010 up to masts rated at 88000 lb/in.^{2} for high-carbon steels and over 100,000 lb/in.^{2} for chrome-moly.

The use of 3-in. mast or of a mast composed of rotating tower sections with guyed bearings is recommended for high stress situations with extreme mast height or antenna area. Rotating tower systems are

Table 7-2
Catalog Examples of Steel Tubing

Diameter	*Wall Thickness*	*Grade*	*Description*
2 in.	0.120 in.	1018	CD
		1010	CDBW
		1020	DOM
		1010	HREW
		4130	CD
		—	STRUCT
2	0.125	1018	CD
		1020	DOM
2	0.188, 0.250	1010	CDBW
		1010	HREW
		1018	CD
		1026	CD
		1020	DOM
		1026	DOM
		4130	CD

available in the amateur market from at least one supplier, and are advertised in the Amateur Radio press. Strength is not the whole picture, as we will see, because the wind force on the area of a larger diameter mast or rotating tower uses up some of the extra capability.

Sec 7.5 Calculation of Mast Bending Moment

In order to determine whether a particular combination of antennas, mast and environment will be safe and survivable, we now need to calculate the bending moment at the base of the mast. The procedure for calculating the bending moment from wind force on the mast itself is the same as the calculation of bending moment of a Yagi element.

The wind force F_{mast} is given by the expression

$$F_{mast} = c_d p A_{mast} \qquad \text{(Eq 7-3)}$$

where

$c_d \approx 0.67$ = drag coefficient
$p = p_o v^2$ = wind pressure
A_{mast} = area (length times diameter) of the mast

We can determine the wind pressure from the expected mean wind speed. This force operates at the midpoint of a mast of uniform diameter, so the bending moment M_{mast} from wind force on the mast itself is given by

$$M_{mast} = \frac{L}{2} F_{mast} = \frac{c_d p L^2 D}{2} \qquad \text{(Eq 7-4)}$$

where

L = mast length
D = mast diameter

To this moment must be added the bending moment from the antenna(s) mounted on the mast. The force for each antenna operates at the point of attachment, so we need to know the distance from the mast support to that point. The maximum bending moment M_{ant} from an antenna of effective area A_{eff} at distance L_a from the base of the mast is given by

$$M_{ant} = c_d p L_a A_a = p L_a A_{eff} \qquad \text{(Eq 7-5)}$$

The total bending moment is the sum of antenna and mast moments. Fig 7-1 shows the location of the mast and antenna forces. As an example, the maximum bending moment on a 2-in. mast having an unsupported length of 10 ft and a Hy-Gain 205BA antenna mounted at 8 ft, in Zone C conditions, is calculated as follows:

$p = 50$ lb/ft^2

$L = 10$ ft

$D = 2$ in. $= 2/12 = 0.167$ ft

$c_d \approx 0.67$

$M_{mast} = (50)(0.67)(0.167)(10^2)/2 = 279.7$ ft-lb $= 3357$ in.-lb

$A_{eff} = 9.0$ ft^2

$L_a = 8$ ft

$M_{ant} = (50)(8)(9.0) = 3600$ ft-lb $= 43200$ in.-lb

$M = 3357 + 43200 = 46557 = 47 \times 10^3$ in.-lb

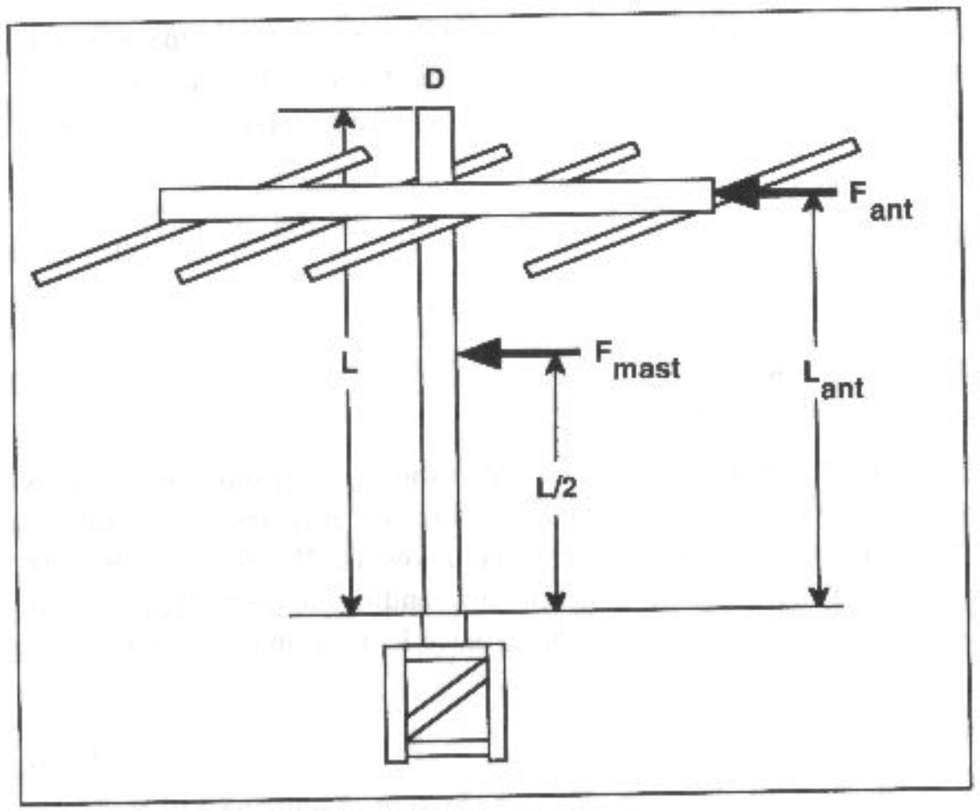


Fig 7-1—Forces on mast and antenna.

The maximum bending moment capability M_{max} of a 2-in. diameter mast with 0.25-in. wall thickness is determined by calculating the section modulus S and multiplying by the material yield strength σ_{max}. For this example, let's choose a 1026 cold drawn mast, so $\sigma_{max} = 65000$ lb/in.^{2}.

$$S = \pi\{2^4 - [2 - 2(0.25)]^4\}/(32 \times 2) = 0.537 \text{ in.}^3$$

$$M_{max} = S\sigma_{max} = 0.537 \times 65000 = 34898 = 35 \times 10^3 \text{ in.-lb}$$

For this example, $M > M_{max}$, so we can say that a 0.25-in.-wall, 1026 cold-drawn 2-in. mast will not safely support this particular antenna at the wind pressure specified.

The effective area, and hence the bending moment from the antenna, is a function of the angle of the wind relative to the boom. In order to see this effect, the moment from the elements and the moment from the boom can be determined separately and then added together.

Using the equations for element and boom force from Chapter 6, we see that the bending moment from the antenna is given by

$$M_{ant} = L_a F_a = c_d p L_a (A_{el}\cos\phi + A_b \sin\phi) \qquad \text{(Eq 7-6)}$$

For the example of the Hy-Gain 205BA given above, the mast bending moment from this particular antenna is a minimum if $\phi = 90°$, for which the antenna effective area is 3.9 ft^2. For this case, the mast moment is calculated as follows.

$M_{mast} = 3357$ in.-lb

$M_{ant} = (50)(8)(3.9) = 1560$ ft-lb $= 18720$ in.-lb for $\phi = 90°$

$M = 3357 + 18720 \approx 22 \times 10^3$ in.-lb

Protecting an undersize mast by placing the boom across the wind is risky at best. This calculation does show that if the antenna can be held stationary in a known wind direction, the forces and moments on the mast are substantially reduced for an antenna whose element area is larger than its boom area. The maximum moment occurs for the same wind angle as the maximum wind force on the antenna.

Alternatively, if we know the mast dimensions and yield strength, as well as the antenna location, element and boom areas, we can easily determine the maximum survival wind velocity, v_{max}. For a mast of diameter D and length L, the moment of area is

$$M_{mast} = \frac{L^2 D}{2} \text{ in.}^3 \qquad \text{(Eq 7-7)}$$

For an antenna at height L_a having element and boom flat (not effective) areas A_{el} and A_b, the maximum moment of area M_a is given by

$$M_a = L_a \sqrt{A_{el}^2 + A_b^2} \text{ in.}^3 \qquad \text{(Eq 7-8)}$$

The total moment of area is given by

$$M = M_{mast} + M_a = \frac{L^2 D}{2} + L_a \sqrt{A_{el}^2 + A_b^2} \text{ in.}^3 \qquad \text{(Eq 7-9)}$$

and the boom angle of maximum moment by

$$\phi_{max} = \arctan\left(\frac{M_{mast} + A_b}{M_{mast} + A_{el}}\right) \qquad \text{(Eq 7-10)}$$

The section modulus of the mast is, as before,

$$S = \frac{\pi(D^4 - d^4)}{32D} \text{ in.}^3 \qquad \text{(Eq 7-11)}$$

where $d = D - 2t$

For a mast of yield strength σ_{max}, the survival wind speed is given by the expression

$$v_{max} = \sqrt{S\sigma_{max} / c_d p_o M} = 231.8\sqrt{S\sigma_{max} / M} \qquad \text{(Eq 7-12)}$$

For the above example of the Hy-Gain 205BA on a 2-in. mast, choosing $\sigma_{max} = 65000$ lb/in.^{2},

$D = 2$ in.

$t = 0.25$ in.

$L = 10$ ft $= 120$ in.

$M_{mast} = (120)^2(2)/2 = 14400$ in.^{3}

$A_{el} = 12.2$ ft$^2 = 1757$ in.^{2}

$A_b = 5.8$ ft$^2 = 835$ in.^{2}

$L_a = 8$ ft $= 96$ in.

$M_a = 96\sqrt{1757^2 + 835^2} = 186{,}624$ in.^{3}

$M = 14400 + 186{,}624 = 201{,}024$ in.^{3}

$S = \pi(2^4 - 1.5^4)/(32 \times 2) = 0.54$ in^3

$\sigma_{max} = 70000$ lb/in^2

$v_{max} = 231.8\sqrt{0.54(65000)/201{,}024} = 96.5$ mi/h

This format lends itself easily to the formulation of a computer spreadsheet. Because published antenna information is normally in terms of effective area, we need to be careful to keep track of the drag coefficient $c_d = 0.67$ so it is not applied twice in error.

Sec 7.6 Stacking

If, as is typical, we succumb to the desire to stack more than one antenna on a mast, we can use the same general techniques to calculate the resultant survival speed at the mast base. Because each antenna will likely have a different variation of effective area with wind angle, we use the above calculation methods. We treat each antenna boom and set of elements separately, then sum the bending moments expressed as a function of the wind angle ϕ. If the mast is strengthened by use of internal reinforcement in the lower half of the mast, the moments at both the reinforcement junction and at the base of the mast must be considered.

Let's use as an example a stack of the three Hy-Gain HF monobanders for the upper frequencies, the 105BA, the 155BA, and the 204BA. We'll stack these on a 2-in. diameter by ¼-in.-wall galvanized high strength ($\sigma_{max} = 88000$ lb/in.^{2}) 1027 steel mast typical of those sold commercially. This is no place for galvanized water pipe!

We can calculate v_{max} and the associated wind angle by the method used above for a single antenna. We calculate separately the bending moment of each set of elements and of each boom, and then add these to the moment of the mast itself. To make a spreadsheet for computer calculations, see the mast parameters in Table 7-3.

Table 7-3
Mast Dimensions (Example 1)

Feature	*Value*
Diameter D	2 in.
Wall t	0.25 in.
Length L	132 in.
σ_{max}	88000 lb/in.^{2}
Section modulus S	0.5369 in.^{3}
Cross-section area A	1.37 in.^{2}
Weight	56 lb

The area A is the cross-sectional area used with the density of steel, $\rho = 0.283$ lb/in.^{3}, to find the weight of the mast, which can be an important cost, shipping and installation factor. We determine the yield strength of the alloy selected and use the expressions

$$S = \frac{\pi[D^4 - (D - 2t)^4]}{32D} \qquad \text{(Eq 7-13)}$$

$$A = 0.25\pi[D^2 - (D - 2t)^2] \qquad \text{(Eq 7-14)}$$

$$w = \rho AL \qquad \text{(Eq 7-15)}$$

Next we assemble the data for the element and boom areas and for the moments of the antennas considered, as indicated in Table 7-4.

Table 7-4
Antenna Effective Areas and Area Moments

Antenna	A_{el}	A_b	*L*	M_{el}	M_b
155BA	4.3 ft^2	3.0 ft^2	114 in.	105,883 in.^{2}	73872 in.^{2}
105BA	2.8	2.8	42	25373	25373
204BA	6.4	3.0	6	8294	3888
Mast	1.2	1.2	132	17511	17511
Total	14.7	10.0		157,061	120,644

Table 7-5
Calculation of v_{max} from M_a and M_{el}

Parameter	*Value*
$\phi_{max} = \arctan(M_b/M_{el})$	38°
$M_a = \sqrt{M_{el}^2 + M_b^2}$	198,048 in.^{3}
$v_{max} = 231.8\sqrt{S\sigma_{max} / M_a}$	113 mi/h

As the antenna data is typically expressed in terms of effective area (including the drag coefficient $c_d \approx 0.67$), the mast effective area is calculated by

$$A_{eff} = c_d DL \qquad \text{(Eq 7-16)}$$

The moments of physical area in the element and boom planes are calculated for the antennas using

$$M_a = \frac{1}{c_d}\frac{A_{eff} L_a}{144} \qquad \text{(Eq 7-17)}$$

and for the mast using

$$M_a = \frac{1}{c_d}\frac{A_{eff} L}{2(144)} \qquad \text{(Eq 7-18)}$$

The factor 144 accounts for the change of units of area from ft^2 to in.^{2}. We use $1/c_d = 1.5$ for this calculation.

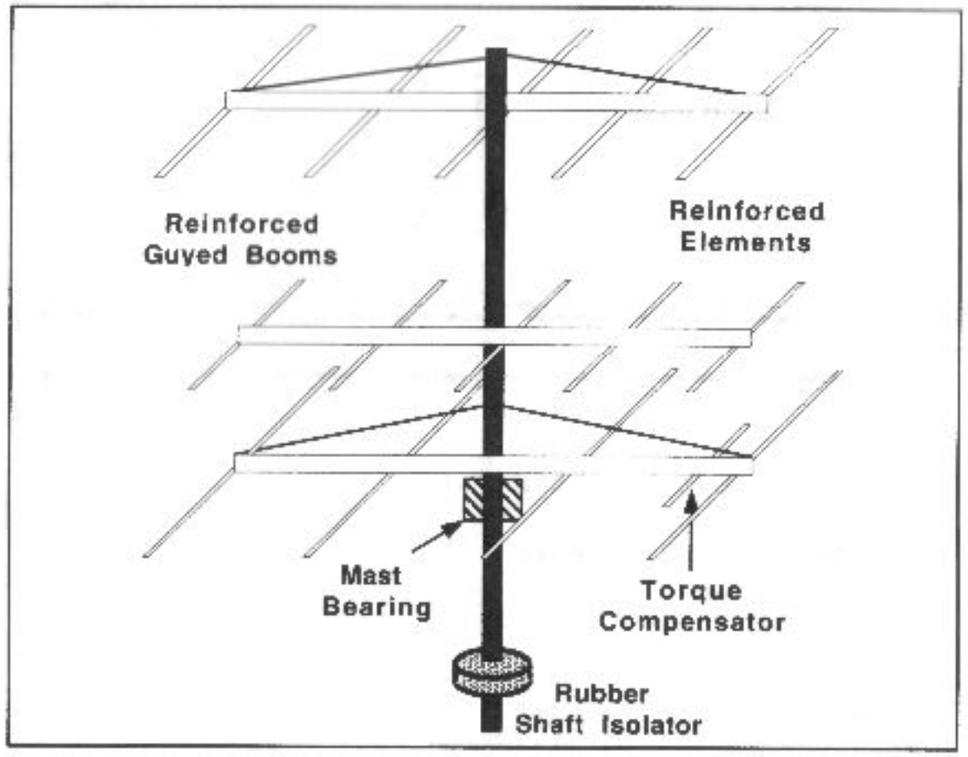


Fig 7-2 — High-strength stack.

The moment of area of the mast is the same in each plane, and is determined by the mast area multiplied by a moment arm of half the mast length. Each moment column is totaled as shown in Table 7-4.

Knowing the element and boom plane moments of area, we can determine the total moment and calculate v_{max}, as shown in Table 7-5. The moment M_a is at its maximum and the survival speed v_{max} is at its minimum at the angle ϕ_{max}. Because of the high winds at my site, this is the configuration I use; it is shown in Fig 7-2. Unfortunately, this close spacing results in substantially reduced gain for the 10-meter antenna (5 dBi as calculated using MN).

As a matter of interest, we can observe the variation of M_a and v_{max} with wind angle ϕ. We calculate $M_a(\phi)$ and v_{max} for $0° \le \phi \le 90°$, as summarized in Table 7-6.

Finally, we can determine the effective area, the angle of maximum side force (a different angle from the angle of maximum moment) and the bending moment expressed as in.-lb at the wind speed v_{max}. These values are given in Table 7-7.

Table 7-6
Area Moment and v_{max}

ϕ	0°	30°	60°	90°
$M_a = M_{el}\cos\phi + M_b\sin\phi$	157,061	196,340	183,017	120,662
$v_{max} = 231.8\sqrt{S\sigma_{max} / M_a}$	127	114	118	145

Table 7-7
Calculation of Area, Force and Moment

Parameter	*Value*
$\varphi_{max} = \arctan(A_b/A_{el})$	34°
$A_{eff} = \sqrt{A_{el}^2 + A_b^2}$	17.8 ft^2
$F = 0.004 v_{max}^2 A_{eff}$	914 lb
$M(v_{max}) = p_o v_{max}^2 M_a$	70302 in.-lb

The tower top and rotator mount must, of course, be capable of supporting the moment and side forces generated by the mast. For this example, a bearing and rotator setup spaced 4 ft (48 in.) would experience a side force at each of 70302/48 = 1465 lb.

With Table 7-7 we have completed the derivation of the spreadsheet to be used to analyze antenna masts. As with other spreadsheets, the use of functions such as IF( ) can be used to make a layout more universal.

The stack used as the example above is mounted on a 70-ft Skyneedle rated for 400 lb side force when fully extended. The mast and its base attachment bolts are thus rated at a total bending moment of (70)(400) = 28000 ft-lb. In order to accommodate the maximum side force of 914 lb at 112 mi/h wind speed, the Skyneedle must be cranked down to (28000)/(914) = 30.6 ft. However, operation is possible in all but the maximum storm conditions, albeit at reduced antenna heights. A self-standing crankup mast or tower should never be guyed unless the manufacturer specifically recommends, as the guying force must be carried out through the erecting cable of the mast; guying could accomplish the opposite of what is intended by overloading the cable to failure.

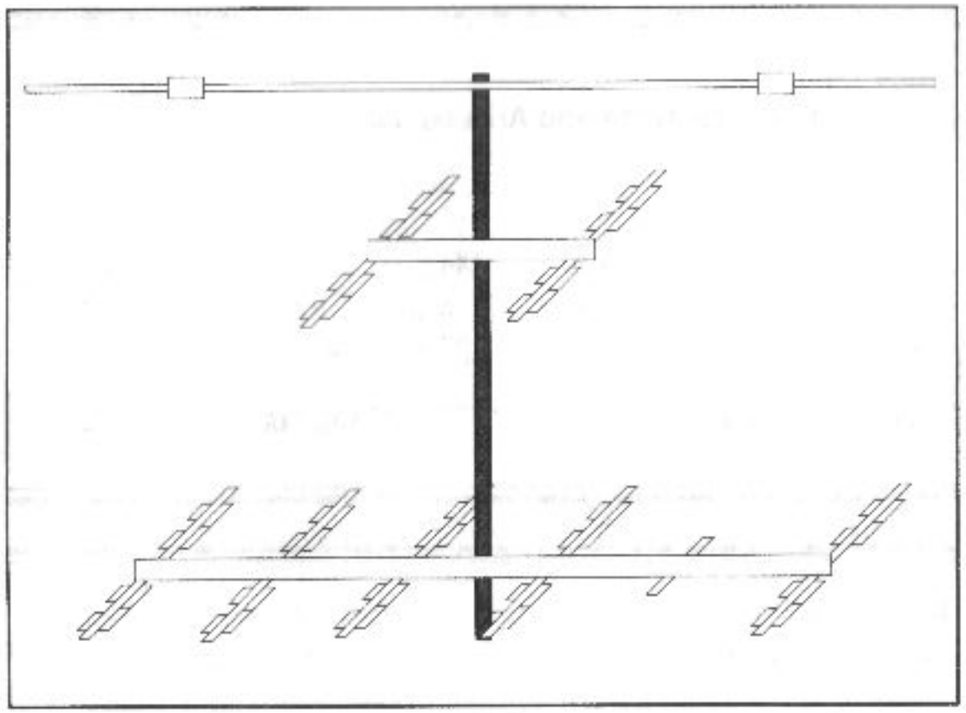


Fig 7-3—Stack of 80-meter dipole, 40-meter 2-element beam and triband beam.

As another example, consider the question of stacking a tribander, a 40-meter 2-element Yagi, and an 80-meter shortened dipole. In this example, a Create CD-78, a KLM 40-2, and a KLM KT34 XA are to be stacked on a 16 ft (192 in.) 4130 chrome-moly mast, with the objective of Zone A survival with an adequate safety margin. The configuration of this antenna stack is shown in Fig 7-3. The mast parameters are summarized in Table 7-8. Using these dimensions, we determine the element plane and boom plane moments as before, as shown in Table 7-9.

Table 7-8
Mast Dimensions (Example 2)

Diameter D	2 in.
Wall t	0.25 in.
Length L	192 in.
σ_{max}	115,000 lb/in^2
Section modulus S	0.5369 in.^{3}
Cross-section area A	1.37 in.^{2}
Weight	75 lb

For both electrical and mechanical reasons, the CD-78 was placed at the top, and in the same plane as the boom of the 40-2, which results in more nearly equal element and boom-plane areas and moments.

Table 7-9
Antenna Effective Areas and Area Moments

Antenna	A_{el}, ft^2	A_b, ft^2	*L, in.*	M_{el}, $in.^2$	M_b, $in.^2$
CD-78	0	2.8	190	0	114,912
KLM 40-2	4.5	3.7	144	139,968	115,085
KT34 XA	7.2	5.4	6	9331	6998
Mast	1.8	1.8	192	37048	37048
Total	14.7	10.0		186,348	274,044

Table 7-10
Calculation of v_{max}

Parameter	*Value*
$\phi_{max} = \arctan(M_b/M_{el})$	56°
$M_a = \sqrt{M_{el}^2 + M_b^2}$	331,399 $in.^3$
$v_{max} = 231.8\sqrt{S\sigma_{max}/M_a}$	100 mi/h

Table 7-11
Area Moment and v_{max}

ϕ	0°	30°	60°	90°
$M_a = M_{el}\cos\phi + M_b\sin\phi$	186.348	298,398	330,504	274,065
$v_{max} = 231.8\sqrt{S\sigma_{max}/M_a}$	133	105	100	110

We calculate $v_{max} = 231.8\sqrt{S\sigma_{max}/M}$ as shown in Table 7-10. Note that the drag coefficient $c_d \approx 0.67$ is contained in the factor 231.8, so we use the physical, rather than the effective area moments. The variation of moment and v_{max} with wind angle ϕ is summarized in Table 7-11. Finally, we can determine the effective area and the bending moment expressed as in.-lb at the wind speed v_{max}. This is shown in Table 7-12.

Table 7-12
Calculation of Area and Moment

Parameter	*Value*
$\phi_{max} = \arctan(A_b/A_{el})$	45°
$A_{eff} = \sqrt{A_{el}^2 \cdot A_b^2}$	19.2 ft^2
$F = 0.004v_{max}^2A_{eff}$	769 lb
$M(v_{max}) - \rho_o v_{max}^2 M_a$	92129 in.-lb

Antenna-area data can be calculated from antenna dimensions or from published specifications. If we know the published effective area A_{eff}, from the length L and diameter D of the boom we can calculate the effective boom area $A_b = c_d LD$ and determine the effective element area from $A_{eff} = \sqrt{A_{el}^2 - A_b^2}$.

Mast limitations can, in part, be overcome by using a rotating tower or rotating tower section as a mast above a fixed base section. The maximum safe moment of restraint for a typical tower design, Rohn 45, is shown on the engineering drawings in the manufacturer's catalog[6] to be 9610 ft-lb, or 115,320 in.-lb. Rohn 55 has a maximum moment of 17160 ft-lb, or 205,920 in.-lb. When compared with the approximately 48000 in.-lb limit for standard 2-in. mast, the advantages of using a tower section as a guyed mast become apparent. Additionally, the advantages of the guyed-tower mast include the ability to guy its center and to climb the entire mast comfortably.

Lattice structures have a substantially higher drag coefficient than the individual cylindrical components.[7] The equivalent face area of the towers corresponds to 5.27-in. and 6.02-in. diameter tube for Rohn 45 and 55, respectively. This means a good portion of the tower strength is used to restrain the wind moment of the tower itself, rather than the antennas. For this reason, the use of a tubular mast should be considered for the upper portion of a tower-mast installation.

To use the strength spreadsheet developed for masts on the problem of unguyed towers, we use Eq 7-12 for v_{max}, substituting the manufacturer's specification for maximum moment (expressed as in.-lb) for the calculated maximum moment $S\sigma_{max}$ in the mast problem. Using the same total moment of physical area, we calculate for the tower mast

Table 7-13
Rotating Tower Data (Force/ft @ 50 lb/ft^2)

Type	*M_{max}*	*Force/ft*	*Weight/section*
Rohn 25G	5130 ft-lb	12.70 lb/ft	40 lb
Rohn 45G	9610	14.63	70
Rohn 55G	17160	16.72	95

Table 7-14
Antenna Effective Areas and Area Moments

Antenna	*A_{el}, ft^2*	*A_b, ft^2*	*L, in.*	*M_{el}, in.^{2}*	*M_b, in.^{2}*
CD-78	0	2.8	234	0	141,523
KLM 40-2	4.5	3.7	156	151,632	124,675
KT34 XA	7.2	5.4	6	9331	6998
Mast	5.9	5.9	240	151,684	151,684
Total	17.6	17.8		312,647	424,881

$$v_{max} = 231.8\sqrt{M_{max} / M_a} \qquad \text{(Eq 7-19)}$$

where M_a = total area moment of the tower and antennas.

As an example, let's look at the same complement of antennas (Fig 7-3) mounted at 19.5 ft, 13 ft and 0.5 ft on 20 ft of unsupported Rohn 45 tower. The tower data from the manufacturer's catalog is summarized in Table 7-13. The effective area, including the effect of aerodynamic coefficients for the tubular lattice structure, is given by the force per foot of tower length at a wind pressure of 50 lb/ft^2.

Using the tower and antenna dimensions, we determine the element plane and boom plane moments as before, as shown in Table 7-14. The effective tower area for Rohn 45G is calculated by $A_{eff} = (F/50)L = 0.293(20) = 5.9$ ft^2, where here F = 14.63, the force/ft at 50 lb/ft^2. Using the same steps as for the mast calculations, we have v_{max} = 108 mi/h.

At this point we are able to calculate the performance of any combination of mast and antennas. We can use the spreadsheet results to

Table 7-15
Section Modulus and Maximum Moment of Masts

Diameter (in.)	*Wall (in.)*	*Mast*	*σ_{max} (lb/in.^{2})*	*TH7 + 40 v_{max} (mi/h)*	*Stack v_{max} (mi/h)*
2	0.125	1010 HREW	35000	50	40
2	0.25	6061-T6	35000	64	52
2	0.125	1018 CD	60000	66	53
2	0.375	1010 HREW	35000	72	58
2	0.25	1026 CD	65000	88	71
2	0.1875	1027 DOM	90000	94	76
2	0.375	1026 CD	65000	98	79
Rohn 25 Tower Section				93	82
2	0.25	1027 DOM	90000	103	83
2	0.5	1026 CD	65000	103	86
2	0.25	4130 CD	115000	117	94
3	0.1875	1026 CD	65000	120	97
3	0.25	1026 CD	65000	134	108
Rohn 45 Tower Section				123	109
3	0.25	1027 DOM	90000	158	128
Rohn 55 Tower Section				159	141

make a meaningful comparison between various mast alternatives. For comparison, we will calculate mast performance with two commonly found stacking setups, a tribander and a small 40-meter 2-element Yagi on a 15-ft mast, and a stack of three monobanders on an 18-ft mast.

The tribander used in the example is the Hy-Gain TH7DX, which has a 9.0-ft^2 element effective area and 2.7-ft^2 boom effective area. It is mounted 0.5 ft above the base of the mast. The 40-meter beam in this example is the Cushcraft 40-2CD, with 4.5-ft^2 element effective area and 2.6-ft^2 boom effective area, mounted at 12 ft.

The monoband stack example is the recommendation given in the Hy-Gain 205BA manual. The antennas and effective areas are those listed in Table 7-4, and the antennas are mounted at 16.5 ft (105BA), 9.5 ft (155BA), and 0.5 ft (204BA). Because it is mounted so low, the bottom antenna can be either the 204BA or 205BA with no change in results. Table 7-15 lists the value of v_{max} for each of these examples for a range of mast types.

Table 7-16
Reinforced Mast Dimensions and v_{max}

Section	*1*	*2*	*3*	*Total*
Length	36 in.	72	108	216
Wall	0.5 in.	0.25	0.12	—
Weight	22 lb	28	24	74
v_{max}	86 mi/h	86	92	86

This set of calculations points up the limitations of a standard 2-in. mast. We can improve survival or load capacity with increased diameter, increased or reinforced thickness, or a stronger alloy or process. High-strength alloys are difficult to drill because of their hardness, which is a good measure of the degree of heat treating. Heat treating is an area of specialized know-how; it is best to be guided by an expert supplier who knows how to use readily available steel tubing that can be galvanized without loss of strength.

Considerable weight and cost advantage is possible if the section modulus of the mast can be reduced away from the base. This is the same issue as with element tapering, except that it is mechanically simpler to taper the wall thickness by reinforcing the lower portion of the mast. The steel tubing supplied in 0.120-in. wall thickness is useful for this telescoping. From the example of other structures, we know that the ultimate strength is the same for a thick-wall mast and for a reinforced mast with the same base wall thickness. The advantage lies in the ability to thin the upper sections where less strength is needed.

As an example, the 2-in.-diameter 0.5-in.-wall mast used in the examples of Table 7-15 weighs 144 lb and provides a v_{max} for the monobander stack of 86 mi/h. A mast can be fabricated by telescoping 0.120-in.-wall tubing of the same alloy to make a reinforced mast with the wall thickness schedule of Table 7-16. This mast has exactly the same v_{max} = 86 mi/h but weighs only 74 lb, almost half the weight and material.

With proper design, this type of reinforced mast can do the job of any of the solid-wall masts. The heavy bottom section should be carried right through the tower bearing down to the rotator, and the strength requirements for the tower top should be checked. Perhaps the ideal

Table 7-17
Capacity of Unsupported Rotating Tower Masts

Antenna		*Antenna*		*Antenna*		*v_{max} mi/h*	
1	*Height*	*2*	*Height*	*3*	*Height*	*45G*	*55G*
40-2CD	19.5 ft	155BA	9.5	205BA	0.5	100	131
CD-78	19.5	KLM 40-2	13	KT34XA	0.5	108	141

reinforced mast would be fabricated from 4130 CD, heat treated after welding, and then galvanized. Such a mast, properly engineered and executed, would be expensive but reliable.

At least one tower manufacturer offers a mast with single or double reinforcing. With this configuration, the moments at the reinforcement junctions, as well as at the base of the mast, must be considered. Before applying such a mast, it is strongly recommended that the user augment his own calculations by obtaining specific alloy information and strength ratings from the manufacturer. Bear in mind that the wall thickness in the bottom section must be the same as for a suitable solid-wall mast.

As mentioned in Sec 7.4, the use of 3-in. mast or of mast composed of rotating tower sections with guyed bearings is recommended for high stress situations with extreme mast height or antenna area. Table 7-15 shows the extreme strength of even a thin-wall 3-in. mast. Strength is not the whole picture, as we have seen, because the wind force on the area of a larger-diameter mast or rotating tower uses up some of the extra capability. Some examples of rotating tower capacity, determined by use of the mast spreadsheet modified for tower sections, are shown in Table 7-17.

A possible negative aspect of rotating-tower installations is the lack of redundant guying; a guy failure will bring down the whole structure. However, redundant guying is a small price to pay for peace of mind in a windstorm. The design of this type of system should be confirmed by the supplier.

Sec 7.7 Boom-to-Mast Attachment

A very strong method of boom attachment to the mast is illustrated in Fig 7-4. As with element-to-boom clamps, it is advisable that

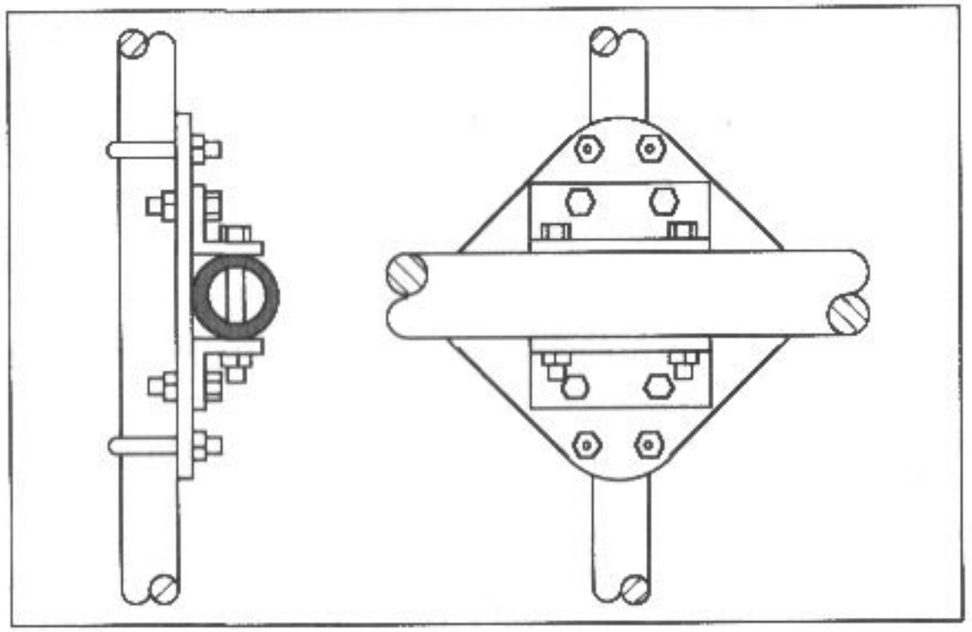


Fig 7-4—Boom-to-mast bracket.

the mast wall thickness under the U bolts equal the diameter of the U bolt. Local reinforcement can be used to avoid crushing the mast.

The boom-to-mast plate is set at a 45° angle to make the best use of its strength for a given weight. For a heavy boom, additional U bolts make an inexpensive safety factor, and a 0.375-in. plate assures strength without deformation. Additional U bolts can also be added around the boom, or bolts can be added through the boom and the plate, with the proviso that they are not so tight as to deform the boom material. The boom material should have a wall thickness at least equal to the bolt or U-bolt diameter.

For installation, the plate is mounted first to the mast, the angle brackets are bolted to the boom, and then the combination is bolted to the plate. This arrangement makes for high strength, ease of attachment at the tower top, and a minimum of hard-to-handle parts. Installing one of the bolts through the bracket and plate allows rotation in the vertical plane of the antenna.

Another convenient boom-mounting method for moderate-size antennas uses the proprietary Hy-Gain mast clamp in connection with an aluminum plate to which the boom is both bolted and U bolted, Fig 7-5. The Hy-Gain clamp uses four 4½-in. through-bolts, so it simulta-

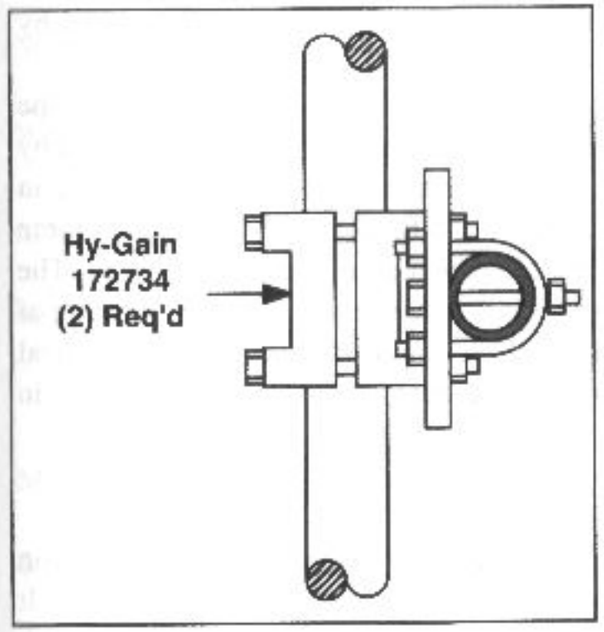


Fig 7-5—Use of Hy-Gain mast clamp.

neously clamps the mast and the boom plate. With this clamp, the mast clamping forces are spread over a larger area than is the case with U bolts. As stainless bolts and nuts tend to gall in this high-tension application, I use plated Grade 5 bolts (5/16 -18) and plated elastic stop nuts, along with SAE size stainless washers under the bolt head and the nut. This method uses a standard mast fixture, so various antennas can be interchanged conveniently.

Hy-Gain's part number for each half of this clamp is 172734. For 2-in. booms, the accompanying no. 172735 boom-bracket plate and no. 172732 boom-to-bracket clamp are convenient, but for high-wind sites it is recommended to carry the boom through the clamp rather than relying only on the strength of the clamp. The booms for the Hy-Gain models 205BA and 7-3 use this method, and the swaged components can be a convenient means of boom fabrication if properly joined.

Sec 7.8 Los Angeles County Specifications

As with any calculation of this type, there is considerable latitude in the area of safety factors and interpretation. Los Angeles County, California, publishes its own method of calculation of antenna and mast forces for antennas mounted higher than 60 ft. This specification appears to be based on the Uniform Building Code (UBC) methods outlined in Chapter 2. It is instructive to review briefly the differences

between the methods described above and the methods prescribed by the Los Angeles County procedure.

A principal element of the Los Angeles County procedure is the specification of a maximum wind velocity of 70 mi/h, compensated by the use of a drag coefficient of unity for tubes with a diameter less than 2 in. In essence, this corresponds to the EIA Zone A (86.6 mi/h) force on cylindrical elements, giving effect to a drag coefficient of 0.67. The Los Angeles procedure also uses a very conservative safety derating of 0.575 applied to the material yield strength. It is suggested that local sources be sought for any other such local codes as may be in force in your area.

Sec 7.9 Rotator Selection

The rotator must have sufficient starting torque to initiate rotation against the inertia and wind torque of the antenna(s) to be rotated. It must also have sufficient running torque to continue the rotation through the most unfavorable wind torque angles, and it must have adequate braking strength and torque to stop and hold the antenna(s) under maximum expected operating and nonoperating wind conditions.

Generally, rotators for amateur service are rated in vague terms of antenna area in square feet. Rotating and braking torques are published for a number of the better rotators.

Sachs[8] provides an exhaustive treatment of rotator torque determined from antenna moments. For amateur Yagi purposes, a much simpler analysis suffices.[9] We can assume that the rotational moment of inertia of an antenna is approximately that of the boom if its weight were evenly distributed, which is given by

$$I = \frac{wL^2}{12} \qquad \text{(Eq 7-20)}$$

where w = boom weight

Of course, it is possible to use the specific element weights, locations, and the actual distribution of boom weight, to calculate the boom moment of inertia more precisely, but this estimate suffices for our purposes. The typical taper of boom wall thickness is somewhat compensated for by the weight of the reflector and last director.

The rotator torque required to accelerate from stopped to a rotating speed of ω radians per second in a time of t seconds is

$$M = \frac{I\omega}{tg} \qquad \text{(Eq 7-21)}$$

where g = acceleration of gravity = 32.2 ft/sec^2

If we assume a rotation speed of 1 revolution (2π radians) per minute (60 seconds), we have

$$\omega = \frac{2\pi}{60} \qquad \text{(Eq 7-22)}$$

Combining, we have

$$M = \frac{wL^2}{12}\,\frac{2\pi}{60\,tg} = \frac{wL^2}{307.5t} \qquad \text{(Eq 7-23)}$$

or by interchanging terms

$$t = \frac{wL^2}{307.5M} \qquad \text{(Eq 7-24)}$$

As long as the motor torque exceeds the friction and wind torque, the antenna will sooner or later reach full motor speed. As an example, for an antenna of boom length L = 32 ft and weight w = 56 lb, for a rotating torque of 750 in.-lb, we have $t = 56(32)^2/(307.5 \times 750) \approx 0.25$ second. If we are willing to wait 1 second for full rotating speed, we need only $M = 56(32)^2/(307.5 \times 1) = 186$ in.-lb.

In the absence of torque balance, the requirement to disengage the brake and start rotation or to brake the antenna(s) against the static wind torque may be a more severe limitation. As we have seen, the unbalanced element distribution of the Hy-Gain 205BA, for example, can require as much as 1600 in.-lb of rotating or braking torque. If the antennas on a mast are balanced for wind torque, the rotator motor and gears must still start, rotate, and hold the antenna(s) against wind-induced rotational oscillations proportional to the antenna rotational moment of inertia. A very large antenna can produce peak oscillating forces that can break the rotator drive components, even though the motor may be able to rotate the antenna. Component heating is rarely a rotator limitation in amateur intermittent service.

It should be noted that each of the antennas on a mast can be torque balanced, or the entire ensemble of antennas can be torque balanced by using a compensating element on just one of the antennas. The compensating element moment is, of course, calculated by deter-

Table 7-18
Comparison of Rotator Torque

Rotator	*Rotating Torque, in.-lb*	*Brake Torque, in.-lb*
Emoto 105	450	2600
Create RC5-1	500	6000
Create RC5-3	500	6000
Emoto 747	600	6000
Emoto 1105	700	8700
Telex Ham IV	800	5000
Yaesu G-800	950	3450
Yaesu G-1000	950	5200
Telex Tailtwister	1000	9000
Create RC5A-2	1400	13000
Create RC5A-3	1400	13000
Emoto 1200	1700	15600
Create RC5B-3	1910	17000
Orion OR-2300	2000	17000
Daiwa	2400	18000
Emoto 1300	2600	21000
Emoto 1800	3450	26000
Telex HDR300	5000	7500
TIC Ringrotor 1032	7500	7500

mining the centroid and wind area of the entire ensemble. This approach has the advantage of simplicity and the disadvantage of requiring a compensator change if any antenna is changed.

Table 7-18 summarizes the manufacturer's rotating and braking-torque ratings for some rotators available in the market. If rotator torque is specified in Newton-meters (n-m), the in.-lb equivalent is obtained by multiplying by 8.85.

Not every rotator can rotate or brake the uncompensated 205BA of the earlier example. I use the HDR300 rotator for the simple reason that I had two of them, and the antennas rotate as desired. I understand the torque rating of prop-pitch motors is approximately 11000 in.-lb; they seem to do a good job for many.

Table 7-19
Roller Chain Characteristics

ANSI Chain No.	*Pitch, in.*	*Width, in.*	*Ultimate Strength, lb*
40	0.5	0.312	3700
50	0.625	0.375	6100
60	0.75	0.5	8500

As can be seen from Table 7-18, a number of the available commercial rotators were designed for light-duty television service and will not survive even the least-severe wind environment for more than a limited time. This fact underlies the popularity of the now hard-to-get prop-pitch motors, and suggests the advantage of making one's own rotator setup.[10] The secret of the prop-pitch motor seems to be gearing down a high-RPM motor.

To do this effectively, you will need to become familiar with gearboxes, and with chain and sprockets. For those whose setups include the use of chain drive, I recommend a quick trip to the local bearing store (they handle all manner of drive components, including chain and sprockets) to get complete catalogs and advice. After breaking no. 40 chain twice, I went to no. 60 and have had no problems since. Chain is ordinarily sold in 10-ft lengths, which is more than is typically needed and seems to result in considerable scrap lengths. For outdoor use, plated chain is preferred to avoid rust. Since it is widely used on motorcycles, no. 50 chain is available in smaller lengths and is a good buy at a motorcycle shop. Learn to use the proper tool to press out the chain pins. (Buy the tool at a bearing shop; it's somewhat expensive—about $14—but worth it.) And learn about master links before you're juggling greasy chain and small parts on the tower.

The standard hubs are a good way to mount sprockets, which should be as large in diameter as convenient to keep the chain force and backlash at a minimum. Split sprockets are available, but they are expensive, so it is well to plan the sprocket installation at the time of mast installation. Table 7-19 summarizes chain facts.

If you do use chain drive, always be sure to keep your fingers absolutely clear. This applies also to any loose clothing that could ultimately pull you into the works; the harm to you is not reversible! An

enclosed guard is most advisable. Safety glasses are also a great investment.

An excellent example of mechanical engineering is the Rotating Tower Systems arrangement to rotate tower sections in lieu of rotating the mast. This setup uses a chain drive with a 2:1 mechanical advantage to apply a commercial rotator, the HDR300, to rotation of an antenna combination with substantial torque requirements.

Another thought I have had is the idea that a washing-machine motor and its gearbox might be adapted to make a superb rotator. Consider the fact that you can buy an entire washing machine for less than the price of the heaviest antenna rotators.

If you use a 120-volt motor, such as the imported heavy-duty Emotator unit, be sure to use a transformer at the tower top so your wiring meets code. In the US, the maximum allowable voltage to run up a tower, for very good safety reasons, is 50 volts peak or 32 volts RMS. Rather than using standard small-gauge rotator cable (another television escapee) for the rotator power, use multiconductor UF Romex cable rated for outdoor use, and be sure to use one with a ground wire.

One way to keep down rotator and drive-train breakage is to use a shock-absorbing mechanism in the drive. I have seen various spring setups, but they all are complicated and of uncertain strength and durability. I use the rubber donut called a "flex-disc" that isolates the drive shaft from the transmission of smaller BMW automobiles. Spending $25 at a foreign-car parts store gets you a beefy rubber donut capable of all the torque expected in an antenna setup. The donut has eight steel tubes cast in it to take four bolts from each direction. The isolation provided seems just enough to protect the rotator from shock loading during high winds. Fig 7-6 shows a sketch of my setup. A machined-flange setup more like the automotive design would be nice, but the results are good with the simple setup. This unit must be supported between suitable bearings, such as the rotator itself and a tower-top sleeve tube. An alternative is to mount the rotator itself on a similar resilient base, although this method carries the shock force through the rotator gears against the inertia of the rotator mass itself.

Let me close this chapter with yet another reminder that safety is paramount in antennas and masts, during design, installation, and subsequent operation. The leverage and sheer weight of a steel mast can be overwhelming, and the caution of a conservative design and installation

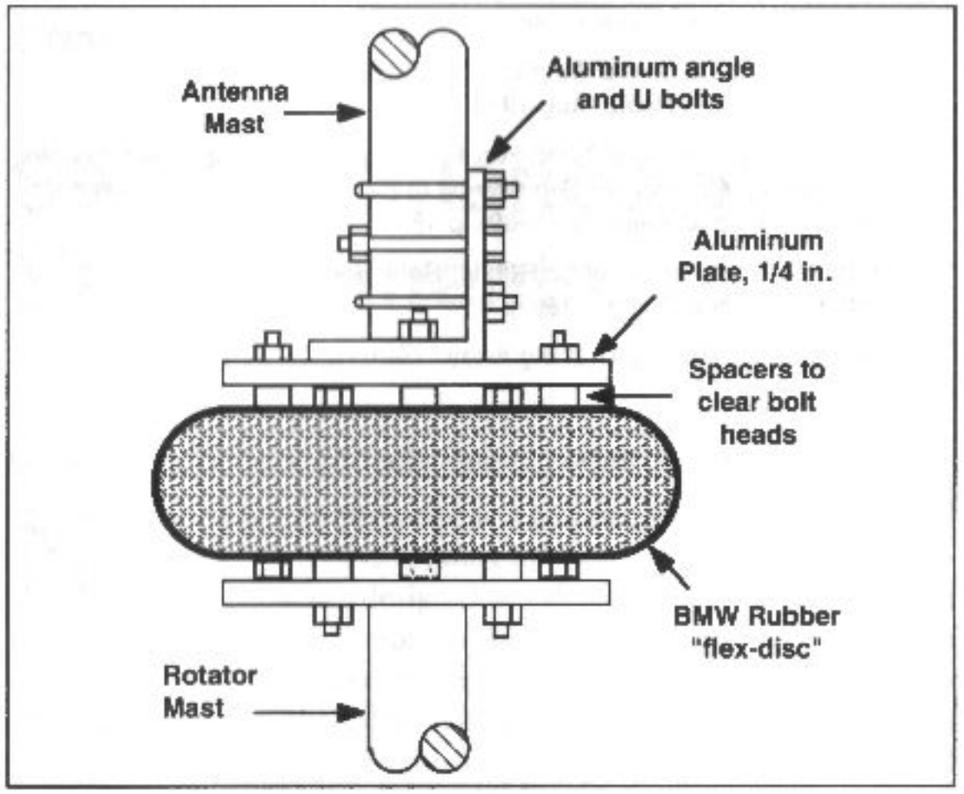


Fig 7-6—Automotive damper used as torsion spring.

are strongly recommended. Removing a bent mast is a difficult and dangerous task. Short cuts or ignorance can result in injury or death.

The services of a professional engineer or installer represent a small portion of the expense of a major antenna installation, and the cost will be repaid many times over during the life of the installation. You and your family (and your neighbors) will sleep better, and you will be able to spend your time contesting or enjoying your radio rather than repairing bent masts or broken antennas.

References

[1] S. Griffiths, "Antenna Mast Design," Part 1, *NCJ*, Sep/Oct 1982, p 11, and Part 2, *NCJ*, Jan/Feb 1983, p 5.

[2] American Society for Testing Materials.

[3] *Steel and Aluminum Stock List and Reference Book* (Earle M. Jorgensen Co, 1986).

[4] H. H. Ryffel, *Machinery's Handbook* (Industrial Press, 1989), pp 430-556.

[5] C. Smith, *Engineer to Win*, (Motorbooks International, 1984), pp 13-84.

[6] *Commercial Products Catalog* (Rohn Corp, 1983).

[7] J. Nagle, "How to Calculate Wind Loading on Towers and Antenna Structures," *Ham Radio*, Aug 1974, p 16.

[8] P. Sachs, "Determination of Wind Forces Acting on Antenna Structures," in C. Richards, *Mechanical Engineering in Radar and Communications* (D. Van Nostrand Reinhold, 1969), p 186.

[9] T. Gentry, "A Primer on Antenna Rotator Selection," in P. O'Dell, *CQ Antenna Buyer's Guide*, 1989, p 65.

[10] V. Mozarowski, "Turning That Big Array," *Ham Radio*, Jun 1986, p 10.

Chapter 8

Self-Impedance of Yagi Elements

Sec 8.1 Reactance of a Dipole Near One-Half Wavelength

Now we turn from mechanical design questions to the electrical issues involved in creating a physical antenna from an underlying computer design. The calculation of Yagi antenna gain and input impedance incorporates the self-impedances of the antenna dipole elements and the mutual impedances among them. These impedances are functions of element length, diameter, and spacing. In order to calculate and optimize Yagi performance, we would like to be able to determine these parameters for dipole elements of arbitrary length and cross-section. We can analyze such dipoles using numerical techniques, but analytic expressions are better suited to synthesis.

For thin dipoles, the properties resulting from radiative effects (the mutual impedances and self-resistances) do not vary greatly with element diameter or shape, and can be modeled well as functions of element length and spacing only. However, the self-reactance of an element is affected to a significant degree by its diameter and the variation of diameter along its length. Because the element self-reactances enter directly into the matrix expressions for Yagi gain and impedance, their determination forms a material part of the process of Yagi electrical design.

The reactance of a near-half-wavelength dipole antenna, no matter what its shape, is a linear function of frequency or length. That is, the reactance can be expressed to a high degree of accuracy for varying length at constant frequency or wavelength for a dipole of length ℓ per side, shown in Fig 8-1, as

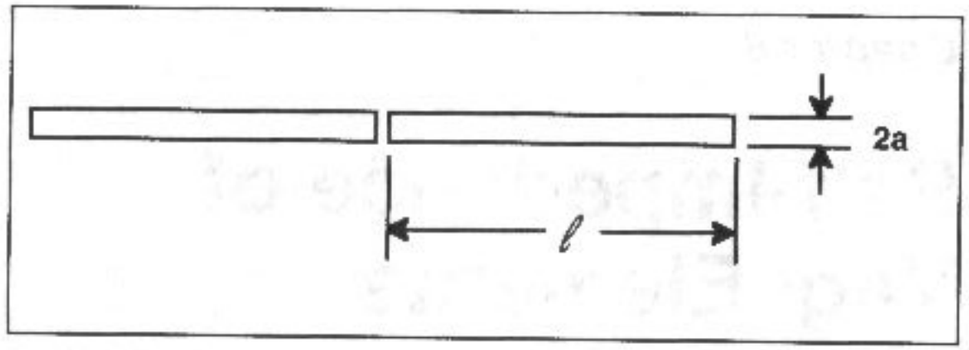


Fig 8-1—Dipole dimensions.

$$X = X_0 + k\frac{\Delta\ell}{\ell_0} \qquad \text{(Eq 8-1)}$$

where

k = a constant

$\Delta\ell = \ell - \ell_0$

X_0 = reactance for $\ell = \ell_0 = \lambda/4$

If instead the length is held constant, we have

$$X = X_0 + k\frac{\Delta f}{f_0} \qquad \text{(Eq 8-2)}$$

where

$\Delta f = f - f_0$

$f_0 = \frac{c'}{\lambda_0} = \frac{c'}{4\ell_0}$

c' = velocity of light in air

The constant is the same in each case. This means we can characterize the reactance near resonance of a dipole of any shape in terms of only two parameters—the reactance for that dipole at exact half-wavelength, and the rate of change of reactance for that dipole with frequency or length. The linear variation of dipole reactance is shown in Fig 8-2.

We note that a near-quarter-wave transmission line also exhibits a linear variation of reactance with frequency or length. For length changes $\Delta\ell$ near $\ell \approx \lambda/4$, the reactance of a $\lambda/4$ transmission line of characteristic impedance Z_0 is

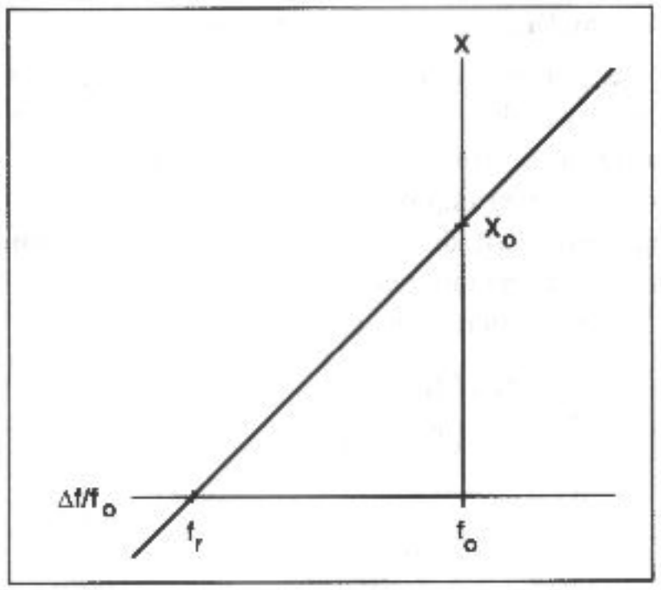


Fig 8-2—Dipole reactance near half-length $\ell = \lambda/4$.

$$X = Z_0 \tan\left(\frac{\pi\Delta\ell}{2\lambda/4}\right) \approx \frac{\pi Z_0}{2} \times \frac{\Delta\ell}{\ell_0} \qquad \text{(Eq 8-3)}$$

This suggests that we can model the reactance of a dipole antenna of length $\ell \approx \lambda/4$ as a quarter-wave transmission line of characteristic impedance Z_0, as shown in Fig 8-3, so that the reactance constant is $\pi Z_0/2$.

The antenna reactance X_0 at $\ell = \lambda/4$ is not zero because of the effect of radiation, and is a function of element shape. A general expression of the reactance of a near-half-wave dipole *of any shape* is

$$X = X_0 + \frac{\pi Z_0}{2}\left(\frac{4\ell}{\lambda} - 1\right) \qquad \text{(Eq 8-4)}$$

where the constants X_0 and Z_0 define any specific dipole

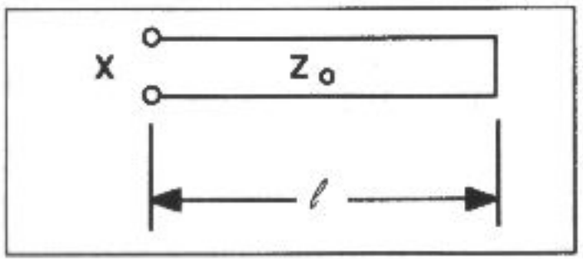


Fig 8-3 —Transmission-line equivalent for the reactance of a dipole at frequencies near resonance.

Sec 8.2 Equivalence Among Dipoles of Various Shapes

The variation of reactance of a dipole of any shape is a linear function of the normalized variable $(4\ell/\lambda - 1)$. This means we can describe any dipole near resonance in terms of an equivalent dipole of different shape and length. For example, we can describe a tapered dipole in terms of an equivalent cylindrical dipole for which Yagi gain and impedance calculations may be more convenient.

The reactance X can be written in the equivalent forms

$$X = X_0 + Z_0\left(\beta\ell - \frac{\pi}{2}\right) = X_0 + \frac{\pi Z_0}{2}\left(\frac{4\ell}{\lambda} - 1\right)$$

$$= X_0 + \frac{\pi Z_0}{2} \times \frac{\Delta f}{f_0} \qquad \text{(Eq 8-5)}$$

For a dipole of length ℓ per side, at $4\ell/\lambda = 1$,

$$\lambda = \frac{1}{4\ell} = \frac{c'}{f_0} \qquad \text{(Eq 8-6)}$$

from which the half-wave frequency is

$$f_0 = \frac{c'}{\lambda} = \frac{c'}{4\ell} \qquad \text{(Eq 8-7)}$$

If for a given dipole we know X_0 and Z_0, we can calculate the resonant frequency for which the reactance X is zero from

$$\frac{\Delta f}{f_0} = \frac{-2X_0}{\pi Z_0} \qquad \text{(Eq 8-8)}$$

If we postulate a dipole of arbitrary shape whose reactance at f_0 is known or calculated to be X_0 and whose reactance at, say, $f1 = 0.95f_0$ is known or calculated to be X_1, from the geometry of Fig 8-4 the resonant frequency f_r for which $X = 0$ is

$$f_r = f_0\left[1 - \frac{X_0\,(f_1 - f_0)}{f_0\,(X_1 - X_0)}\right] = f_0\left(1 - \frac{X_0}{\Delta X} \times \frac{\Delta f}{f_0}\right) \qquad \text{(Eq 8-9)}$$

where

$\Delta f = f_1 - f_0$

$\Delta X = X_1 - X_0$

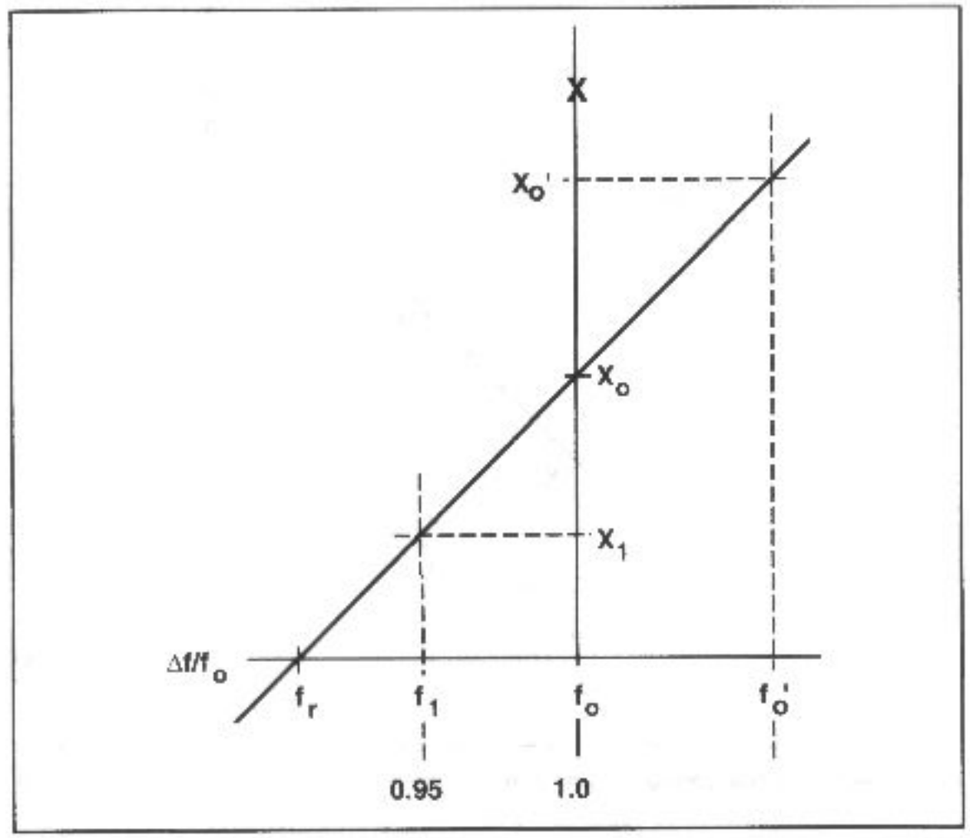


Fig 8-4—Dipole reactance for frequency normalized to $f_0 = c'/4\lambda$

For this dipole, we can define the reactance slope by a characteristic impedance

$$Z_0 = \frac{2\Delta X f_0}{\pi \Delta f} \qquad \text{(Eq 8-10)}$$

and the per-unit shortening at resonance can be expressed as

$$\partial = \frac{2X_0}{\pi Z_0} \qquad \text{(Eq 8-11)}$$

such that

$$f_r = f_0(1 - \partial) \qquad \text{(Eq 8-12)}$$

Suppose now that we wish to find another dipole of a different shape (say, cylindrical) for which the reactance as a function of frequency is identical to the example above. Because it has a different shape, this dipole will have a different reactance at its exact physical

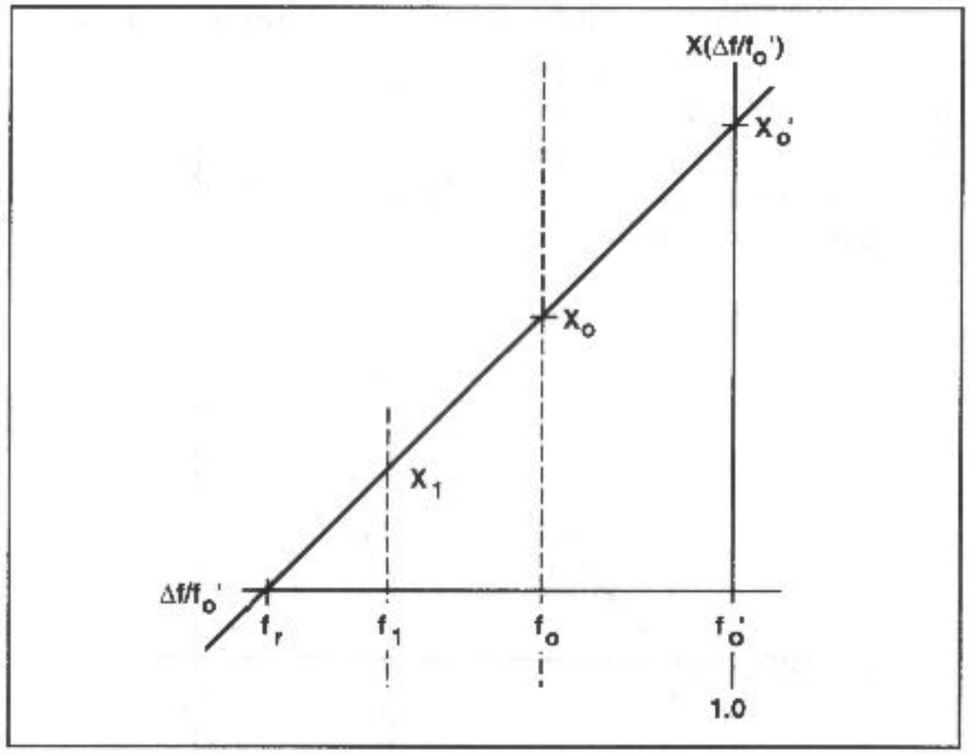


Fig 8-5—Dipole reactance for frequency normalized to $f_0' = c'/4\ell'$.

half-wavelength, which I designate X_0' at frequency f_0'. Again from Fig 8-4, by simple geometry we have

$$f_0' = f_0\left[1 - \frac{0.05(X_0 - X_0')}{\Delta X}\right] = f_0\left[1 - \frac{2(X_0 - X_0')}{\pi Z_0}\right] \qquad \text{(Eq 8-13)}$$

But consider now the reactance of this second dipole as a function of frequency normalized instead to f_0'. The reactance curve normalized to f_0' must pass through X_0 at f_0, and through $X = 0$ at f_r. Diagrammed in Fig 8-5, this is similar in appearance to Fig 8-4 if we set its characteristic impedance (reactance slope) Z_0' equal to

$$Z_0' = Z_0 \frac{f_0'}{f_0} \qquad \text{(Eq 8-14)}$$

where $f_0' = c'/\ell'$ = the half-wave frequency of the cylindrical dipole of length ℓ'

Because

$$\frac{f_0'}{f_0} = \frac{\ell}{\ell'} \qquad \text{(Eq 8-15)}$$

we have found the length $\ell' = \ell f_0 / f_0'$ and characteristic impedance $Z_0' = Z_0 f_0'/f_0$ of a cylindrical dipole with the exact same resonant frequency and reactance variation with frequency as the arbitrary dipole of length ℓ and characteristic impedance Z_0.

It remains to find the appropriate cylindrical radius that provides the characteristic impedance Z_0'. We will return to this question later.

Sec 8.3 Effect of Element Shape

The resonant frequency of a dipole antenna depends upon its shape as well as its length. Lord Rayleigh[1] was the first to point out this fact, and its implications for antennas have been explored by Schelkunoff,[2,3] and by Uda and Mushiake.[4]

Because the modeling and design of Yagi antennas is more convenient under the assumption of cylindrical elements, it is necessary to determine the equivalence between a tapered element and a cylindrical element. Lawson, in his classic work on Yagi antennas,[5] developed a method for treating dipole elements with multiple section diameters, using Uda's calculated element impedances as a basis.

I direct your attention to footnote 12 of Lawson's first chapter, which describes the approximations inherent in the assumption that the mutual impedances of thin dipoles are not strongly affected by shape, so that a tapered dipole can be represented by a cylindrical dipole of the same self-impedance. Beezley, in connection with his Yagi design computer programs MN and YO,[6] suggests that Lawson's algorithms can, by comparison to numerical calculations of tapered dipoles, be modified for improved accuracy.

Schelkunoff describes a method to determine the self-impedance of a dipole of arbitrary shape. His development proceeds from the fact that a conical dipole can be modeled exactly, and that the effect of radiation on the terminal impedance of the dipole can be easily quantified. From the exact results of the conical dipole shown in Fig 8-6, dipoles of arbitrary shape can be treated by approximation.

Schelkunoff shows that, because the ratio of thickness to distance from center is constant, the conical dipole can be modeled by a uniform transmission line of *constant* characteristic impedance

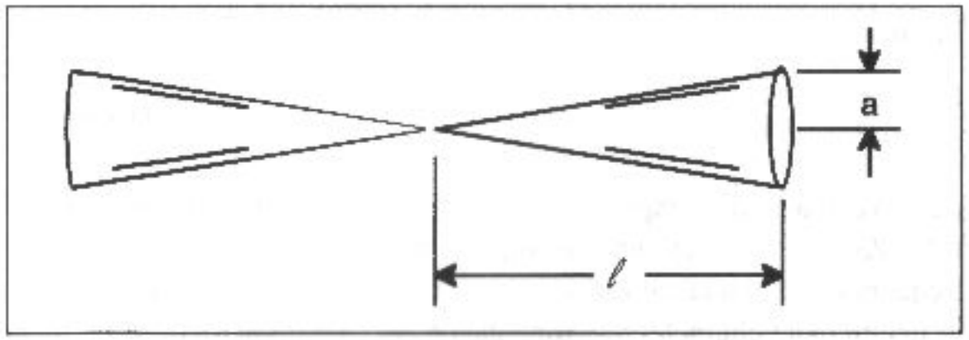


Fig 8-6—Dimensions of a conical dipole.

$$Z_0 = 120 \ln \frac{2\ell}{a} \qquad \text{(Eq 8-16)}$$

where

ln = natural logarithm
$\ell \approx \lambda/4$ = the length of each dipole half
a = radius at the tip

As an example, a conical dipole of length per side of $\ell = 250a$ has a characteristic impedance of $Z_0 = 120 \ln[(2)(250)] \approx 746\ \Omega$.

The effect of radiation results in a terminal impedance Z_a, the real part of which is the familiar radiation resistance of a thin half-wave dipole, $R_a = 73.13\ \Omega$. The imaginary part of Z_a results in the nonzero reactance of an exact half-wave dipole, which for a conical shape is approximately $X_a = 153\ \Omega$. The variation of $X_a(\beta\ell)$ and $R_a(\beta\ell)$ with the electrical length $\beta\ell$ of the antenna half is given exactly by Schelkunoff in terms of sines, cosines, sine integrals and cosine integrals that are functions of $\beta\ell$. These functions are tabulated or can be calculated from power series expressions.[7] For an exact half-wave antenna, $\beta\ell = \pi/2$ and $\ell = \lambda/4$, so $\beta = 2\pi/\lambda$.

To calculate the wavelength λ or wave number β, we use the velocity of light c' in air, calculated as

$$c' = \frac{c}{\sqrt{e}} \qquad \text{(Eq 8-17)}$$

where

c = speed of light in a vacuum (299,792,500 m/s or 11802.85 in./μs)
e = the relative dielectric constant of air[8] (1.0006)

This yields a velocity in air of 11799.32 in./μs, units which are convenient for use with frequencies expressed in MHz. From this, an accurate calculation of wavelength in air is

$$\lambda = \frac{c'}{f} = \frac{11{,}799.32}{f_{MHz}} \text{ in.} \qquad \text{(Eq 8-18)}$$

where f = frequency in MHz

As an example, for f = 14.2 MHz, $\lambda = 830.94$ in.

The effect of variation of impedance with frequency or antenna length is modeled near resonance to the first order by the reactance of the transmission line Z_0, which results in a reactance of

$$\Delta X(\beta\ell) = Z_0 \Delta(\beta\ell) = \frac{\pi Z_0}{2} \times 4\Delta\left(\frac{\ell}{\lambda}\right) = \partial \frac{\pi Z_0}{2} \qquad \text{(Eq 8-19)}$$

where

$\beta = 2\pi/\lambda$
$\partial = 4\ell/\lambda - 1$

As was shown earlier, resonance occurs when $\Delta X = X_a(\pi/2) = X_0$, so that $\partial_r = 2X_0 / \pi Z_0$. For the conical dipole, $X_0 \approx 153\ \Omega$, so the fractional offset of resonant frequency is given by $\partial = (2)(153)/(\pi Z_0) = 97.4/Z_0$. In the example above of $Z_0 = 746\ \Omega$, the resonant length is offset from $\ell = \lambda/4$ by a per-unit shortening of $\partial_r = 97.4/746 = 0.131 = 13.1\%$.

Schelkunoff extends this result to thin dipoles of arbitrary shape by defining the nominal characteristic impedance at any point on a dipole of arbitrary shape, corresponding to a nonuniform transmission line of arbitrary characteristic impedance, to be

$$K_z = 120 \ln\left[\frac{2z}{a(z)}\right] \qquad \text{(Eq 8-20)}$$

where

z = distance from dipole center
a(z) = radius at that point

The average characteristic impedance K_a is approximated to first order by the integral

$$K_a = \frac{1}{\ell}\int_0^{\ell} K_z dz = \frac{120}{\ell}\int_0^{\ell} \ln\left[\frac{2z}{a(z)}\right] dz \qquad \text{(Eq 8-21)}$$

Separating terms, we have

$$K_a = \frac{120}{\ell}\left(\int_0^{\ell} \ln 2z\, dz \; - \int_0^{\ell} \ln a(z)\, dz \right) \qquad \text{(Eq 8-22)}$$

Note the ease with which integrals of logarithms are separated into terms which can be treated individually. For cylindrical antennas with a constant radius a(z) = a, the integral is

$$K_a = 120\left[\ln\left(\frac{2\ell}{a}\right) - 1 \right] \qquad \text{(Eq 8-23)}$$

Schelkunoff points out that for antennas of any shape, K_a is given by this same equation if a is the logarithmic mean radius defined by

$$\ln a = \frac{1}{\ell} \int_0^{\ell} \ln a(z)\, dz \qquad \text{(Eq 8-24)}$$

where a(z) = radius at a typical point z

This means we can determine the equivalent mean radius a of any shape dipole if we can perform this integration, and because the same integral also appears in the equation for K_a (Eq 8-22), we can determine the equivalent average characteristic impedance of that dipole. If the dipole consists of step-tapered cylindrical sections, we can perform the above integrals section by section. If we can determine the dipole reactance at $\ell = \lambda/4$, we can calculate the resonant frequency and find the cylindrical dipole with the same resonant frequency and terminal-impedance variation.

An indication of the importance of correcting the resonant length of a tapered element can be gained from the analytic solution for the resonant length of the spheroidal dipole, which tapers in an elliptical shape. The parameters for a spheroidal tapered dipole of half-length ℓ and center radius a are

$$K_a = 120 \ln\left(\frac{\ell}{a}\right) \qquad \text{(Eq 8-25)}$$

and

$$\partial = \frac{5040}{(K_a + 83)^2} \qquad \text{(Eq 8-26)}$$

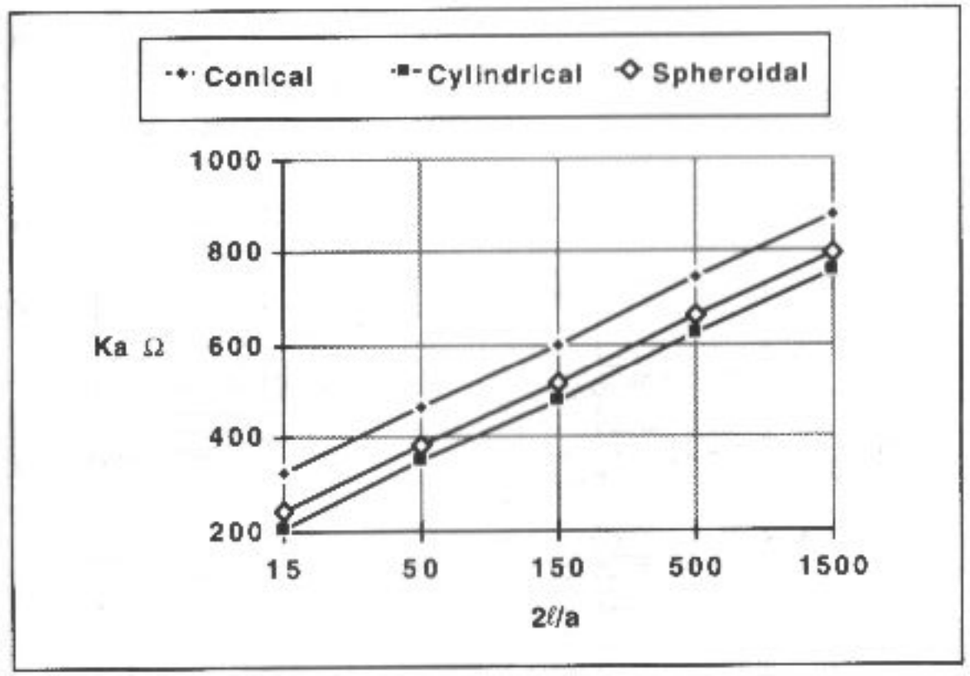


Fig 8-7—Dipole average characteristic impedance K_a versus its total length to radius ratio, 2 ℓ / a.

This simple calculation yields the result that, at least for this particular taper, the resonant length is substantially longer than a cylinder of the same diameter at its center. As we shall see, this agrees with the results of general tapered dipoles. The variation of K_a with the ratio $2\ell/a$ is shown for conical, cylindrical and spheroidal dipole shapes in Fig 8-7, and the variation in the percent shortening from an exact half-wave at resonance for the same antennas is shown in Fig 8-8. The degree of difference relative to a cylindrical element is more than enough to move the resonance of the tapered dipole entirely out of an amateur band, and indicates the significance of accurately modeling the resonant length of a tapered element.

Schelkunoff accounts for the effect of the departure of the actual radius from the mean radius (the effect of the difference between the nominal characteristic impedance at each point and the average characteristic impedance) by two functions $M(\beta\ell)$ and $N(\beta\ell)$ which are

$$M(\beta\ell) = \beta \int_0^{\ell} \left(K_a - K_z\right) \sin 2\beta z\, dz \qquad \text{(Eq 8-27)}$$

and

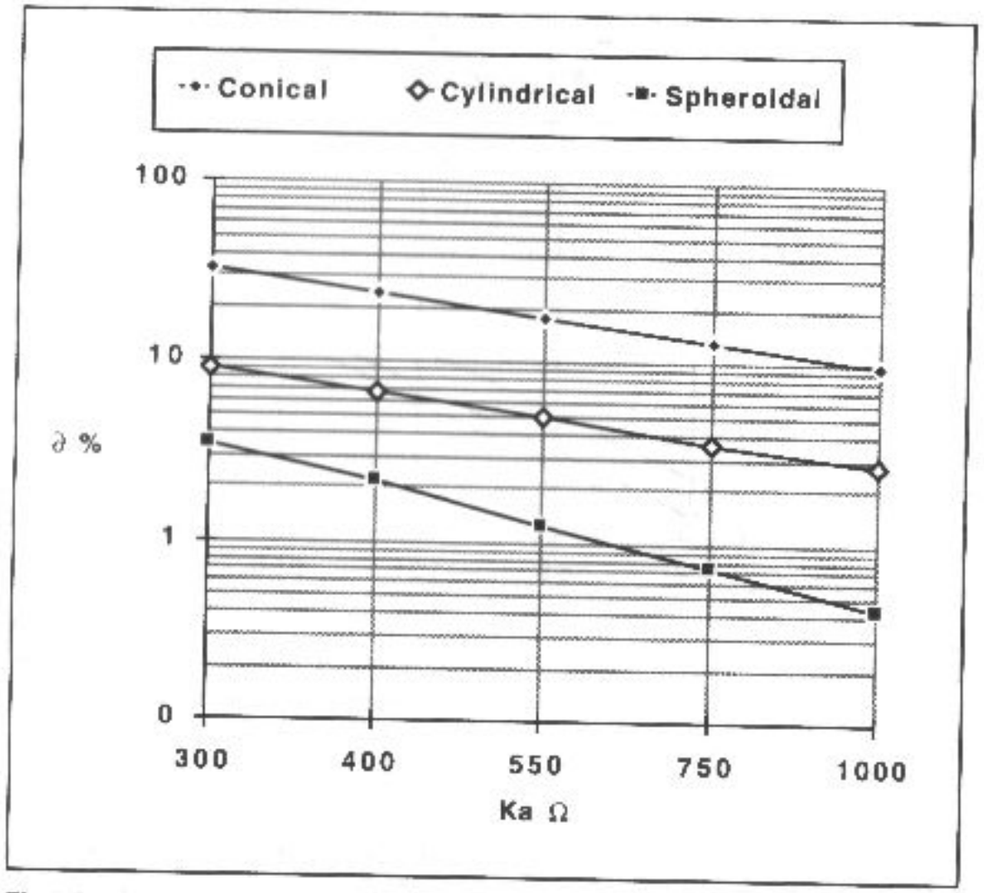


Fig 8-8—Percent shortening at resonance ∂_r versus K_a.

$$N(\beta\ell) = \beta\int_0^{\ell}\left(K_a - K_z\right)\cos 2\beta z\, dz \qquad \text{(Eq 8-28)}$$

These integrals can be evaluated by separating the terms, using the convenient property of logarithms that the logarithm of a product is the sum of the logarithms of the factors.

Near resonance ($\beta\ell = \pi/2$), the input impedance of a half-wave dipole is to first order approximately

$$R_i \approx R_a(\pi/2) = 73\ \Omega \qquad \text{(Eq 8-29)}$$

and

$$X_i(\beta\ell) \approx X_a(\pi/2) - N(\pi/2) + K_a\left(\beta\ell - \frac{\pi}{2}\right) = X_0 + \frac{\pi K_a \partial}{2} \qquad \text{(Eq 8-30)}$$

where

$$X_0 = X_a - N$$

$$\partial = \frac{4\ell}{\lambda} - 1$$

and

$$K_z = \frac{1}{\ell}\int_0^{\ell} K_z\, dz \qquad \text{(Eq 8-31)}$$

For a cylindrical dipole of $\ell = \lambda/4$ or $\beta\ell = \pi/2$, $X_a(\pi/2) \approx 153.64\ \Omega$ from the conical dipole analysis and $N(\pi/2) \approx 111.09\ \Omega$, so $X_0 = X_a - N = 153.64 - 111.09 = 42.55\ \Omega$. The percent shortening at resonance is

$$\partial_r = \frac{2X_0}{\pi Z_0} = \frac{27.09}{Z_0}$$

For a dipole of dimension $\ell = 250a$, the average characteristic impedance is given to first approximation ($Z_0 \approx K_a$) by

$$K_a = 120[\ln(500) - 1] = 626\ \Omega$$

and the percent shortening at resonance is

$$\partial_r = \frac{27.09}{K_a} = 4.3\%$$

Uda,[9] for the same cylindrical dipole dimensions ($\lambda/a = 4\ell/a = 1000$), gives values of $X_0 = 39.64\ \Omega$ and $X(\partial{=}5\%) = -8.57\ \Omega$. The percent shortening at resonance of $\partial_r = 4.11\%$, and the equivalent average characteristic impedance is $Z_0 = 614\ \Omega$.

Using MININEC numerical methods with 80 segments, the calculated parameters for the same dipole dimensions are $X_0 = 44.6\ \Omega$ and $X(\partial{=}5\%) = -3.1\ \Omega$, so by calculation $Z_0 = 607\ \Omega$ and $\partial_r = 4.68\%$.

The agreement with numerical methods, while excellent, suggests that we can improve the accuracy of the Schelkunoff estimate by including terms of higher order. The accuracy of the approximation may be improved by including second-order terms to yield

$$R_i \approx \frac{K_a}{K_a + M(\beta\ell)} \times R_a(\beta\ell) \qquad \text{(Eq 8-32)}$$

and

$$X_i(\beta\ell) = \left(X_a(\beta\ell) - N(\beta\ell)\right)\frac{K_a}{K_a + M} + \frac{K_a(K_a - M)}{K_a + M} \times \left(\beta\ell - \frac{\pi}{2}\right)$$

(Eq 8-33)

Near resonance we can use K_a, $M(\pi/2)$, $N(\pi/2)$, $X_a(\pi/2)$ and their first derivatives to estimate the terminal reactance variation with frequency. This yields an estimate of the input reactance for $\beta\ell = \pi/2$ as

$$X_i(\beta\ell) = X_0 + \left(\beta\ell - \frac{\pi}{2}\right)\frac{dX_i(\beta\ell)}{d(\beta\ell)} = X_0 + \frac{\pi Z_0 \partial}{2} \qquad \text{(Eq 8-34)}$$

where $\frac{\pi\partial}{2} = \beta\ell - \frac{\pi}{2} = \frac{4\ell}{\lambda} - 1$

We can take account of the variation of M, N and X_a (K_a is constant for constant ℓ) in Eq 8-34 for X_i to estimate the effective value of Z_0 to be

$$Z_0 = \frac{dX_i(\beta\ell)}{d(\beta\ell)} \approx K_a - 2M + \left(1 - \frac{M}{K_a}\right)\frac{d(X_a - N)}{d(\beta\ell)} - \frac{X_0}{K_a} \times \frac{dM}{d(\beta\ell)}$$

(Eq 8-35)

The exact expressions for M, N, X_a and K_a for a cylindrical dipole are

$$M(\beta\ell) = 60[\mathrm{Cin}(2\beta\ell) + \cos(2\beta\ell) - 1] \qquad \text{(Eq 8-36)}$$

$$N(\beta\ell) = 60[\mathrm{Si}(2\beta\ell) - \sin(2\beta\ell)] \qquad \text{(Eq 8-37)}$$

$$X_a = 60\mathrm{Si}(2\beta\ell) - 30[\mathrm{Cin}(4\beta\ell) - \ln 4] - 30\mathrm{Si}(4\beta\ell)\cos(2\beta\ell) \qquad \text{(Eq 8-38)}$$

where Cin(x) and Si(x) are integral functions (see reference 7)

$$K_a = 120\left[\ln\left(\frac{2\ell}{a}\right) - 1\right] \qquad \text{(Eq 8-39)}$$

Using tabulated values of Cin(x) and Si(x), the numerical values at resonance ($\beta\ell = \pi/2$) are

$$M(\pi/2) = 60(\mathrm{Cin}\,\pi - 2) = 60(1.65 - 2) = -21.13\ \Omega$$

$$X_a(\pi/2) - N(\pi/2) = 30\mathrm{Si}(2\pi) = 30(1.42) = 42.54\ \Omega$$

$$\frac{d(X_a - N)}{d(\beta\ell)} = 30(-2)[\ln 4 - \mathrm{Cin}(2\pi) + 2] = -60(1.39 - 2.44 + 2)$$

$$= -56.93\ \Omega/\text{radian}$$

$$\frac{dM}{d(\beta\ell)} = 60\left[\frac{2(1-\cos\pi)}{\pi}\right] = \frac{240}{\pi} = 76.39\ \Omega/\text{radian}$$

For a cylinder

$$Z_0 = K_a + 42.26 - \left(1 + \frac{21.13}{K_a}\right)\left(56.93 + \frac{3250}{K_a}\right)$$

$$= K_a - 14.67 - \frac{4253}{K_a} - \frac{68673}{K_a^2} \qquad \text{(Eq 8-40)}$$

For the example above of $\ell/a = 250$, including terms of the order of M/K_a and X_0/K_a, we have

$K_a = 626\ \Omega$

$$X_0 = \left(1 - \frac{-21.1}{626}\right) \times 42.54 = 44.0\Omega$$

$$Z_0 = 626 - 14.67 - \frac{4253}{626} - \frac{68673}{626^2} = 604\,\Omega$$

and

$$\partial_r = \frac{2 \times 44}{604\,\pi} = 4.64\%$$

From this we can calculate

$$\Delta X(\partial = 5\%) = \frac{0.05 \times X_0}{\partial} = 47.4\,\Omega$$

and

$$X(\partial = 5\%) = X_0 - \Delta X = 44.0 - 47.4 = -3.4\ \Omega$$

These quantities are in good agreement with the MININEC calculations summarized earlier. In this higher order approximation, the effective value of Z_0 is approximately $K_a - 21.7$ for a cylindrical element of this length-to-radius ratio, as can be seen from Fig 8-9.

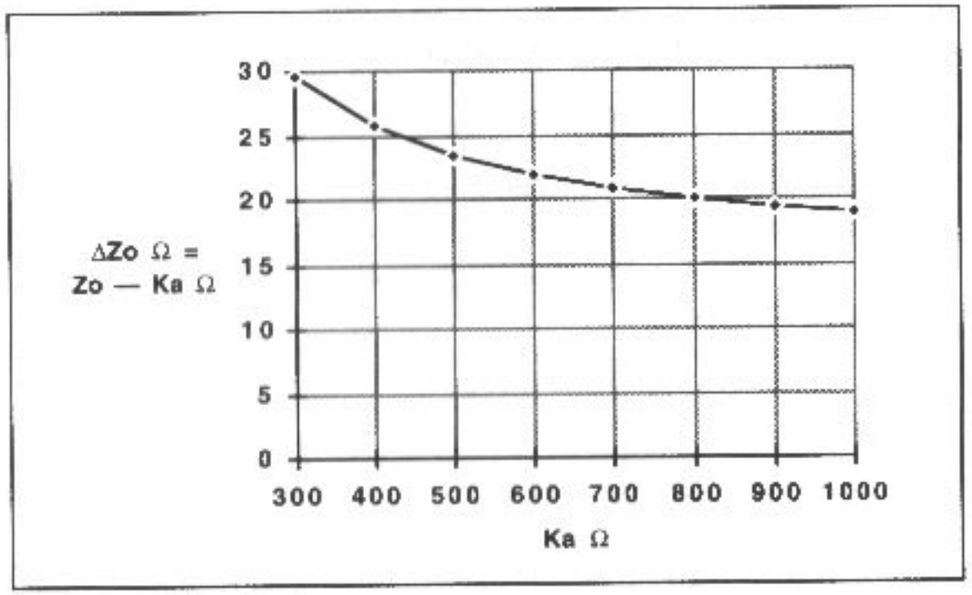


Fig 8-9—ΔZ_0 versus K_a for cylindrical dipoles.

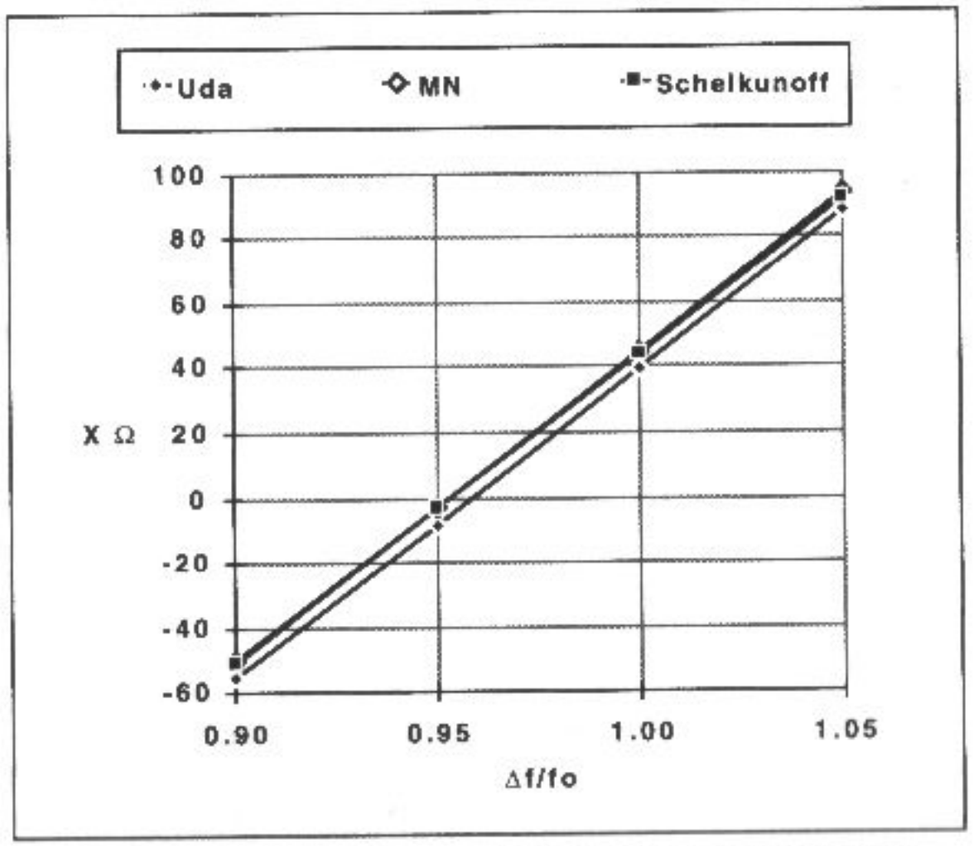


Fig 8-10—Reactance versus frequency for a cylindrical dipole, $4\ell/\lambda = 1000$.

Table 8-1
Uda, MN and Schelkunoff Model Data for $2\ell/a = 500$

	Uda	*MN(80)*	$Z_0 \approx K_a$	$Z_0 = K_a - \Delta Z_0$
$X_0\ \Omega$	39.64	44.6	42.6	44.0
$X_{\partial=5\%}\ \Omega$	–8.57	–3.1	–6.6	–3.4
$\Delta X_{\partial=5\%}\ \Omega$	48.2	47.7	49.2	47.4
$Z_0\ \Omega$	614	607	626	604
∂_r %	4.11	4.68	4.33	4.64

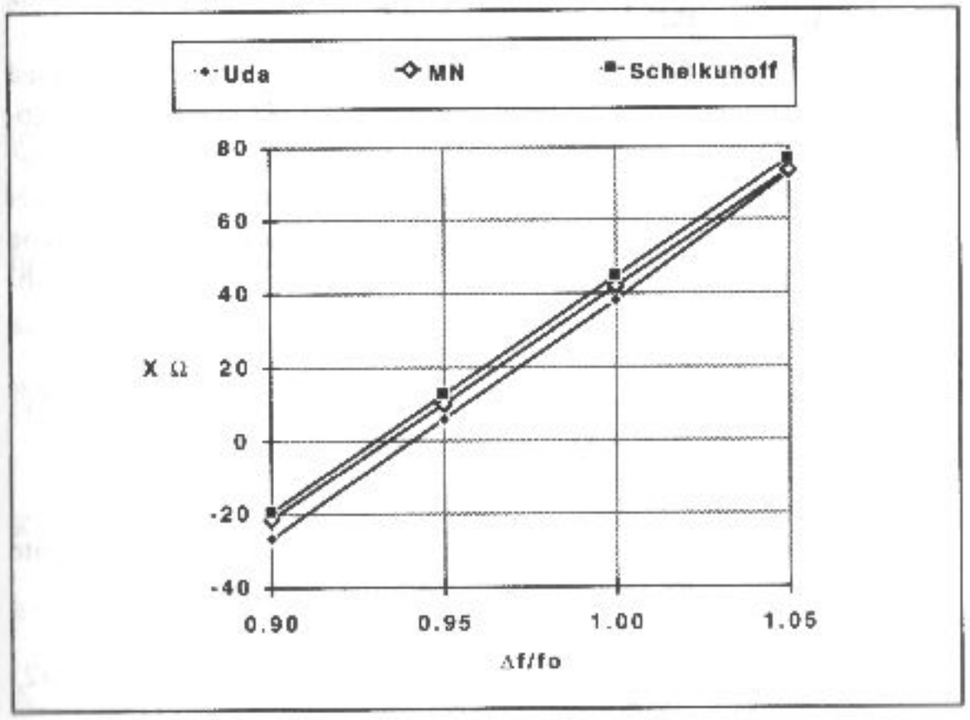


Fig 8-11—Reactance versus frequency for a cylindrical dipole, $4\ell/\lambda = 200$.

Table 8-1 compares Uda's calculated values with MININEC calculations for 80 segments, and with the results of the first and second approximations using Schelkunoff's method. An interesting comparison is the relationship among Uda's tabulated reactance data and that calculated by MN and by the method of Schelkunoff. Figs 8-10 and 8-11 show this comparison for $4\ell/a = 1000$ and $4\ell/a = 200$, where a is the radius of the cylindrical element. For Fig 8-10, which is representative

of the dimensions appropriate to a high-frequency Yagi, the dimensions used were length $\ell = 100$ in. and radius a = 0.4 in.

The extremely good agreement among all three methods is evident, and the especially close agreement between MN and the Schelkunoff method is remarkable. The MN calculation of reactance is based on the use of 80 segments per dipole. The frequency for which this length is an exact half-wave is 29.498 MHz.

The agreement is also excellent for much thicker elements, as evidenced by Fig 8-11, for which $\ell = 100$ in. and a = 2 in. The frequency for which this length is an exact half-wave is, of course, also 29.498 MHz.

Sec 8.4 An Algorithm for Step-Tapered Radius

Although solutions for cylindrical, spheroidal, diamond-shaped and linear-taper dipoles have been published,[10] I believe that the algorithm derived here to determine the effect of step-tapered radius is new.

The self-impedance of a dipole of step-tapered radius is of immediate interest in Yagi elements constructed from telescoping tubing. The dimensions are defined in Fig 8-12. To begin, we calculate the logarithmic mean radius by

$$\ln a_e = \frac{1}{\ell}\int_0^{\ell} \ln a(z)\, dz \qquad \text{(Eq 8-41)}$$

where a(z) = radius at a typical point z

For each section n, the radius $a = a_n$, a constant. We can evaluate the integral section by section, using the nomenclature of Fig 8-12.

$$\ln a_e = \frac{1}{\ell}\left[\int_{x_0}^{x_1} \ln a_1\, dz + \int_{x_1}^{x_2} \ln a_2\, dz + \int_{x_2}^{x_3} \ln a_3\, dz + \ldots\right] \qquad \text{(Eq 8-42)}$$

Because $\ln a_n$ is a constant for each section n, the value of each term is

$$\int_{x_{n-1}}^{x_n} \ln a_n\, dz = \ln a_n \int_{x_{n-1}}^{x_n} dz = \ln a_n \left(x_n - x_{n-1}\right) = \ln a_n \ell_n \qquad \text{(Eq 8-43)}$$

and the value of the integral is

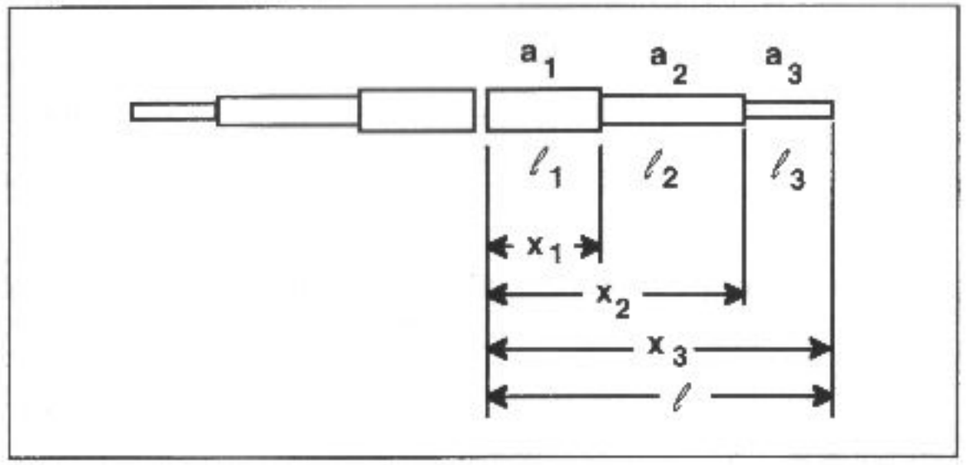


Fig 8-12—Dimensions of step-tapered dipole.

$$\ln a_e = \sum \frac{\ln a_n \ell_n}{\ell} \qquad \text{(Eq 8-44)}$$

The appropriate equivalent radius for the first-order approximation is then

$$a_e = e^u \qquad \text{(Eq 8-45)}$$

where $u = \ln a_e$ from Eq 8-44

We can now calculate the average characteristic impedance K_a by

$$K_a = 120\left[\ln\left(\frac{2\ell}{a}\right) - 1\right] = 120(\ln 2\ell - \ln a_e - 1) \qquad \text{(Eq 8-46)}$$

and, substituting the expression for $\ln a_e$,

$$K_a = 120\left(\ln 2\ell - \sum \frac{\ln a_n \ell_n}{\ell} - 1\right) \qquad \text{(Eq 8-47)}$$

The same integration procedure allows the calculation of Schelkunoff's functions M and N for the case of a tapered element. We have

$$M = M_{cyl} + 60\int_0^{\ell} \ln \frac{a(z)}{a_e} \sin u\, du \qquad \text{(Eq 8-48)}$$

where $u = 2\beta\ell$

Separating the integral by sections, we have

$$\int_0^{\ell} \ln \frac{a(z)}{a_e} \sin u \, du = \int_{x_0}^{x_1} \ln \frac{a_1}{a_e} \sin u \, du + \int_{x_1}^{x_2} \ln \frac{a_2}{a_e} \sin u \, du + \ldots$$

(Eq 8-49)

$$M = M_{cyl} + 60\left[-\ln \frac{a_1}{a_e}\left(\cos 2\beta x_1 - \cos 2\beta x_0\right) - \ln \frac{a_2}{a_e}\left(\cos 2\beta x_2 - \cos 2\beta x_1\right) - \ldots\right]$$

(Eq 8-50)

Similarly,

$$N = N_{cyl} + 60\left[\ln \frac{a_1}{a_e}\left(\sin 2\beta x_1 - \sin 2\beta x_0\right) + \ln \frac{a_2}{a_e}\left(\sin 2\beta x_2 - \sin 2\beta x_1\right) + \ldots\right]$$

(Eq 8-51)

Last, we can determine the values of the derivatives $M'(\beta\ell)$ and $N'(\beta\ell)$, which are

$$M'(\beta\ell) = \frac{dM}{d(\beta\ell)} = \frac{dM_{cyl}}{d(\beta\ell)} + 60\left[\ln \frac{a_1}{a_e}\left(2x_1 \sin 2\beta x_1 - 2x_0 \sin 2\beta x_0\right) + \ln \frac{a_2}{a_e}\left(2x_2 \sin 2\beta x_2 - 2x_1 \sin 2\beta x_1\right) + \ldots\right]$$

(Eq 8-52)

and

$$N'(\beta\ell) = \frac{dN}{d(\beta\ell)} = \frac{dN_{cyl}}{d(\beta\ell)} + 60\left[\ln \frac{a_1}{a_e}\left(2x_1 \cos 2\beta x_1 - 2x_0 \cos 2\beta x_0\right) + \ln \frac{a_2}{a_e}\left(2x_2 \cos 2\beta x_2 - 2x_1 \cos 2\beta x_1\right) + \ldots\right]$$

(Eq 8-53)

Paralleling the example of Lawson, we can now make a table for a given element by section n, beginning with the section radii a_n and the section lengths ℓ_n. We total ℓ_n from the half-element center to obtain x_n and thus ℓ, the total half-element length. Now we calculate $\ln a_n$ for each section, then calculate the quantity $\ln a_n \ell_n / \ell$ for each section, and total over all sections to derive $\ln a_e$.

Next we calculate $\sin 2\beta x_n$ and $\cos 2\beta x_n$ for each section including $n = 0$. Using $\ln (a_n/a_e) = \ln a_n - \ln a_e$, we calculate the values of

$\ln(a_n/a_e)(\sin 2\beta x_n - \sin 2\beta x_{n-1})$ and $\ln(a_n/a_e)(\cos 2\beta x_n - \cos 2\beta x_{n-1})$ for each section. We total these quantities and multiply by 60 to determine ΔM and ΔN relative to the cylindrical element. Last, we calculate the derivatives $\Delta M'$ and $\Delta N'$ of ΔM and ΔN by calculating $2x_n \cos 2\beta x_n$ and $2x_n \sin 2\beta x_n$ to be multiplied by $60 \ln(a_n/a_e)$ and summed over all sections.

The value of ΔN provides $X_0 = X_a - N_{cyl} - \Delta N$, the reactance for a step-tapered element of length equal to a physical half-wave. The values of ΔM and the derivatives allow a more accurate estimate of Z_0 from K_a.

Once we know X_0 and Z_0, we know how to determine the length ℓ and the characteristic impedance Z_0 of the equivalent cylinder. We can then determine the cylindrical radius a' that corresponds to a characteristic impedance Z_0 for a cylinder of length ℓ.

First, we assume that the cylindrical radius $a' \approx a_e$, so $K_a' = K_a$. Thus we calculate

$$X_0' = 42.54(1 + 21.13/K_a) \qquad \text{(Eq 8-54)}$$

From X_0', X_0 and Z_0, we calculate

$$\frac{f_0'}{f_0} = \frac{1 - 2(X_0 - X_0')}{\pi Z_0} \qquad \text{(Eq 8-55)}$$

From this we calculate

$$Z_0' = \frac{f_0'}{f_0} Z_0 \qquad \text{(Eq 8-56)}$$

and

$$\frac{\ell'}{\ell} = \frac{f_0}{f_0'} \qquad \text{(Eq 8-57)}$$

Now that we know Z_0' and ℓ', we calculate $K_a' \approx K_a$ for a cylindrical dipole from

$$K_a' = Z_0' + 14.67 + 4253/K_a + 68673/K_a^2 \qquad \text{(Eq 8-58)}$$

from which we calculate a' from K_a' and ℓ' by

$$\ln a' = \ln(2\ell') - K_a'/120 - 1 \qquad \text{(Eq 8-59)}$$

Table 8-2
Section Calculations for Taper Algorithm

Section n	*1*	*2*	*Total*
Diameter $2a_n$	0.8 in.	0.4 in	
Length ℓ_n	50 in.	50 in.	100 in.
$\ell = \Sigma \ell_n$ totaled over all sections = 100 in.			
Distance $x_n = \Sigma \ell_n$ totaled over sections 1 to n	50 in.	100 in.	
ln(radius) = $\ln(a_n)$	–0.92	–1.61	
Length × ln(radius) = $\ell_n \ln(a_n)$	–45.8	–80.5	–126.3
$\ln(a_e) = \Sigma \ell_n \ln(a_n/\ell)$ totaled over all sections = –1.26			
$\Delta N = 60[\ln(a_n) - \ln(a_e)][\sin(\pi x_n/\ell) - \sin(\pi x_{n-1}/\ell)]$	20.79	20.79	41.59
$\Delta M = -60[\ln(a_n) - \ln(a_e)][\cos(\pi x_n/\ell) - \cos(\pi x_{n-1}/\ell)]$	20.79	–20.79	0
$(\Delta N)' = 120[\ln(a_n) - \ln(a_e)][x_n\cos(\pi x_n/\ell) - x_{n-1}\cos(\pi x_{n-1}/\ell)]/\ell$	0	41.59	41.59
$(\Delta M)' = 120[\ln(a_n) - \ln(a_e)][x_n\sin(\pi x_n/\ell) - x_{n-1}\sin(\pi x_{n-1}/\ell)]/\ell$	20.79	20.79	41.59

Thus we have the length ℓ' and radius a′ of a cylindrical element equivalent to the tapered element. As an approximation, we can use $K_a' \approx Z_0' + 21.7$ for $K_a' \approx 600\ \Omega$.

We now are ready to calculate the equivalent cylindrical element for an arbitrary tapered element. As we have seen, tapering the element is a consequence of mechanical requirements. The dimensions of the tapered element are specified in a tabular summary of the schedule of section diameters and lengths. The taper schedule can be that of an actual physical antenna, or it can be the outcome of the mechanical-design process for element strength detailed earlier. The resulting algorithm for step-tapered dipole elements is summarized in Table 8-2 for a two-step taper. The algorithm adapts readily to a computer spreadsheet format, and can use the element dimension data from the strength spreadsheet of Chapter 4. Again, the IF( ) spreadsheet command is useful to accommodate a variable number of element sections. Because x_0 equals 0, $\sin(\pi x_0/\ell)$ equals 0 and $\cos(\pi x_0/\ell)$ equals 1.

Table 8-3
Tapered Dipole Equivalent Cylindrical Element

Tapered Element

Equation	*Quantity*
$M = -21.13 + \Delta M$	–21.13 Ω
$K_a = 120[\ln(2\ell) - \ln(a_e) - 1]$	667 Ω
$X_0 = (42.545 - \Delta N)(1 - M/K_a)$	0.99 Ω
$Z_0 = K_a - 2M - (1 - M/K_a)(56.93 + \Delta N') - (X_0/K_a)(76.39 + \Delta M')$	608 Ω
$f_r/f_0 = 1 - 2X_0/\pi Z_0$	0.9990
$f_0 = 11799.32/4\ell$	29.498 MHz
$\ell = \Sigma \ell_n$	100 in.

Equivalent Cylindrical Element

Equation	*Quantity*
$M_{cyl} = -21.13$	–21.13 Ω
$K_a' = Z_0' + 14.67 + 4253/K_a + 68673/K_a^2$	656 Ω
$X_0' = 42.545(1 + 21.13/K_a)$	43.89 Ω
$Z_0' = Z_0 f_0'/f_0$	635 Ω
$f_0'/f_0 = [1 - 2(X_0 - X_0')/(\pi Z_0)]$	1.0449
$f_0' = f_0(f_0'/f_0)$	30.824 MHz
$\ell' = \ell/(f_0/f_0')$	95.70 in.
$d' = 2a' = 4\ell' e^{-u}$, where $u = K_a'/120 + 1$	0.594 in.

Now that we have the key parameters ΔN, ΔM, ΔN′ and ΔM′, we can derive the tapered dipole self-impedance parameters and the equivalent cylindrical dipole impedances and dimensions as summarized in Table 8-3.

For any arbitrary frequency f, we can determine the reactance X_f. We use f_0 to calculate $f_r = f_0(f_r/f_0)$, and use f and Z_0 to find

$$X_f = \pi Z_0(f - f_r)/2f_0 \qquad \text{(Eq 8-60)}$$

As an example, using the data from Table 8-3 and choosing f = 28.25 MHz, we have $f_r = f_0(f_r/f_0) = 29.468$ MHz and $Z_0 = 608\ \Omega$. So for f = 28.25 MHz, $X_f = \pi Z_0(f - f_r)/2f_0 = -39.42\ \Omega$.

Although they look complex, these calculations are straightforward and can be extended to any number of dipole sections. The use of a computer spreadsheet makes quick work of the calculations.

Sec 8.5 Comparison of the Algorithm with MININEC

In order to test the accuracy of the above algorithm, a comparison was made with calculations derived from the MN version of MININEC. As an example, let us treat the simple step-tapered dipole example from Uda.[11] We choose $a_2 = a_1/2$ and $4\ell/a_1 = 1000$, and we let $x_1 = \ell_1/\ell$ range from 0 to 1 in steps of 0.1. The dimensions are shown in Fig 8-13. For each of the stepped dipoles we can determine the reactance and equivalent cylinder. The cases for $\ell_1/\ell = 0$ and $\ell_1/\ell = 1$ are, of course, cylindrical dipoles without taper.

MN data was generated using 80 segments per dipole for each of the 11 cases by running MN in each case to calculate X_0, the reactance at the frequency f_0 for which the dipole length was an exact half-wave, and X_1, the reactance at the frequency $f_1 = 0.95f_0$. The dipole dimensions chosen are $\ell = 100$ in., $a_1 = 0.4$ in., and $a_2 = a_1/2 = 0.2$ in. The half-wave frequency is $f_0 = c'/4\ell = 29.498$ MHz, so $f_1 = 0.95f_0 = 28.023$ MHz.

For each value of x_1 from 0 to 1 in steps of 0.1 we tabulate $X_0 = X(f_0)$ and $X_1 = X(f_1)$. From these MN data points we derive, for each x_1 value, $\Delta X = X_0 - X_1$ and, because $\Delta f/f_0 = 0.05$, $Z_0 = 2\Delta X/(0.05\pi)$. The resonant frequency is $f_r = f_0(1 - 0.05X_0/\Delta X)$. We next calculate $\ln a_e = (\ell_1/\ell)\ln a_1 + (\ell_2/\ell)\ln a_2$, which is used to calculate $K_a = 120(\ln 2\ell - \ln a_e - 1)$.

The half-wave reactance of the equivalent cylinder is $X_0' = 42.54(1 + 21.13/K_a)$. This gives us $f_0' = f_0[1 - 0.05(X_0 - X_0')/\Delta X]$. Thus

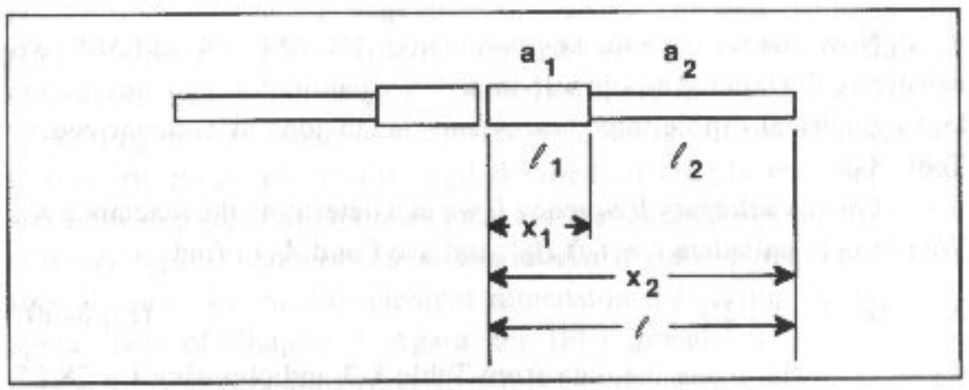


Fig 8-13—Dipole dimensions for comparison example.

we have $Z_0' = Z_0 f_0'/f_0$ and $\ell' = \ell f_0/f_0'$. Finally we can estimate, for a cylinder, $K_a' \approx Z_0' + 21.7$, so we have $\ln a' = \ln 2\ell' - (K_a'/120 + 1) \approx \ln 2\ell' - [(Z_0' + 21.7)/120 + 1]$.

For comparison, we use the algorithm described here to calculate, for the same stepped dipoles, $\ln a_e$ and K_a, as well as ΔN and ΔM plus their derivatives. From these we can derive calculated values for X_0 and Z_0, and hence ΔX and X_1 for each x_1 from $x_1 = 0$ to 1 in steps of 0.1. Knowing X_0 and X_1, we can determine the remaining parameters f_r, f_0', ℓ', Z_0' and $\ln a'$.

Glisson and Wilton have conducted an extensive study of stepped-radius wire junctions as they occur in tapered elements.[12] Subsequent work by Logan (K6BRF) and Rockway[13] indicates excellent agreement between MININEC calculations and the results of the Glisson-Wilton work. The comparison between the algorithm presented here and data from the MN version of MININEC for the above dipoles is shown in the accompanying figures. First, Fig 8-14 compares the values of X_0 and X_1 derived from MN and from the algorithm. The notation "calc" refers to the values derived from the algorithm. The

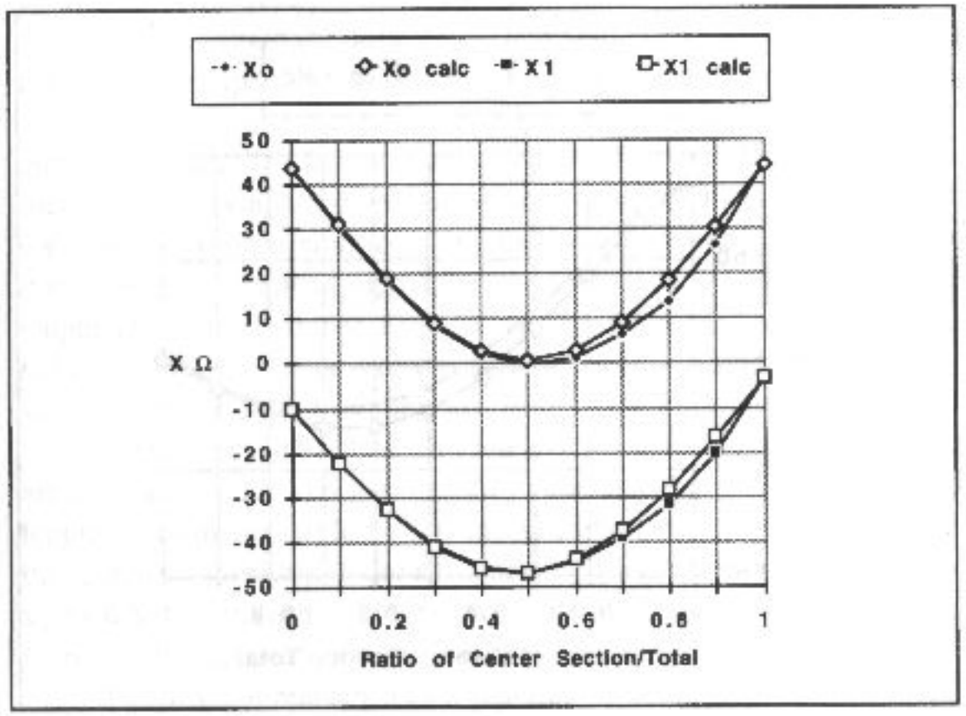


Fig 8-14—MN and Schelkunoff-algorithm reactance comparison; "calc" indicates results of Schelkunoff algorithm.

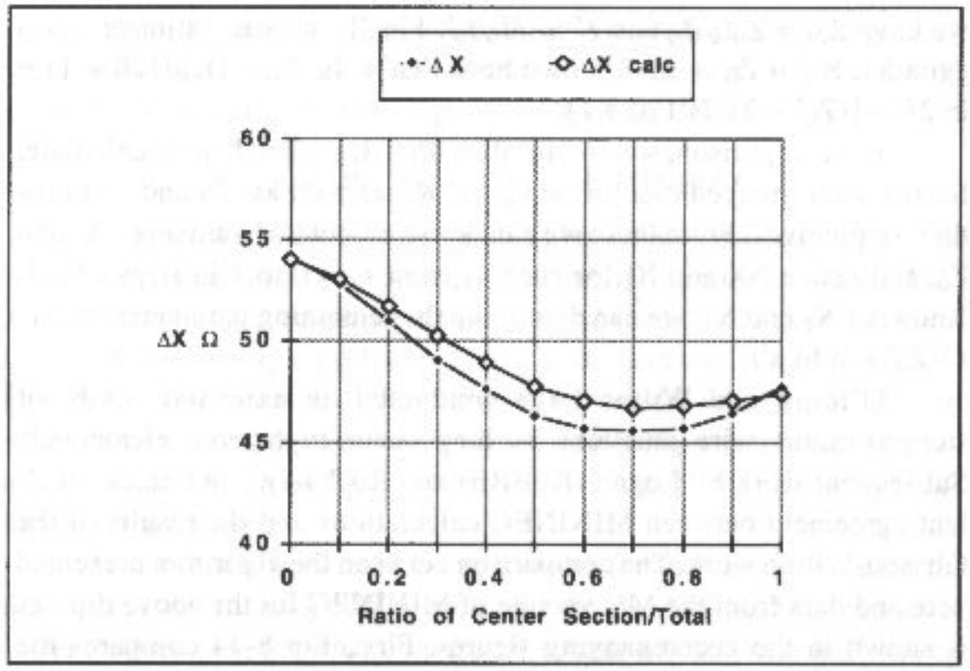


Fig 8-15—MN and Schelkunoff-algorithm ΔX comparison; "calc" indicates results of Schelkunoff algorithm.

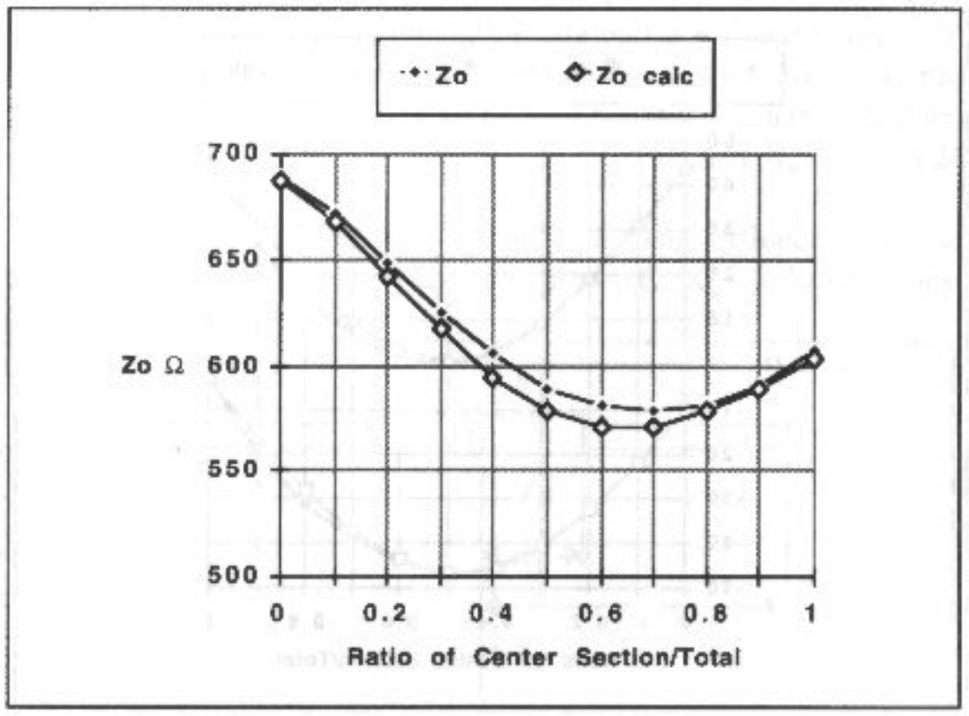


Fig 8-16—MN and Schelkunoff-algorithm Z_0 comparison; "calc" indicates results of Schelkunoff algorithm.

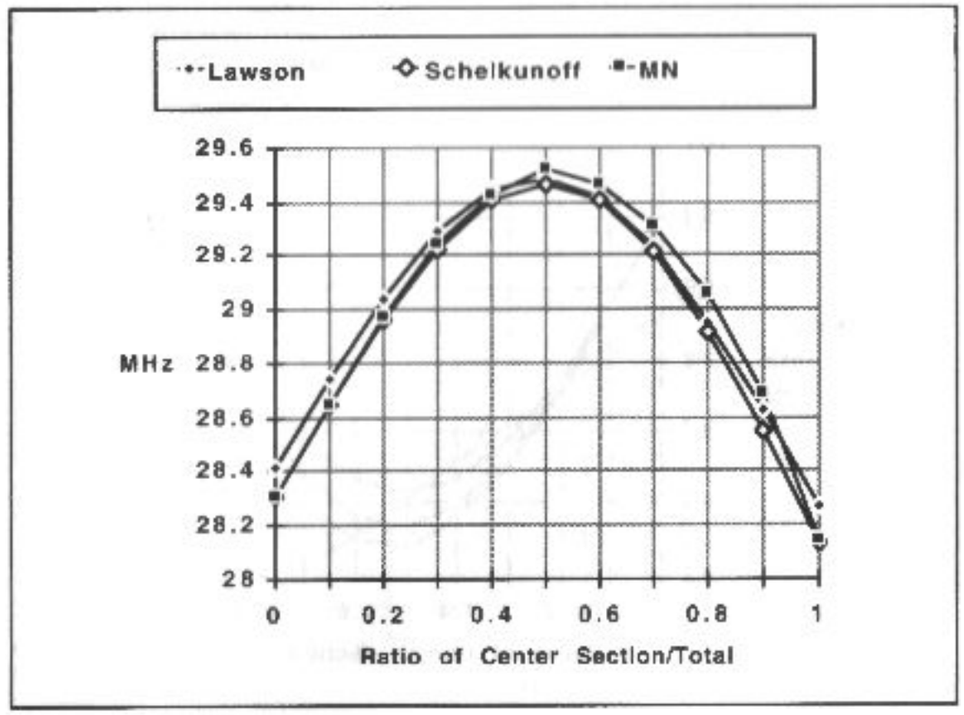


Fig 8-17—Lawson, Schelkunoff, and MN resonant-frequency comparison.

agreement is apparent. Next, Fig 8-15 compares the MN and algorithm values of ΔX. Again the agreement seems excellent. Related to the data for ΔX is the expanded-scale comparison of Z_0, shown in Fig 8-16.

Lawson's algorithm also provides an estimate of resonant frequency and length of the equivalent cylinder. I have improved Lawson's method by applying the dipole impedance concepts discussed in Sec 8.4. Calculations were performed using Lawson's method (as modified) for the same set of stepped dipoles, at the frequency for exact half-wavelength and with a cylinder diameter equal to twice the logarithmic mean radius. Fig 8-17 compares resonant frequency f_r derived from the Lawson and Schelkunoff algorithms and from the MN version of MININEC.

Finally, in Fig 8-18 the Lawson and Schelkunoff algorithm results are compared with MN data for ℓ'/ℓ, expressed here as the equivalent length ℓ' for the example $\ell = 100$ in. As can be seen from this example, the Lawson algorithm underestimates the length effect of the taper at $x_1/\ell = 0.5$ by 1 in., while the improved algorithm based on Schelkunoff increases the accuracy by a factor of 4, assuming that MN is exact.

Plotting the reactance as a function of normalized frequency f/f_0 for different values of x_1/ℓ is a different comparison between the modified

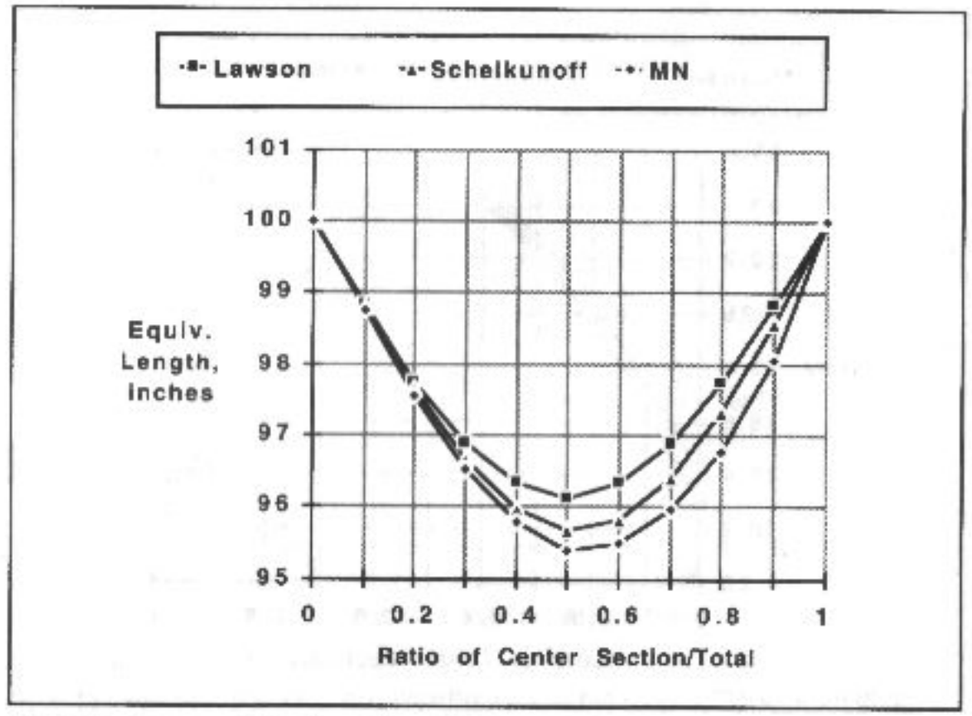


Fig 8-18—Lawson, Schelkunoff, and MN equivalent-length comparison.

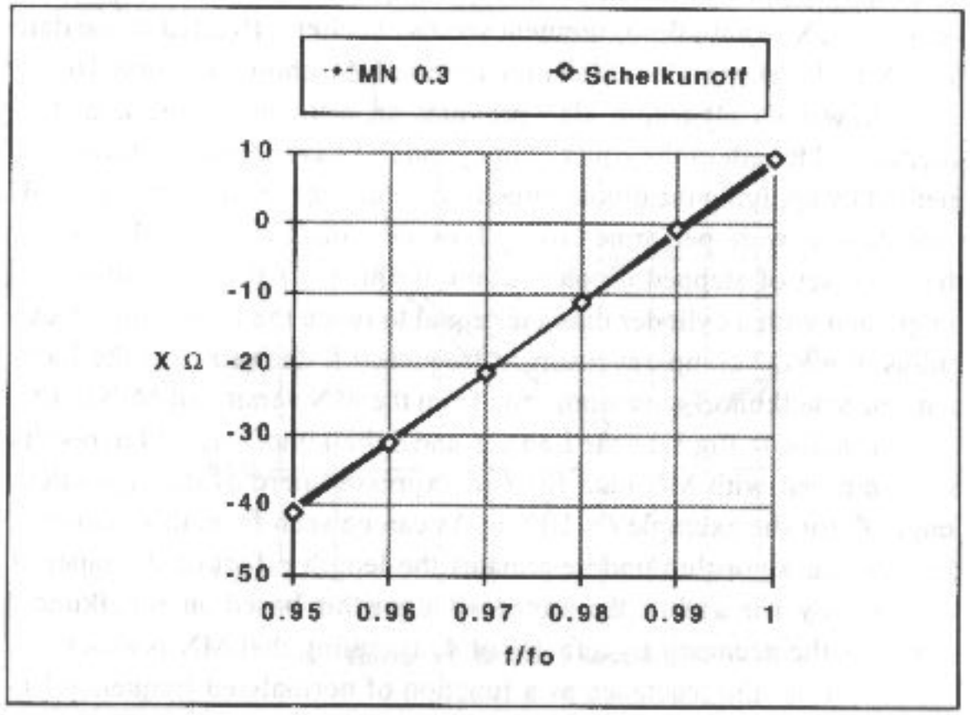


Fig 8-19—MN and Schelkunoff algorithm reactance comparison for $x_1/\!/ = 0.3$.

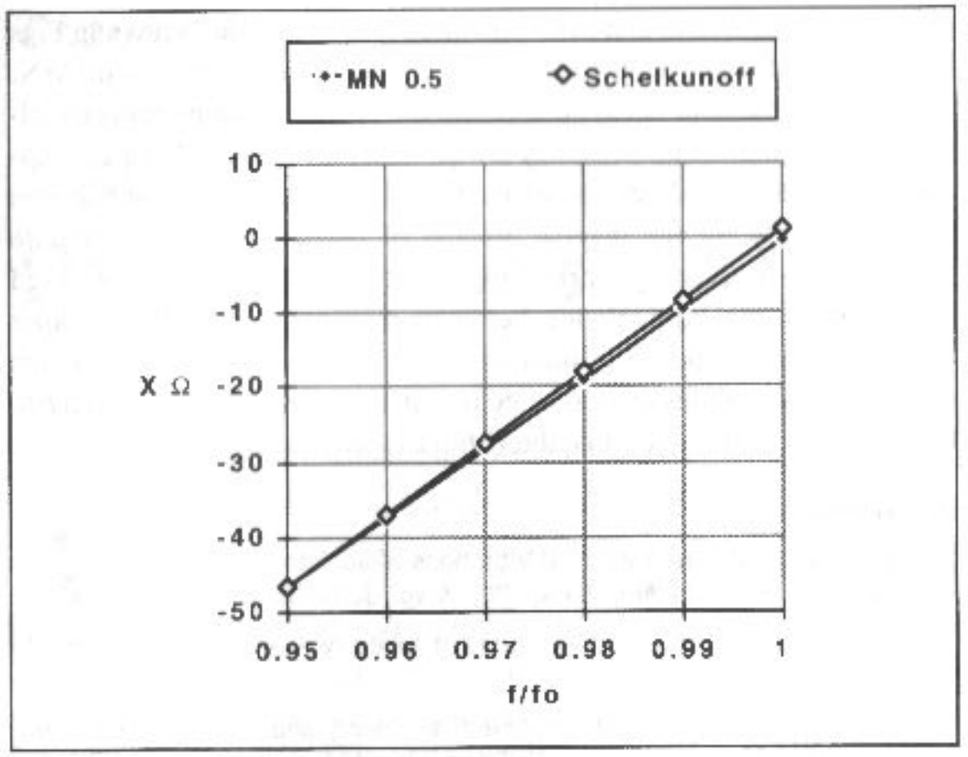


Fig 8-20—MN and Schelkunoff-algorithm reactance comparison for $x_1/\!/ = 0.5$.

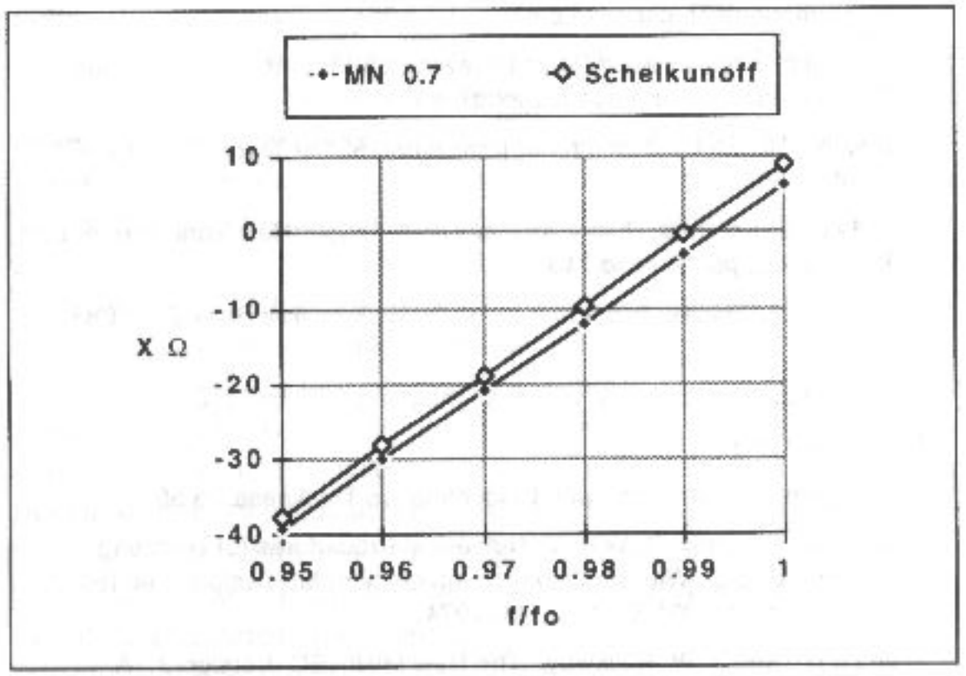


Fig 8-21—MN and Schelkunoff-algorithm reactance comparison for $x_1/\!/ = 0.7$.

Schelkunoff algorithm and MN. The three reactance plots shown in Figs 8-19 through 8-21 show the agreement between the algorithm and MN.

The agreement between MN and the new algorithm seems excellent, and the algorithm provides a flexible tool for the design of step-tapered dipoles and Yagi antennas. The algorithm spreadsheet can be used iteratively for the inverse process of finding the taper schedule equivalent to a given cylindrical dipole. The process can also be used to determine the equivalence between dipoles with different taper schedules, as is required to maintain the electrical design of elements whose taper schedules are modified in order to provide greater strength. In Chapter 9 we'll apply this algorithm to some practical examples.

References

[1] Lord Rayleigh, "On the Electrical Vibrations Associated with Thin Terminated Conducting Rods," *Phil. Mag.*, Jul 1904, p 104.

[2] S. A. Schelkunoff, "Theory of Antennas of Arbitrary Size and Shape," *Proc IRE*, Sep 1941, p 493.

[3] S. A. Schelkunoff and H. T. Friis, *Antennas Theory and Practice* (New York: John Wiley & Sons, Inc, 1952), Chap 13, p 425.

[4] S. Uda and Y. Mushiake, *Yagi-Uda Antenna* (Sendai: Sasaki Printing and Publishing Co, Ltd, 1954), Chap 5, "Theory of Antenna with Discontinuous Thickness," p 46.

[5] J. L. Lawson, *Yagi Antenna Design* (Newington: American Radio Relay League, 1986), "Taper Corrections," p 7-5.

[6] B. Beezley, *MN* and *YO* (computer programs), 507½ Taylor St, Vista, CA 92084, 1989.

[7] C. A. Balanis, *Antenna Theory Analysis and Design* (New York: Harper and Row, 1982), pp 124 and 743.

[8] J. D. Kraus, *Antennas*, 2nd ed. (New York: McGraw-Hill Book Co, 1988), p 851.

[9] See reference 4, "Numerical Values of Input Impedance," p 24.

[10] See reference 3.

[11] See reference 4, "Antenna with Discontinuous Thickness," p 58.

[12] A. W. Glisson and D. R. Wilton, "Numerical Procedures for Handling Stepped-Radius Wire Functions," University of Mississippi final report for contract N66001-E-0156, Mar 1974.

[13] J. C. Logan and J. W. Rockway, "The New MININEC (Version 3): A Mini-Numerical Electromagnetic Code," *Technical Document 938* (San Diego: Naval Ocean Systems Center, Sep 1986), pp 33-37.

Chapter 9

Yagi Element Electrical Design Issues

Sec 9.1 Equivalent Diameter of Element Clamps

The concept of equivalent radius is useful in the solution of antenna problems that involve electrically thin wires of arbitrary cross-section. Generally, the solution is first calculated for circular wires of arbitrary radius, and this radius is then replaced by the equivalent radius of the noncircular wire or tube. In this way the original problem is reduced to that of finding the equivalent radius. The concept of equivalent radius apparently originated with Hallen[1] and is treated by many antenna texts,[2] including Uda[3] and Lawson.[4]

The equivalent radius of a noncircular wire or rod is equal to the radius of a circular wire or rod whose capacitance as the center conductor of a coaxial capacitor is identical to that of the noncircular center conductor. The analytical calculation of equivalent radius involves a double integration over the perimeter (circumference) of the noncircular shape. This integration has been accomplished for certain simple shapes, but is extremely difficult for complex shapes such as element clamps, which may typically involve both rectangular and cylindrical cross-sections.[5] Published values for equivalent radius of several geometric cross-section shapes are shown in Fig 9-1.

In an uncharacteristic oversight, Lawson incorrectly applied the equation derived by Uda for parallel conductors, which is appropriate only if the distance between them is large compared to the effective thickness of the two conductors.[6] We would like a simple expression for the equivalent radius of, for example, the Hy-Gain element clamp and the plate element clamp. It is also of interest to know the equivalent radius of an element clamp made from angle or channel, as these are inherently strong shapes.

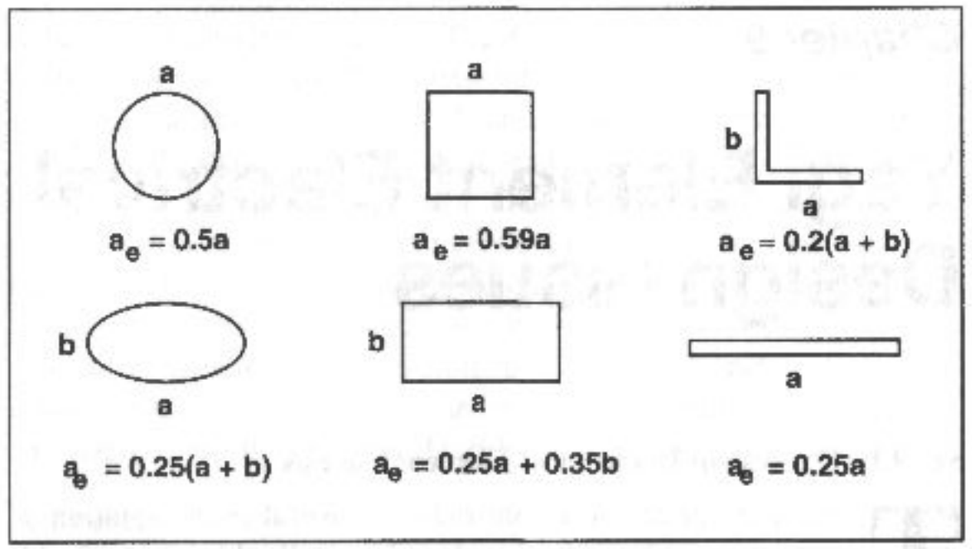


Fig 9-1—Equivalent radii for noncircular cross-sections.

This problem is resolved in a short paper by Jaggard,[7] from which this section is derived. Jaggard shows that the equivalent radius a_e of a noncircular shape must lie between the radii a_i and a_c of the inscribed and circumscribed circles which geometrically bound the noncircular conductor. Further, he shows that these bounds can be narrowed by the use of radii a_{in} and a_{out} which are the radii of circles of the area A and perimeter P of the cross-section shape of the conductor,

$$a_{in} = \sqrt{A/\pi} \qquad \text{(Eq 9-1)}$$

and

$$a_{out} = P/2\pi \qquad \text{(Eq 9-2)}$$

The equivalent radius a_e is bounded by $\sqrt{A/\pi} \le r_e \le P/2\pi$, and a satisfactory estimate for the equivalent radius is the mean of the two bounding radii. Hence,

$$a_e \approx \frac{\sqrt{A/\pi} + P/2\pi}{2} \qquad \text{(Eq 9-3)}$$

As examples, consider a cylinder with rectangular flanges (the Hy-Gain clamp) and a cylinder mounted on a rectangular plate (as described by Lawson). The dimensions are shown in Fig 9-2. The area of the Hy-Gain clamp of diameter D, flange width W, and flange total thickness t, is

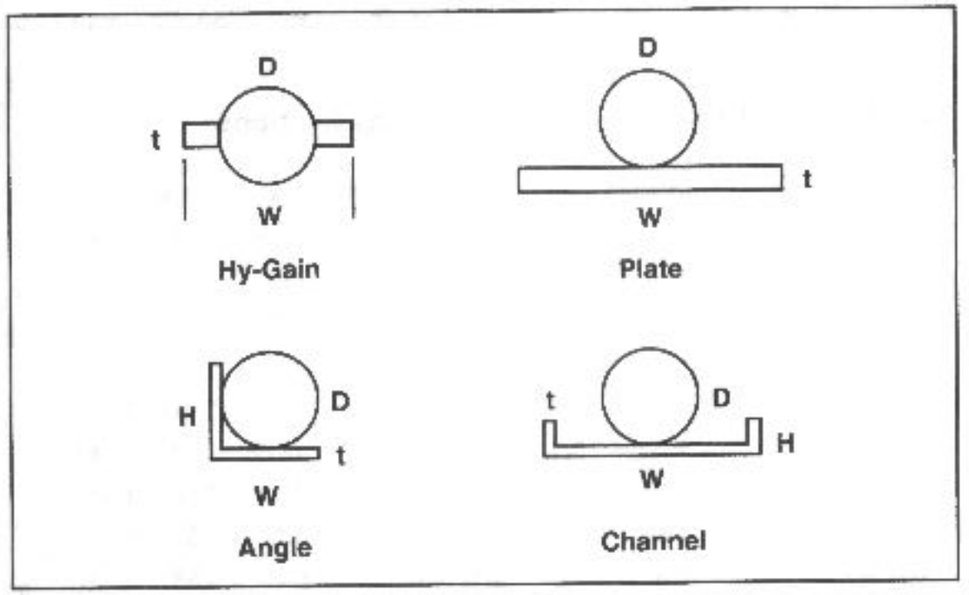


Fig 9-2—Dimensions of element clamp cross-sections.

$$A = \frac{\pi D^2}{4} + t\,(W - D) \qquad \text{(Eq 9-4)}$$

The perimeter, noting that the flange edge is the same length as the "missing" part of the cylinder, is

$$P = \pi d + 2(W - D) \qquad \text{(Eq 9-5)}$$

As an example, consider the Hy-Gain 204BA clamp, for which D = 1.5 in., W = 3.25 in., and t = 2 × 0.125 = 0.25 in. The area, perimeter and equivalent radius are

$$A = \frac{\pi(1.5)^2}{4} + 0.25\,(3.25 - 1.5) = 1.77 + 0.44 = 2.20 \text{ in.}^2$$

$$P = \pi(1.5) + 2(3.25 - 1.5) = 4.71 + 3.5 = 8.21 \text{ in.}$$

$$a_e = \frac{\sqrt{2.20/\pi} + 8.21/2\pi}{2} = \frac{0.83 + 1.31}{2} = 1.07 \text{ in.}$$

The equivalent diameter $d_e = 2a_e = 2.14$ in.

For the plate clamp with element diameter D, plate width W, and thickness t, the area is

Table 9-1
Calculation of Equivalent Radius of Cross Sections

	Circle	*Ellipse*	*Square*	*Rect*	*Angle*	*Hy-Gain*	*Plate*	*Angle*	*Channel*
D	1	1			0.5	1.5	1.25	1.25	1.25
W		2	1	1	1	3.25	6	1.5	3
t			1	0.5	0.1	0.25	0.375	0.25	0.25
h								1.5	0.75
A			1.00	0.50	0.14	2.20	3.48	1.91	2.23
P			4.00	3.00	3.00	8.21	16.68	7.70	12.43
a_e	0.50	0.75	0.60	0.44	0.34	1.07	1.85	1.00	1.41
d_e	1.00	1.50	1.20	0.88	0.69	2.14	3.71	2.01	2.82
$\frac{a_e}{a+b}$	0.25	0.25	0.30	0.29	0.23	0.27	0.26	0.33	0.33

$$A = \frac{\pi D^2}{4} + tW \qquad \text{(Eq 9-6)}$$

and the perimeter is

$$P = \pi D + 2(W + t) \qquad \text{(Eq 9-7)}$$

Consider the example offered by Lawson for his 14-MHz Yagis, for which D = 1.25 in., W = 6 in., and t = 0.375 in. The area, perimeter and equivalent radius are

$$A = \frac{\pi(1.25)^2}{4} + 0.375(6) = 1.23 + 2.25 = 3.48 \text{ in.}^2$$

$$P = \pi(1.25) + 2(6 + 0.375) = 3.93 + 12.75 = 16.68 \text{ in.}$$

$$a_e = \frac{\sqrt{3.48/\pi} + 16.68/2\pi}{2} = \frac{1.05 + 2.65}{2} = 1.85 \text{ in.}$$

$$d_e = 2a_e = 3.71 \text{ in.}$$

Note that this is considerably larger than the value of 2.58 in. used in Table 8-6 of Lawson's calculations. However, because the impedance is determined by the logarithm of the equivalent radius, the error is not overly serious.

Table 9-2
Hy-Gain and Plate Element Clamps

	204BA	*105BA*	*204DR*	*105DR*	*2x204DR*	*Plate*
D	1.5	1.045	1.75	1.5	1.75	1.25
W	3.25	2.5	3.25	3.25	3.75	6
t	0.25	0.17	0.25	0.25	0.5	0.375
A	2.20	1.11	2.78	2.20	3.41	3.48
P	8.21	6.19	8.50	8.21	9.50	16.68
a_e	1.07	0.79	1.15	1.07	1.28	1.85
d_e	2.14	1.58	2.29	2.14	2.55	3.71
$\frac{a_e}{a+b}$	0.23	0.22	0.23	0.23	0.23	0.26

Table 9-1 summarizes the equivalent radius of various cross-sections. The values for the circle, ellipse, square, rectangle and angle are essentially the same as published values for exact calculations.

For the specific element clamps of interest, Table 9-2 summarizes the equivalent radius and diameter, calculated by the method of Jaggard. It is interesting to note that the equivalent diameter is very close in all cases to the average of the two outside dimensions a and b of the cross-section, as the equivalent radius is close to

$$0.25(a + b) = \frac{a/2 + b/2}{2}$$

This is a safe first-order approximation in the absence of the more accurate estimates provided here.

Lawson suggests the effect of the boom is to reduce the electrical length of the clamp section by the amount of 6% of the boom diameter in the case of the plate clamp, and to remove from the center or clamping section some 70% of the boom diameter in the case of an element through the boom. In the case of the Hy-Gain antennas, the element lengths are measured from the boom surface rather than the boom center line; in this case the external longitudinal dimension of the element clamps should be considered to be electrically longer by 30% of the 2-in. boom diameter, or an additional $(0.3 \times 2)/2 = 0.3$ in. per side.

Sec 9.2 Algorithm Spreadsheets for Tapered Elements

Tables 9-3 and 9-4 summarize the Schelkunoff and Lawson derivations of the equivalent dipole length for the Hy-Gain 204BA reflector, with reactance calculated at 14.075 MHz. Lawson's methods are summarized in *The ARRL Antenna Book*.[8]

The concepts relating to the equivalence of dipoles of various shapes can be used to improve the scope of Lawson's taper algorithm, which as published includes no provision for calculating the characteristic impedance or diameter of the equivalent cylinder. In order to characterize a dipole, we need two parameters such as length and diameter or Z_0, not just the length alone. With this improvement, Lawson's algorithm is summarized in Table 9-4.

For comparison, the reactance of the same element is calculated using the computer program MN at f = 14.075 MHz. The results of the comparison are shown in Fig 9-3.

Both the new Schelkunoff algorithm (see Sec 8.4) and the improved Lawson algorithm appear to yield the reactance variation with frequency comparable to that determined by use of numerical methods.

Table 9-3
Schelkunoff Spreadsheet for Hy-Gain 204BA

Section	*0*	*1*	*2*	*3*	*4*	*5*	*Total*
Diameter $2a_n$	2.14	1.25	1.125	0.875	0.625	0.4375	
Length ℓ_n	2.925	41.875	46	50.5	24	53	218.3
x_n	2.925	44.8	90.8	141.3	165.3	218.3	
$\sin \pi x_n/\ell$	0.04	0.60	0.97	0.89	0.69	0.00	
$\cos \pi x_n/\ell$	1.00	0.80	0.26	–0.45	–0.72	–1.00	
$\ln a_n$	0.07	–0.47	–0.58	–0.83	–1.16	–1.52	–0.8986
$\ell_n(\ln a_n)$	0.20	–19.68	–26.47	–41.75	–27.92	–80.55	–196.16
ΔN	2.45	14.37	7.07	–0.30	3.24	25.75	52.57
ΔM	0.05	5.14	10.44	3.05	–4.39	–10.33	3.96
$\Delta N'$	1.56	7.75	–2.15	–3.43	8.21	33.74	45.67
$\Delta M'$	0.07	6.31	10.79	1.53	1.78	39.00	59.48

Tapered		*Equivalent Cylinder*		*Reactance*	
M	–17.17 Ω	M_{cyl}	–21.13 Ω	f	14.075 MHz
K_a	717 Ω	K_a'	704 Ω	f_r	13.649 MHz
X_o	–10.26 Ω	X_o'	43.80 Ω	X(f)	32.12 Ω
Z_0	649 Ω	Z_0'	683 Ω		
f_r/f_o	1.0101	f_o'/f_o	1.0531		
f_o	13.513 MHz	f_o'	14.230 MHz		
ℓ	218.3 in.	ℓ'	207.30 in.		
d_{avg}	0.814 in.	d′	0.866 in.		

Table 9-4
Lawson Spreadsheet for Hy-Gain 204BA

Section	*0*	*1*	*2*	*3*	*4*	*5*	*Total*
Diameter $2a_n$	2.14	1.25	1.125	0.875	0.625	0.4375	0.814
Length ℓ_n	2.925	41.875	46	50.5	24	53	218.3
x_n	2.925	44.8	90.8	141.3	165.3	218.3	
log K	2.91	3.15	3.19	3.30	3.45	3.60	3.33
ℓ_n log K	8.52	131.71	146.79	166.66	82.71	190.86	727.24
m	0.64	0.93	0.94	0.99	1.05	1.11	
ϕ	1.21	18.47	37.43	58.25	68.15	90.00	
f(θ)	1.00	0.93	0.55	-0.10	-0.59	-0.91	
Eff. length	2.44	39.04	44.64	50.56	23.40	48.42	208.51

Tapered		*Equivalent Cylinder*		*Reactance*	
ℓ	218.3 in.	ℓ'	208.51 in.	f	14.075 MHz
ℓ'/ℓ	0.9551	c'/f_o'	0.48180 λ	f_r	13.632 MHz
$\Delta X_{0.9}$	104.70	$\Delta X'_{0.9}$	109.6 Ω	X(f)	34.29 Ω
X_o	–9.27 Ω	X_o'	39.9 Ω		
Z_0	667 Ω	Z_0'	698 Ω		
f_r/f_o	1.0089	K_a'	719 Ω		
f_o	13.513 MHz	f_c'	14.147 MHz		
d_{avg}	0.814 in.	d′	0.769 in.		

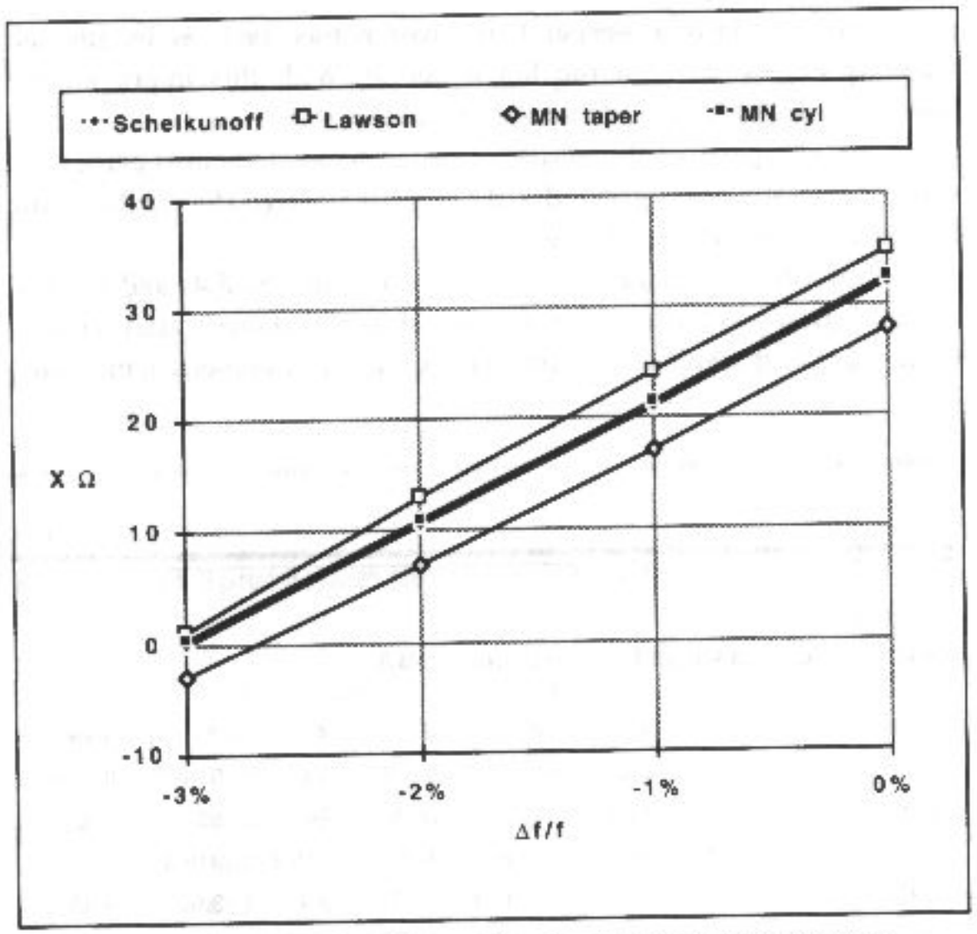


Fig 9-3—Comparison of algorithms and method-of-moments computer program MN.

Sec 9.3 The Hy-Gain Insulated Clamp

For the case of the insulated driven element, the length of the clamp shields the element center section and is subtracted from the element physical length. This has two effects—the reactance of the element is decreased by the length change, and there is a physical capacitance from the clamp. The physical capacitance of each clamp is in parallel with half the antenna impedance, and is not part of the radiation impedance of the antenna itself. In the case of an inductive matching network (such as a beta match), the series combination of the two clamp capacitances (twice the reactance of each) appears across the matching inductor.

The capacitance is calculated by the equation for coaxial capacitors, which is

$$C = \frac{2L\pi\varepsilon}{\ln\ (D / d)} \qquad \text{(Eq 9-8)}$$

where

d = inside diameter of the metallic clamp
D = diameter of the driven element
L = length of the capacitor

The dielectric constant ε is given by

$$\varepsilon = k\varepsilon_o \qquad \text{(Eq 9-9)}$$

where

k ≈ 2.3 for the dielectric in the Hy-Gain insulator
ε_o = 8.854 pF per meter

The shielding of the element center section results in the clamp capacitances shown in Table 9-5.

Table 9-5
Reactance of Hy-Gain Driven Element Clamp

	204BA	*155BA*	*105BA*
Type	#14	#13	#13
Diameter	1.75 in.	1.5 in.	1.5 in.
Length	3.75 in.	2.625 in.	2.625 in.
Element shortening per side	3.75 in.	2.625 in.	2.625 in.
Clamp capacitance per side	67 pF	24 pF	24 pF
Reactance per side	−169 Ω	−315 Ω	−235 Ω
Clamp total reactance	−339 Ω	−630 Ω	−470 Ω

Sec 9.4 Comments on Beta Matching

Although the beta-matching method requires a split and insulated driven element, it has the advantage of easy adjustment and it enhances wind survival by reducing the required length of the driven-element halves. In many applications, the series inductance arising from the wire length between the balun and the driven-element connection must be accounted for in matching calculations. Also, it is convenient to be able

to estimate the driven-element reactance change resulting from a known adjustment in either center section or tip.

The beta match works by transforming the Yagi input resistance through the input reactance to yield an admittance with its real part equivalent to the desired source impedance. This is typically accomplished by shortening the driven element to create a capacitive reactance at the operating frequency, in series with the driving-point resistance. Canceling the reactive part of the input admittance by means of a parallel inductor leaves the desired real part. The inductance can take any form, and comparisons among various inductor formats can be made by determining the resonant frequency of the inductor in parallel with a test capacitor.

Because of the narrow-band nature of Yagi antennas, it is unlikely that a matching network of any type will be narrower than the antenna itself. Hence, it is also unlikely that an ordinary matching network can markedly increase the bandwidth of the antenna.

Assume that the input impedance of the Yagi antenna is expressed as

$$Z_a = R + jX \qquad \text{(Eq 9-10)}$$

This can be inverted to admittance form to yield

$$Y_a = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} \qquad \text{(Eq 9-11)}$$

To achieve a match, we desire that the real part of Y_a equal $1/Z_0$, so

$$\frac{R}{R^2 + X^2} = \frac{1}{Z_0} \qquad \text{(Eq 9-12)}$$

which yields

$$X^2 = RZ_0 - R^2 = R(Z_0 - R) \qquad \text{(Eq 9-13)}$$

Hence,

$$X = \sqrt{R(Z_0 - R)} \qquad \text{(Eq 9-14)}$$

We can then select a shunt reactance to cancel the reactive part of Y_a thus

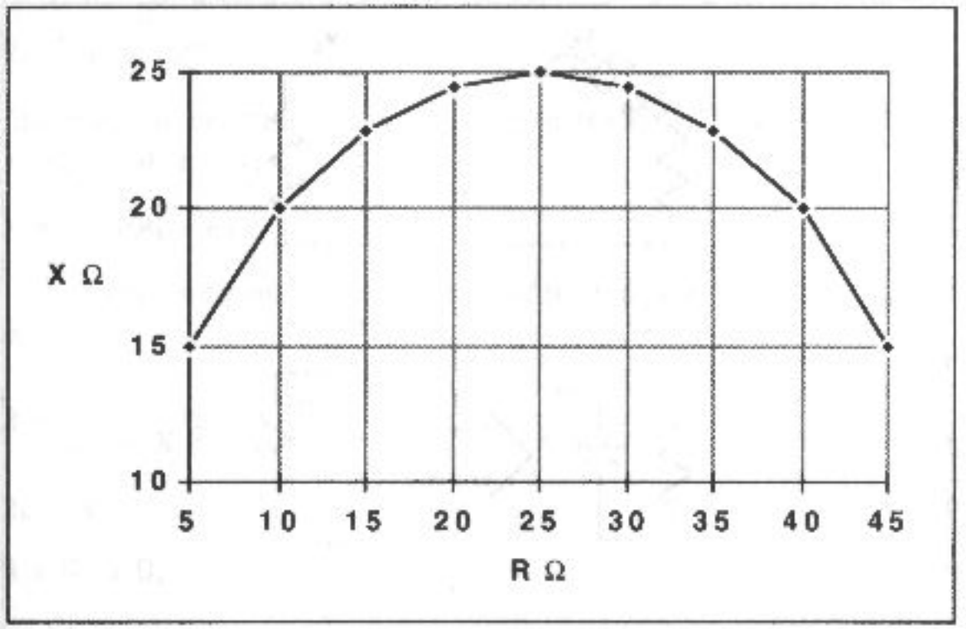


Fig 9-4—Reactance required to match input resistance.

$$X_\beta = \frac{R^2 + X^2}{X} = \frac{RZ_0}{X} \qquad \text{(Eq 9-15)}$$

From these equations, we can determine the input reactance and shunt reactance necessary to match a given input resistance, or we can determine the antenna impedance from the parameters of the beta network and driven-element shortening. An interesting characteristic of the beta match is that the curve of X versus R is very flat, peaking at R = 25 Ω as shown in Fig 9-4. Thus, a single adjustment of the driven element to attain the desired value of X should not be critical.

A typical installation includes inductance in series with the source and capacitance across the feed point. As an example, the lead lengths from the balun to the driven-element halves on a Hy-Gain 204BA account for approximately 25 Ω of inductive reactance in series with the source impedance, and the driven-element mounting brackets place approximately 339 Ω of capacitive reactance in shunt with the beta inductor. The equivalent circuit, taking account of this lead inductance X_L, is shown in Fig 9-5.

Thus, we can determine that we desire a match to a driving admittance

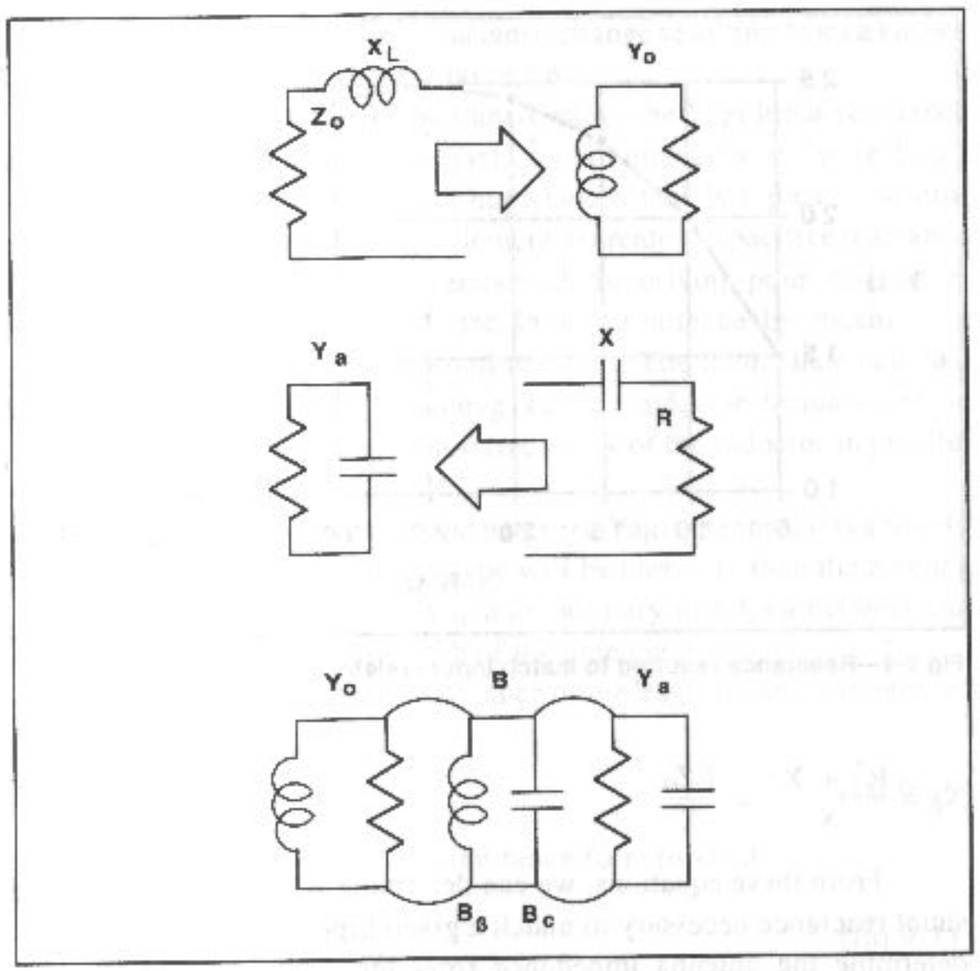


Fig 9-5—Beta-match equivalent circuits.

$$Y_o = \frac{1}{Z_0 + jX_L} = \frac{Z_0}{Z_0^2 + X_L^2} - j\frac{X_L}{Z_0^2 + X_L^2} = \frac{1}{R_0} - j\frac{X_L}{Z_0 R_0} \qquad \text{(Eq 9-16)}$$

where we have defined

$$R_0 = \frac{Z_0^2 + X_L^2}{Z_0} \qquad \text{(Eq 9-17)}$$

For the example here, the real part of this admittance corresponds to a source resistance level of $R_0 = (50^2 + 25^2)/50 = 62.5\ \Omega$.

To achieve a match, we first equate the real parts of Y_a and Y_o.

$$\frac{1}{R_0} = \frac{R}{R^2 + X^2} \qquad \text{(Eq 9-18)}$$

Choosing the negative sign for capacitive reactance by element shortening, we then have

$$X = -\sqrt{R(R_0 - R)} \qquad \text{(Eq 9-19)}$$

Next we equate to zero the sum of the imaginary parts of Y_{in}, Y_o, and B

$$B = \frac{X_L}{Z_0^2 + X_L^2} + \frac{X}{R^2 + X^2} = \frac{X_L}{Z_0 R_0} + \frac{X}{R R_0} \qquad \text{(Eq 9-20)}$$

Because

$$B = B_\beta + B_c \qquad \text{(Eq 9-21)}$$

we have

$$B_\beta = B - B_c = \frac{X_L}{Z_0 R_0} + \frac{X}{R R_0} - \frac{1}{X_c} \qquad \text{(Eq 9-22)}$$

where both X and X_c are negative, and

$$X_\beta = -\frac{1}{B_\beta} \qquad \text{(Eq 9-23)}$$

If $X_L = 0$ and $X_c = 0$, then $R_0 = Z_0$ and these equations reduce to the simpler expressions above. If we know X_L, Z_0 and B for a matched condition, to take the example of a manufactured antenna, we can determine the input resistance of the antenna from the expression

$$R = \frac{R_0}{1 + \left(R_0 B - \frac{X_L}{Z_0}\right)^2} \qquad \text{(Eq 9-24)}$$

and we can determine X from Eq 9-19,

$$X = -\sqrt{R(R_0 - R)}$$

Ideally, we can measure X_L and X_β, or we can estimate them by measurement of their resonance with known capacitors. We can approximate X_L and X_β by transmission-line segments. For the lead-wire

reactance, we assume each side is a separate transmission line, one side of which is the antenna boom.

$$Z_{0L} = 2(120)\ln \frac{2s}{\sqrt{d_L d_b}} \qquad \text{(Eq 9-25)}$$

where

s = spacing from lead wire to boom center
d_L = diameter of the lead wire
d_b = diameter of the boom

$$X_L \approx 2\pi Z_{0L} h \qquad \text{(Eq 9-26)}$$

where h = length of lead wire, wavelengths

Similarly, for the beta inductor

$$Z_{0\beta} = 2(120)\ln \frac{2s}{\sqrt{d_\beta d_b}} \qquad \text{(Eq 9-27)}$$

where

s = spacing from beta wire to boom center
d_β = diameter of the beta wire
d_b = diameter of the boom

If the inductor is clear of the boom so the two wires can "see" each other, a more accurate estimate may be

$$Z_{0\beta} = 120 \ln \frac{4s}{d_\beta} \qquad \text{(Eq 9-28)}$$

If we know $Z_{0\beta}$,

$$X_\beta \approx 2\pi Z_{0\beta} h \qquad \text{(Eq 9-29)}$$

where h = beta transmission line length, in wavelengths

These equations can be assembled into a spreadsheet for calculation of beta-match parameters of an antenna of known input impedance.

Consider the case of matching an antenna of input resistance R = 31.3 Ω to a feed line of Z_0 = 50 Ω. Assume it is fed through h = 6.5 in. of d_L = 0.1-in. diameter wire, spaced 1 in. from a d_b = 2-in. diameter boom. Assume further that the capacitive reactance of the split driven element mount totals X_c = –169 Ω. Last, consider a beta-match inductor made from d_β = 0.25-in. diameter wire spaced 4 in. from the 2-in.

diameter boom. We choose the frequency f = 14.075 MHz, so the wavelength is λ = 11799.32/14.075 = 838.3 inches.

For the input lead wire, the center-to-center spacing s = 2 in., d_L = 0.1 in. and d_b = 2 in. Thus, from Eq 9-25

$Z_{0L} = 2(120)\ln \frac{2(2)}{\sqrt{0.1(2)}} = 526\ \Omega$, and from Eq 9-26

$X_L = 2\pi(526)\frac{6.5}{838.3} = 25.6\ \Omega$. From Eq 9-17

$$R_0 = \frac{50^2 + 25.6^2}{50} = 63.1\ \Omega$$

We require the element reactance (Eq 9-19) to be

$X = -\sqrt{32.8(63.1 - 31.3)} = -31.6\ \Omega$. From Eq 9-20

$B = \frac{25.6}{50(63.1)} + \frac{-31.6}{31.3(63.1)} = -0.008$. From Eqs 9-20 and 9-22

$$B_\beta = B - B_c = -0.008 - \frac{1}{339} = -0.011$$

$X_\beta = -1/B_\beta = 92.6\ \Omega$

For the beta transmission line, s = 5 in., and d_β = 0.25 in., so from Eq 9-28

$$Z_{0\beta} = 120 \ln \frac{4(5)}{0.25} = 526\ \Omega$$

The beta transmission-line length, in wavelengths, is

$$h_\beta = \frac{X_b}{2\pi Z_{0\beta}} = \frac{92.6}{2\pi(526)} = 0.028\ \lambda$$

$h_\beta = 0.028(838.3) = 23.5$ in.

Thus, for this example with series inductance, the element reactance must be X = –31.6 Ω and the beta-match length is h = 23.5 in. By comparison, if the feed-line reactance is zero, the required element reactance is X = –24.2 Ω, and the beta-match length is h = 13.8 in., a substantial difference. The problem of series inductance can be reduced by means of the connection format shown in Fig 9-6. This beta match

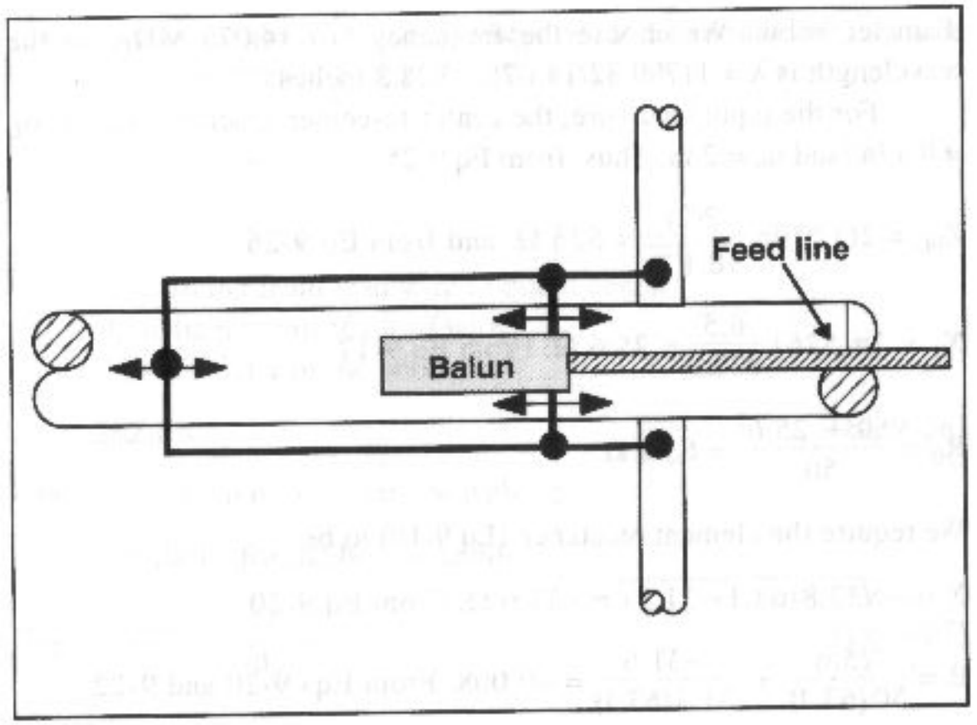


Fig 9-6—Beta match with minimum series inductance.

allows for adjustment of resonance and match at the boom, without requiring retuning the element tips.

Sec 9.5 Element Reactance

The element reactance required to accommodate various matching schemes can be calculated by use of the taper algorithm. As a shortcut, the element length adjustment necessary to achieve the required input reactance can be estimated to a satisfactory degree of accuracy by the following equations.

For a 1-inch change in the length of both center sections of a near-half-wave element, the change in reactance is approximately

$$\Delta X = \frac{\pi K_c}{2} \times \frac{4\Delta\ell}{\lambda} = \frac{2\pi K_c \Delta\ell}{\lambda} \qquad \text{(Eq 9-30)}$$

where K_c = characteristic impedance of the center section

$$K_c = 120\left(\ln\frac{2\ell}{a_c} - 1\right) \qquad \text{(Eq 9-31)}$$

where a_c = *radius* (not diameter) of the center section

For a 1-inch change in the length of both tip sections of a near-half-wave element, the change in reactance is approximately

$$\Delta X = \frac{\pi K_a^2}{2K_t} \times \frac{4\Delta\ell}{\lambda} = \frac{2\pi K_a^2}{K_t} \times \frac{\Delta\ell}{\lambda} \qquad \text{(Eq 9-32)}$$

where

K_a = average impedance of the element

K_t = characteristic impedance of the tip section

$$K_t = 120\left(\ln\frac{2\ell}{a_t} - 1\right) \qquad \text{(Eq 9-33)}$$

where a_t = *radius* (not diameter) of the center section

Consider as an example the Hy-Gain 204BA element. From the element spreadsheets, K_a = 717 Ω, $2a_c$ = 1.25 in., $2a_t$ = 0.4375 in., and ℓ = 218.3 in. For f = 14.075 MHz, λ = 11799.32/14.075 = 838.32 in. Further, from Eqs 9-31 and 9-33, K_c = 666 Ω and K_t = 792 Ω. Thus,

$\Delta X_c = \frac{2\pi K_c}{\lambda}$ = 5.0 Ω/inch change to both center sections, and

$\Delta X_t = \frac{2\pi K_a^2}{K_t\lambda}$ = 4.9 Ω/inch change to both tip sections

For the beta-match example above, we desire an input reactance of X = –31.5 Ω. This means we would *shorten* the center sections by $\Delta\ell$ = 31.5/5.0 = 6.3 in. per side from the dimension that would provide zero reactance in the Yagi antenna design. We could accomplish the same end by shortening the tips by $\Delta\ell$ = 31.5/4.9 = 6.4 in. This is possible, of course, because the gain of a Yagi antenna is essentially independent of driven-element length.

References

[1] E. Hallen, "Theoretical Investigations into the Transmitting and Receiving Qualities of Antennae," *Nova Acta Regiae Soc Sci Upsal*, Ser IV, Vol II, 1938.

[2] C. A. Balanis, *Antenna Theory Analysis and Design* (New York: Harper and Row, 1982), p 338.

[3] S. Uda and Y. Mushiake, *Yagi-Uda Antenna* (Sendai: Sasaki Printing and Publishing Co, Ltd, 1954), "Theoretical Expression of the Equivalent Radius," p 13.

[4] J. L. Lawson, *Yagi Antenna Design* (Newington: American Radio Relay League, 1986), p 7-12.

[5] C. Su and J. German, "The Equivalent Radius of Noncircular Antennas," *Microwave Journal*, Vol 9, 1966, p 64.

[6] See "Equivalent Radius of Parallel-two-conductors," p 19 of reference 3.

[7] D. Jaggard, "On Bounding the Equivalent Radius," *IEEE Trans on Antennas and Propagation*, Vol AP-28, May 1980, p 384.

[8] G. Hall, ed., *The ARRL Antenna Book*, 16th Ed. (Newington: ARRL, 1991), page 2-24ff.

Chapter 10

Siting and Installation of Yagi Antennas

Sec 10.1 Ground Reflection and Antenna Performance

The relationship between the height of a radio antenna and its communications effectiveness is determined by the laws of physics.[1] Communicating over great distances requires that the energy from the transmitter be radiated at angles close to the horizon. The strength of the antenna radiation at low elevation angles is directly related to the height of the antenna. The higher the antenna, the greater the signal strength at lower radiation angles and, thus, the greater the communications distance possible.

Short-wave communications can be effectively accomplished only if the antenna is at a physical height that is a substantial portion of a wavelength. For the three Amateur Radio bands usable throughout the sunspot cycle for worldwide communications (40, 30 and 20 meters), an antenna height above ground of approximately 70 feet, while not optimum, permits an amateur licensee to enjoy reliable worldwide communications.

Additionally, antennas which are placed within structures suffer substantial loss of effectiveness. Placement within a structure also increases the opportunity for interference with inadequately designed consumer electronics equipment, and increases the potential for other radiation effects upon equipment and inhabitants.

The effect of ground reflection on antenna performance is dramatic. The ground immediately beneath the antenna affects the antenna resonant frequency and efficiency. More significantly, the reflection of radiated signals from the ground in the foreground of a Yagi antenna determines the vertical radiation pattern of the antenna over ground. For a horizontal antenna over flat (alluvial) ground, this results

in a complete cancellation of signal at the horizon caused by the efficient reflection below the Brewster angle.[2]

The reflection is from the conductive or dielectric discontinuity between the air and the surface of the ground (giving effect to skin depth), not from some magical underground water layer. For antenna heights above ½ λ, there are also cancellations possible at other, higher radiation angles. For a typical flat-ground installation, the ground is busy at work canceling our hard-earned signal in the very range of vertical angles we want most! On the other hand, the peaks of the vertical pattern offer a 6-dB gain over the free-space gain of the antenna at the same elevation angles, so the ground reflection can be most valuable.

The vertical pattern of the antenna over ground is perhaps the most significant variable among antennas used for long-distance high-frequency transmission. The propagation modes employing angles near zero degrees, including chordal-hop modes, encounter the lowest reflection loss, and hence are the most effective (lowest path loss) modes for transmission over long distances[3] if the receiving-end antenna is capable of responding to the low-angle mode.[4]

As we shall see, this means that antenna height above flat ground is a key parameter, and, more important, that a sloping foreground can be employed for substantial enhancement of low-angle radiation,[5,6] at no sacrifice of higher angle radiation. To realize this benefit, it is important that the first Fresnel zone in the foreground not be shadowed (be fully illuminated) and that the foreground surface roughness or vegetation not reduce the reflection coefficient.

The Rayleigh criterion[7] characterizes a surface as smooth if

$$\Delta h < \frac{\lambda}{8 \sin \alpha} \qquad \text{(Eq 10-1)}$$

where α = angle of incidence

As an example, for $\lambda = 800$ inches and $\alpha = 6°$, we have a smooth surface if the roughness is less than $\Delta h = 800/(8 \times 0.1) = 1000$ inches or 83 feet. For radio waves in the HF band this shows that ground nearly always behaves as a perfectly smooth surface, particularly for low angles of incidence.

Reflection from a smooth surface is termed specular, and is directional, obeying the laws of classical optics, and is phase coherent.

Radiation from a rough surface is termed diffuse; diffuse reflection has little directivity and phase coherence, and has large amplitude variations. When the roughness of a surface is less than the Rayleigh criterion, the reflection is adequately described by the reflection coefficient for smooth ground. The reflection does not change abruptly from specular to diffuse, but the greater are the irregularities above the Rayleigh limit, the smaller is the reflection coefficient.

For long-distance communication, particularly over the full range of propagation conditions and modes, we would like to obtain effective radiation with no nulls over the range of elevation angles from 1° or 2° (chordal hop) up to 15° for 28 MHz, 17° for 21 MHz, and 20° for 14 MHz.[8,9] In my experience, angles all the way down to 1° seem to be significant in chordal hop and skew-path propagation situations on the higher bands. The often-referenced 1928 data for the UK to New Jersey path does not tell the entire story.

On the lower frequencies, the lower angles result in excessive D-region absorption, so the useful range is centered on somewhat higher angles (5° to 23°). Of course there will be times when propagation is best at much higher angles, particularly for communications over a shorter distance.

We can express the ground reflection gain over perfect, sloping ground by the expression

$$\frac{A}{A_f} = 2 \sin\left[\frac{2\pi h}{\lambda} \sin (\alpha + \alpha_o)\right] \qquad \text{(Eq 10-2)}$$

where

A = radiated amplitude
A_f = free-space radiated amplitude
h = height above the ground plane
λ = wavelength, same units as h
α = elevation angle
α_o = ground-slope angle

The ground-reflection gain (or loss) can be expressed in dB relative to free space by

$$dB = 20 \log_{10} \frac{A}{A_f} \qquad \text{(Eq 10-3)}$$

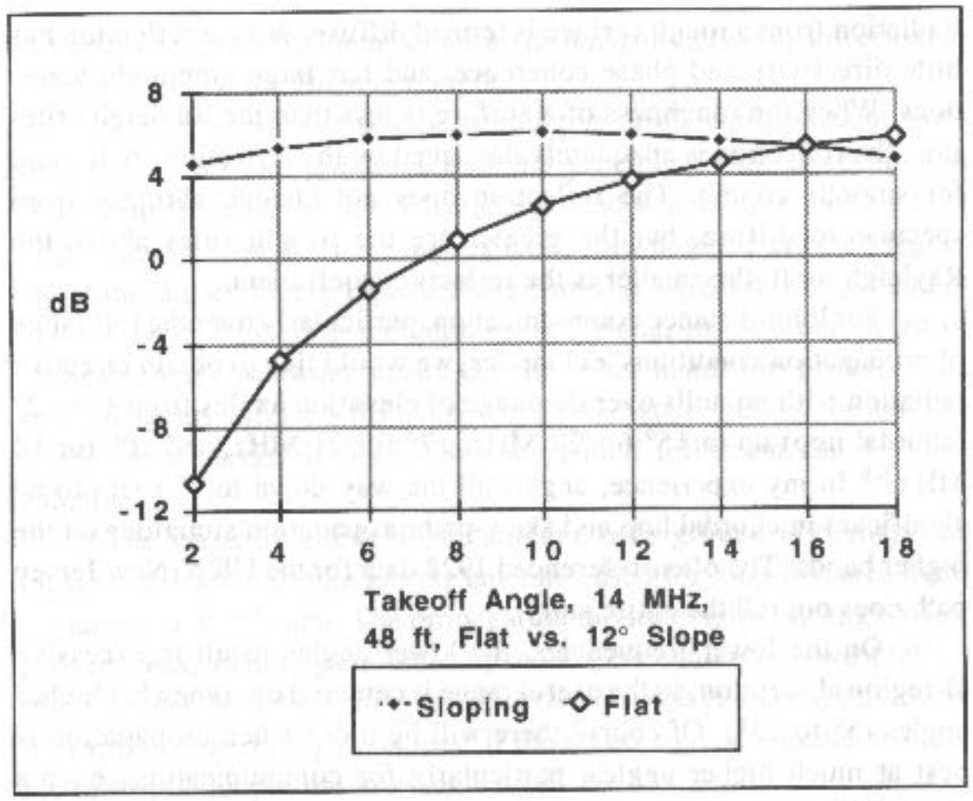


Fig 10-1—Ground-reflection gain for 14-MHz Yagis at 48 feet over sloping and flat ground.

The advantageous effect of ground-slope can be seen in Fig 10-1, which compares the ground-reflection gain of a 14-MHz antenna 48 ft above a downslope of 12° with that of the same antenna at the same height above flat ground.

Another interesting comparison is the reflection gain of the antenna 48 ft above a sloping foreground compared to that of the same antenna 100 ft above flat ground. Fig 10-2 shows the narrowing of the elevation pattern for the flat-ground case; the reflection gain is everywhere greater for the sloping foreground installation.

If, in our eagerness to respond to the instinct that higher is better, we position an antenna too high above a sloping foreground, we run the risk of bringing one of the higher angle nulls right down into the useful range of elevation angles. This is illustrated in Fig 10-3, comparing ground reflection gain for antennas at 100 ft above sloping and flat ground. A slope of 12° is fairly steep but not uncommon in hilly terrain.

This explains the occasional disappointment experienced in hilltop installations. Under these circumstances we might hope for

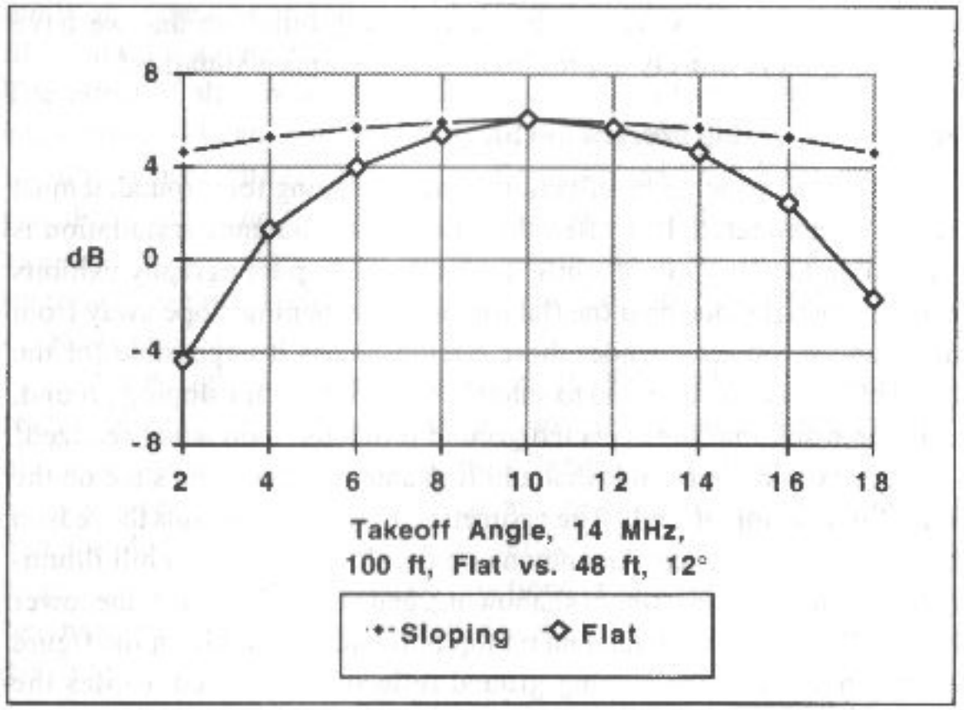


Fig 10-2—Antenna radiation angle for 14-MHz Yagis at different heights over sloping and flat ground.

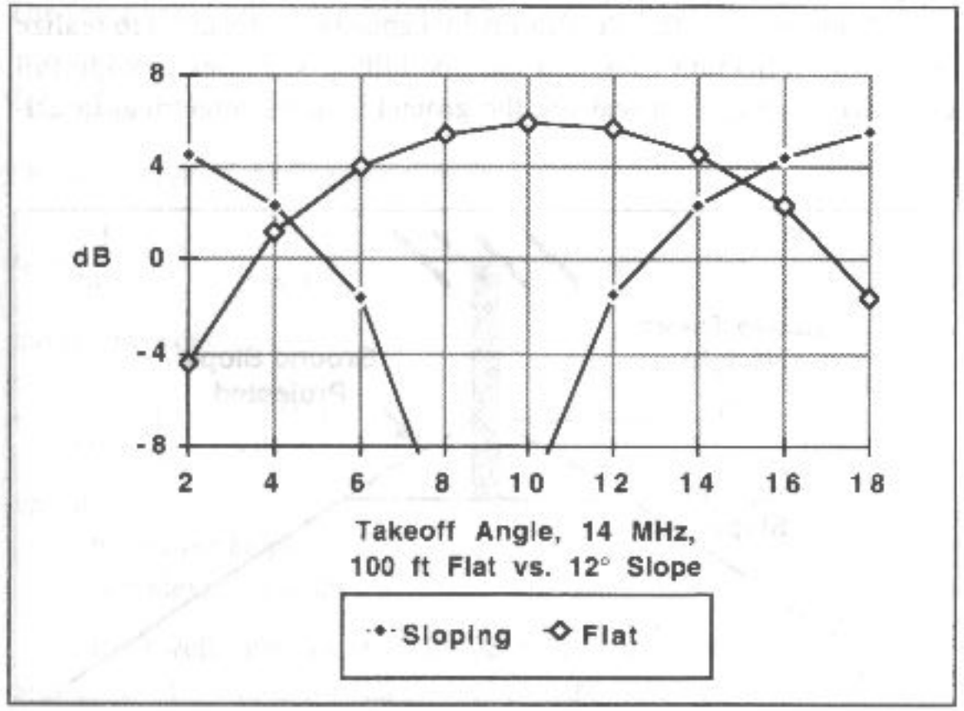


Fig 10-3—Ground-reflection gain for 14-MHz Yagis at 100 feet over sloping and flat ground.

substantial vegetation attenuation in the foreground, so that we have something more closely approximating a free-space situation!

Sec 10.2 Avoiding Foreground Shadowing

In order to achieve reflection from the sloping foreground, it must be fully illuminated. It is often the case that the antenna installation is located on the very top of a hill. Typically, hilltop topography exhibits a more gradual slope near the flat top, with steepening slope away from the rounded top area. Under these circumstances it is possible for the brow of the near foreground to shadow the more distant sloping ground, with the result that the expected ground reflection gain is not realized.

Moxon recommends that a hilltop antenna should be sited on the *side*,[10] not the top of a hill. The geometry of Fig 10-4 explains the reason behind this experience. An antenna on the sloping side of a hill illuminates the foreground without shadowing, and makes full use of the tower height. By contrast, an antenna on top of the idealized hill in the figure loses a portion of the sloping ground-reflection area, and it loses the effect of that portion of the tower below the projection of the ground-slope onto the tower. If the tower is too low or the edge too abrupt, the foreground illumination is lost completely.

Note however that the full Fresnel zone is not required to realize the ground reflection gain, and that the hilltop site does provide full azimuthal coverage. In real life the ground is not symmetrical in azimuth, so some compromises arise. Subject to adequate tower height to illuminate the slope, this siting situation has proved extremely effective. The effect of shadowing can result in dramatic changes for relatively small changes in antenna height, and it complicates the optimization of multiband installations. Subject to the understanding of the effect of possible foreground shadowing, we can expect a hilltop antenna to be as effective as a slope-sited antenna, with the bonus of substantially increased azimuthal coverage. I have been very satisfied with the results of this type of siting.

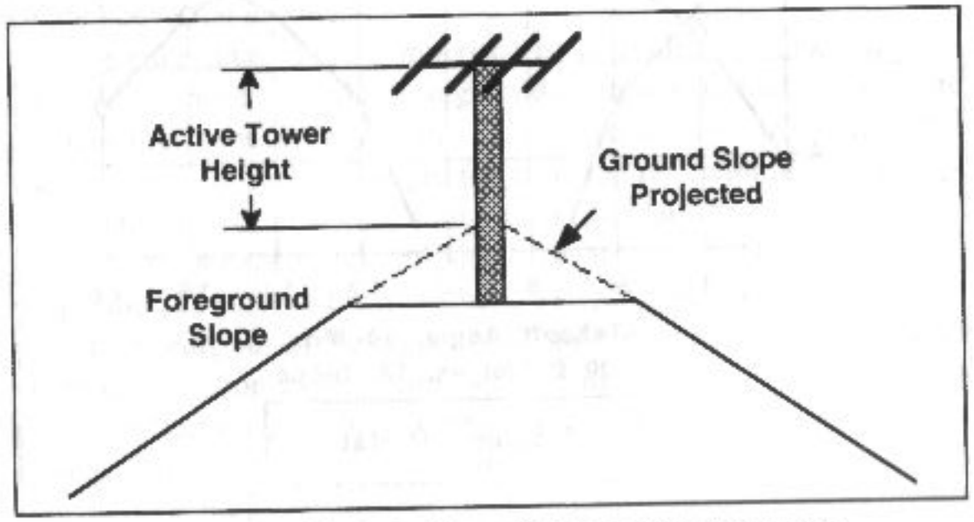


Fig 10-4—This drawing illustrates the partial loss of the hilltop-site reflection area and tower height.

A 1966 *QST* article by Paul Rockwell, W3AFM,[11] is an excellent review of siting issues, and concludes, as I have, that hilltop locations are well worth the wind exposure, offering a 10-20 dB advantage. Rockwell confirms an additional advantage of a sloping foreground—that the dimensions of the Fresnel reflection are much smaller (and hence can be located to avoid obstructions or absorption). The antenna height required for a given elevation angle of maximum radiation α is given by

$$h = \frac{\lambda}{4 \sin \alpha} \qquad \text{(Eq 10-4)}$$

The foreground reflection point for an antenna at this height is given by

$$d_r = \frac{h}{\tan \alpha} \qquad \text{(Eq 10-5)}$$

The near edge of the Fresnel zone is

$$d_n = \frac{h}{\tan \alpha}\left(3 - \frac{2\sqrt{2}}{\cos \alpha}\right) \qquad \text{(Eq 10-6)}$$

and the far edge is

$$d_f = \frac{h}{\tan \alpha}\left(3 + \frac{2\sqrt{2}}{\cos \alpha}\right) \qquad \text{(Eq 10-7)}$$

where

h = tower height
α = elevation angle

Rockwell gives as an example the requirements for a main-lobe peak at $\alpha = 1°$, which for flat ground requires an antenna height at 14.2 MHz of h = 992 feet; the reflection point is 10.8 miles out, with the near

and far edge of the ground-reflection elliptical area at 1.8 miles and 62.7 miles. It is likely that there will be major foreground disruptions to the uniformity of the reflection zone under these circumstances. By contrast, with a 12° foreground slope the required effective tower height for α = 1° is h = 77 feet, and the ground-reflection elliptical area boundaries are at 32 feet and 1970 feet, with the reflection point at 333 feet.

An elevation angle of α = 1° is an extreme example, and because of the broad lobe possible with sloping foreground, an elevation angle of α = 10° is a better choice, as shown in Fig 10-1. For this elevation angle, with a foreground slope of 12°, the effective tower height as defined in Fig 10-4 should be h = 47 feet or 0.67 λ. In any event, it is clear that the substantial reduction in the size of the ground-reflection ellipse makes it much more likely that the entire active reflection area will be uniform and effective. With substantial foreground slope, everything gets better.

Sec 10.3 Antenna Comparison Using Fading Statistics

A sky-wave signal received on an HF receiver exhibits a statistical variation of signal strength. This statistical variation can be expressed in terms of its cumulative probability distribution, which is typically defined as a Rayleigh distribution. If the signal is modulated by voice or code, the resulting received signal exhibits a probability distribution that includes the effect of the cumulative probability distribution of the modulation as well as the probability distribution related to the propagation path.

If we are interested in comparing two antennas located close enough so they can both feed the same receiver, we can use the characteristics of propagation statistics to measure their relative gains for a given distant signal and the propagation modes involved. Assume we have two similar antennas, and we wish to compare their operating gain, including the effect of siting and foreground slope. We connect the two antennas through a switch to a typical HF receiver or transceiver, and tune in a signal from a remote location. If we switch back and forth, we see a seemingly complex variation of S-meter readings caused by both modulation and fading statistics. It seems to require a good imagination to make a meaningful comparison.

But wait. Switch to one antenna. Switch the receiver AGC to a fast time constant, and reduce the RF gain until the received signal just causes the S-meter to jump a few times per second as the signal (and its

modulation peaks) just exceed the threshold set by the reduced receiver gain. Now switch to the other antenna. Does the S-meter jump more often or less? This is a direct comparison of the actual antenna gain for this mode, using the cumulative probability distribution of the received signal. In a short time, a conclusion can be formed about the relative gains of the two antennas.

The time rate of a signal exceeding a given level is a measure of the fraction of the time a signal exceeds that level. If the time rates are identical for two antennas, the gains (for this path, direction and distance) are identical. If we are careful not to transmit, we can place a variable attenuator in one antenna feed line and adjust for equal received-signal level. I have found this to be a powerful method of antenna and site comparison. I try not to take down my reference standards in my eagerness to build and put up new antennas, and I have so far avoided inadvertently transmitting through my variable attenuator.

Note that this comparison is possible simply by the knowledge that the same cumulative probability density function exists for propagated radio waves received by both antennas. If we know the specific characteristics of HF fading, we could develop an answer to another question of great interest to radio amateurs: How much is 1 dB worth?

Imagine a miniature pileup, consisting of two signals of absolutely equal radiated power divided absolutely equally among whatever propagation modes are active. Because of fading, we would expect one to be stronger than the other 50% of the time at a distant receiver. Now increase the power of one station by exactly 1 dB. This station will now be stronger more than 50% of the time. For Rayleigh fading this percentage becomes approximately 54%, so the ratio becomes 54:46 and the station with the 1 dB advantage is stronger 1.17 times as often as the weaker.

If instead we ask the same question with a 3-dB difference, the result is that the stronger station is louder 62% of the time, or 1.66 times as often as the weaker signal. A 6-dB difference results in a 75% dominance, a factor of 3 in time.

We can get an idea of how this works by using an amateur-band receiver. Tune in a steady, but fading, HF signal such as the Northern California DX Foundation beacons on 14.100 MHz.[12] Set the AGC as above to a fast time constant, and observe the range of S-meter readings during the steady tones. Under some conditions the signals will fade 10 to 20 dB during the time of each power step tone, while on others the fading will be much less. However, the fading statistics are still

Rayleigh, and, although the excursions are greater, the ratio of times remains the same. Note that longer paths tend to show greater variation because of multipath fading.

It is the statistical variation of propagation that is the great leveler in Amateur Radio; the variations in path statistics are greater than the differences among stations, so no one station dominates all the time. Also, a real-life pileup involves many signals, rather than two, and the results are dependent upon how the receiver AGC affects demodulated audio levels and upon the modulation characteristics of the various competitors. Last, of course, timing is key, as the fraction of time that any one station is transmitting at peak amplitude is quite small. Even so, the advantages of improved signals accrue over time, and are worth the effort to achieve.

Sec 10.4 Experimental Verification of Taper Algorithm

A rough measurement, made to verify the new Schelkunoff taper algorithm (see Sec 8.4), was at the same time a good demonstration of the effect of reflection from imperfect ground. For use with a simple antenna noise bridge, I constructed a dipole center section, each half of which consisted of a 1-in. length of ⅞-in. aluminum tubing. To this I attached either a tapered element totaling 100 in. or a cylindrical ¾-in. diameter element totaling 96 in. Including the center fixture, the tapered element consisted of 24 in. of ⅞-in. tubing, 23.5 in. of ⅝-in. tubing, and 52.5 in. of 7/16-in. tubing.

In each case, the reactance was measured over the range of 28 to 30 MHz. Keeping the same center section allowed a substitution measurement, with all variables the same for each type of antenna. Using the new taper algorithm from Chapter 8, the equivalent cylindrical dipole for the tapered element is $L' = 96.42$ in. and $d' = 0.577$ in. The calculated free-space resonant frequency of the tapered dipole is $f_r = 29.266$ MHz and of the cylindrical dipole, $f_r = 29.319$ MHz.

The dipole was mounted horizontally above a ladder at a height of 87 in. over the ground. The work area is paved with asphalt over typically rocky mountaintop soil. I measured the input impedance with the noise bridge and found the two dipoles to be essentially identical. The reactance variation with frequency for the two configurations is shown in Fig 10-5. I'm sure that the slight departure from linear reactance variation with frequency is the result of the simple instrumentation used, although I did modify the noise bridge to account for the

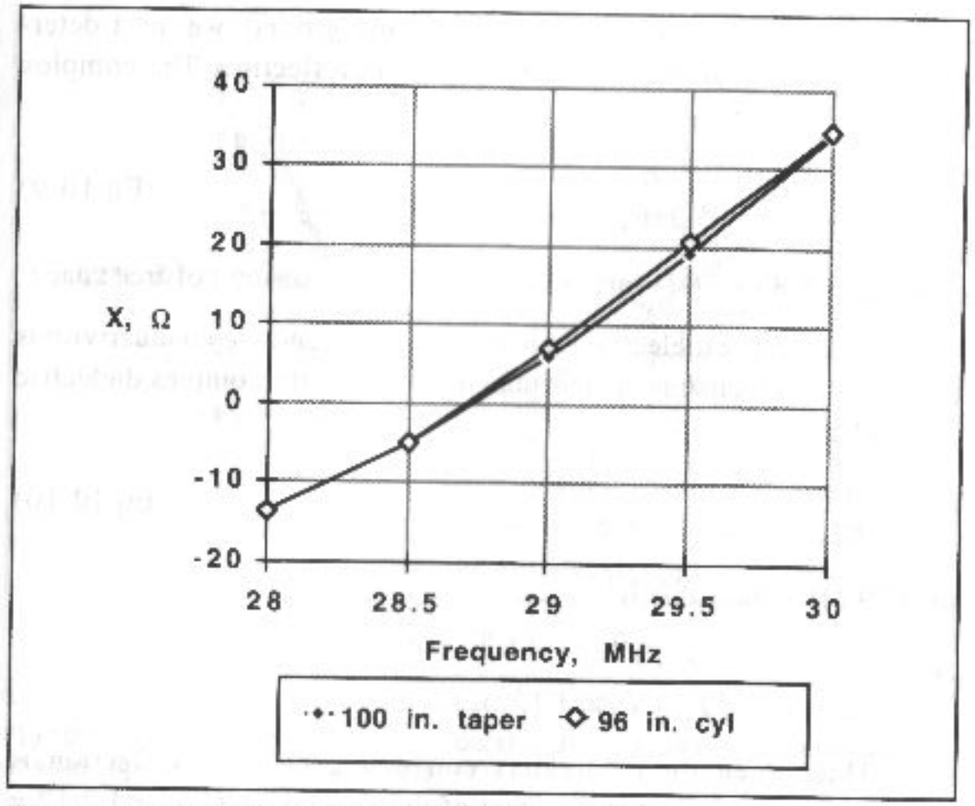


Fig 10-5—Measured reactance of 100-in. tapered element and 96-in. cylindrical element.

short length of transmission line between the bridge and the antenna center. The use of a substitution measurement provides confidence in the equivalence of the two dipoles.

As a further comparison, I used the taper algorithm of Chapter 8 to calculate the reactance of the tapered and cylindrical dipoles. To do this, the effect of the ground beneath the antenna must be taken into account. For the case of a dipole over a conductive (reflecting) surface, the input impedance is given by

$$Z_{in} = Z_{11} - rZ_{21} \qquad \text{(Eq 10-8)}$$

where

Z_{11} = self impedance of the dipole
Z_{21} = mutual impedance between two parallel side-by-side dipoles at a distance of twice the height
r = ground-reflection coefficient, determined as follows

For the case of imperfectly conducting ground, we must determine the phase and amplitude of the ground reflection. The complex dielectric constant is[13]

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon' - j\,\frac{\sigma}{2\pi f \varepsilon_o} \qquad \text{(Eq 10-9)}$$

where $\varepsilon_o = 8.85 \times 10^{-12}$ farads/m, the dielectric constant of free space

For rocky soil, the dielectric constant is $\varepsilon' \approx 12$, and the conductivity is $\sigma \approx 2 \times 10^{-3}$ siemens/meter. The imaginary part of the complex dielectric constant is

$$\varepsilon'' = \frac{\sigma}{2\pi f \varepsilon_o} \qquad \text{(Eq 10-10)}$$

so at 29 MHz the value is

$$\varepsilon'' = \frac{2 \times 10^{-3}}{2\pi(29 \times 10^6)\ (8.85 \times 10^{-12})} = 1.2$$

Thus, even for moderately conductive soil, the reflection is largely from the dielectric constant of the ground, as $\varepsilon''/\varepsilon' = 1.2/12 = 0.1$. The amplitude of the reflection coefficient r for vertical incidence is given by

$$r = \frac{\sqrt{\varepsilon} - 1}{\sqrt{\varepsilon} + 1} \qquad \text{(Eq 10-11)}$$

For the case of $\varepsilon = 12 + j0$ this is

$$r = \frac{\sqrt{12} - 1}{\sqrt{12} + 1} = \frac{2.46}{4.46} = 0.55$$

By comparison, $r \approx 1$ for grazing angles.

For a height of 87 in., the spacing in wavelengths is $2(87)/406 \approx 0.43\ \lambda$. From Fig 10-6, the chart of X_{21},[14] for a spacing of $0.43\ \lambda$ and a length of $0.48\ \lambda$, we would expect $X_{21} \approx -32\ \Omega$, so $rX_{21} = -17\ \Omega$. Thus, at the dipole free-space resonant frequency $f_r \approx 29.3$ MHz for which self-reactance $X_{11} = 0$, we would expect to measure $X_{in} = -17\ \Omega$. From the measurements we do in fact measure a value of $rX_{21} \approx -17\ \Omega$. The resonant frequency is offset downward by the effect of the adjacent dielectric ground reflection.

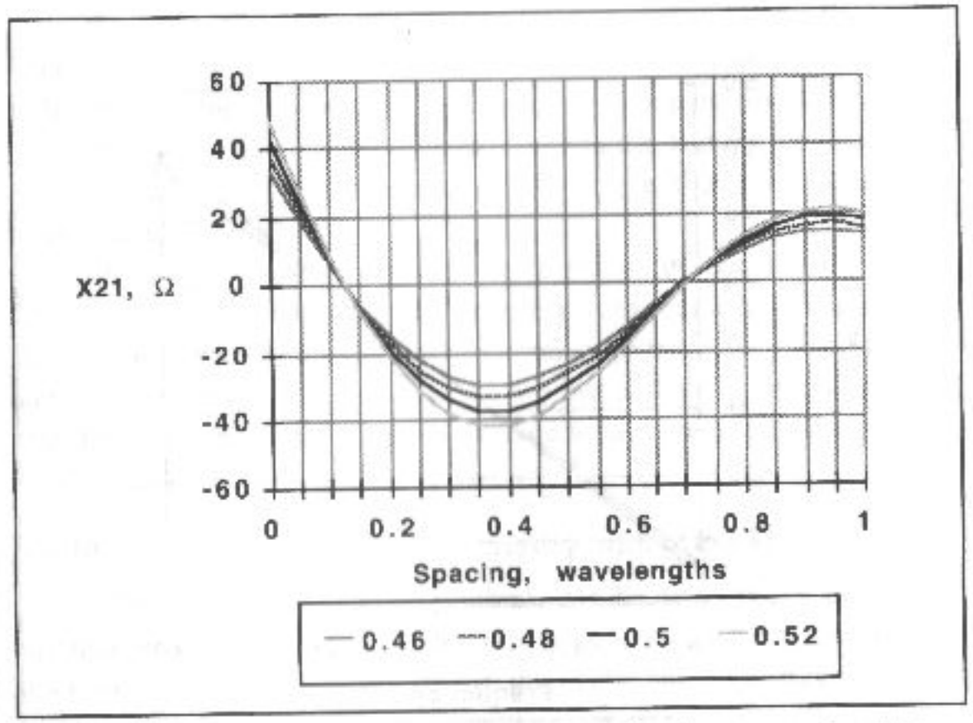


Fig 10-6—Mutual reactance between parallel dipole antennas as a function of spacing and length. *(After Kraus)*

The calculated reactance variation with frequency for the tapered and cylindrical dipoles above real ground is shown in Fig 10-7. This simple measurement and calculation seems to confirm that the cylinder of $\ell = 96.4$ in. length per side is equivalent to the tapered $\ell = 100$-in. element, as predicted by the new taper algorithm. It also shows the effect of ground, to reduce the resonant frequency of dipoles near it. In this case, the measured resonance is at $f_r = 28.8$ MHz, while the free-space resonance is at $f_r = 29.3$ MHz. This detuning affects all of the elements of a Yagi close to ground. Similar detuning is expected in a stack of monoband antennas.

The same comparison was made using MN, which showed that the tapered dipole has identical impedance characteristics as a cylindrical dipole of 96.3-in. length and 0.6-in. diameter. Note that for calculations above real ground, MN uses an infinitely conductive ground model to calculate antenna currents, so the effect of ground dielectric constant and conductivity is not seen in the MN calculations of resonant frequency above ground.

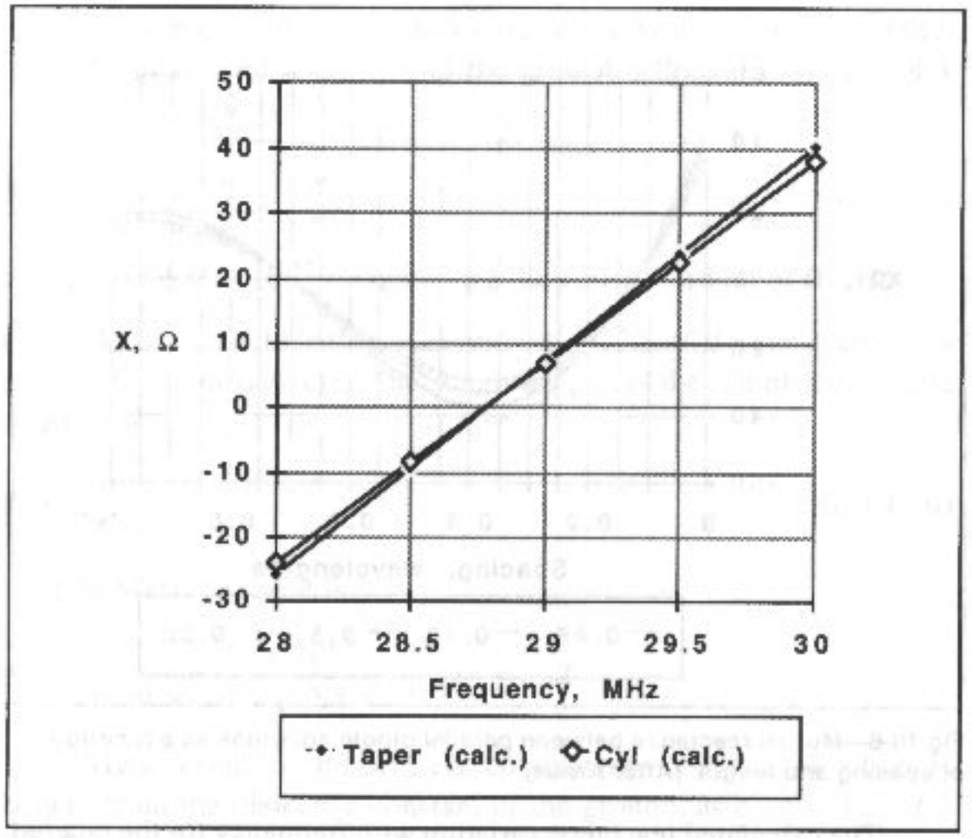


Fig 10-7—Calculated reactance of 100-in. tapered dipole element and 96-in. cylindrical element.

Sec 10.5 Stacking Yagis—Equivalent to Foreground Slope?

As one who has long admired those radio amateurs whose energy and dedication extended to the construction of stacked monoband Yagis, I have always been surprised by the lack of startling gain increases in "upper-lower-stacked" comparisons. An enlightening discussion at the 1989 Dayton HamVention[15] shed some light on the issues at work here.

Essentially, for two antennas to create signals which add constructively at a desired elevation angle, the signals must have amplitudes and phases that are closely matched. This is almost impossible to achieve, given the fact that the elevation patterns of the individual Yagis at substantially different heights above ground are quite different. The opportunity for substantial gain improvement is limited to those elevation angles at which the patterns of the two antennas provide comparable signal amplitudes.

This generally means that the gain improvement from stacking is only 1 dB or so. The pattern of the stacked combination is more likely to be the envelope of the two individual antenna patterns, so the result is a very good fill of the useful elevation angles. But, as we have seen, we can accomplish the same result with a single Yagi at modest height over sloping foreground.

A more interesting opportunity for stacking gain with sloping foreground may be two identical antennas at the same height above the slope, placed in echelon (side-by-side) and spaced ½ to 1 λ to achieve adequate stacking gain. In this case we are adding identical patterns, and the drive should be phased to create amplitude-additive signals in the desired directions.[16]

Sec 10.6 Suggestions for Further Studies with MININEC

The power of numerical methods can be applied to a number of antenna problems of interest. Among these are the questions of close stacking of antennas for different frequencies, and a thorough analysis of issues relating to optimizing placement over physically realistic sloping foreground. This last analysis can be extended to behavior with sloping foreground of stacked antennas for the same frequency.

Another problem worth studying in greater detail is the question of creating full-gain multiband antennas on a single boom. This is a problem that has not been resolved successfully; there is always a substantial compromise involved because of the impact on the higher frequency antenna of reflection from the lower frequency directors.[17] It is possible that this can be resolved only by using open-sleeve[18] or some other multifrequency element construction. Because the mutual impedances between half-wave and full-wave elements are at a minimum, it is possible that duoband antennas with a 2:1 frequency ratio may prove more tractable.

A last problem of interest is the question of the interaction of a tower structure with a side-mounted antenna. Side mounting is often used in stacked antenna setups, as are rotating tower masts. But there is a question about the real impact of the transverse dimension of the tower on adjacent Yagi elements. This numerical analysis may shed some light on some as yet unexplained disappointments.

Sec 10.7 A Broadband Matching Suggestion

Yagi antennas are limited in bandwidth, even compared to a thin dipole. The question of increasing the SWR bandwidth by some form of compensating matching network has attracted some interest in the amateur community.[19] In order to have any compensating effect, a matching network must exhibit bandwidth comparable to that of the antenna itself, and it must exhibit impedance levels that result in substantial reactive currents across the feed point.

As we did with dipoles in Chapter 8, we can model the feed-point impedance of a Yagi as a transmission line of characteristic impedance Z_0 plus a series resistance R equal to the real part of the feed-point impedance. We chose Z_0 such that the reactance variation with frequency of the Yagi feed point is the same as the reactance variation of the transmission-line model. We can achieve broadbanding if we place another resonant circuit or shorted quarter-wave feed line across the input terminals, so long as the characteristic impedance Z_0' of the compensating resonator is of the order of $Z_0' = R^2/Z_0$. For an example, where the Yagi input impedance is equivalent to R = 25 Ω and Z_0 = 700 Ω, we require $Z_0' = R^2/Z_0 = 25^2/700 \approx 0.9$ Ω!

This matching technique results in substantial circulating reactive currents, and hence is likely to be quite lossy. We can achieve some compensation with values of Z_0' in the range of 3 to 5 Ω, but we still have mechanical problems as well as very high currents. When I tried this approach on a 75-meter dipole it worked, but the circulating currents were high enough to melt the electrical connections.

It is possible to achieve the effect of low-impedance line by tapping well down onto a quarter-wave resonator (shorted ¼-λ line section) placed along the boom of a Yagi antenna. The high currents and voltages would seem to pose unusual problems. Although I intend to explore this further, it appears that broadbanding the Yagi electrical design itself using computer modeling is far more fruitful. In many cases, simply shortening the director immediately adjacent to the driven element results in much increased bandwidth with minimal penalty in gain and pattern, particularly with five or more elements.

Sec 10.8 Some Comments on Coaxial Cable

After all the effort to maximize the gain of a Yagi antenna, it seems foolish to throw away all the gain through the use of cheap, lossy coax. The electrical performance of coaxial cable is well documented

in the amateur literature.[20] Good-quality cable is worth the investment. Once installed, the cable should be checked periodically for SWR and power loss. This is especially true for the case of the newer flexible cables with partial air dielectric, because they are quite susceptible to water leaking into the air space. Professional communications installations include pressurized air-drying systems to prevent this type of problem in the very expensive microwave cables employed.

One point often overlooked is the requirement for mechanical mounting adequate to withstand the expected wind environment. All cables should be installed so there is no tension on the connectors themselves. A short service loop is a good idea. The cables also cannot be bent sharply.

The best method for retaining flexible coaxial cable at the upper end of a tower is to use a metal web hanger that works like a child's "Chinese finger torture" device. These hangers are available at electrical supply houses for the purpose of suspending drop-cables from ceiling boxes. They cost less than a cable replacement. The cable hangers to use for RG-8 are Pass & Seymour or Slater FC52.

Supporting the remaining cable is no place for hose clamps, which can cut the outer cover. The best bet, although not inexpensive, is to use the same metal net hangers at intermediate points (say, every 50 feet). A satisfactory method is to use the very large, wide plastic tie-wraps, wrapping them three times around the cable to increase the frictional holding power and distribute the force. Vinyl tape is the last choice, and it must achieve holding power by multiple spiral wraps around the cable, not by tight overlapping wraps.

A last point is weatherproofing coaxial cables at connections. A rubber insulating tape exists for sealing the underwater electrical connections at submersible well pumps. Any tape that can insulate line voltage under water should be perfect for coaxial connectors; ask about it at a pump supplier. This tape is then covered with ordinary black vinyl tape, and ultimately by a painted-on layer of insulating coating (3M Scotchkote electrical coating). This approach results in a connection that is absolutely waterproof for the life of the pump, typically many years. I recommend the use of these same materials to weatherproof coaxial-cable connectors.

Sec 10.9 Where is the Power Measured?

An interesting issue is the definition, under Part 97 of the FCC regulations, of where the transmitter power is measured. The question is whether the lossy feed line is part of the amplifier or part of the antenna. It seems reasonable to assume that the rationale underlying the regulations has to do with the interference potential of the power that will ultimately be radiated. Accordingly, the feed line and its loss would seem to be properly considered as part of the amplifier.

I posed this question informally to several members of the FCC staff. I think the answers I got mean that the power is measured at the antenna, as it is, for example, at AM broadcast stations. It appears that if the rules are silent on a subject, the FCC does not quarrel with the licensee's reasonable interpretation of them.

In order to keep track of transmitter output power and coax condition, I would enjoy a device that could measure the transmitted and reflected power at the antenna feed point and telemeter back the results diplexed over the same coax for monitoring in the shack. Bird offers an HF directional coupler that could form the basis of such an instrument.

Sec 10.10 Some Contest Tips

In addition to preparing your antennas and station for complete reliability and operating convenience, two additional points may prove helpful. First, it seems that bad weather and contests go together. During periods of precipitation, charged rain or snow particles hitting the antenna elements result in a substantial noise level caused by precipitation static. This static can be avoided by using a radome surrounding the receiving antenna. I accomplish the same thing with an indoor antenna during rain-static periods.

The second point is more far-reaching. When I was racing a car, I could not help noticing that last-minute pre-race changes (demon tweaks) always seemed to have unexpected and undesirable side effects. If you have ever watched the Indianapolis 500 race and wondered why so many million-dollar race cars could fail in the first lap, you've witnessed this phenomenon. It also happened to many of my racing competitors, but, after enough bad experiences of my own, I avoided it by a rule I swear by to this day: "Don't change spark-plugs between practice and the race!"

This does not mean you must not fix broken equipment. It means you must not attempt last-minute improvements that are not completely tested before the prime time of the contest. In relation to Amateur Radio contests, it means that each element of a contest station must be completed while there is still time to recover from any unexpected problems or side effects, tested in place, and then left alone, well before the start of the contest. This rules against last-minute attempts to improve antennas or equipment, and also means the widespread practice of installing a new antenna or bringing in a new transceiver at the last minute is a very risky practice. If something fails, it is likely to be the last thing you touched.

Sec 10.11 Some Suggestions for Antenna Manufacturers

There are a number of respected manufacturers of HF Yagi antennas. After learning the benefits of structural analysis to Yagi antenna survival, I would offer to a manufacturer the following suggestions:

- Specify antenna survival more clearly in terms of EIA RS-222 zones, and offer antennas that meet at least Zone A requirements.
- Obtain certification of mechanical design by a registered engineer, and provide copies of the certification with each antenna.
- Design and offer reinforcement and guying kits for users whose environmental requirements dictate a stronger antenna (and provide certification of these, also).
- Design or compensate antennas for wind-torque balance.
- Do not make the integrity of electrical connections dependent upon mechanical properties of plastic insulators or compressible tubing shapes.
- Provide guidance about stacking, mast strength, rotators and installation.
- Incorporate the best of current computer-generated pattern results and back this up with meaningful range measurements (provide a copy with the antenna).

These suggestions are offered with the understanding that these steps must make business sense, and that a product should generally be aimed at the center of the market, not at the extremes. Most commercially made Yagi antennas provide good value for the price. There is no magic to reduce the price of an antenna below the price of its weight in aluminum—antennas cost by the pound.

However, I for one would pay more for an antenna that I knew was going to last through the wind and ice conditions I expect at my particular site. I believe that installations rated for less than EIA Zone A conditions are doomed to premature failure anywhere in the world. The steps above may constitute meaningful differentiation for a manufacturer, or they may become the minimum demanded by knowledgeable consumers. The amateur antenna customer should have a chance to make the choice armed with real information and knowledge.

Sec 10.12 Installation and Safety

It is assumed that the reader intends to mount the antennas studied in this book on a tower structure to obtain suitable height. The subject of towers is outside the scope of this book. The reader is earnestly referred to the catalogs of suppliers whose towers are specified under EIA RS-222-C.[21] As we have seen from wind statistics, the 50-mi/h ratings normally provided are not adequate to ensure survival in even the most benign wind locations. The tower must be strong enough to handle the moments and forces that result from our efforts to design survivable antennas and masts.[22]

Don't assume you know everything about safe installation. Read everything you can get your hands on about towers.[23] Finally, weatherproof electrical and mechanical connections, be sure to provide lightning grounding, and inspect your installation on a regular basis.[24]

Among various hints and kinks, do install a standard automotive grease fitting in the top sleeve tube section of a tower, before installation. In this way, drilling, tapping and inserting to an inside flush position can be accomplished in relative comfort. For a real thrill, try handling a greased 20-ft steel mast atop a 140-ft tower!

Galvanized surfaces should be inspected reasonably often. Any rust spots should be treated to stop the spread of corrosion, with one of the cold-galvanizing sprays now on the market. Note that these sprays have limited shelf life after first use, so plan your work to use them up.

An anti-icing paint is available for commercial antenna installations. I am sorry I cannot give a reference for a source, but this might be an interesting approach for areas where icing is an unusually severe problem.

As a practical issue, I have found that balancing the weight of the antenna about the mast-attachment point makes installation much easier. I use an appropriate length of PVC pipe inserted typically inside the

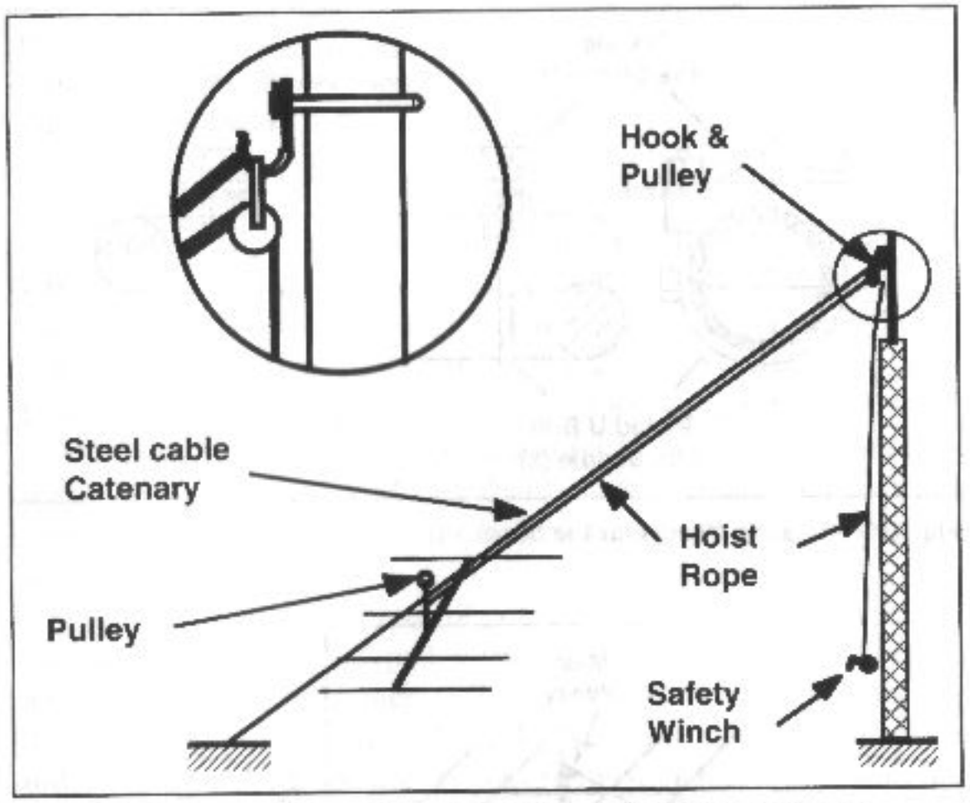


Fig 10-8—Catenary safety method of installing large HF Yagi antennas.

director end of the boom, held in place by the element-locating bolt through the boom. For shorter boom antennas, it is often convenient to locate the outermost elements with ¼-in. diameter forged eye-bolts, which are also a convenient attachment point for boom guying. (I have found that any method other than a bolted guy attachment will ultimately slip in a storm.)

For safe installation, I always include a forged or welded eye-bolt (cheap eye-bolts are "widow-makers") at the boom center. As shown in Figs 10-8 and 10-9, we use a self-braking winch at the mast base for one person to pull up the antenna. The antenna hangs from an upside-down pulley on a temporary wire-rope catenary, stretched from a firm ground attachment to a point on the mast above the mounting location. I learned this technique from Mike Staal, K6MYC, who used this approach with complete safety on his 7- and 3.5-MHz Yagis.

In order to preclude damage in really serious wind storms, I use a rope on the end of the boom to tie off the antennas against rotation. Figs 10-10 and 10-11 show how to pull that rope out of the way of

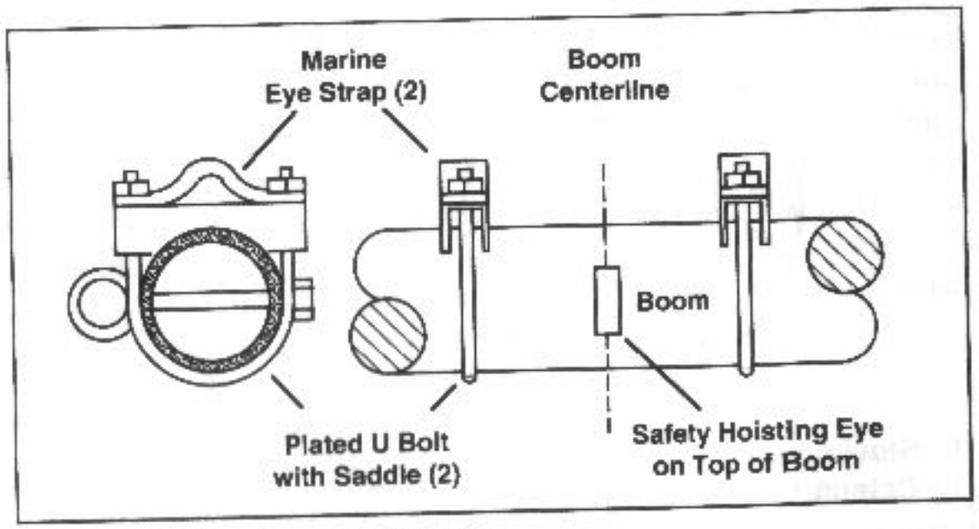


Fig 10-9—Hoisting clamps for the boom.

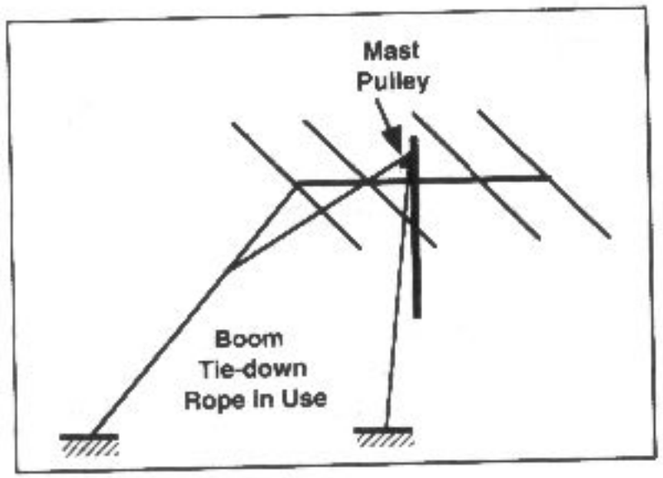


Fig 10-10—Wind safety tie-down for large HF Yagi antennas.

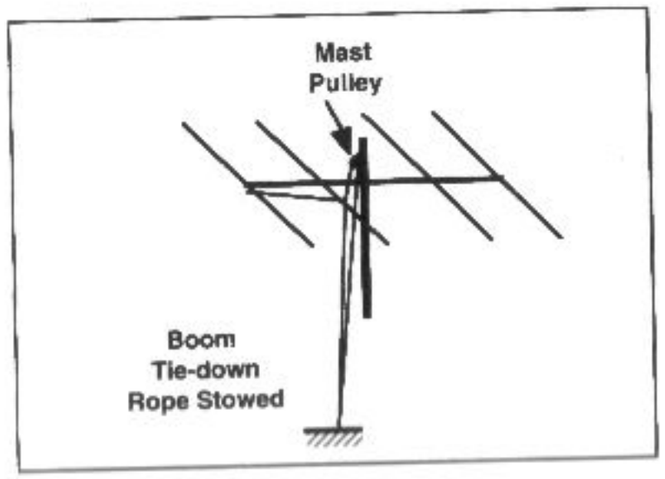


Fig 10-11—Stowing the tie-down rope to permit rotation.

rotation when it is not in use. If this method is not followed, the wind typically wraps the rope around the antenna and tower before the mischief is noticed.

Sec 10.13 Concrete Basics for Tower Installation

I have observed that many amateurs use concrete in tower installations without having an adequate knowledge of concrete and reinforcement. The simple drawings in tower catalogs are a good place to start, but a little more information can be helpful. Typical tower specifications call for concrete that can provide 3000 lb ultimate compression strength after a 28-day drying time, and for ½-in. steel reinforcing bar (rebar) meeting ASTM A-615 Grade 40 standards. Concrete consists of portland cement, fine aggregates such as sand, course aggregates such as gravel, and water.

The two principal types of concrete useful for tower bases and guy anchors are ASTM Type I, normal portland cement, and Type IA, air-entraining portland cement that provides extra resistance to freezing damage. Cement is sold in 94-lb sacks. Whether you mix your own or order it ready-mixed, you need to convert the strength requirements into how many sacks of cement, how much sand and gravel, and how many gallons of water per cubic yard.

The ultimate strength of concrete decreases as the water used in the mix increases. Mixes with less than 6½ gallons of water per sack of cement will provide the required 3000 lb/in.^{2} strength. A typical mix per cubic yard of concrete might include 5½ sacks of cement (5 sacks is the absolute minimum for this service), 36 gallons of water, 1730 lb of coarse aggregate (maximum size ¾ in.), and 1250 lb of average-size sand.[25]

For some sites, the concrete can be delivered directly to the excavated holes. For others, pumping is advisable, so the mix must be more workable. Concrete consistency is measured in the field by the slump test. The concrete sample is tamped into a conical metal form 12 in. high, 4 in. diameter at the top, and 8 in. diameter at the bottom. The number of inches of slump following removal of the form is the measure of the consistency of the concrete. Slump may not exceed 6 in. It is typically in the 3- to 4-in. range, and is affected by the ratio of water and the other concrete components.

Concrete develops its full strength over a 28-day curing period, and the ultimate strength is greatest if the concrete is moist-cured by

maintaining a wet covering for the entire time. Air drying for the entire time can reduce the ultimate strength by as much as 50%. For tower installations, backfilling immediately after the curing begins can provide the required moist cover.

The rebar typically specified for tower-base service provides 40000 lb/in.^{2} yield strength. Rebar of ½-in. diameter is termed no. 4, and the number is stamped on the rebar, along with letters indicating the mill and type of steel used. Higher strength rebar also carries a stamp indicating the yield strength ("60" for 60000 lb/in.^{2}, and "75" for 75000 lb/in.^{2}). The diameter of the inner edge of a 90° bend must be at least seven times the rebar thickness. Typically the rebar must be covered by at least 2 in. of concrete to prevent exposure to moisture, and spacing is as specified in the manufacturer's tower-base or guy-anchor drawing.

References

[1] G. Hall, *Antenna Height and Communications Effectiveness* (Newington: ARRL, 1987); private distribution, now out of print.

[2] F. E. Terman, *Electronic and Radio Engineering* (New York: McGraw-Hill, 1955), p 809.

[3] G. Hagn, "HF Receiving Antenna Directivity Patterns and Gain for Ionospheric Propagation Model Predictions for Short-Wave Broadcasting," *IEEE Trans on Broadcasting*, Jun 1988, p 221.

[4] N. M. Maslin, *HF Communications* (New York: Plenum Press, 1987), p 106.

[5] S. Baily, K. Beeke, and R. Manton, "The Performance of H.F. Curtain Antennas with Irregular Foregrounds," *Proc Fourth International Conference on HF Radio Systems and Techniques*, IEE (UK), Apr 1988.

[6] W. Utlaut, "Siting Criteria for HF Communication Centers," *NBS Technical Note 139* (Boulder, CO: National Bureau of Standards, Apr 1962).

[7] See p 45 of reference 4.

[8] J. Breakall, "Computer Modeling of Propagation and its Relation to Antenna Design," Antenna Forum at Dayton HamVention, 1989.

[9] F. Donovan, "How to Prepare Your Antenna Farm for the New Sunspot Peak," Antenna Forum at Dayton HamVention, 1989.

[10] L. A. Moxon, *HF Antennas for All Locations* (Potters Bar, Herts: RSGB, 1982), p 132.

[11] P. D. Rockwell, "Station Design for DX, Part I—Antenna Topics and Siting," *QST*, Sep 1966, pp 50-54.

[12] S. Ford, ed., *The ARRL Operating Manual* (Newington: ARRL, 1991), p 2-17.

[13] J. D. Kraus, *Antennas*, 2nd ed. (New York: McGraw-Hill Book Co, 1988), p 716.

[14] See p 426 of reference 13.

[15] See reference 9.

[16] A. Clement, "LARAE—Line Array of Rotary Antennas in Echelon," in G. Hall, ed, *The ARRL Antenna Compendium, Volume 1* (Newington: ARRL, 1985), p 72.

[17] D. Rodman, "A High Performance Duoband Yagi for 15 and 20 Meters," *Ham Radio*, Aug 1989, p 41.

[18] R. A. Cox, "The Open-Sleeve Antenna," in G. Hall, ed, *The ARRL Antenna Book*, 16th Ed (Newington: ARRL, 1991), p 7-4ff.

[19] See reference 18, p 9-5.

[20] S. Katz, "Selecting the Right Transmission Line," in P. O'Dell, *CQ Antenna Buyer's Guide*, 1989, p 68.

[21] Standard RS-222-C, *Structural Standards for Steel Antenna Towers and Antenna Supporting Structures* (Washington, DC: Electronic Industries Association, 1976.

[22] R. Cox, "Match Your Antenna to Your Tower," *Ham Radio*, Jun 1984, p 14.

[23] J. Haerle, "Tower Installation: Make it Sturdy, Make it Safe," *Ham Radio*, Jun 1984, p 22.

[24] S. Makulec, "Tower Maintenance: Keep Your Tower Up," *Ham Radio*, Jun 1984, p 26.

[25] T. W. Love, *Construction Manual: Concrete and Formwork*, 4th ed. (Solana Beach, CA: Craftsman Book Co, 1981), p 22.

Chapter 11

Yagi Antenna Examples

Sec 11.1 Applying the Techniques of This Book

In order to show by example the application of the techniques of this book, I thought it would be helpful to review the steps I have taken to design antenna installations currently in use at my own location. The active interest of my wife Barbara, KK6QM, makes it possible to consider a site and antenna installations that might be somewhat extreme for many radio amateurs, but that is why I developed the techniques presented here!

Let's review how the various chapter elements are used in determining the survival speed of Yagi antennas.

1) We use the standards information of Chapter 2 to determine the expected design wind velocity at a given installation.

2) For a selected antenna, we interpret element strength using the element spreadsheet concept developed in Chapter 4, with clamp strength integrated into the calculation by the method of Chapter 5. In general, tip sections will have a relatively larger effect on ice survival, so any design changes focus on balancing the wind and the ice-loading performance by calculating both.

3) The boom strength and torque balance are determined with the boom spreadsheet developed in Chapter 6. I've expanded the spreadsheet to consider the effect on boom torque balance of asymmetrical boom lengths, the coaxial feed line, the balun, phasing lines, and larger diameter plastic pipe sleeves over the outer boom sections. If the coax is run below the boom so that it increases the boom wind area, the effect is much greater than I had thought. This can be a problem (see Sec 11.3.1), but it can also be put to good use as we'll see in Sec 11.3.8.

4) It is convenient to strengthen and torque balance an existing Yagi antenna without changing the electrical design. If we are starting

with a monotaper (cylindrical element) computer-generated electrical design, or if we need new, stronger taper dimensions electrically equivalent to an existing antenna, we incorporate into the element spreadsheet the taper algorithm of Chapter 8 and the clamp electrical equivalent diameter calculation of Chapter 9.

5) The driven element shortening for the reactance (typically 25 to 35 Ω capacitive) required for a beta match can be calculated directly from the taper spreadsheet of Chapter 8. The design of the matching system proceeds using the equations of Chapter 9.

6) The resulting antenna is mounted on a mast whose survival strength is determined by the mast spreadsheet developed in Chapter 7.

7) Choosing a site for optimum radiation angle is summarized in Chapter 10.

The element design information in this chapter was generated by using a combined element spreadsheet that incorporates the element strength, taper algorithm, and clamp electrical and mechanical parameters. From the input of element and clamp dimensions, this spreadsheet derives element and clamp survival speed as well as equivalent cylindrical dimensions and reactance at a given frequency. Equivalent alternate taper schedules are defined as those which result in the same element reactance at the antenna design frequency.

The element dimensions and locations are then incorporated along with boom-section dimensions into the boom spreadsheet to determine torque balance and boom survival speed. Boom design improvements and guying can be evaluated with this spreadsheet. The boom spreadsheet also calculates antenna equivalent area for use in the mast spreadsheet, which determines mast survival speed from the area and location parameters of single or stacked antennas.

Sec 11.2 Calculating Expected Wind Conditions

We currently have three towers with Yagi antennas that will serve as examples. One is a 70-ft telescoping Tri-ex Skyneedle with stacked monobanders for 10, 15 and 20 meters. The second is a 120-ft guyed Rohn 45G tower with stacked 40- and 20-meter monobanders. The third is a 40-ft Rohn 25G with a large tribander. I've used the techniques of this book to assist in the physical design of 10- and 15-meter monobanders on 45-ft booms, which are now sited on separate new towers at the top edge of a long slope favoring Japan, Europe, Africa, the Caribbean

and the US. Later we'll add rotating top sections and upgrade the tribander tower.

The site is at the top of a 3400-ft hill, which slopes down at approximately 12° to a foreground some 2800 feet below. Because of high wind conditions, surface forestation is minimal so this site would be considered open country land. The location is approximately 10 miles inland from Monterey Bay and the Pacific Ocean, so storm winds come right off an extended fetch of water, even though the RS-222-C zone map indicates Zone A (86.6 mi/h or 30 lb/ft^2).

Using the methods of Chapter 2, let's calculate the expected fastest mile of wind at heights of 70 ft and 130 ft for this site. We'll use the BSI topography and ground roughness factors to multiply the EIA RS-222-C wind speed for this area. The slope is

$$\psi = \tan 12° = 0.21$$

and the topography factor is

$$S_1 = 1 + 1.2\psi = 1.26$$

Thus the expected basic wind speed at a height of 33 ft is 1.26 × 86.6 = 109 mi/h. As shown in Chapter 2, if we use EIA-222-D we would calculate a different wind velocity, but would end up with about the same force in a given location.

The ground roughness and height factors, from Table 2-5, are 1.06 for 70 ft and 1.12 for 130 ft. Thus we can expect fastest-mile wind speeds of 1.06 × 109 = 115 mi/h at 70 ft and 1.12 × 109 = 122 mi/h at 130 ft. These correspond to wind pressures of 53 lb/ft^2 and 59 lb/ft^2, respectively. The 70-ft mast is located near the house, and seems to experience additional gusting resulting from the presence of the structure.

Other local data includes destruction of antennas rated at 80 mi/h, damage to antennas rated at 100 mi/h, readings in excess of 100 mi/h on our own wind gauge, and reported readings of 118 knots or 123 mi/h on a weather-service anemometer on another mountain peak about 1 mile away. The model seems to correlate well with observed data, and I have concluded that a wind speed of 122.5 mi/h or 60 lb/ft^2 should be used as the design standard for our site. You will want to develop your own estimate of the strongest winds expected in the worst storms at your location.

Sec 11.3 Antenna Examples

Now that we have been through the theory of the mechanical design of elements, booms and masts, and through the electrical modeling of tapered elements, we are ready to combine that information into the physical design of specific examples of Yagi antennas. First we look at the task of evaluating and perhaps mechanically improving existing antennas without making any electrical changes. Of interest to many is the expected mechanical performance of a popular 7-MHz Yagi with shortened elements, the Cushcraft 40-2CD. Because of the complexity of electrical changes, we will restrict our strengthening to element internal reinforcement or guying. We will explore the possibility of a stronger boom to complement element improvements.

As additional examples, we will look at expected mechanical performance and structural improvements (without electrical changes) for the Hy-Gain TH5 and TH7 triband antennas. Next, we will explore mechanical performance and improvements for the popular Hy-Gain 105BA, 155BA, 204BA and 205BA monoband Yagi antennas, including electrical redesign of the elements to accommodate increased mechanical requirements. Analysis of these antennas indicates that all but the 155BA meet the manufacturers' advertised mechanical performance (typically EIA RS-222-C Zone B), but that they can also be strengthened to survive more rigorous wind and ice exposure. Although there are a number of respected amateur antenna manufacturers, I have had good success with the Hy-Gain antennas because of the manufacturer's apparent attention to mechanical issues, and I preferred to create these examples from direct experience.

Last, we will explore how to realize a computer-generated design in a robust physical construction. For this purpose, we will rely on published designs for 4- and 5-element monobanders for the 14, 21 and 28-MHz bands. In some cases, exploration of the impact of accurate clamp and taper modeling has revealed some electrical improvements, particularly in the area of SWR bandwidth. The result of these efforts is a group of high-performance Yagi antennas that promise survival under extreme environmental stress.

Ultimately, one would hope to incorporate into electrical design programs such as YO and MN the improved accuracy offered by the Jaggard equivalent diameter and Schelkunoff taper algorithms of Chapters 8 and 9. It would also be desirable to include the site wind speed,

element, boom and mast mechanical spreadsheets in commercially available Yagi design software.

Sec 11.3.1 Cushcraft 40-2CD

The Cushcraft 40-2CD is a widely used 2-element 40-meter Yagi with shortened elements. The use of capacitive end loading promotes a higher radiation resistance and hence wider bandwidth than similar antennas that rely only on inductive loading.[1] The manufacturer rates the antenna for "a steady 80-mi/h wind." Calculations show that the elements and boom can easily be strengthened.

Giving account to the area of the insulating tube for the driven element, the element torque balance seems excellent. The boom guying struts do not provide any side support against wind force, so their use is primarily to remove vertical deflection.

The weaker points of the antenna are the rather long single-thickness element inner sections, the boom center joint, and the driven-element mounting plate. The boom has a junction right at the center between the two U bolts. The moment here is the same as immediately outside the U bolts, and the insert is the only support. The boom center insert is only 18 in. of a single thickness of 2-in. tubing with 0.058-in. wall thickness.

The boom survival speed is calculated to be 86 mi/h, with the center joint confirmed as the weakest point. Boom survival speed can be increased to 101 mi/h by replacing the 2 × 18-in. center joint with a 2 × 36-in. length of 6063-T832 tubing with the same 0.058-in. wall thickness, plus an additional 1.875 × 0.058 × 12-in. center inner reinforcement of the same material. If instead we use a 2 × 0.058 × 84-in. center reinforcement along with the same additional center inner

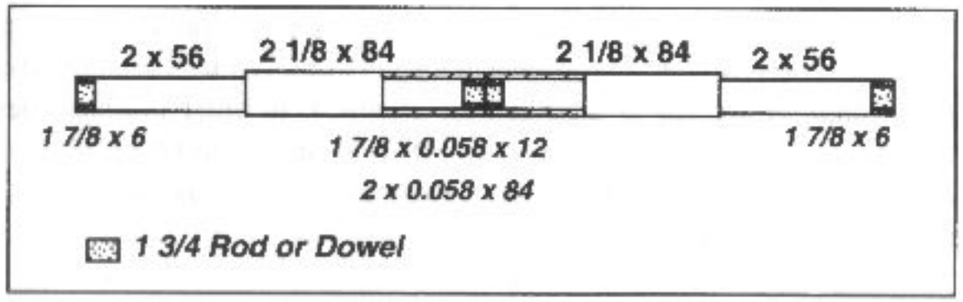


Fig 11-1—Reinforcing the Cushcraft 40-2CD boom.

reinforcement piece, we have a boom v_{max} = 117 mi/h. The method of reinforcement is detailed in Fig 11-1.

The inner center tube is further reinforced by a solid 1.75 × 12-in. wood dowel or fiberglass rod, which prevents crushing of the boom by the U bolts used to mount it. The stock boom tips are reinforced with 1.875 × 0.058-in.-wall tubing, which should also be reinforced with solid dowel inside to prevent crushing by the element-mounting U bolts.

Each of the two boom-section joints should be fastened with two ¼ × 2½-in. stainless bolts at right angles to each other, spaced 1½ in. along the boom. The holes provided by the manufacturer for smaller bolts can be drilled out to ¼ in. to make a light drive fit, and a drill guide can be made using 1½-in.-wide paper tape with lines drawn across it spaced πD/4 = 1.67 in. for the 2⅛-in. boom sections. (See Fig 6-24.) Use stainless elastic stop nuts. I also bolted the element-boom plates to the boom and bolted the boom to the boom-mast plate, which I redrilled to mount to the Hy-Gain 102734 cast mast clamp, as shown in Fig 11-2. Because the ¼-in.-thick boom-mast plate deforms if the U bolts are tightened snugly, it would be helpful to make a thicker plate with the same hole pattern.

The boom center joint also requires two ¼ × 2½-in. stainless bolts. The mast plate is attached to the boom by using two of the U bolts supplied, but without the V blocks supplied. I also drilled the plate and boom for two 5/16 × 3-in. through-bolts with elastic stop nuts. Last, I also drilled one more hole centered in the top of the plate to hoist the antenna and to measure the weight balance during construction. Used on a 2-in. mast, the Hy-Gain clamp requires four 5/16 × 4½-in. bolts and two 5/16 × 2½-in. bolts. Because of the stress here, I use Grade 5 plated bolts with elastic stop nuts in preference to the nuts and bolts supplied by Hy-Gain, which are threaded over their full length and are longer than convenient for a 2-in. mast.

The straps for the support tubes (which are not really necessary after the strengthening) are held in place with two rivets. The support-tube joints are riveted as well as hose-clamped. In order to place the antenna at the very top of the mast, I attached the support tubes to the mast below, rather than above the boom, as the boom is now strong enough not to need them.

The element plate for the driven element is increased in thickness, either by obtaining a second one from the manufacturer or by drilling out a ⅛-in. (Zone B) or ¼-in. (to exceed Zone C) plate to double with

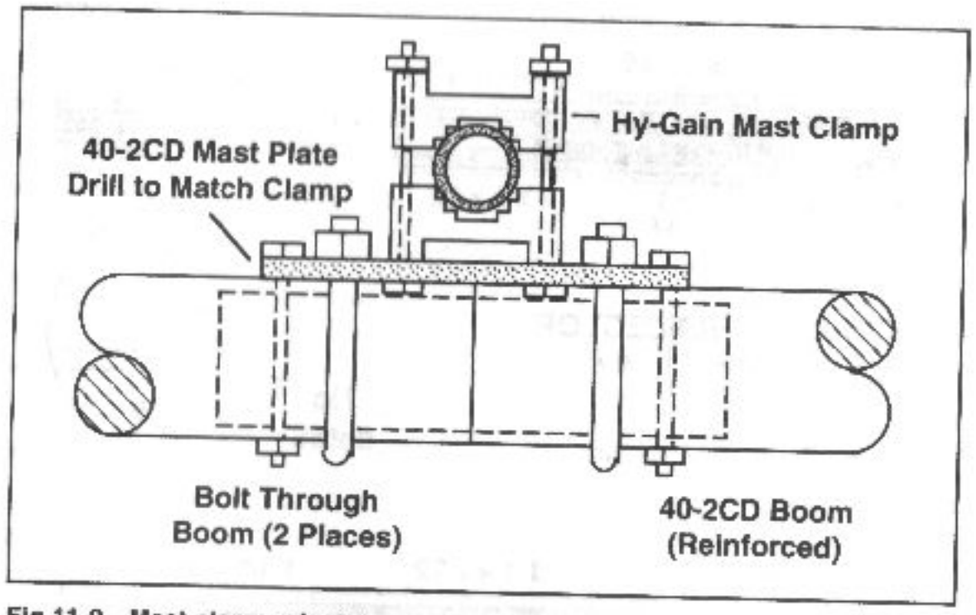


Fig 11-2—Mast-clamp adapter.

the original. Using a dummy mast, I mount the boom and use bubble levels to get the element plates horizontal. After mounting with the supplied U bolts, each plate is drilled for two ¼-in. stainless bolts to go through the boom. The boom is drilled using the plate as a template, first on top and then on the bottom. The ¼-in. bolts are centered 0.781 in. in from each edge of the plate, and the bolt heads are at the plate end under the elements. The reflector requires two ¼ × 2½-in. stainless bolts and stop nuts, and the driven element requires two ¼ × 3-in. stainless bolts and stop nuts. Last, the inner boom reinforcement is riveted to the outer section, using two ⅛-in. stainless rivets on each side, so the center reinforcing sections can contribute to resisting rotation of the through-bolts.

The elements are reinforced substantially, using the dimensions shown in Figs 11-3 and 11-4. Each joint is held with the supplied all-stainless hose clamps and with at least one ⅛-in. stainless rivet. The rivet holes can be filled with silicone caulk for weatherproofing. I found it easiest to assemble the antenna without the loading coil and hat tip sections, which are easy to bend or scar. Until the antenna is being raised, the loading coils should be kept separate and protected with shop rags to prevent tearing the heat-shrink tubing, which would result in water problems. I am uncertain about the strength of the loading-coil

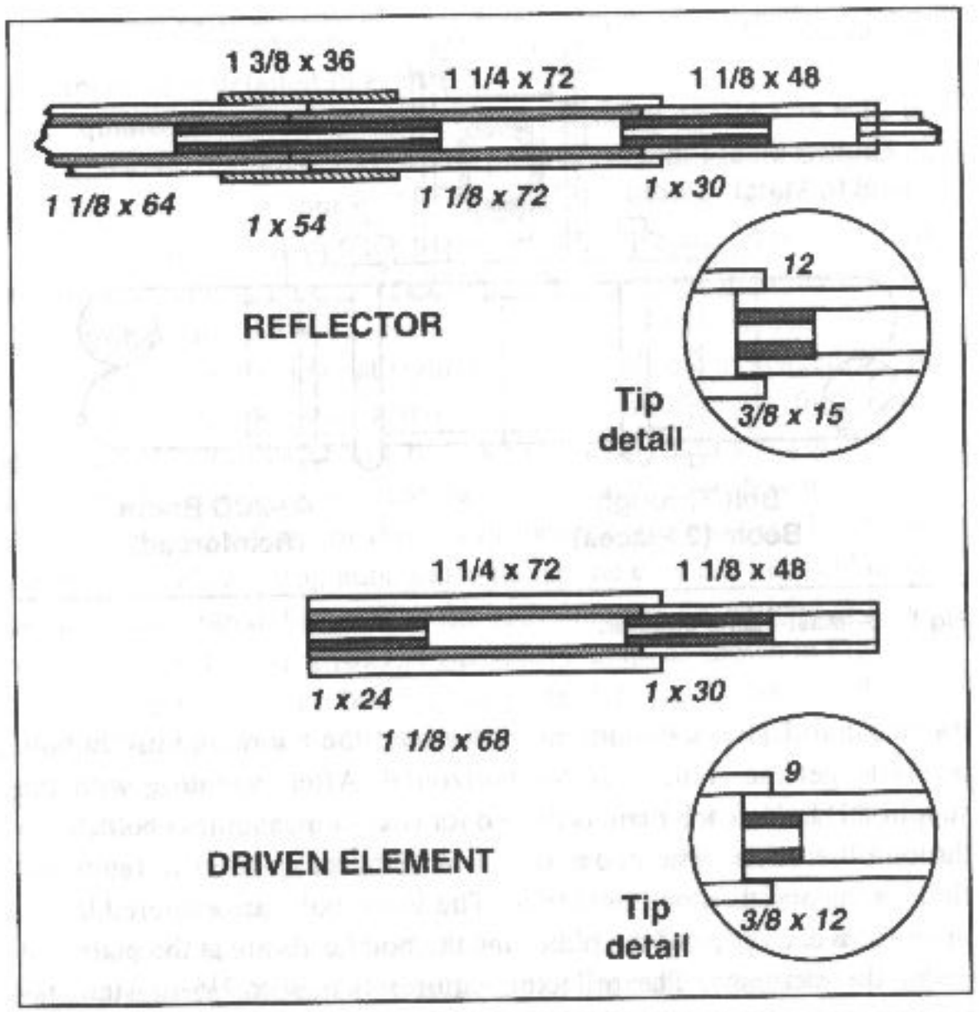


Fig 11-3—Detail of Cushcraft 40-2CD element assembly for v_{max} = 100 mi/h.

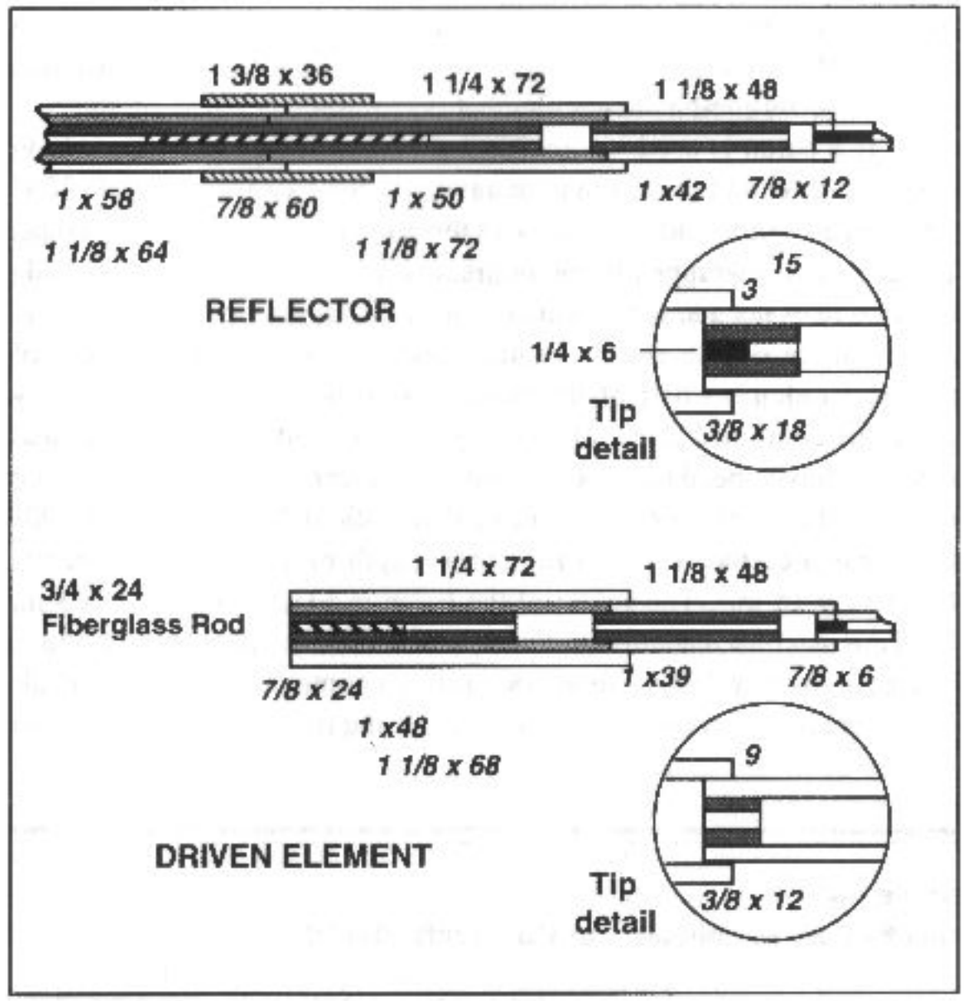


Fig 11-4—Detail of Cushcraft 40-2CD element assembly for v_{max} = 118 mi/h.

sections, but as with tribanders, they are quite far out and away from the concentrated moment region of the center sections.

At assembly, insert into the center insulator a ¾ × 24-in. fiberglass rod, half into each side of the driven-element center sections. Spacing between the driven-element halves at the center inside insulator is 2 inches. Note the offset for overlap at the reflector center joint between the 1⅛ and 1-in. tubes inside.

I replaced the plated terminal loading-coil self-tapping screws with 10-32 × 1¼-in. stainless machine screws in holes tapped through the aluminum tube, secured with safety nuts. To do this, it was necessary to cut a small circular flap in the heat-shrink tubing that covers the loading coils. When I had replaced the terminal screws, I used large-diameter heat-shrink tubing to cover the openings and to double the covering over the coils. I was not able to measure any change in inductance (12.93 µH), Q (161), or series resistance (3.26 Ω) because of the additional heat-shrink tube, and I am confident now that the connections will not loosen or corrode.

The driven element is held in its 2-in. insulating section by drilling the bolt holes to take ¼-in. stainless bolts and nuts. The electrical connection is made immediately outboard of the insulating section with machine screws tapped into the element wall, or a clamp made from a 1-in. length of externally telescoped aluminum tubing, slit lengthwise with a raised dimple for a screw and retained with two hose clamps. Dick Weber, K5IU, suggests that large-diameter hose clamps

around each end of the insulator will strengthen it against tearing failures. Because of the inherent symmetry and small size of this antenna, no torque-balancing element is required.

If a balun is used, an addition of similar area could be made to the reflector end of the boom to maintain torque balance. I use a W2DU ferrite bead balun, and attach it with three or four long black tie-wraps. The coax is tie-wrapped to the boom, and is routed around to the side so that it does not add to the wind resistance or upset the torque balance. If instead the coax is run below the boom, it adds to the wind area of the driven-element half of the boom. Even though the boom is very short, the combined effect of RG-8 type coax (0.405 in. diameter) and a W2DU ferrite bead balun at the driven element creates an unbalanced moment of area of 7660 in.^{3} or more than 1400 in.-lb at 100 mi/h. This balun has a diameter of 1¼ in. and a length of 12 in., so the area is 1¼ × 12 = 15 in.^{2}. The center of the balun is 131.5 – 12/2 = 125.5 in. from the mast, so the balun moment of area is 15 × 125.5 = 1882.5 in.^{3}, or approximately 350 in.-lb at 100 mi/h. To compensate for this unbalance, the boom can be extended 6.84 in. at the reflector end (it can't be

Table 11-1
Mechanical Parameters for Cushcraft 40-2CD

	Unmodified	*Reinforced+*	*Reinforced++*
Elements			
v_{max}	71 mi/h	100 mi/h	118 mi/h
v_{ice}	47 mi/h	66 mi/h	70 mi/h
Center joint v_{max}	71 mi/h	104 mi/h	135 mi/h
DE clamp thickness	0.25 in.	0.375 in.	0.5 in.
Boom			
v_{max}	96 mi/h	101 mi/h	117 mi/h
Center joint v_{max}	86 mi/h	117 mi/h	117 mi/h
2 × 0.058-in.-wall center insert	18 in.	36 in.	84 in.
1.875 × 0.058-in.-wall center insert		12 in.	12 in.
Boom extension to compensate balun		6.84 in.	6.84 in.

shortened at the driven-element end), or a dummy balun made of plastic pipe can be mounted at the reflector end of the boom. To compensate for both the balun and the coax, an extension of 26 in. is required at the reflector end of the boom.

Table 11-1 compares the unmodified and reinforced versions of this popular antenna. Although this antenna does not provide the opportunity for further reinforcement without guying the elements or changing the electrical design, these improvements make a big difference in the wind and ice survival of this useful antenna. The higher strength configuration has survived and operated well during this year's winter storms at our site.

Sec 11.3.2 Hy-Gain TH5

The Hy-Gain TH5, TH6 and TH7 triband Yagi antennas have been popular with amateur users for a number of years. As an example of how to model and improve mechanical performance, let us explore the TH5. To begin, Table 11-2 shows the element section diameters and lengths, including the mechanical dimensions of the traps. We will use the element mechanical spreadsheet (as shown in Table 4-3) to determine the wind survival speed v_{max} and the wind survival speed with 0.5 in. of radial ice. The data is from measurements on the antenna.

In order to calculate the element areas and clamp strength, Table 11-3 lists the element-to-boom clamp dimensions. The Hy-Gain clamps have the advantage of spreading force over a wide area of the boom, and so will not crush the boom. However, they will eventually rotate out of alignment unless bolted through the boom.

Table 11-2
TH5Mk2 Element Taper Schedule

Values are in inches.

Section	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Diameter	1.25	1.125	1.63	1	1.63	0.875	0.625	0.4375
Reflector	83	48	0	1	12	8		35
Reflector 2						55	24	29
Rear Driven	48.5	33	12	9	12	8		29.5
Front Driven	48.5	26	12	7	12	8		25
Director	48	27	12	7	12	8		24.75

Table 11-3
TH5 Clamp Dimensions

Values are in inches.

	Length	*Diameter*
Reflector	2.625	1.5
Reflector 2	2.5	1.045
Rear driven	3.75	1.75
Front driven	3.75	1.75
Director	2.625	1.5

Table 11-4
TH5 Element Survival Speeds and Centroid of Moments

	v_{max}, *mi/h*	v_{ice}, *mi/h*	v_{clamp}, *mi/h*
Reflector	100	69	109
Reflector 2	124	76	140
Rear Driven	102	72	156
Front Driven	111	78	169
Director	110	78	143
Totals	100	69	109

	Spacing, in.	*Station, In.*	*Location, in.*	*Area, in.*^{2}	*Moment, in.*^{3}
Reflector	–	0	–109.25	403	–43979
Reflector 2	36	36	–73.25	153	–11169
Rear Driven	53.25	89.25	–20	335	–6706
Front Driven	55	144.25	35	312	10907
Director	68	212.25	103	310	31926
Totals		Centroid:	–12.6	1512	–19022

From these dimensions we can calculate the survival speed of each element separately (using $\sigma_{max} = 35000$ lb/in.^{2}), and can determine the area of each element so that the element moments and their centroid can be calculated. The element clamps are made from 3003-H14 alumi-

Table 11-5
TH5 Boom Dimensions and Survival Speed

Section	*1*	*2*	*Total/half*	*Total*
Diameter, in.	2	2		
Thickness, in.	0.1	0.049		
Length, in.	41	71	112.75*	225.5
Weight, lb	2.44	2.46	4.9	9.8
Area, in.^{2}	83.5	142	225.5	451
ΣArea, in.^{2}	225.5	142		
Moment, in.^{3}	12713	5041		
Section modulus, in.^{3}	0.27	0.17		
v_{max}, mi/h	125	120	120	

* Boom sections are spaced 1.5 in. (0.75 in./side) in the boom center joint.

num, which has a yield strength of $\sigma_{max} = 17000$ lb/in.^{2}. Table 11-4 lists the results.

Next we list the boom dimensions, from which the boom survival speed is calculated section by section. The typical section overlap at the swaged joints is 4 in., which is subtracted from the length of the tip section. Boom swage joints should be designed, as they are in this antenna, so that the inner section inserts the thicker-wall swage into the inner end of the outer section. The boom survival calculation is summarized in Table 11-5.

The boom is symmetrical, so the boom moment centroid is zero. The last step is determining the survival speed of the boom-to-mast clamp, which acts as the joint between the two boom halves. This clamp is made from 5052-H32 aluminum, which has a yield strength of σ_{max} = 22000 lb/in.^{2}. At the worst-case azimuthal angle, the peak boom moment is 35332 in.^{3} We can see from Table 11-6 that the center joint does not limit the boom strength. We summarize the TH5 parameters as in Table 11-7. We can see that this antenna meets the manufacturer's specifications, and should survive EIA RS-222-C Zone B wind speeds.

The effect of the element moment imbalance, evidenced by the nonzero centroid of element moments, can result in rotator stress. Let's see what can be done about this unbalanced mast torque. If we could

Table 11-6
TH5 Boom Joint Strength

	$\theta = 0°$	$\theta = 15°$	$\theta = 30°$
S, in.^{3}	0.5608	0.4935	0.4691
σ_{max}, lb/in.^{2}	22000	22000	22000
M_{max}, in.^{3}	12338	10856	10320
v_{max}, mi/h	137	128	125

move the element ensemble toward the director end by 12.58 in., the element moment would be zero. However, we can move the elements only until the director is 109.25 in. from the mast, which is a net change of 109.25 – 103 = 6.25 in. This leaves us with a torque imbalance corresponding to the total element projected area of 1512 in.^{2} acting at a centroid of –6.33 in. Even this modest step is a substantial reduction in the worst-case rotator torque required, as shown in Fig 11-5.

Table 11-7
TH5 Summary

Elements	
v_{max}	100 mi/h
v_{ice} (0.5 in. ice)	69 mi/h
Clamp v_{max}	109 mi/h
Moment centroid	–12.6 in.
Effective area	7.0 ft^2
Boom	
v_{max}	120 mi/h
Center joint v_{max}	125 mi/h
Moment centroid	0 in.
Effective area	2.1 ft^2
Total effective area	7.3 ft^2

To compensate the mast torque completely for the elements after we have moved them as far as possible toward the director end, we require a compensator moment of area M_c = 9573 in.^{3}. We arbitrarily pick a location 12 in. inboard from the director, so the distance from the mast is L = 109.25 – 12 = 97.25 in. Thus, the compensator area with offset elements must be A = M_c/L = 9573/97.25 = 98.4 in.^{2}, or 49.2 in.^{2} per side.

This compensating element can be made in a number of different ways, so long as the area per side is 49.2 in.^{2}. Perhaps the most convenient way is to use the same Hy-Gain no. 13 clamp as is used for

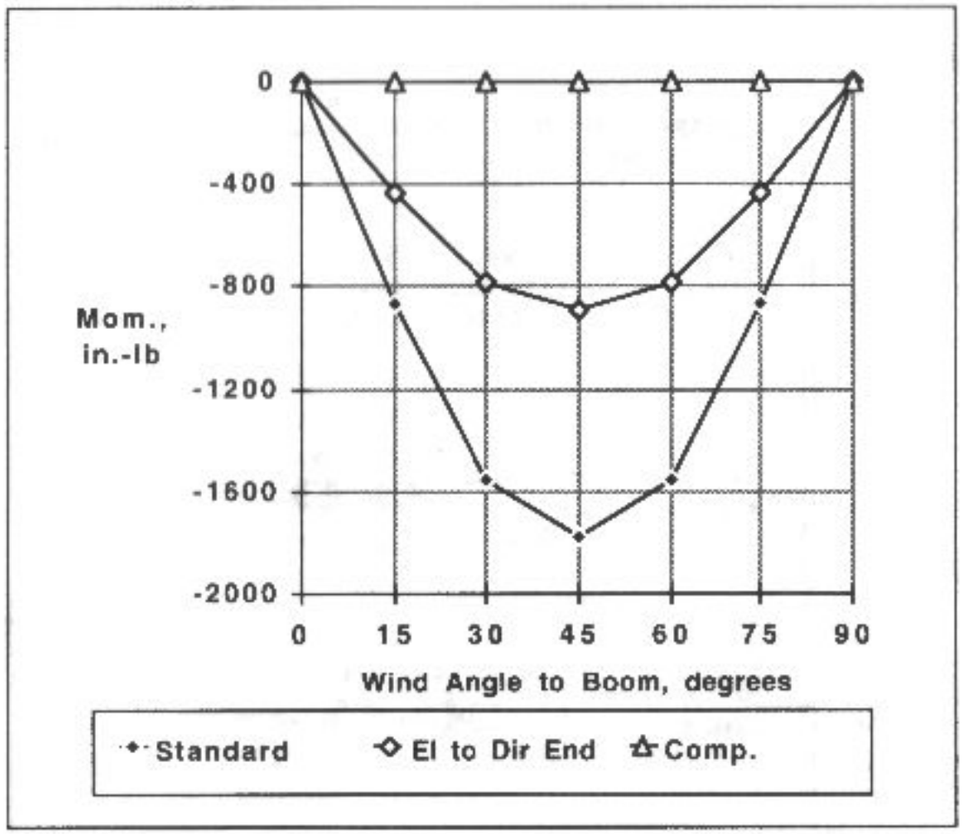


Fig 11-5—Wind angle ϕ versus rotator torque required for the Hy-Gain TH5 at v = 100 mi/h.

element mounting. This clamp will hold a 1.25-in. diameter fiberglass tube. The clamp area is 2.625 × 1.5 = 3.94 in.^{2}, so the exposed length of fiberglass tubing must have an area of 49.2 – 3.9 = 45.3 in.^{2}.The required exposed length is 45.3/1.25 = 36.2 in. per side, and the total length including the portion inside the clamp is 36.2 + 2.6 = 38.8 in. If it proves undesirable to move the elements on the boom, the corresponding total compensator area is 209 in.^{2}, which requires 83 in. per side for a 1.25-in.-diameter compensating element. Different clamping arrangements can be treated in the same way.

In addition to moving the elements and adding the compensating element, we can reinforce the elements as shown in Chapter 4, and can use the stronger driven-element clamp for the reflector to improve its wind survival under conditions of vertical gusting. As an alternative, we can carry the internal reinforcement of the reflector right through new holes drilled in the boom, which will strengthen the smaller element

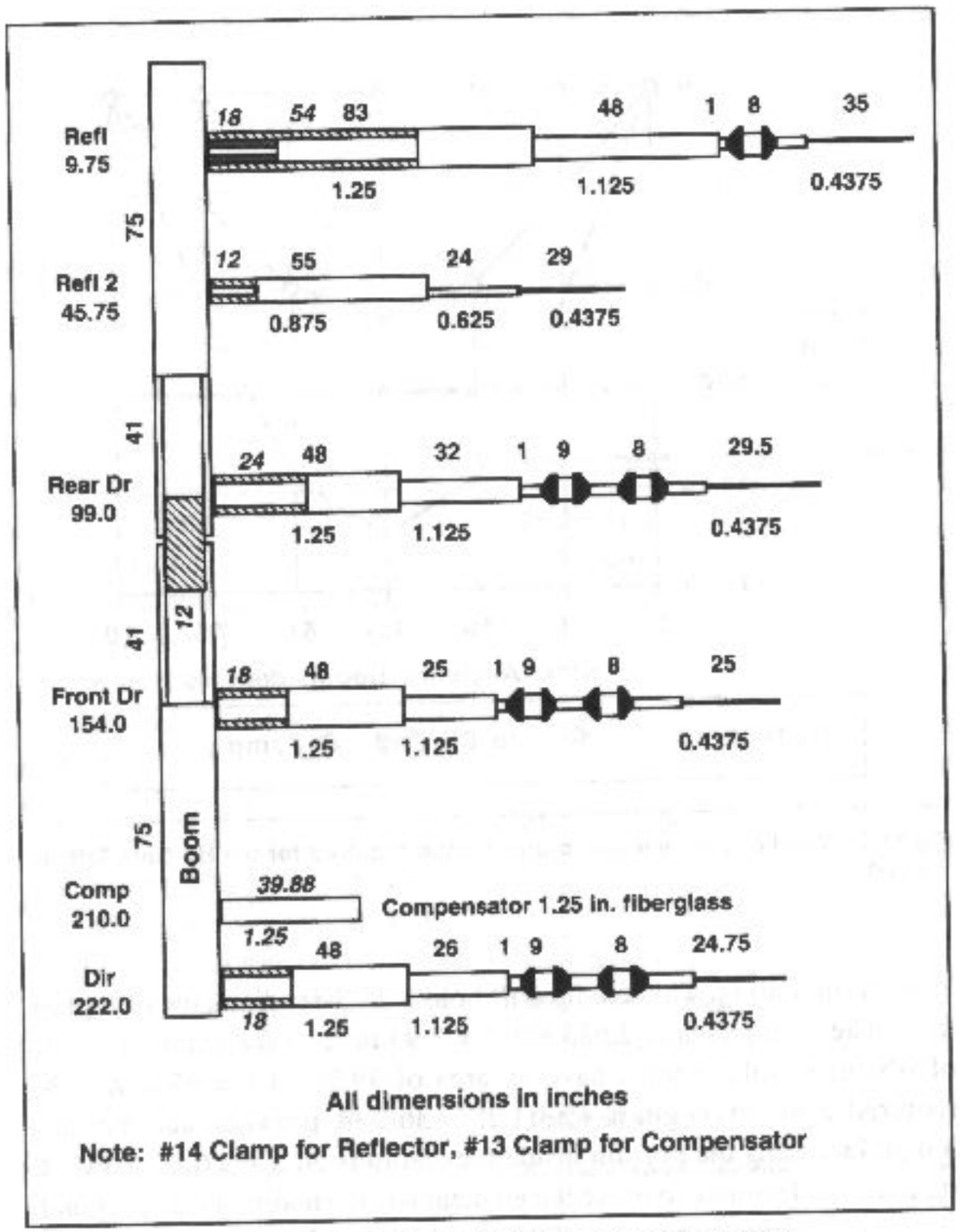


Fig 11-6—Reinforced, torque-balanced Hy-Gain TH5 dimensions.

clamp if we desire to retain it. Incidently, the reflector clamp is also the weakest link in Hy-Gain's shortened 40-meter Yagi, which can benefit from the same treatment. The resulting element locations and reinforcement are shown in Fig 11-6.

The element and compensator parameters resulting from these changes are listed in Table 11-8. Because of the slightly increased wind

Table 11-8
Element Survival Speeds and Compensator, TH5+

	V_{max}, mi/h	V_{ice}, mi/h	V_{clamp}, mi/h
Reflector	124	84	131
Reflector 2	142	86	144
Rear driven	124	86	159
Front driven	130	91	173
Director	129	90	145
Totals	124	84	131

	Spacing, in.	Location, in.	Area, in.^{2}	Moment, in.^{3}	Boom dist., in.
Reflector	–	–103	405	–41714	9.75
Reflector 2	36	–67	152	–10216	45.75
Rear driven	53.25	–13.75	335	–4610	99
Front driven	55	41.25	312	12854	154
Director	68	109.25	311	34006	222
Centroid location/totals		–6.49	1516	–9680	
Compensator	–12	97.25	101	9680	210
Total			1617	0	

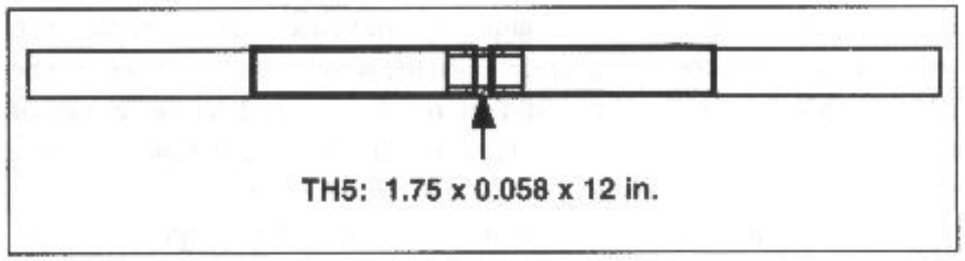


Fig 11-7—Hy-Gain TH5 boom-joint reinforcement.

area of the stronger reflector clamp, the compensating-element area has increased to 101 in.^{2}, which requires a 39.88-in. length for a 1.25-in.-diameter compensating element. The larger reflector clamp requires a 1.5-in. slit bushing. Reinforcement sections should be retained with rivets.

Although the boom strength is already commensurate with the increased element strength, we may wish to reinforce the boom center clamp. This is done by adding a 12-in. length of 1.75 × 0.188-in.-wall 6061-T6 or 6061-T832 aluminum tubing for internal reinforcement, as shown in Fig 11-7. The reinforcing tube must be drilled to match the holes in the boom and boom clamp, so the through-bolts can be inserted. This reinforcement will raise the survival speed of the center joint to 135 mi/h, and the shift of elements toward the director end raises the boom survival speed to 126 mi/h.

Table 11-9
TH5+ Summary

Elements	
V_{max}	124 mi/h
V_{ice} (0.5 in. ice)	84 mi/h
Clamps V_{max}	128 mi/h
Moment centroid	0 in.
Compensator area	101 in.^{2}
Effective area	7.5 ft^2
Boom	
V_{max}	126 mi/h
Moment centroid	0 in.
Effective area	2.1 ft^2
ΔLength, balun & coax	2.48 in.
Total effective area	7.8 ft^2

Hy-Gain uses a similar reinforcement approach on the boom of their 3-element 40-meter Yagi. The reinforcement part number is 171541, 1.75 × 0.083 × 71⅛ in., used on the Hy-Gain 7-3 antenna.

The typical balun used with the TH5 triband antenna is the Hy-Gain BN-86, which is mechanically convenient but susceptible to burnout if power is fed to the antenna on the wrong frequency band. The BN-86 adds 1.5 in. to the width of the boom, and has a length of approximately 5.75 in. It is mounted on the unmodified antenna at a point 32.13 in. from the mast, and at a point 38.38 in. from the mast if the elements are moved toward the director end to improve torque balance. In the first case, a 2.44-in. extension at the reflector end will balance the already small 558-in.^{3} boom-area moment unbalance caused by the balun, coax and compensator end area. If the elements are moved as described above, the addition required is 2.48 in., which compensates the boom-area moment of 565 in.^{3}.

Table 11-9 summarizes the strengthened TH5 with these reinforcements. We can see from the data that this antenna is already very robust, and requires only a minimum of element and boom strengthening to survive 120-plus mi/h winds. Only the center sections of the elements require additional strengthening. The strength of the traps is an unknown, but is assumed to be equal to that of 0.058-in.-wall tubing of the same outside diameter. This may be an incorrect assumption, but the traps are located away from the high-moment center section. The element reinforcement dimensions are summarized in Table 11-10. All compensator dimensions in this chapter are listed per side, including the effect of the Hy-Gain clamp area.

Table 11-10
TH5+ Reinforcement

Element	*Center Reinforcement*
Reflector	54 × 1.125, 18 × 1 in.
10 reflector	12 × 0.75 in.
Rear driven	24 × 1.125 in.
Front driven	18 × 1.125 in.
Director	18 × 1.125 in.
Compensator	39.88 × 1.25 in. (per side)
Boom	1.75 × 0.058 × 12-in. center joint

One final problem in trap tribanders such as the TH5 and TH7 is the effect of rain driven by wind through the vent holes of the traps. Also, these traps tend to make an annoying whistling noise in light wind. Both problems are solved by wrapping the traps in fine-weave water-permeable cloth such as is used to pass water and keep out silt in building-foundation water-drain systems. Taping on this covering has been very successful for me.

Sec 11.3.3 Hy-Gain TH7

The Hy-Gain TH7 is a popular tribander that is somewhat larger than the TH5. Hy-Gain specifies this antenna at 100 mi/h survival. We can analyze this antenna by the same procedures used earlier. The physical dimensions are listed in Table 11-11.

We find the survival speed for the elements and clamps, and the centroid of the element moments, as in Table 11-12. Each half of the TH7 boom consists of two sections; the inner section is reinforced with a 36-in. length of telescoping tubing inside. The calculated boom cen-

Table 11-11
TH7 Element Dimensions

Values are in inches.

Section	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Diameter	1.25	1.125	1.63	1	1.63	0.875	0.625	0.4375
Reflector	95.5	33.5	0	1	12	8		42
10 reflector						55	23.5	38.5
Rear driven	48.5	31	12	8.5	12	8		32.5
Front driven	35.5	40	12	6.5	12	8		24.75
10 director						53		43.5
15 director						55	23.5	51
Director	48	27.5	0	1	12	21		53.5

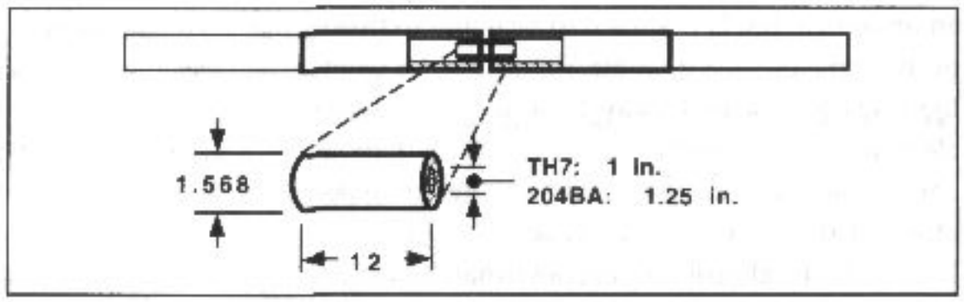


Fig 11-8—Reinforcement of Hy-Gain TH7 and 204BA boom-center joint.

ter-joint survival speed is 102 mi/h, under conservative treatment of vertical gusting. The survival speed of the boom sections is 129 mi/h, as shown in Table 11-13.

The calculated TH7 parameters are shown in Table 11-14. The boom clamp can be strengthened by internal reinforcement as on the TH5, or this antenna can benefit from 3-way boom guying. The joint should be reinforced with a 12-in. length of 6061-T6 or 6063-T832 heavy-wall tube with an outside diameter to fit the 1.584-in. inside diameter of the reinforced center section. For a clearance of 0.005 in. per inch of diameter, we require approximately 0.008-in. clearance, which yields a diameter of 1.584 – 2(0.008) = 1.568 in. for the reinforcing element. A suitable reinforcement can be made from 1¾ × 0.375-in. 6061-T6 tube with an outside diameter machined to 1.568 in. The

Table 11-12
TH7 Element Survival Speeds and Centroid of Moments

	V_{max}, mi/h	V_{ice}, mi/h	V_{clamp}, mi/h
Reflector	101	70	110
10 reflector	117	71	132
Rear driven	102	71	156
Front driven	110	78	167
10 director	126	69	161
15 director	108	64	122
Director	108	70	140
Totals	101	64	110

	Spacing, in.	*Station, in.*	*Location, in.*	*Area, $in.^2$*	*Moment, $in.^3$*
Reflector	—	0	–142.75	407	–58143
10 reflector	48	48	–94.75	160	–15175
Rear driven	58.5	106.5	–36.25	332	–12050
Front driven	55	161.5	18.75	309	5801
10 director	44.5	206	63.25	132	8328
15 director	37.5	243.5	100.75	171	17238
Director	39.5	283	140.25	308	43179
Centroid location/totals			–5.95	1820	–10823

Table 11-13
TH7 Boom Dimensions and Survival Speed

Section	*1*	*2*	*3*	*Total/half*	*Total*
Diameter, in.	2	2	2		
Wall, in.	0.203	0.12	0.049		
Length, in.	36	43.5	65.5	145.75	291.5
V_{max}, mi/h	127	133	129		

reinforcement piece must be drilled to match the existing boom section and center clamp holes. Thus strengthened as shown in Fig 11-8, the boom system provides v_{max} = 119 mi/h, calculated on the basis that includes the strength of the unreinforced center joint assembly.

If instead we triple-guy the boom at the joint between the two sections, we realize the survival speed of the outer section, which is 129 mi/h. Let us use reinforced elements and the larger driven-element clamp on the reflector.

If the element locations are unchanged, the compensation element required at a location 12 in. inboard of the 10-20 meter director is 84.4 in.^{2}, or a length of 33.2 in. per side using 1.25-in. fiberglass tube. However, if the element locations are all shifted 2.5-in. closer to the director end of the boom, the compensator requirement drops to 50.6 in.^{2}, or 19.7 in. long for 1.25-in. diameter. This latter configuration results in a 0.5-ft^2 smaller wind area and in a higher boom-survival wind speed. If the boom were only 3.2 in. longer at each end, the element ensemble could be relocated 6.2 in. toward the director end and no torque compensation would be required. As an alternative, larger diameter 10- and 15-meter directors could accomplish the same end.

The relatively longer element tips are the limiting factor in ice-loading survival. For the TH7 antenna, replacement of three director tip sections is required; this is accomplished on the 10-20 meter and 15-meter directors with 7/16-in. tubing with a 0.049-in. wall, and the removed 7/16 × 0.035-in.-wall tubing is used to replace the thinner tip sections of the 10-meter director. The element reinforcement dimensions for the TH7 are summarized in Table 11-15. Reinforcement sections should be riveted at both ends.

Table 11-14
TH7 Parameter Summary

Elements	
v_{max}	100 mi/h
v_{ice} (0.5 in. ice)	64 mi/h
Clamp v_{max}	110 mi/h
Moment centroid	–6.0 in.
Effective area	8.5 ft^2
Boom	
v_{max}	129 mi/h
Center joint v_{max}	102 mi/h
Moment centroid	0 in.
Effective area	2.7 ft^2
Total effective area	8.9 ft^2

Table 11-15
TH7+ Reinforcement

Element	*Center Reinforcement*	*Tip Replacement*
Reflector	54 × 1.125, 18 × 1 in.	
10 reflector	18 × 0.75 in.	
Rear driven	30 × 1.125 in.	
Front driven	18 × 1.125 in.	
10 director		Use 0.035-in. wall
15 director	24 × 1.125, 12 × 0.5 in.	Use 0.049-in. wall
Director	18 × 1.125 in.	Use 0.049-in. wall
Compensator	19.7 × 1.25 in. per side	
Boom center	1.5625 × 12 in., 1 in. ID	

Table 11-16
TH7+ Summary

Elements	
v_{max}	125 mi/h
v_{ice} (0.5 in. ice)	73 mi/h
Clamp v_{max}	122 mi/h
Moment centroid	0 in.
Compensator area	50.6 in.^{2}
Effective area	8.7 ft^2
Boom	
v_{max}	119 mi/h*
Center reinforcement	1.568 × 12 in., 1 in. ID
Moment centroid	0 in.
ΔLength, balun & coax	8.7 in.
Effective area	2.7 ft^2
Total effective area	9.1 ft^2

* v_{max} = 129 mi/h if guyed

The compensator is 1¼-in. fiberglass tube as on the TH5, described in the preceding section. The length shown is for the case of elements moved 2.5 in. toward the director end.

The BN-86 balun at the front driven element plus the coax, phasing lines and compensating element, combine to create a boom-area moment unbalance that can be compensated by adding 7.85 in. (8.7 in. if the elements are moved 2.5 in. toward the director end) to the reflector end of the boom. The same amount can be removed instead from the director end.

One problem I have experienced with my TH7 is twisting of the boom by rotational oscillation of the reflector and director. This has caused fatigue failures in the matching strap crossovers, so I have replaced them with well-insulated flexible braid conductor. This is a fix that might also be considered for the KLM tribanders and loaded-element Yagi antennas.

The parameters of the reinforced TH7 are summarized in Table 11-16. If guying is chosen, the three guys are attached to the boom at the junction 79.5 in. from the center, and are supported at least 24 in. away from the boom at the mast. This configuration will survive 129-mi/h winds, at which wind speed the guy force is 587 lb and the support compressional force is 168 lb. The guying and support material must be chosen accordingly, adding a safety factor for the static guy tension. Methods of guy attachment and support are shown in Chapter 6.

Sec 11.3.4 Hy-Gain 105BA

The Hy-Gain 105BA is a popular 10-meter monoband antenna that provides a calculated 9.7 dBi gain on a 24-ft boom. This antenna and its sister 155BA and 204BA designs for 15 and 20 meters offer the advantage of being torque balanced as built. The weakest element sections are the relatively thin-walled tips, which result in a lower v_{ice} than might be desirable. Each boom half consists of two sections of 2-in. diameter and 0.049-in. wall thickness.

The boom has a calculated v_{max} = 100 mi/h, and the center joint provides v_{max} = 143 mi/h in this application. Each of the boom inner sections can be reinforced with a 1.875 × 0.058 × 30-in. 6061-T6 tube to provide a boom v_{max} of 124 mi/h. We can reinforce the inner sections and the boom clamp at the same time by using a continuous 1.875 × 0.058 × 60-in. reinforcement tube, as shown in Fig 11-9. This raises the clamp v_{max} above 150 mi/h. Bolt holes must be drilled in these reinforcements to match the boom sections and center joint. Fig 11-9 also shows the dimensions for reinforcement of a higher gain, 5-element, 10-meter Yagi that is shown later in this chapter.

Table 11-17
105BA Element Taper Schedule (28.25 MHz)

Values in columns headed 0 through 3 are in inches.

Section	*0*	*1*	*2*	*3*	*Vmax*	*Vice*
Diameter	1.045	0.875	0.625	0.4375		
Thickness	0.085	0.058	0.035	0.035*		
Reflector	2.5	44	21.5	42.5	128 mi/h	71 mi/h
Driven	2.625	24	23.5	54	110	61
Director 1	2.5	24	23.5	52.25	114	63
Director 2	2.5	24	23.5	47	122	69
Director 3	2.5	24	23.5	45.75	119	66

* R and D3 = 0.028-in. wall

Table 11-18
105BA Elements (28.25 MHz)

Section	*Spacing, Inches*	*Location, Inches*	*Length, Inches*	*Cyl Length, Inches*	*Cyl Diam, Inches*	*X, Ω*
Reflector	—	10.38	108.30	103.53	0.651	+35.48
Driven	93.25	103.63	98.875	95.47	0.550	–42.83
Director 1	56.75	160.38	100.05	96.12	0.562	–36.24
Director 2	52.75	213.13	94.80	90.94	0.573	–84.44
Director 3	72	285.13	93.55	89.71	0.576	–95.67

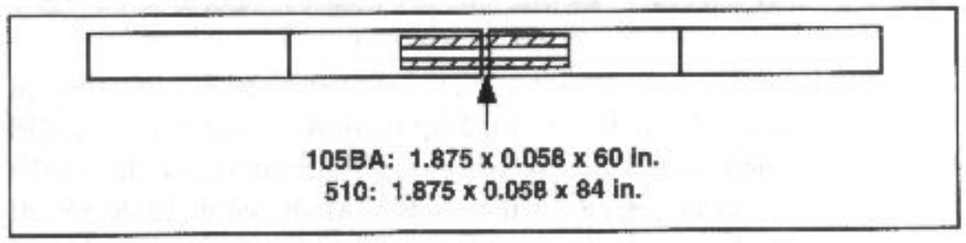


Fig 11-9—Hy-Gain 105BA boom-joint reinforcement.

Table 11-19
105BA+ Elements (28.35 MHz)

Section	*Spacing, Inches*	*Location, Inches*	*Length, Inches*	*Cyl Length, Inches*	*Cyl Diam, Inches*	*X, Ω*
Reflector	—	10.38	107.92	103.17	0.652	+35.48
Driven	93.25	103.63	98.52	95.12	0.550	–42.83
Director 1	56.75	160.38	99.70	95.78	0.563	–36.24
Director 2	52.75	213.13	94.46	90.61	0.574	–84.44
Director 3	72	285.13	93.21	89.38	0.576	–95.67

Table 11-20
105BA+ Element Taper Schedule (28.35 MHz)

Values in columns headed 0 through 3 are in inches.

Section	*0*	*1*	*2*	*3*	*Vmax*	*Vice*
Diameter	1.045	0.875	0.625	0.4375		
Thickness	0.085	0.058	0.035	0.049†		
Reflector	2.5	44	21.5	42.12	130 mi/h	72 mi/h
Driven	2.625	24	23.5*	53.65	125	69
Director 1	2.5	24	23.5*	51.90	129	71
Director 2	2.5	24	23.5	46.66	123	69
Director 3	2.5	24	23.5	45.41	125	70

†DR and D1 = 0.049-in. wall; D2 and D3 = 0.035-in. wall; R = 0.028-in. wall
* Section 2 reinforcements: DR = 7.5 in.; D1 = 5.5 in.

Table 11-17 gives 105BA dimensions for a center frequency of 28.25 MHz; the wall thickness of the tip section of Director 3 is 0.028 in. Using the new Schelkunoff element-taper algorithm, we derive the underlying electrical design for the 105BA, as shown in Table 11-18. This is derived from the element schedules and locations as measured on the unmodified antenna. The element location is measured from the reflector end of the boom, and all dimensions are inches.

In order to design stronger elements, mechanical analysis of element strength and electrical analysis of taper effect were combined to

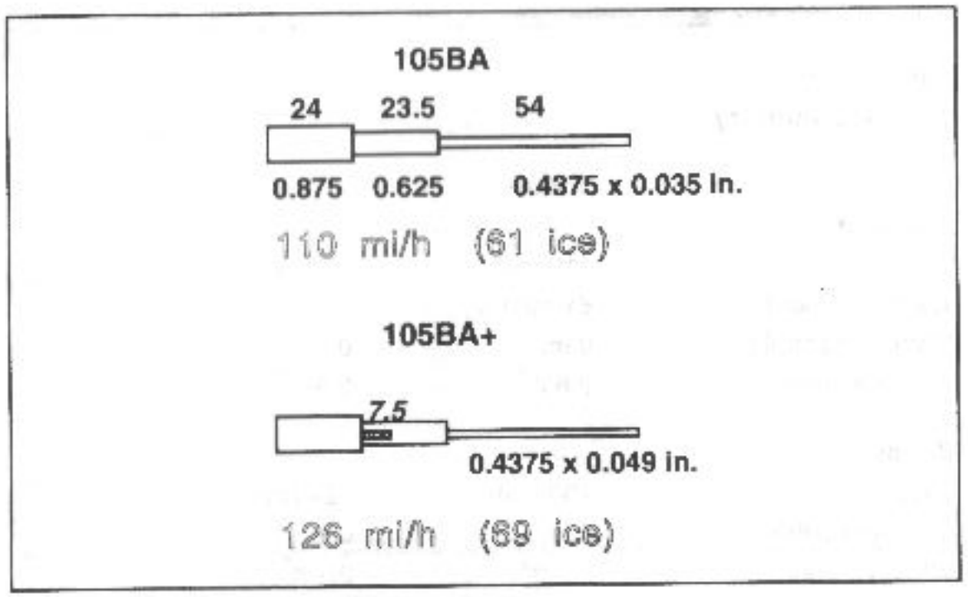


Fig 11-10—Hy-Gain 105BA element development.

yield the element dimensions shown here. A compromise center frequency of 28.35 MHz was chosen to be useful in both CW and SSB contesting.

The compromise antenna uses the element spacings shown by Hy-Gain for the CW frequency tuning, and achieves nearly complete torque balance at the same element locations. Fine adjustments are made in the tip lengths to achieve the design reactances at the chosen frequency, as shown in Table 11-19.

The element dimensions in Table 11-20 provide the same calculated reactance at 28.35 MHz as do those above at 28.25 MHz. The driven-element and first-director tip sections are replaced with 0.049-in.-wall tubing, and the original third-director tips are replaced with the 0.035-in.-wall tips just removed from the driven element. The reflector- and second-director tips are unchanged. For compromise tuning centered on 28.35 MHz, the tip lengths are adjusted to the values shown. If desired, the factory dimensions for CW or SSB operation can be retained. The driven-element section 2 is internally reinforced with a 0.5 × 0.58 × 10-in. length of 6063-T832, and the first-director section 2 is reinforced by an 8-in. length of the same tubing. A useful alternative is ¼-in. IPS pipe, which has an outside diameter of 0.540 in., a better fit to the 0.555-in. inside diameter of this section. This same method can be applied to the 204BA and 205BA antennas that use this component. The development of the 105BA elements is shown in Fig 11-10.

Table 11-21
105BA Summary

	105BA	*105BA+*
Elements		
v_{max}	110 mi/h	123 mi/h
v_{ice} (0.5 in. ice)	61 mi/h	69 mi/h
Moment centroid	0 in.	0 in.
Effective area	2.8 ft^2	2.8 ft^2
Boom		
v_{max}	100 mi/h	124 mi/h
Moment centroid	0 in.	0 in.
Effective area	2.7 ft^2	2.7 ft^2
ΔLength, balun & coax	–3.25 in.	–3.25 in.
Total effective area	3.9 ft^2	3.9 ft^2

With the boom and element modifications shown, the Hy-Gain 105BA can survive our local weather and icing conditions, and should be capable of doing so in any severe-weather amateur application. The antenna in either configuration is sufficiently well torque balanced so that no compensation is required. If desired, the W2DU balun and coax can be compensated by removing approximately 3.25 in. from the director end of the boom. Alternatively, a 2.375 × 19.75-in. compensating sleeve of ABS 2-in. IPS plastic pipe can be secured over the boom at the director end. Table 11-21 summarizes the wind performance of the 105BA.

Sec 11.3.5 Hy-Gain 155BA

The Hy-Gain 155BA 15-meter monobander elements are of an unusual design, with just two sections of 0.875 in. and 0.4375 in. joined by a swage. The elements provide v_{max} = 87 mi/h, and because of the thin-wall tip sections, v_{ice} = 49 mi/h. The element taper schedule is shown in Table 11-22.

The 26-ft 155BA boom is similar in design to the TH7, with outer sections 2 × 0.049 × 79 in., and inner sections 2 × 0.12 × 81 in., with the inner 36 in. reinforced with an additional 0.083-in. wall thickness.

Table 11-22
155BA Element Taper Schedule

Values are in inches.

Section	*0*	*1*	*2*
Diameter	1.045	0.875	0.4375
Thickness	0.085	0.049	0.035
Reflector	2.5	84	66.5
Driven	2.625	72	65
Director 1	2.5	72	65
Director 2	2.5	72	57

Table 11-23
155BA+ Element Reinforcement Schedule

Values are in inches.

Section	*0*	*1a*	*1b*	*2a*	*2b*
Diameter	1.045	0.875	0.875	0.4375	0.4375
Thickness	0.085	0.107	0.049	0.096	0.035
Reflector	2.5	19	65	7.5	59
Driven	2.625	6	66	6	59
Director 1	2.5	6	66	6	59
Director 2	2.5	0	72	0	57
Director 3	2.5	0	72	0	58

The boom itself is quite strong, with a survival speed of 155 mi/h. The boom v_{max} is limited only by the center joint, which will survive 124-mi/h winds. Although not generally required, the boom center can be reinforced with the aluminum plug shown in Fig 11-8 for the 204BA, which will result in a boom capable of surviving wind speeds of 139 mi/h.

The antenna design is inherently torque-balanced, because of the relatively larger reflector spacing which compensates for the smaller director areas. As on other monoband antennas, the use of the W2DU

Table 11-24
155BA++ Element Reinforcement Schedule

Values are in inches.

Section	*0*	*1a*	*1b*	*2a*	*2b*
Diameter	1.045	0.875	0.875	0.4375	0.4375
Thickness	0.085	0.107	0.049	0.096	0.035
Reflector	2.625	37*	47	16.68	50
Driven	2.625	24	48	15	50
Director 1	2.625	24	48	15.19	50
Director 2	2.5	24	48	7	50
Director 3	2.5	24	48	8	50

* Inner 7 in. double reinforced with 0.625 × 0.058-in.-wall tubing

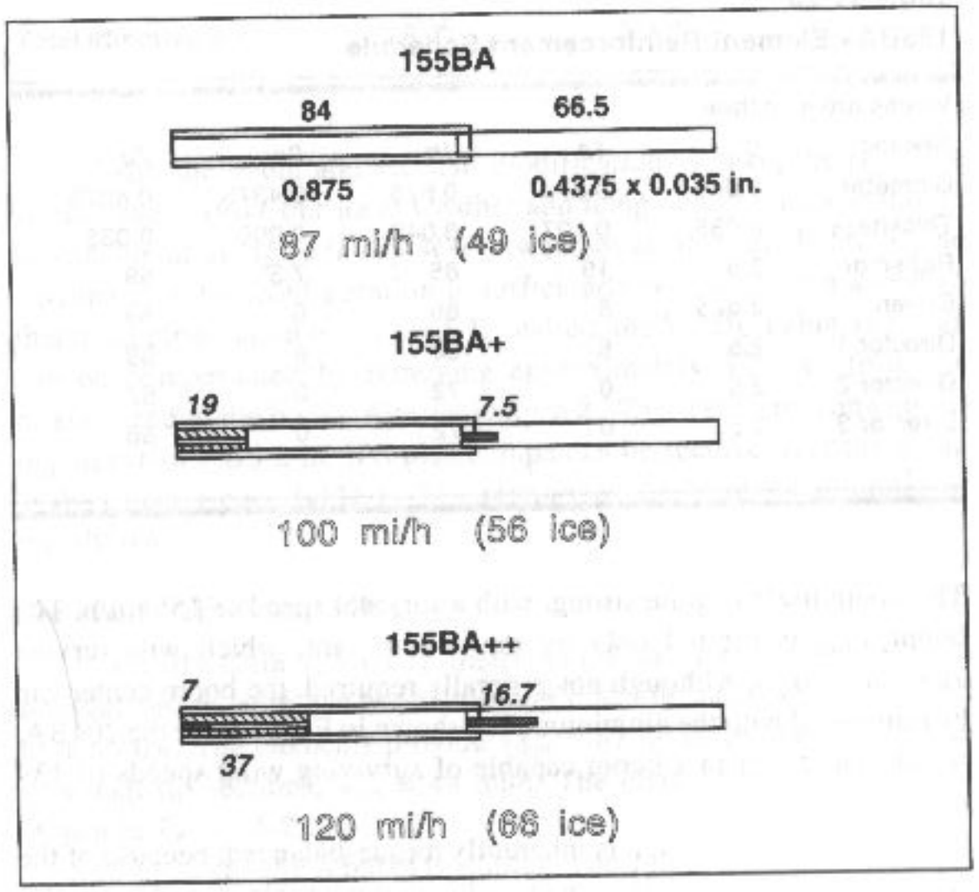


Fig 11-11—Hy-Gain 155BA element development.

ferrite-bead balun will prevent some of the inevitable burnouts as with the Hy-Gain balun when loading up on the wrong band; the boom-area moment unbalance from this balun and the coax can be compensated by removing approximately 5.5 in. from the reflector end.

The 155BA elements can be reinforced simply as summarized in Table 11-23 to provide $v_{max} = 100$ mi/h and $v_{ice} = 56$ mi/h. The reflector, driven-element and first-director tip sections are reinforced at their inner ends with 0.375 × 0.065-in.-wall tubing with the outer diameter turned down to 0.358 in. to fit the 0.3675-in. inner diameter of the 0.4375 × 0.035-in. tips. Alternatively, these tip sections can be replaced entirely with 0.049-in.-wall tubing. The inner sections of the same three elements are reinforced with 0.75 × 0.058-in.-wall tubing as indicated.

The 155BA elements can be reinforced for higher winds in the same fashion, as summarized in Table 11-24 and Fig 11-11. This provides $v_{max} = 120$ mi/h and $v_{ice} = 66$ mi/h. Note that the reflector and first director require the larger element clamp.

In order to realize this capability, it is necessary to use the larger driven-element clamp, bushed to fit, on the reflector and first director. These tip lengths are adjusted using the new taper algorithm to compen-

Table 11-25
155BA Summary

	155BA+	*155BA++*
Elements		
v_{max}	100 mi/h	120 mi/h
v_{ice} (0.5 in. ice)	56 mi/h	66 mi/h
Moment centroid	0 in.	0 in.
Effective area	4.3 ft^2	4.3 ft^2
Boom		
v_{max}	124 mi/h	124 mi/h
Moment centroid	0 in.	0 in.
Effective area	2.9 ft^2	2.9 ft^2
ΔLength, coax & balun	–5.5 in.	–5.25 in.
Total effective area	5.2 ft^2	5.2 ft^2

sate for the different equivalent diameter of the larger clamp. All center sections are internally reinforced at their inner ends with 0.058-in.-wall 6061-T6 or 6063-T832 tubing. Tips are reinforced with 0.375 × 18-in. tube with the outer diameter turned down to 0.358 in. Reinforcements should be dimpled or riveted at their outer ends.

Last, to improve the already excellent torque balance, the element ensemble is relocated 1.75 in. toward the director end of the boom, so that the reflector is 9.75 in. from the boom end. In this configuration, the W2DU balun and coax can be compensated by removing approximately 5.25 in. from the reflector end. The results of these improvements are summarized in Table 11-25.

Sec 11.3.6 Hy-Gain 204BA

The Hy-Gain 204BA is a widely used 4-element 20-meter monoband Yagi that is often considered a standard of comparison. It provides a calculated 8.2 dBi gain and over 20 dB front-to-back pattern parameter. It is rated by the manufacturer for 100 mi/h survival under EIA RS-222-C.

The calculated element v_{max} = 100 mi/h, and boom v_{max} = 126 mi/h. Boom survival is limited primarily by the center joint v_{max} = 106 mi/h, a very conservative number derived for a 30° wind angle from the horizontal. The element moment centroid is –3.08 in., resulting in an element moment of area of 30.09 in.^{3}, which can be compensated by a 1.25-in.-diameter compensator with a length of 11.51 in. per side. The torque balance is good enough that many applications will not require element torque balance. The balun and coax can be compensated by adding 1.75 in. to the director end of the boom, assuming the use of the W2DU ferrite-bead balun.

For heavy-duty applications, the elements and boom of the 204BA can be reinforced in the same fashion as the 105BA and 155BA, described in previous sections. The boom can be strengthened through internal reinforcement at the boom center, or it can be guyed. The boom center joint should be reinforced with the aluminum plug shown in Fig 11-8, which will provide boom v_{max} = 124 mi/h. Alternatively, the boom can be triple-guyed from the boom ends to 24-in. supports to achieve v_{max} = 157 mi/h, with a guy force of 762 lb and support force of 115 lb at that wind speed.

The 204BA++ element design derived in Chapter 4 is applied here to provide improved strength. This reinforcement also cures a tendency

Table 11-26
204BA++ Taper and Reinforcement Schedule

Values are in inches.

Section	*0*	*1a*	*1b*	*2a*	*2b*
Diameter	1.5	1.25	1.25	1.125	1.125
Thickness	0.125	0.174	0.095	0.116	0.058
Reflector	3.75	30.5	14	27.5	18.5
Driven	3.75	21	14	27	19
Director 1	2.625	7	14	27	19
Director 2	2.625	2.75	14	27.25	18.75

Section	*3a*	*3b*	*4a*	*4b*	*5*
Diameter	0.875	0.875	0.625	0.625	0.4375
Thickness	0.084	0.049	0.131	0.035	0.049
Reflector	19.5	31	10	14	53.17
Driven	19.5	31	10	14	53
Director 1	19.5	31	10	17	53
Director 2	19.5	31	10	20	53

for the elements to unplug at the joint between sections 1 and 2. A rivet or bolt at this point is also recommended for added security. If 0.058-in.-diameter tip sections are not available, using the more easily found 0.049-in.-diameter tubing reduces v_{ice} from 72 mi/h to 70 mi/h, with no penalty in v_{max}.

The improved element-section schedule is shown in Table 11-26. Because the smaller no. 13 clamp provides a v_{max} of only 108 mi/h for this element, the reflector-element clamp is replaced by the stronger driven-element clamp with a 1.5-in. slit tube of matching length for mounting the 1.25-in. element. The reflector tip dimension is calculated by the new Schelkunoff algorithm for use with the stronger no. 14 clamp.

The reinforcement schedule is shown in Table 11-27, with section numbers keyed to Table 11-26. The reinforcement of section 4 should be located with a dimple or rivet at its outer end. The reinforcement tube includes sufficient length to extend inboard of the joint. Center-section reinforcements should be similarly dimpled or riveted to hold them in

Table 11-27
Reinforced 204BA++

Values are in inches.

Section	*1, 2*	*3*	*4*	*5*
Diameter	1	0.75	0.5	0.4375
Reflector	72	24	12	t = 0.049
Driven	62	24	12	t = 0.049
Director 1	48	24	12	t = 0.049
Director 2	44	24	12	t = 0.049

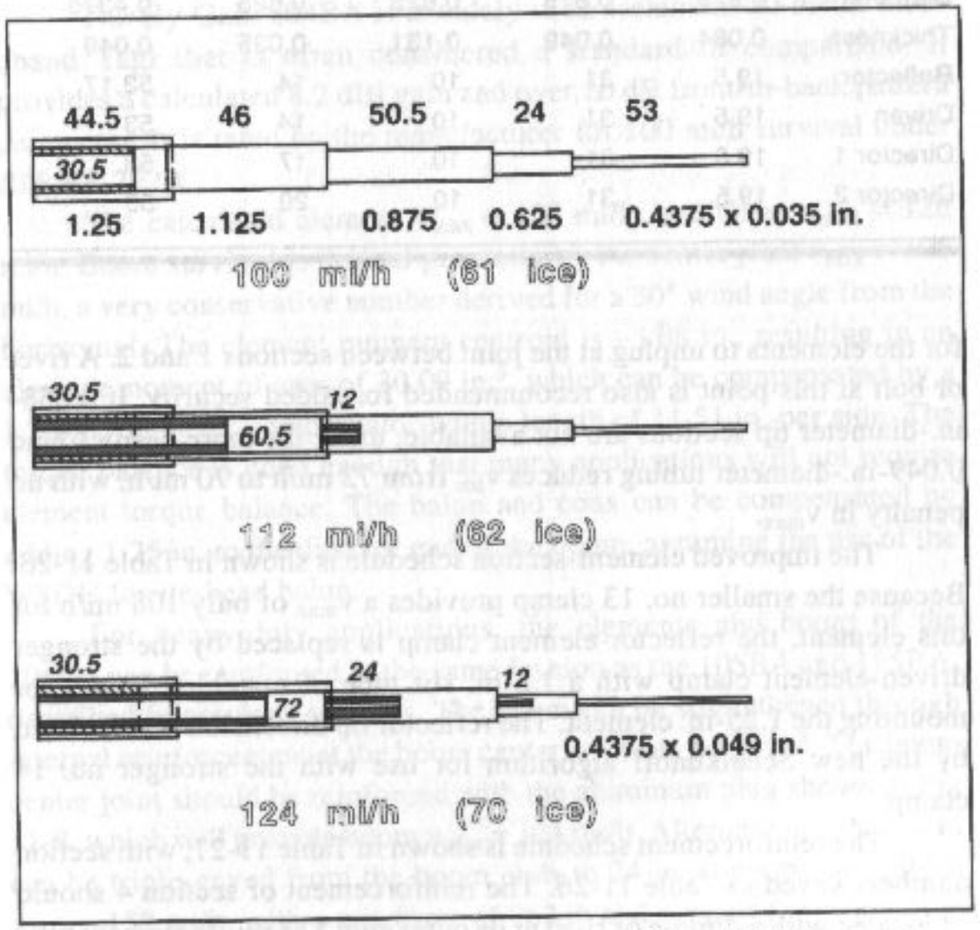


Fig 11-12—Element reinforcement for Hy-Gain 204BA and 205BA antennas.

Table 11-28
204BA Summary

	204BA	*204BA++*
Elements		
v_{max}	100 mi/h	123 mi/h
v_{ice} (0.5 in. ice)	61 mi/h	70 mi/h
Moment centroid	–3.08 in.	–2.37 in.
Compensator area	30.09 in.^{2}	22.99 in.^{2}
Compensator length	11.5 in.	5.2 in.
Effective area	6.5 ft^2	6.5 ft^2
Boom		
v_{max}	126 mi/h	126 mi/h
Center joint v_{max}	106 mi/h	124 mi/h
Moment centroid	0 in.	0 in.
Effective area	2.9 ft^2	2.9 ft^2
ΔLength, balun & coax	+1.75 in.	–1.5 in.
Total effective area	7.0 ft^2	7.1 ft^2

place. Fig 11-12 illustrates the method of reflector-element reinforcement.

The steps outlined here result in a very robust, compact monoband Yagi that has survived the wind environment at my site. The elements can be moved 1.625 in. toward the director end to improve the torque balance. This results in a boom-to-driven-element dimension of 30 in., rather than 31.625 as shown in the manufacturer's manual. With this configuration, a compensating element with a length of 5.2 in. per side offsets the residual element moment unbalance, and the removal of 1.5 in. from the reflector end of the boom compensates the boom-moment unbalance from the balun and coax. The summary of physical parameters for the reinforced 204BA is shown in Table 11-28.

If a compensator is used, it is mounted 12 in. inboard of the outermost director. The effect of the compensating element is minimal on this relatively small and already well-balanced antenna, so it can be

eliminated if a robust rotator is chosen. The small effective area of this antenna lends itself well to stacking in high-wind areas.

Sec 11.3.7 Hy-Gain 205BA

The Hy-Gain 205BA is a very popular 20-meter monoband Yagi. Computer calculations of forward gain and pattern front-to-back ratio (for the entire range from 90° to 180°) are 9.15 dBi gain and 15 dB front-to-back ratio. The antenna as delivered by the manufacturer is rated for 80 mi/h survival. This antenna is closely related to the 204BA, and uses very similar elements that can be strengthened by the same techniques. The boom is substantially longer, and features a center section carried continuously through the boom-joint clamp. There is substantial torque imbalance that requires correction.

The original elements provide a calculated v_{max} = 100 mi/h and v_{ice} = 61 mi/h. As with the other Hy-Gain elements discussed in this chapter, the effect of the swage sections is ignored for simplicity; this results in a minor increase in area and moment, and a corresponding decrease in calculated v_{max}.

The dimensions for reinforced elements are listed in Table 11-29. The dimensions of the tip sections are calculated using the Schelkunoff taper algorithm of Chapter 8 to provide the same electrical length as the original unreinforced elements. The center frequency is chosen as 14.125 MHz, and the reflector dimensions account for the stronger element clamp used. If desired, the antenna can be tuned to 14.075 MHz using a reflector tip dimension of 53.17 in., directors 1 and 2 at 53 in., and driven and director 3 elements at 50 in. These elements are essentially the same as the 204BA elements, and offer v_{max} = 123 mi/h and v_{ice} = 72 mi/h if the tip section is 0.058-in. wall and 70 mi/h if 0.049-in. wall.

Table 11-30 shows the reinforcement schedule. The reinforcement of section 4 should be located with a dimple or rivet at its outer end, and the reinforcement tube includes sufficient length to extend inboard of the joint. Center-section reinforcements should be similarly retained.

The 34-ft 205BA boom is essentially the 204BA boom with a 110-in. center extension through the mast clamp. Boom survival speed is calculated to be v_{max} = 106 mi/h except for the swaged joint between the boom center section and the middle section, which yields a calculated v_{max} = 95 mi/h. If the center section is further strengthened by the

Table 11-29
205BA++ Element Taper and Reinforcement Schedule

Values are in inches.

Section	*0*	*1a*	*1b*	*2a*	*2b*
Diameter	1.5	1.25	1.25	1.125	1.125
Thickness	0.125	0.174	0.095	0.116	0.058
Reflector	3.75	30.5	14	27.5	18.5
Driven	3.75	21	14	27	19
Director 1	2.625	7	14	27	19
Director 2	2.625	2.75	14	27.25	18.75
Director 3	2.625	2.75	14	27.25	18.75

Section	*3a*	*3b*	*4a*	*4b*	*5*
Diameter	0.875	0.875	0.625	0.625	0.4375
Thickness	0.084	0.049	0.131	0.035	0.049
Reflector	19.5	31	10	14	52.37
Driven	19.5	31	10	14	49.24
Director 1	19.5	31	10	17	52.27
Director 2	19.5	31	10	20	52.28
Director 3	19.5	31	10	20	49.29

Table 11-30
Reinforced 205BA+

Values are in inches.

Section	*1, 2*	*3*	*4*	*5*
Diameter	1	0.75	0.5	0.4375
Reflector	72	24	12	t = 0.049
Driven	62	24	12	t = 0.049
Director 1	48	24	12	t = 0.049
Director 2	44	24	12	t = 0.049
Director 3	44	24	12	t = 0.049

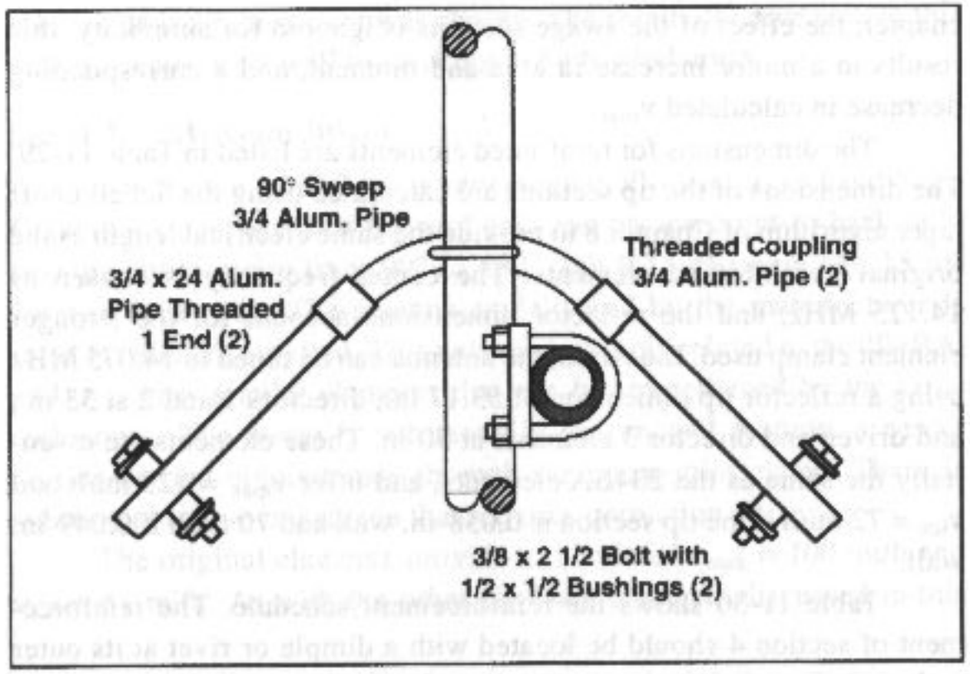


Fig 11-13—Boom guy mount with 90° pipe bend. All dimensions are in inches.

addition of an inner tube 110 × 0.083-in. wall, the boom survival increases to v_{max} = 112 mi/h, limited now by the strength of the second boom sections. If the boom is guyed at the joint between the middle and tip sections, v_{max} = 126 mi/h is achieved for 24-in. guy supports. At that speed, the guy force is 795 lb and the support force is 142 lb. This configuration has worked extremely well for me, using the three-way 120° boom-guying method shown in Chapter 6.

If a 90° aluminum pipe bend is used to mount the boom guy struts at the mast, the mount can be offset upward to bring the ends to the 120° points. The mount is shown in Fig 11-13. The struts are attached to the pipe bend using threaded couplers. Through-bolts are advisable to retain the strut in case of guy breakage. This configuration has two advantages—all of the hardware is located above the boom, which can then be mounted just above the tower top, and the system can be tensioned at installation simply by pushing down the support before tightening the U bolt to the mast.

We need to determine the dimensions required to provide the desired amount of support offset d, say 24 in. The amount of offset is determined by the geometry of Fig 11-14. We also need to know the upward offset Δh from the boom centerline to the connection point

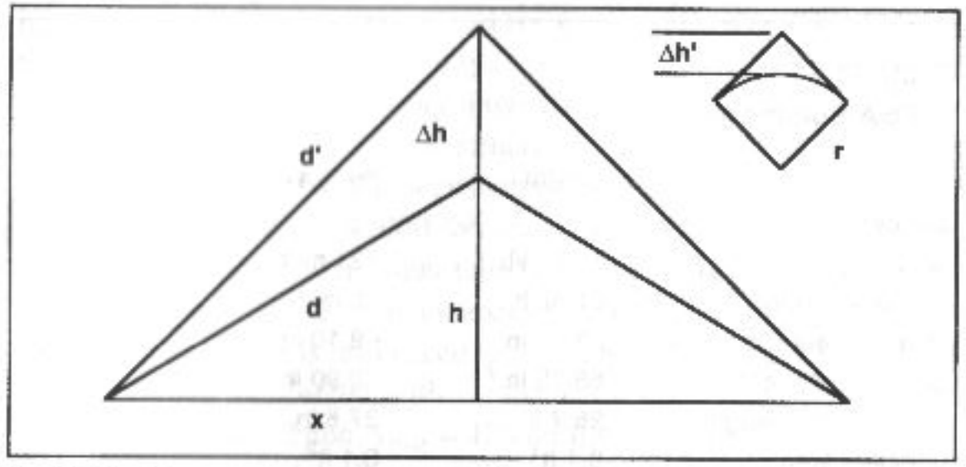


Fig 11-14—Boom guy mount geometry. See text.

between the support and the mast, and the length d′ of the arms to place the ends at the 120° points.

The angle between d and x is 30° and the angle between d′ and x is 45°. The boom is mounted at the point of intersection of d and h, and the struts extended intersect the mast at the point of intersection of d′ and Δh. The horizontal distance x is given by

$$x = d\cos 30° = d'\cos 45° \qquad \text{(Eq 11-1)}$$

so we have

$$d' = d\frac{\cos 30°}{\cos 45°} = 1.22d \qquad \text{(Eq 11-2)}$$

Also

$$h = x\tan 30° = d\cos 30°\tan 30° \qquad \text{(Eq 11-3)}$$

If we define $h' = h + \Delta h$, we determine h′ from

$$h' = x\tan 45° = d\cos 30°\tan 45° \qquad \text{(Eq 11-4)}$$

and now we have

$$\Delta h = h' - h = d\cos 30°(\tan 45° - \tan 30°) = 0.37d \qquad \text{(Eq 11-5)}$$

To allow for the radius of the bend, we see that the diagonal of the square of side r is $r\sqrt{2}$, so the difference in height Δh′ is given by

$$\Delta h' = (\sqrt{2} - 1)r = 0.41r \qquad \text{(Eq 11-6)}$$

Table 11-31
205BA Summary

	205BA	*205BA++*
Elements		
v_{max}	100 mi/h	121 mi/h
v_{ice} (0.5 in. ice)	61 mi/h	70 mi/h
Moment centroid	–7.79 in.	–8.10 in.
Compensator area	68.15 in.^{2}	70.90 in.^{2}
Compensator length	26.7 in.	27.8 in.
Effective area	8.1 ft^2	8.1 ft^2
Boom		
v_{max}	95 mi/h	120 mi/h
Moment centroid	0 in.	0 in.
Effective area	3.8 ft^2	3.8 ft^2
ΔLength, balun & coax	6.75 in.	6.75 in.
Boom guying		
v_{max} guyed		126 mi/h
Location on boom		132 in.
Support distance		24 in.
Guy force		795 lb
Support force		142 lb
Total effective area	9.0 ft^2	9.0 ft^2

The total upward offset Δh to the center line of the pipe bend is

$$\Delta h - \Delta h' = 0.37d - 0.41r \qquad \text{(Eq 11-7)}$$

For the 205BA example here, for d = 24 in. and r = 6 in., the support mounting point is offset upward from the boom center line by 6.42 in.

Because of the strength of the pipe bend, we can simply mount it with a plated U bolt and saddle to fit the mast. The lower boom guys can be tied to the tips of the struts outboard of the through-bolts. On installation, the ropes are tied to the struts as tightly as possible and the

upper guy (I use steel cable and turnbuckles here) is tensioned to bring the boom up against the lower guys.

As a boom-strengthening alternative, the center section can be replaced by a composite tube structure. The section is built using telescoping 0.120-in.-wall tube, which is generally available in 2024-T4 material having σ_{max} = 42000 lb/in.^{2}. (Calculations were made using σ_{max} = 35000 lb/in.^{2}.) The outer tube is 2 × 0.12 × 102 in., and the next inner tube is 1.75 × 0.12 × 140 in., extending 19 in. into the second boom sections. The center is reinforced with a tube 1.5 × 0.12 × 36 in. Each joint is further reinforced with a tube 1.5 × 0.12 × 24 in. With this alternative center section, v_{max} = 120 mi/h with no guying.

Table 11-31 lists the resulting mechanical parameters for the reinforced antenna, as well as the compensating element and boom guying dimensions. The compensator is located 12 in. toward the mast from director 3. This antenna represents a good compromise between gain and survival, and the modified elements and boom have given a good account of themselves in contest service.

Sec 11.3.8 QHS N6ND510

An interesting 10-meter electrical design has been developed and published by Dick Craig, N6ND.[2] This design is one of several, including the N2FB,[3] K8CC[4] and the commercial KLM designs, that obtain extraordinary gain from a design that closely groups the reflector, driven element and first director. Unfortunately, this design results in substantial torque unbalance, but that can be dealt with by a compensating element.

In response to a question from a friend, I have improved the design using the YO computer program[5] and by incorporating more accurate information about the electrical effect of the element clamps and the boom. Also while working with the design, I determined that shortening the first director with this spacing results in dramatically wider bandwidth without any noticeable sacrifice in gain or pattern. The antenna, which I've named the QHS N6ND510, provides an SWR of 1.3:1 from 28.0 to 28.7 MHz.

This antenna provides a calculated 10.4-dBi forward gain and a 23-dB front-to-back pattern parameter. It can be built using parts from the Hy-Gain 105BA, which, although better torque balanced, provides only 9.6 dBi gain with somewhat less front-to-back ratio on exactly the same boom length. The first contest outing of the antenna described here

resulted in a national high 10-meter score by Dave Siddall, K3ZJ, in the 1990 ARRL DX contest.

A beta match has the benefit of improved torque balance because the driven element is shorter than resonant length. To determine the driven-element dimensions, we calculate the beta-match parameters using the methods of Chapter 9. We know we will need the reactive part of the antenna input impedance to be in the range of –30 Ω. We start with the known or calculated resistive part of the antenna input impedance (in this case R_{in} = 25.2 Ω for X_{in} = –25 to –35 Ω) and make some assumptions about the effect of the inductance of the leads from the balun to the driven elements. We also include the effect of the capacitance between the driven element and the element clamp, which we determine from the insulator dimensions and assumed insulator dielectric constant.

The inductance between the balun and driven element is modeled as a short (4-in.) length of transmission line, one side of which is the 0.081-in.-diameter no. 12 lead wire, and the other side of which is the 2-in.-diameter boom itself. The effect of spacing on characteristic impedance is not critical, so an approximation of 1-in. spacing from the surface of the boom (2 in. to the boom center line) will suffice. The expression for the line impedance Z_{line} is

$$Z_{line} = 2(120) \ln (s / \sqrt{d_1 d_2}) \qquad \text{(Eq 11-8)}$$

where

s = spacing
d_1 = wire diameter
d_2 = boom diameter

The calculation proceeds as follows.

Frequency, MHz	28.35 MHz
Wavelength = c′/f	416.2 in.
Input length	4 in.
Length	0.010 wavelength
Spacing to boom center	2 in.
Wire diameter	0.081 in.
Boom diameter	2 in.
Z_{line}	551 Ω
Z_0	50 + *j* 0 Ω
$Z_0' = R_0 + jX_0$	50.2 + *j* 33.1 Ω

Thus, the effect of the series inductance is to add approximately 33 Ω of inductive reactance to the impedance to be matched. The required reactance X_{in} at the antenna input is calculated by

$$X_{in} = -\sqrt{\frac{R_{in}(R_o^2 + X_L^2)}{R_o} - R_{in}^2} \qquad \text{(Eq 11-9)}$$

which will be used in the computer program YO to determine the driven-element dimension. For R_{in} = 25.2 Ω, we calculate the required X_{in} = –34.3 Ω. By comparison, without the effect of the series inductance, we would require X_{in} = –25 Ω.

It remains to calculate the required reactance of the beta element, the dimensions of which can then be determined. The susceptance B required at the antenna terminals is given by

$$B = \frac{R_o X_{in}}{R_{in}(R_o^2 + X_L^2)} + \frac{X_L}{(R_o^2 + X_L^2)} \qquad \text{(Eq 11-10)}$$

The shunt capacitive reactance from the insulated clamp is approximately 235 Ω per side, so the required susceptance of the beta line alone is $B_\beta = B + B_c = -0.012$; hence, the required reactance $X_\beta = 1/B_\beta$ = 84 Ω. One means of setting the beta reactance is to use a dip meter and determine the beta length that resonates with a capacitor having this reactance at the band center frequency. We can calculate the approximate length of the beta line as follows.

B	–0.0098
$X_C = 1/B_C$	235 Ω per side
$B_\beta = B + B_C$	–0.012
$X_\beta = 1/B_\beta$	84 Ω
Spacing (line to center)	4 in.
Diameter	0.25 in.
Boom diameter	2 in.
$Z_o = 2(120)\ln(2s / \sqrt{d_1 d_2})$	582 Ω
Length for 84 Ω	0.024 wavelength
β length	9.5 in.

We then use the YO calculations to set the driven-element dimensions to bring the input impedance to $Z_{in} = R_{in} + jX_{in} = 25.2 - j34.3$ Ω at band center (28.35 MHz). The SWR rises to 1.3 at 28.0 and 28.7 MHz.

Table 11-32
QHS N6ND510 Taper Schedule (28.35 MHz)

Values in columns headed 0 through 3 are in inches.

Section	*0*	*1*	*2*	*3*	v_{max}	v_{ice}
Diameter	1.045	0.875	0.625	0.4375		
Thickness	0.085	0.058	0.035	0.049†		
Reflector	2.5	44	21.5	44	124 mi/h	68 mi/h
Driven	2.625	24	23.5*	55.5	121	67
Director 1	2.5	24	23.5*	53.125	122	68
Director 2	2.5	24	23.5*	52.25	124	69
Director 3	2.5	24	23.5	47.125	122	68

†DR, D1, D2 = 0.049-in. wall; D3 = 0.035-in. wall; R = 0.028-in. wall

* Section 2 reinforcements: DR 7.5 in.; D1, D2 5.5 in.

The construction and element measurements are shown in Fig 11-15 and Table 11-32. No internal reinforcement is required for the element sections. If the original 105BA elements are used without reinforcement, the element v_{max} = 108 mi/h and v_{ice} = 60 mi/h.

A more robust design, adapting the 105BA+ element shown in Table 11-20, employs the same element electrical dimensions. In this case, however, the second director also requires internal reinforcement of section 2 and a 0.049-in.-wall tip. For greater strength, 0.058-in.-wall tips can be used if available. A center frequency of 28.35 MHz was chosen to be useful in both CW and SSB contesting. Table 11-33 summarizes the underlying monotaper (cylindrical element) design and the calculated element reactance at 28.35 MHz.

To balance the boom-plane moment of the balun and coax and to offset the element ensemble to reduce the size of the required compensating element to 1.25 × 25 in. per side, the '510 boom is an asymmetrical derivative of the 105BA boom. The 105BA boom consists of four 75-in. sections, with 1.5-in. spacing at the center joint and 4-in. overlap at the section swage joints. For the '510, the director end section is ideally 83.5 in., which can be realized by an 8.5-in. tip extension or a

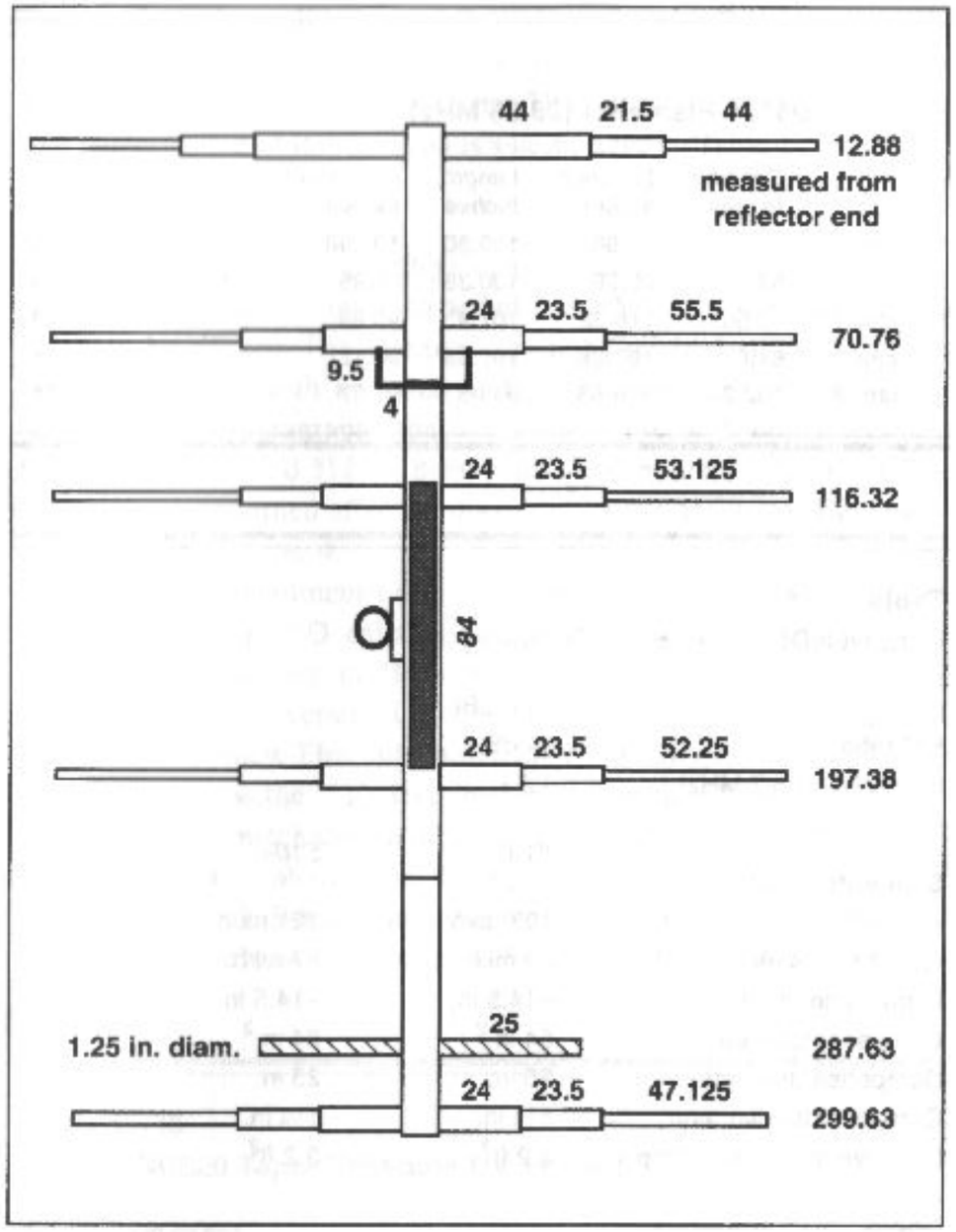


Fig 11-15—Dimensions for the QHS N6ND510 10-meter monoband array. All values are in inches. The figures along the boom are boom-section lengths.

complete replacement 83-in. tip section (no. 171538) used on the Hy-Gain 7-2.

The element locations given in Fig 11-15 and Table 11-33 are derived for this boom configuration, which is an example of the use of the coax and balun moments to permit an asymmetrical boom with

Table 11-33
QHS N6ND510+ Elements (28.35 MHz)

Section	*Spacing, Inches*	*Location, Inches*	*Length, Inches*	*Cyl Length, Inches*	*Cyl Diam, Inches*	*X, Ω*
Reflector		12.88	109.80	104.98	0.646	+52.86
Driven	57.88	70.76	100.38	96.95	0.547	−25.38
Director 1	45.56	116.32	101.93	96.99	0.560	−24.73
Director 2	81.06	197.38	100.05	96.12	0.562	−32.95
Director 3	102.25	299.63	94.93	91.07	0.573	−80.24

Table 11-34
QHS N6ND510 and 510+ Summary

Gain	10.4 dBi
F/B ratio	24 dB
SWR (28.0-28.7 MHz)	1.3

	510	*510+*
Elements		
V_{max}	107 mi/h	121 mi/h
V_{ice} (0.5 in. ice)	59 mi/h	67 mi/h
Moment centroid	−14.5 in.	−14.5 in.
Compensator area	64 in.^{2}	64 in.^{2}
Compensator length	25 in.	25 in.
Compensator diameter	1¼ in.	1¼ in.
Effective area (incl. comp.)	3.2 ft^2	3.2 ft^2
Boom		
V_{max}	91 mi/h	121 mi/h
Center joint V_{max}	132 mi/h	143 mi/h
Moment centroid	−4 in.	−4 in.
Effective area	3.0 ft^2	3.0 ft^2
ΔLength, balun & coax	8.5 in.	8.5 in.
Total effective area	4.4 ft^2	4.4 ft^2

elements offset toward the director end. The center boom joint can profitably be reinforced internally as on the 105BA, in this case with a 1⅞ × 0.058 × 84-in. 6061-T6 or 6063-T832 tube, as shown in Fig 11-9. The performance of this antenna is summarized in Table 11-34.

Sec 11.3.9 QHS N6ND520

The same electrical design can be applied to 15 and 20 meters. Here is an example designed to be built on a 51.5-ft boom, which I call the QHS N6ND520. The antenna is no lightweight, but it will survive 118-mi/h winds. Both unguyed and guyed boom configurations are explored in the paragraphs that follow. For use with larger diameter boom sections, 3 × 0.3125 × 8-in. plate element clamps are used. The elements are patterned after the maximum-survival 20-meter elements introduced in Chapter 4.

The element dimensions are derived from monotaper dimensions that resulted from YO calculations for the 14.175-MHz center frequency. The accuracy of the dimensions is thus controlled by the accuracy of the YO version I used; newer versions may have improved frequency accuracy. This antenna yields essentially the same 10.4-dBi calculated gain as the '510 described in the previous section.

The beta-match design proceeds as with the previous '510 example. From the YO calculations, the input resistance for $X_{in} \approx -25$ to -35 Ω is $R_{in} = 25.1$ Ω. For this antenna, let us take the inductance between the balun and driven element to be 5 in. of 0.081-in.-diameter no. 12

Table 11-35
QHS N6ND520 Taper Schedule (14.175 MHz)

Values are in inches.

Section	*0*	*1a*	*1b*	*1c*	*2*	*3*	*4*	*5*
Diameter	1.5	1.25	1.25	1.25	1.125	1	0.75	0.5
Thickness	0.125	0.174	0.116	0.058	0.058	0.058	0.058	0.058
Reflector	4	28	38	12	16	32	34	61.3
Driven	4	14	38	12	16	32	34	61.3
Director 1	4	12	38	12	16	32	34	62.1
Director 2	4	10	38	12	16	32	34	61.7
Director 3	4	—	38	12	16	32	34	61.9

lead wire, 1 in. over the 2.25-in. boom. The calculation proceeds as follows.

Frequency, MHz	14.175 MHz
Wavelength = c'/f	832.4 in.
Input length	5 in.
Length	0.008 wavelength
Spacing to boom center	2.125 in.
Wire diameter	0.081 in.
Boom diameter	2.25 in.
Z_{line}	551 Ω
$Z_0' = R_0 + jX_0$	50.1 + *j*20.7 Ω
R_{in}	25.1 Ω
X_{in} required	−29.0 Ω
B	0.013
X_c	135 Ω per side
$B_\beta = B + B_c$	0.0164
$X_\beta = 1/B_\beta$	61.1 Ω

We calculate the approximate dimensions of the beta inductor as follows.

Spacing (line to center)	5 in.
β diameter	0.25 in.
Boom diameter	2.25 in.
$Z_o = 2(120)\ln(2s / \sqrt{d_1 d_2})$	622 Ω
Length for 61.1 Ω	0.016 wavelength
β length	13.0 in.

We then use the YO calculations to set the driven-element dimensions to bring the input impedance to $Z_{in} = R_{in} + jX_{in} = 25.1 - j29\ \Omega$ at band center (14.175 MHz). The SWR rises to 1.39 at 14.0 and 14.35 MHz. The element taper and reinforcement schedule for this antenna is shown in Table 11-35.

For these elements, v_{max} = 122 mi/h and v_{ice} = 73 mi/h. Details of the element construction are shown in Fig 11-16. Because of the strength of the clamp, elements can be made in separate halves or with continuous center sections, at the choice of the constructor. By comparison with the Hy-Gain 20-meter element, the choice of a thicker tip section enhances the ice survival.

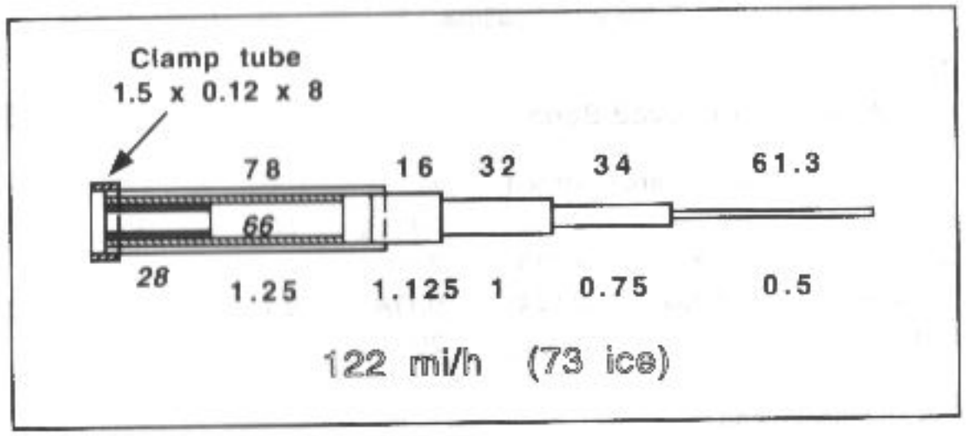


Fig 11-16—QHS N6ND520 element construction details. All values are in inches.

Table 11-36
QHS N6ND520 Monotaper (Cylindrical Element) Design (14.175 MHz)

Section	*Spacing, Inches*	*Location, Inches*	*Length, Inches*	*Cyl Length, Inches*	*Cyl Diam, Inches*	*X, Ω*
Reflector	—	26.38	221.23	210.40	0.923	+55.54
Driven	115.24	141.62	203.29	193.66	0.887	−29.37
Director 1	91.51	233.13	206.01	195.69	0.886	−19.22
Director 2	161.12	394.25	203.69	193.47	0.882	−30.38
Director 3	205.37	599.62	193.83	184.04	0.857	−77.75

Table 11-36 summarizes the underlying monotaper (cylindrical element) design. The "Location" column is the distance from the reflector end of the boom to the element centerline. In order to demonstrate the design techniques, I chose to use telescoping 6061-T6 aluminum pipe and tube for the very large boom. The sections are telescoped and then soundly bolted together. This is particularly important for the guyed-boom configuration with its axial compression force. The dimensions for the unguyed-boom configuration are listed in Table 11-37, and the boom construction is shown in Fig 11-17. Note the internal reinforcement at the boom tips for mounting the elements and compensator.

The boom survival speed can be increased substantially by using a triple-guyed configuration similar to that detailed for the Hy-Gain

Table 11-37
QHS N6ND520 Unguyed Boom

v_{max} = 112 mi/h; values are in inches.

Section	*1*	*2*	*3a*	*3b*	*3c*
Diameter	2.875	2.375	2	2	2
Thickness	0.203	0.154	0.174	0.116	0.058
Length	55	114	20	48	72*

*56 on reflector end

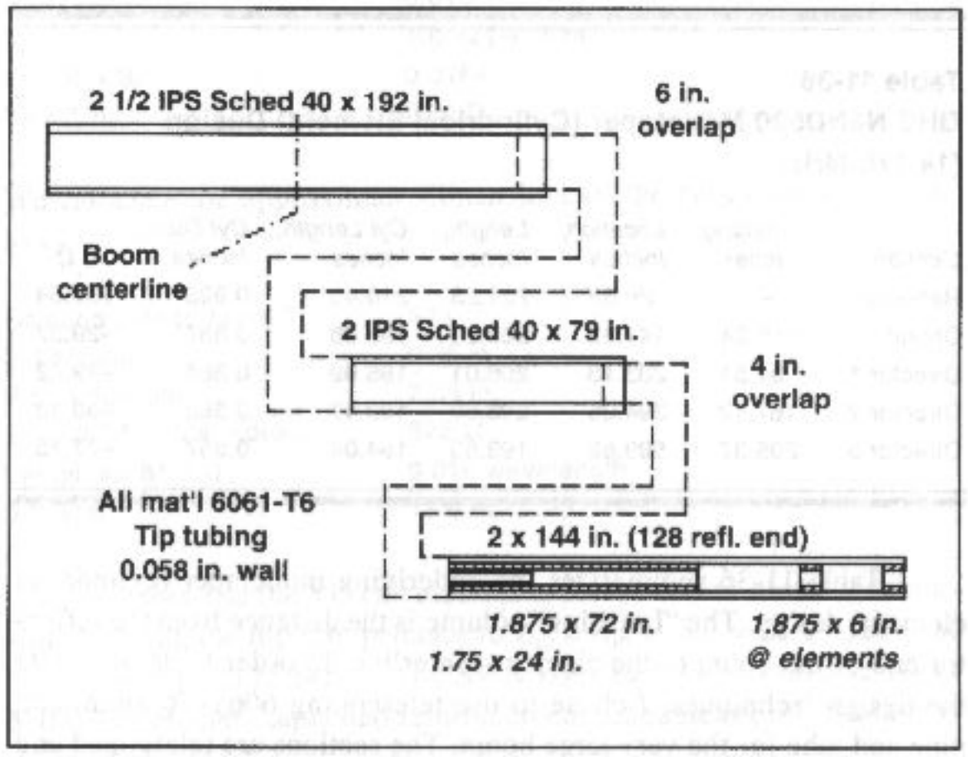


Fig 11-17—Construction details for the unguyed QHS N6ND520 boom.

205BA in Sec. 11.3.7. The boom dimensions are shown in Table 11-38, and the construction is detailed in Fig 11-18. The guy centers are supported 32 in. from the boom, and the guy points are 169 in. from the boom center, at the outer end of the second section.

Table 11-38
QHS N6ND520 Guyed Boom

v_{max} = 125 mi/h; values are in inches.

Section	*1*	*2*	*3a*	*3b*	*3c*
Diameter	2.875	2.375	2	2	2
Thickness	0.203	0.154	0.174	0.116	0.058
Length	96	73	20	48	72*

*56 on reflector end

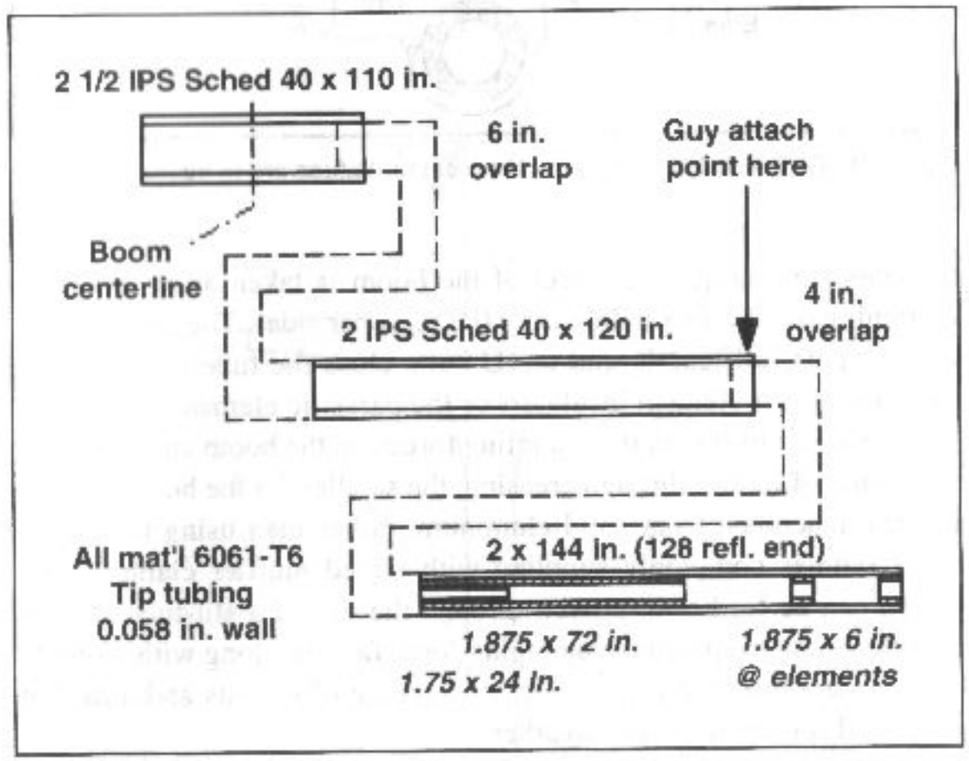


Fig 11-18—Construction details for the guyed QHS N6ND520 boom.

Because the boom diameter varies, the element clamps shown in Fig 11-19 are employed to attach the elements. The 3 × 0.3125-in. plates and 1.5-in.-diameter mounting sections have an equivalent electrical diameter of 2.73 in., and provide strength commensurate with the

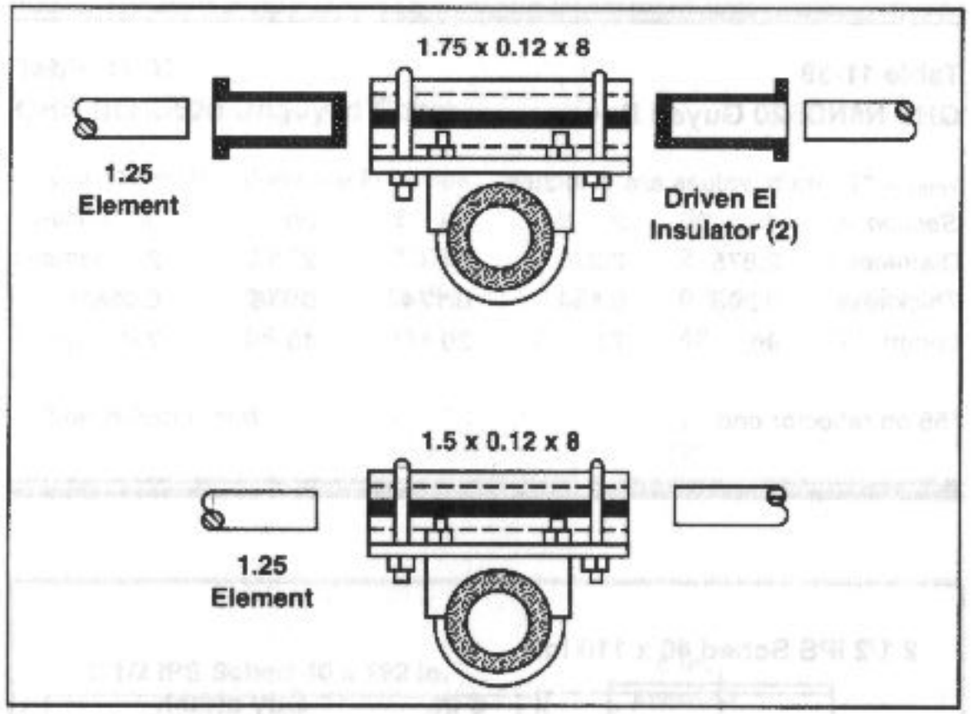


Fig 11-19—QHS N6ND520 element clamp detail. Values are in inches.

elements and boom. The effect of the boom is taken as an effective shortening of 6% of its diameter ($0.03D_{boom}$ per side). The large tube is slit along its full length, and the U bolts close the tube to clamp the Hy-Gain driven-element insulators or the parasitic elements.

In order to spread the mounting forces on the boom and avoid the possibility of failure through creasing, the saddles for the boom U bolts are best machined from solid aluminum, rather than using the sheet-metal saddles commonly supplied with plated muffler clamps. This design can be further improved through the use of a slit-tube section over the boom to spread the clamping force farther, along with larger U bolts. Tower manufacturers are a good source of U bolts and nuts that are plated for use in severe weather.

In order to increase the boom survival speed by reducing the compensator area required, I chose a boom length such that the element ensemble could be offset by 20 in. toward the director end. This is reflected in the element locations as measured from the reflector end of the boom before any shortening to compensate for the balun moment. The compensating element is located 12 in. inboard of Director 3. This antenna is shown in Fig 11-20.

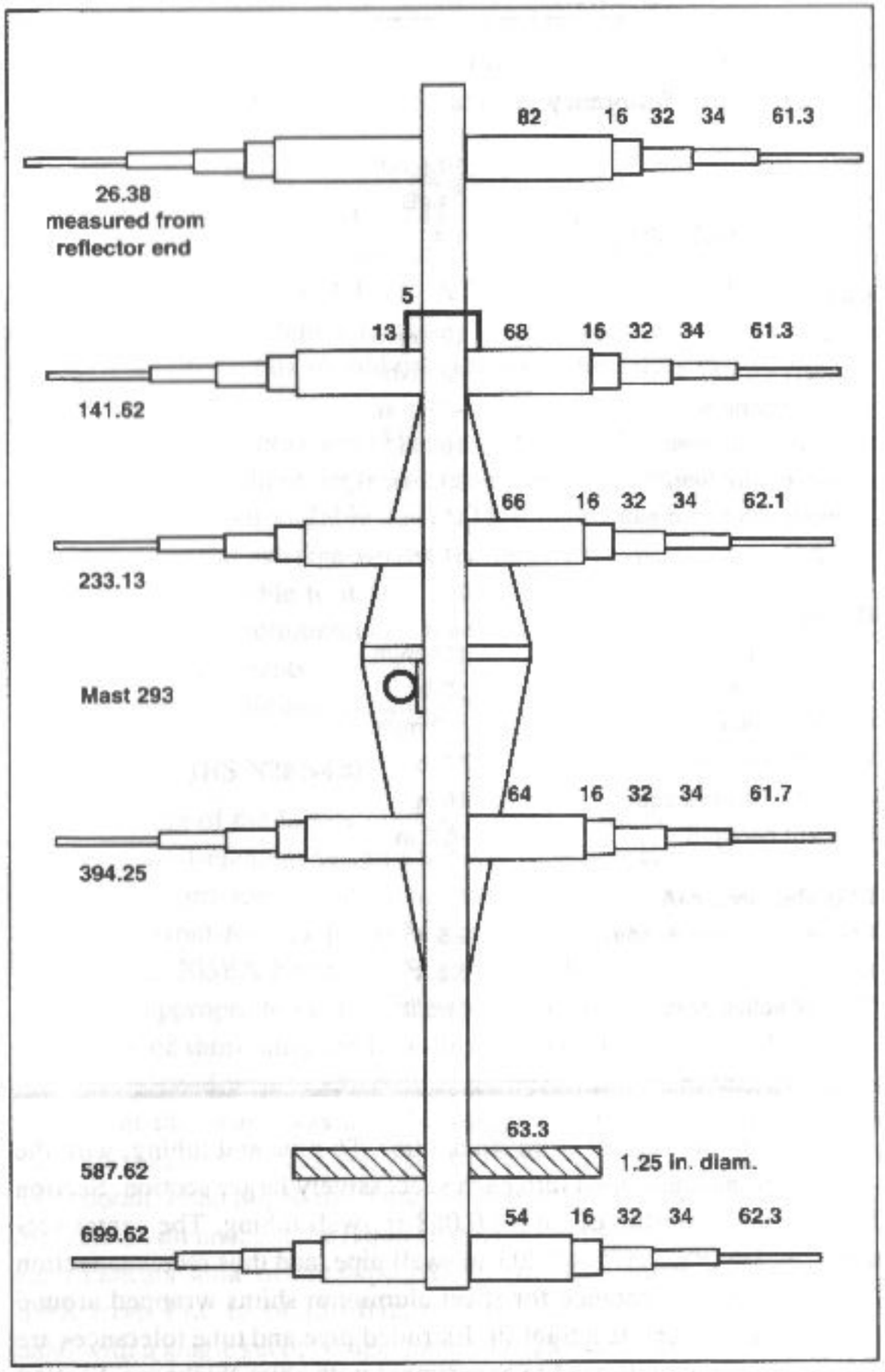


Fig 11-20—Dimensions for the QHS N6ND520 20-meter monoband array. Its calculated gain is 10.4 dBi, and v_{max} = 121 mi/h. All values are in inches.

Table 11-39
QHS N6ND520 Summary

Gain	10.4 dBi
F/B ratio	24 dB
SWR (14.0-14.35 MHz)	1.4
Elements	
v_{max}	121 mi/h
v_{ice} (0.5 in. ice)	72 mi/h
Moment centroid	–25.1 in.
Compensator area	160 in.^{2}
Compensator length	63.3 in.
Compensator diameter	1¼ in.
Weight	51 lb
Boom	
v_{max} guyed	125 mi/h
Weight, guyed	57 lb
v_{max} unguyed	112 mi/h
Weight, unguyed	62 lb
ΔLength, balun & coax	16 in.
Moment centroid	–6.8 in.
Effective area c_dA	
Elements + compensator	9.5 ft^2
Boom	6.8 ft^2
Total effective area	11.6 ft^2

The boom is built from stock 6061-T6 pipe and tubing, with the outer sections telescoped into each successively larger section. Section 3 is internally reinforced, using 0.058-in.-wall tubing. The center section uses 2½ IPS, 2.875 × 0.203-in.-wall pipe, and thus receives section 2 with adequate clearance for sheet aluminum shims wrapped around the inner tube to create a tight fit. Extruded pipe and tube tolerances are not exact, so you will need to experiment with shim thickness. Section 2 uses 2 IPS, 2.375 × 0.154-in.-wall pipe, which receives 2-in. tubing with the same type shim spacer. This boom weighs 57 lb in the guyed (125 mi/h) configuration and 62 lb in the unguyed (112 mi/h) configuration. The 20-in. longer director-end tip section compensates the unbalanced moment of the coax and balun, and makes the element offset possible.

The compression force in the guyed boom at v_{max} is nearly 1100 lb, and the guy force is 1110 lb. The guy supports must be capable of withstanding the compression force from both of the attached guys, which is approximately 420 lb. A 32-in. length of ¾ IPS (1.05 × 0.113-in.) aluminum pipe can support a calculated compression force of over 8800 lb, so this should provide adequate safety margin against support failure.

This antenna provides substantial gain while maintaining torque balance and a very high degree of wind survival. The antenna parameters are summarized in Table 11-39. This antenna is the largest I have considered for the extreme wind conditions we experience at our site. No doubt it is possible to design and construct much larger antennas, and I have seen commercially made log-periodic arrays that have 6-in.-diameter elements, each guyed separately. The same techniques described here can be used for any size antenna the builder can imagine.

Sec 11.3.10 QHS N2FB420

Another of the family of close-spaced reflector and first director designs is the 4-element design attributed to Wayne Hillenbrand, N2FB. This antenna provides a calculated gain of 9.5 dBi on a 0.5-λ boom with reasonable front-to-back performance. I was curious to see if it would work on the 205BA boom, and if more bandwidth could be obtained. Using the appropriate element clamp and boom correction, I tried the same trick of shortening the first director to improve bandwidth.

The broadbanding effect was not as marked as on the 5-element Yagis, but the result was a 4-element 20-meter Yagi that provides a calculated gain of 9.2 dBi and nearly 20 dB F/B on the Hy-Gain 205BA 34-ft boom. I had hoped the wind area would be somewhat less than the 5-element antenna, but the requirement for a substantial torque compensator cancels some of the expected benefit. This antenna offers a 1.55 SWR from 14.0 to 14.35 MHz, so it is usable over the full 20-meter band with a single set of dimensions. YO calculations show that, for X_{in} = –25 to –30 Ω, R_{in} = 25.3 Ω. For 5-in. balun leads and a 2-in. boom, we calculate the required X_{in} = –29 Ω. This value is then used in YO to

Table 11-40
QHS N2FB420 Taper Schedule (14.15 MHz)

Values are in inches.

Section	*0*	*1a*	*1b*	*2a*	*2b*
Diameter	1.5	1.25	1.25	1.125	1.125
Thickness	0.125	0.174	0.095	0.116	0.058
Reflector	3.75	34	14	27	19
Driven	3.75	21	14	27	19
Director 1	3.75	21	14	27	19
Director 2	2.625	7	14	27	19

Section	*3a*	*3b*	*4a*	*4b*	*5*
Diameter	0.875	0.875	0.625	0.625	0.4375
Thickness	0.084	0.049	0.131	0.035	0.049
Reflector	19.5	31	10	14	53.58
Driven	19.5	31	10	14	54.10
Director 1	19.5	31	10	14	49.13
Director 2	19.5	31	10	14	51.39

Table 11-41
QHS N2FB420 Elements (14.15 MHz)

Section	*Spacing, Inches*	*Location, Inches*	*Length, Inches*	*Cyl Length, Inches*	*Cyl Diam, Inches*	*X, Ω*
Reflector	—	10.46	222.38	211.03	0.869	+56.99
Driven	109.79	120.25	205.85	195.68	0.833	–21.76
Director 1	108.19	228.44	204.93	194.49	0.852	–27.47
Director 2	190.11	418.55	193.19	183.45	0.815	–83.49

set the driven-element length to provide $Z_{in} = 25.3 - j29\ \Omega$. The beta inductor is 5 in. × 13 in.

The element dimensions for EIA Zone C application are shown in Table 11-40. The elements are scaled to the proper length using the taper and clamp electrical diameter algorithms of Chapters 8 and 9. These elements, based on the 205BA++ design of Sec 11.3.7, provide

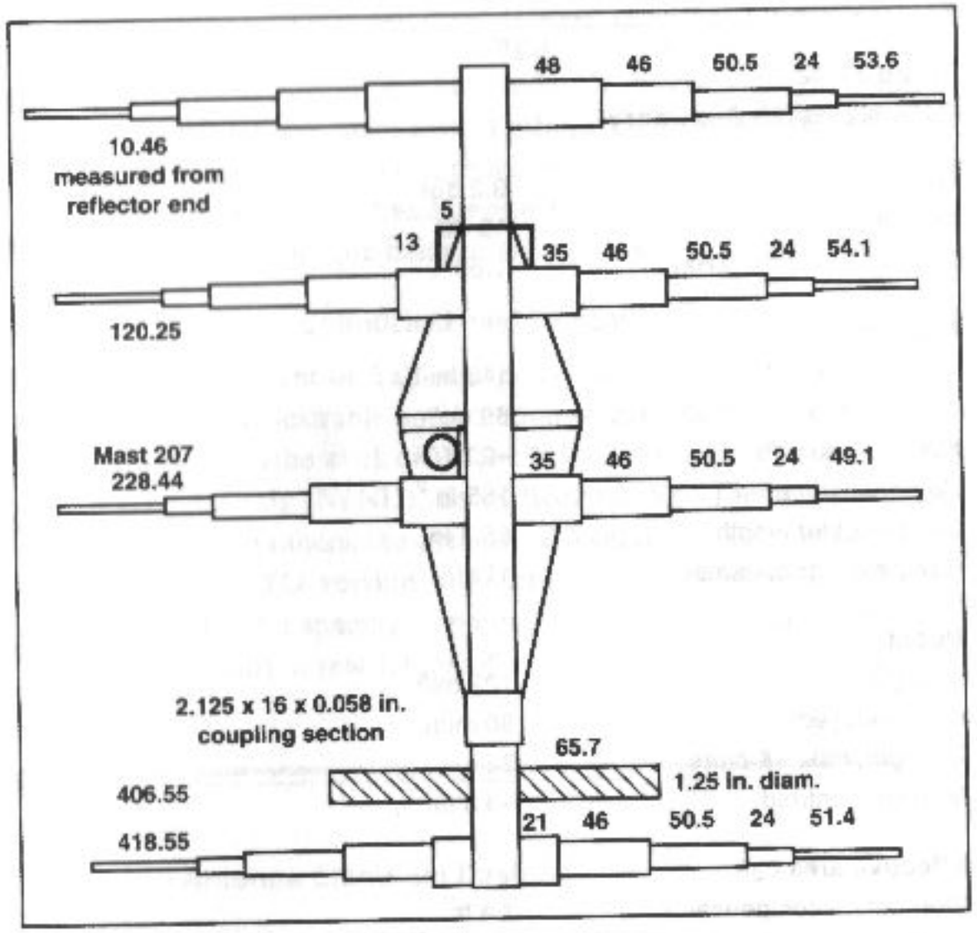


Fig 11-21—Dimensions for the QHS N2FB420 20-meter monoband array. The theoretical gain is 9.2 dBi, and v_{max} = 122 mi/h. All values are in inches.

v_{max} = 118 mi/h and v_{ice} = 69 mi/h. The larger element clamps are required on the reflector and first director. The equivalent cylindrical element lengths and reactances are shown in Table 11-41. The general layout of this antenna can be seen from Fig 11-21.

The boom is based on the Hy-Gain 205BA, with the director end lengthened 7 in. and strengthened by a 2⅛ × 0.058 × 18-in. sleeve bolted around the swage joint and inner end of the director-end boom tip section. The sleeve overlaps the inner section by 2 in., and the stock 79-in. tip section is bolted to the sleeve such that this end of the boom is lengthened by 7 in. The extension at the director end of the boom allows the elements to be offset in that direction to reduce the required area for the torque balancing element, and the 7-in. shorter length of the stock reflector end compensates for the moment of the balun and coax.

Table 11-42
QHS N2FB420 Summary

Gain	9.2 dBi
F/B ratio	19 dB
SWR (14.0-14.35 MHz)	1.55
Elements	
v_{max}	118 mi/h
v_{ice} (0.5 in. ice)	69 mi/h
Moment centroid	–23.1 in.
Compensator area	165 in.^{2}
Compensator length	65.7 in.
Compensator diameter	1¼ in.
Boom	
v_{max} guyed	122 mi/h
v_{max} unguyed	90 mi/h
ΔLength, balun & coax	7 in.
Moment centroid	–3.5 in.
Effective area c_dA	
Elements + compensator	7.4 ft^2
Boom	4.1 ft^2
Total effective area	8.5 ft^2

If this antenna is built without boom guying, v_{max} = 90 mi/h, and the swage at the outer end of the center section is the weak link. The boom requires triple guying to meet EIA RS-222-C Zone B and Zone C wind-speed survival. The guyed boom will survive v_{max} = 111 mi/h, and v_{max} = 122 mi/h with the addition of the tip joint strengthening sleeve. Extending the boom will result in increased gain, and boom lengths from 42 to 48 ft have given good results for 4-element 20-meter Yagis similar to this one. The parameters of this antenna are listed in Table 11-42. It may be difficult to find fiberglass tubing strong enough to make the required compensator lengths.

The additional area of the compensator removes some of the benefit sought in the 4-element antenna, but the result is indicative of the mechanical problems of gain maximization. It is possible that a

slightly reduced reflector length, using a small transmission-line inductor at the boom similar to the beta inductor, might reduce the torque imbalance without too much loss of efficiency. Several manufacturers are now introducing computer-optimized versions of their Yagi antennas; it is hoped that the electrical improvements will not be accompanied by loss of favorable torque balance and wind-survival capability.

Sec 11.4 Advanced Cylindrical Yagi Designs

With the advent of Yagi-design computer software, a family of designs offering a desirable combination of gain, pattern and SWR has emerged. Based on the work of Wayne Hillenbrand (N2FB), Dean Straw (N6BV), Dick Craig (N6ND), Brian Beezley (K6STI) and Mike Staal (K6MYC), among others, the cylindrical Yagi designs shown here were developed using YO version 4.0 software. All of these designs use nonuniform element spacing to provide the following characteristics.

1) Gain only a few tenths of a dB below the maximum for the boom length.

Table 11-43
Advanced Antenna Electrical Designs

10 Meters	*10M5*	*10M6*	
Boom length	26 ft 5 in.	35 ft 4 in.	
Gain	10.74 dBi	11.62 dBi	
SWR (28-28.7 MHz)	1.39	1.32	
F/B ratio	24 dB	26 dB	
15 Meters	*15M5*	*15M5L*	*15M6S*
Boom length	31 ft 11 in.	35 ft 4 in.	35 ft 4 in.
Gain	10.55 dBi	10.73 dBi	10.72 dBi
SWR (21-21.45 MHz)	1.23	1.29	1.29
F/B ratio	23 dB	24 dB	26 dB
20 Meters	*20M5M*	*20M5S*	*20M5*
Boom length	35 ft 4 in.	41 ft 11 in.	47 ft 9 in.
Gain	9.64 dBi	10.03 dBi	10.53 dBi
SWR (14-14.35 MHz)	1.34	1.26	1.33
F/B ratio	20 dB	23 dB	24 dB

Table 11-44
Element Dimensions for Advanced Antenna Designs

All values are in inches.

	Location	*Length*	*Location*	*Length*	*Location*	*Length*
10 Meters						
	——10M5——		——10M6——			
Diameter		0.625		0.625		
Reflector	0.00	104.843	0.00	105.577		
Driven	37.47	97.280	37.47	97.084		
Director 1	98.03	95.726	79.96	96.869		
Director 2	208.51	95.163	182.14	94.708		
Director 3	317.44	90.923	305.89	94.145		
Director 4			424.00	89.482		
15 Meters						
	——15M5——		——15M5L——		——15M6S——	
Diameter		0.625		0.625		0.625
Reflector	0.00	140.225	0.00	140.042	0.00	139.688
Driven	48.00	130.452	50.05	130.451	50.05	134.413
Director 1	112.00	129.645	130.94	128.462	108.24	129.327
Director 2	242.24	127.983	278.50	127.738	200.09	125.330
Director 3	382.97	123.209	424.00	122.270	296.40	126.797
Director 4					423.86	121.652
20 Meters						
	——20M5M——		——20M5S——		——20M5——	
Diameter		0.875		0.875		0.875
Reflector	0.00	211.237	0.00	211.237	0.00	209.962
Driven	48.00	195.510	73.00	195.510	74.94	195.497
Director 1	119.00	195.253	148.00	194.998	167.70	194.010
Director 2	261.30	193.491	310.00	193.491	362.73	191.837
Director 3	424.00	183.998	503.05	183.998	573.45	184.748

2) Extremely clean patterns with a front-to-back ratio of 20-27 dB.

3) Very broad SWR curves, covering the entire band with the SWR typically less than 1.35.

4) At least one design for each of the 10, 15 and 20-meter bands for construction on a 35-ft 8-in. boom (three 12-ft sections) designed to survive 120 mi/h wind speeds with typical element and compensator configurations.

The features of each of these designs are listed in Table 11-43. The 10M5 can be constructed on the Hy-Gain 155BA boom, and the 10M6 is designed to be built on the 35-ft 8-in. boom described later in this section. The 15M5 can be constructed on the Hy-Gain 204BA boom, and the 15M5L and 15M6S are designed to be built on the 35-ft 8-in. boom. The 20M5M is designed to be built on the 35-ft 8-in. boom. The 20M5S is designed for a 42-ft boom, and the 20M5 for a 48-ft boom. The development of the mechanical element design proceeds using the methods outlined earlier in this chapter.

An exploration of 10- and 20-meter antennas with the same element spacing as the 15M6S revealed that useful antennas can be built for all three bands with exactly the same element positions. This holds promise for the development of a large tribander, which I want to explore in the future.

The element spacings and lengths for the cylindrical element designs are given in Table 11-44. Conversion to physical elements is accomplished as shown in the examples earlier in this chapter. All dimensions are in inches; the first column for each design is the element location, and the second is the element length per side. These dimensions include driven-element shortening to the approximate length required for beta-match reactance. If desired, similar designs can be derived for zero input reactance by increasing the driven-element length. The 10- and 15-meter designs are based on 0.625-in.-diameter cylindrical elements in order to approximate the typical average diameter of the smaller elements used on these bands.

The smallest 20-meter design is a similar size to the commercially available Hy-Gain 205CA, and the midsize antenna is comparable to the KLM 20-5, but provides substantially more gain and broader SWR bandwidth without the complexity of dual feed. The largest 20-meter design here is quite similar to the N6ND520 design shown earlier in this chapter, but offers a slightly superior electrical design.

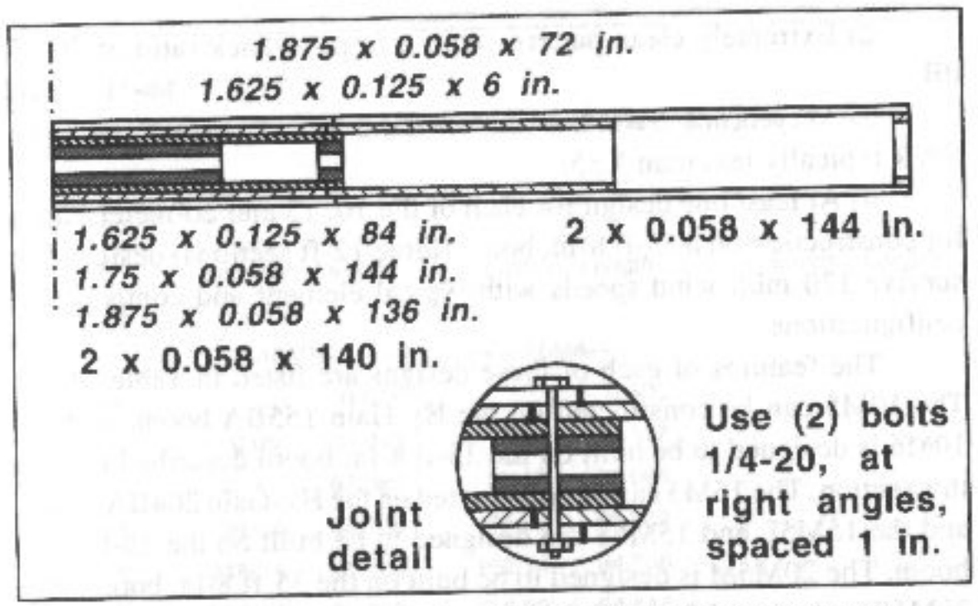


Fig 11-22—35-ft 8-in. × 2-in. boom for 10M6, 15M5L or 20M5M antennas. These booms are rated at 120 mi/h. For the 20M5M, the boom is guyed at the outer end of the double-wall section (140 in.).

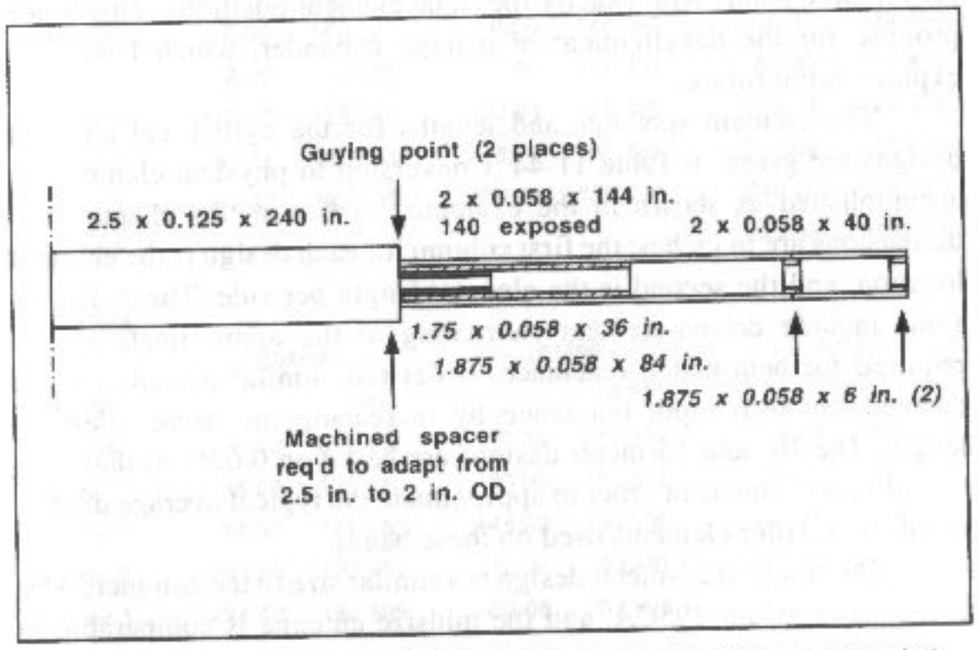


Fig 11-23—48-ft × 2.5-in. boom for the 20M5 antenna. The boom is guyed at the outer end of the triple-wall section (152 in.).

Table 11-45
Boom Materials

All dimensions are in inches. Materials are 6061-T6 or 6063-T832 aluminum.

Diameter	*Wall Thickness*	*Length*	*Quantity*
36-ft Boom			
1.625	0.125	84	1
1.625	0.125	6	2
1.75	0.058	144	1
1.875	0.058	136	1
1.875	0.058	72	2
1.875	0.058	4	2
2.0	0.058	140	1
2.0	0.058	144	2
48-ft Boom			
1.75	0.058	36	2
1.875	0.058	84	2
1.875	0.058	6	4
2.0	0.058	144	2
2.0	0.058	40	2
2.5	0.125	240	1
2-2.5 in. machined spacer			2

A 35-ft 8-in. × 2-in.-diameter unguyed boom design was developed for use with these antennas. For 20 meters, with the 20M5M design for example, the same boom can be guyed to provide the same 120-mi/h survival. If unguyed, the survival for 20-meter elements is 106 mi/h. The details of this boom are shown in Fig 11-22, which shows half of the symmetrical boom construction. Materials for this boom are listed in Table 11-45.

Also useful is a high-strength 48-ft boom, constructed by using a machined adapter to fit 2-in.-diameter tubing into an extruded 2.5 × 0.125 × 240-in. center section. This boom provides 120-mi/h survival for 20-meter elements when guyed as shown in Fig 11-23. It can be further strengthened by using a 0.188-in.-wall center section and a 2.75

× 0.12 × 96-in. sleeve at the center to provide unguyed 121-mi/h survival for 15-meter elements. The required materials for this boom are also listed in Table 11-45.

Even if not required for survival, the use of a steel upper boom guy removes droop and improves the visual appearance of antennas with these long booms. A suitable compensator material is 2½-in. IPS ABS black plastic drain pipe, which can be stiffened by inserting 2-in. IPS ABS pipe inside. Internal end caps are available from plumbing suppliers.

An M^2 10M7 and a 15M6 antenna that Mike Staal, K6MYC, built for me on booms of this 48-ft design have survived their first winter storms with no problems. They employ an element ensemble offset to the director end and reflector-end boom tails for compensation (with coax and balun aiding).

Sec 11.5 Manufacturer's Mechanical Antenna Parts

An excellent source of mechanical parts such as castings and stampings is the parts bin of a major manufacturer. Most manufacturers sell individual parts as spares, although the pricing is of course not as attractive as pricing for complete antennas.

Over the years I have made use of parts available from Telex/Hy-Gain, and with their permission have included a brief listing of useful parts. This list appears in Table 11-46. The parts can be ordered from the manufacturer, and most can be delivered by UPS.

Sec 11.6 Mast Design Examples

Now that we have explored some specific antennas, the next step is to determine the capabilities of the masts we use. The mast I use with the 120-ft Rohn 45 tower is a 15 ft × 2 × 0.25-in.-wall 1026 DOM mast rated by the supplier at a yield strength of σ_{max} = 88000 lb/in.^{2}. The mast extends 10 ft above the tower bearing support.

I wanted to know the survival strength of a stack on this mast of the 40-meter two-element beam over the 20-meter five-element monobander. The 20-meter monobander, a Hy-Gain 205BA, is mounted at the tower top, so the question was how high on the mast could I mount the Cushcraft 40-2CD antenna without exceeding the strength limits of the mast? I intended to mount the 205BA just above the tower top and wanted to mount the 40-2CD as close as possible to the mast top.

For reference in this and related calculations, the approximate element and boom-plane effective areas of a number of commercially

Table 11-46
Partial List of Hy-Gain Parts

Hy-Gain reserves the right to modify parts specifications or to discontinue any items listed.

Type	*Item*	*PN*	*Where Used*
Mast clamp	Cast (2)	102734	All
Mast clamp	Casting-boom bracket	172735	All
Mast clamp	Boom-bracket clamp	172732	All
El-Bm clamp	⅞ in., el no. 4 (2)	163764	105/155 (8)
El-Bm clamp	1¼ in., el no. 13 (2)	165919	205 (8), 105/155 (2)
El-Bm clamp	1½ in., el no. 14 (2)	165920	205 (2)
Boom	2 × 0.049 × 69½ in.	171166	TH7
Boom	2 × 0.049 × 75 in.	173754	105
Boom	2 × 0.049 × 79 in.	171029	204, 205, 155
Boom	2 × 0.049 × 83 in.	171538	7-2
Boom, swaged	2 × 0.049 × 75 in.	175661	105
Boom, swaged	2 × 0.12 × 54¾ in.	171539	7-2
Boom, swaged	2 × 0.12 × 81 in.	170424	205
Boom, swaged	2 × 0.12 × 81 in.	171540	7-3
Boom, swaged	2 × 0.12 × 79½ in.	872098	TH7
With insert	1¾ × 0.083 × 36 in.		
Boom, swaged	2 × 0.12 × 81 in.	871048	204, 155
With insert	1¾ × 0.083 × 36 in.		
Boom, swaged	2 × 0.12 × 110 in.	871017	205
With insert	1¾ × 0.083 × 96 in.		
Boom insert	1¾ × 0.083 × 71 in.	171541	7-2
Element	7/16 × 0.028 × 48 in.	174865	105
Element	7/16 × 0.035 × 55 in.	174939	105
Element	7/16 × 0.035 × 58 in.	171533	105
Element	7/16 × 0.035 × 68 in.	174868	155
Element, swaged	⅝ × 0.035 × 24 in.	190004	105
Element, swaged	⅝ × 0.035 × 26 in.	190006	105
Element, swaged	⅞ × 0.049 × 84 in.	174862	155
Element, swaged	⅞ × 0.049 × 72 in.	190100	155
Element, swaged	⅞ × 0.049 × 55 in.	190206	205
Element, swaged	⅞ × 0.049 × 55 in.	191007	7-2
Element, swaged	1⅛ × 0.058 × 48 in.	190300	205
Insulator, DE	⅞ × 1¼ in.	463767	155, 105
Insulator, DE	1¼ × 1½ in.	465833	7-2, 204, 205
Insulator, EI	1 × ⅞ in.	460316	7-2
Insulator	⅛-in. rod support	463642	7-2
Caplug	7/16 in.	455644	All elements
Caplug	2 in.	455625	All booms
Manual	Instructions	801965	7-2
Manual	Instructions	801827	205
Manual	Instructions	801998	155
Manual	Instructions	800041	105
Manual	Instructions	801832	204

available antennas are listed in Table 11-47. Using the mast spreadsheet of Chapter 7, we find that the setup with the 40-meter antenna at the top of the mast is good for 123 mi/h, and will generate a side-force of 1206 lb at the tower top at that wind speed. This calculation is summarized in Table 11-48. The strength of the tower top itself is a concern, and the tower installation should be properly engineered to survive.

Table 11-47
Antenna Effective Areas

c_d = 0.67

Antenna	*El A_{eff}*	*Boom A_{eff}*
105BA	2.8 ft^2	2.8 ft^2
155BA	4.3	3.0
204BA	6.4	3.0
205BA	8.1	3.9
CD78	2.8	0.0
KLM 40-2	4.5	3.7
KT34 XA	7.2	5.4
40-2CD	4.5	2.6

The same configuration could be used to mount a large tribander at the tower top, and the mast can be extended substantially higher for lower wind ratings. For example, the same mast material can support the 40-2CD at 15 ft above the tower top at a wind speed of 98 mi/h, at which speed it generates 780 lb side-force. By way of comparison, the same 15-ft mast dimensions using a σ_{max} = 35000 lb/in.^{2}

Table 11-48
Dimensions for 40-2CD over 205BA or Tribander

Mast Dimensions

	Diam, in.	*Wall, in.*	*Length, in*	*σ_{max}, lb/in.^{2}*	*S, in.^{3}*	*A, in.^{2}*	*Wt, lb*	*v_{max}, mi/h*
One section	2	0.25	120	88000	0.54	1.37	47	123

Antenna Dimensions and Area Moments

Ant No.	*Antenna*	*El A_{eff}, ft^2*	*Boom A_{eff}, ft^2*	*Height, in.*	*El moment, in.^{3}*	*Boom moment, in.^{3}*
1	40-2CD	4.5	2.6	117	113724	65707
2	205BA	8.1	3.9	6	10498	5054
Mast		1.1	1.1	120	14472	14472
Total		13.7	7.6		138694	85233

material such as HREW will fail at a relatively low wind speed of 63 mi/h.

Now let's look at the mast for the 70-ft Skyneedle. With the same mast material as the example above, we will not be able to stack the three monobanders at the 7-ft and 9-ft spacings recommended by the manufacturer. The calculated RS-222-C survival speed using this mast in a 16-ft stack is only 83 mi/h. I solved the problem initially by stacking the 10-meter Yagi between the 20- and 15-meter Yagis in a short stack, which results in a 111-mi/h survival speed. The calculations for this case are summarized in Table 11-49.

This is a marginal setup for my site, but I resolved to park the antennas at 90° to the wind during the storm season; at that angle the short-stack mast will survive 144-mi/h fastest-mile winds and will generate a side-force of 600 lb at a wind speed of 122 mi/h. The Skyneedle will fail if fully extended at 122 mi/h, as it is rated at only 400 lb side-force at the top when fully extended. Ignoring the reduction of its own area and moment when partially retracted, we calculate that the Skyneedle must be kept below 400/600 × 70 = 46 ft during the storm season.

Hy-Gain recommends stacking these antennas on a 16-ft mast. As we learned in Chapter 7, steel tubing is available in 0.120-in. wall

Table 11-49
Dimensions for Short Stack

Mast Dimensions

	Diam, in.	*Wall, in.*	*Length, in*	*σ_{max}, lb/in.^{2}*	*S, in.^{3}*	*A, in.^{2}*	*Wt, lb*	*v_{max}, mi/h*
One section	2	0.25	132	88000	0.54	1.37	47	111

Antenna Dimensions and Area Moments

Ant No.	*Antenna*	*El A_{eff}, ft^2*	*Boom A_{eff}, ft^2*	*Height, in.*	*El moment, in.^{3}*	*Boom moment, in.^{3}*
1	155BA	4.3	3.0	120	111456	77760
2	105BA	2.8	2.8	42	25402	25402
3	204BA	6.4	3.0	6	8294	3888
Mast		1.2	1.2	132	17511	17511
Total		14.7	10.0		162663	124561

Table 11-50
Antenna Areas and Locations on Mast

Ant No.	*Antenna*	*El A_{eff}, ft^2*	*Boom A_{eff}, ft^2*	*Height, in.*
1	105BA	2.8	2.8	198
2	155BA	4.3	3.0	114
3	204BA	6.4	3.0	6

Table 11-51
Mast Dimensions for 16-Foot Hy-Gain Stack

Section	*Diam, in.*	*Wall, in.*	*Length, in.*	*σ_{max}, lb/in.^{2}*	*S, in.^{3}*	*A, in.^{2}*	*Wt, lb*	*v_{max}, mi/h*
Top Sec	2	0.12	108	88000	0.31	0.71	22	107
Section 2	2	0.24	48	88000	0.53	1.33	18	101
Section 3	2	0.36	24	88000	0.65	1.85	13	101
Section 4	2	0.48	12	88000	0.73	2.29	8	102
Section 5	2	0.73	9	88000	0.78	2.91	7	100
Total			201				68	100

thickness for telescoping. Using the spreadsheets for mast strength, we can design a composite mast that will permit stacking 20-, 15- and 10-meter monobanders at 9 ft and 7 ft spacing while surviving 100 mi/h winds. The locations and effective areas of the antennas are summarized in Table 11-50.

One internally tapered mast example, which has a survival speed equal to that easily realized for the antennas themselves, is constructed of unannealed 1026 DOM with a yield strength of $\sigma_{max} = 88000$ lb/in.^{2}, which is welded at all joints before galvanizing. As shown in Chapter 4, because of the cylindrical symmetry of the structure, the welds are not required to carry shear force from bending, but are for the purpose of preventing slip of the various sections; the outermost section can easily carry the axial component of mast and antenna weight. Galvanizing should be accomplished so as to preclude seepage of moisture between the sections, and a reliable top cap will be helpful as well. The

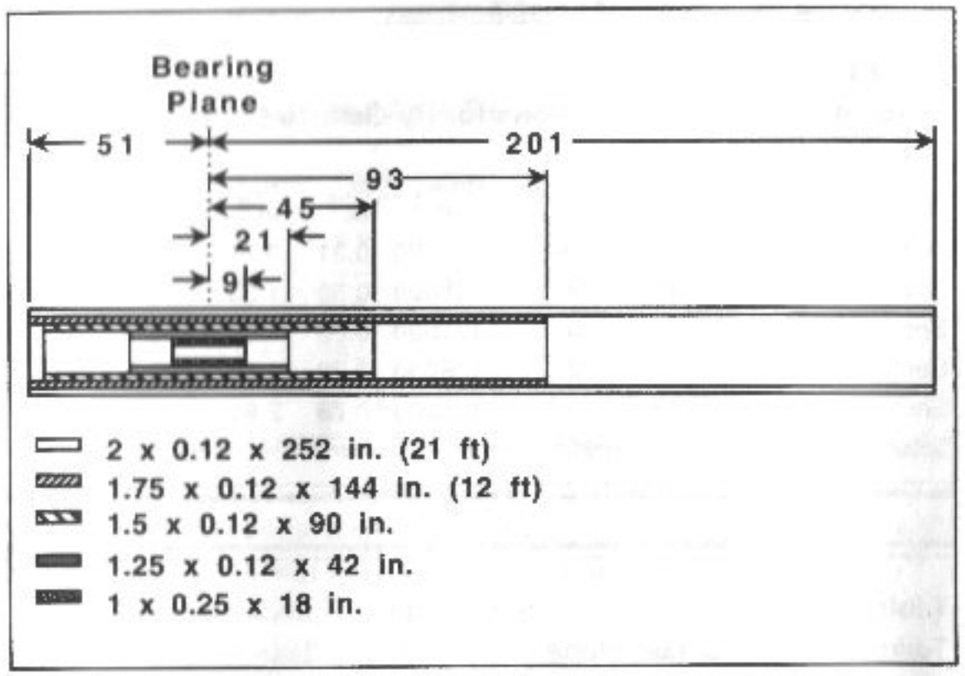


Fig 11-24—Reinforced mast for Hy-Gain stack.

mast dimensions and wind survival speeds are summarized in Table 11-51.

The mast must be equally strong below the tower bearing, and should have as much length as possible below the bearing plane. Tubing is typically available in lengths of 21 to 22 ft, and the dimensions for a 21-ft overall-length mast are shown in Fig 11-24. If the same mast is constructed of 4130 CD chrome-moly and is hardened to a yield strength of $\sigma_{max} = 125{,}000$ lb/in.^{2} after welding but before galvanizing, it will survive 119-mi/h winds with the 16-ft Hy-Gain stack. The calculations for this material are summarized in Table 11-52. In either case the mast will weigh approximately 69 lb. This second mast will be constructed for future use at my site, as it offers wind survival speeds commensurate with the reinforced antennas described earlier in this chapter.

Even this very strong mast configuration, if constructed from a material such as HREW with a yield strength of $\sigma_{max} = 35000$ lb/in.^{2}, will survive only a 63 mi/h fastest-mile wind speed. The importance of high-strength materials in aggressive mast applications is clear from these examples, and the use of the far stronger 3-in. mast design should be considered.

Table 11-52
Chrome-Moly Mast Dimensions for Hy-Gain 16-Foot Stack

Section	Diam, in.	Wall, in.	Length, in.	σ_{max}, lb/in.^{2}	S, in.^{3}	A, in.^{2}	Wt, lb	v_{max}, mi/h
Top Sec	2	0.12	108	125000	0.31	0.71	22	128
Section 2	2	0.24	48	125000	0.53	1.33	18	120
Section 3	2	0.36	24	125000	0.65	1.85	13	121
Section 4	2	0.48	12	125000	0.73	2.29	8	121
Section 5	2	0.73	9	125000	0.78	2.91	7	120
Total			201				68	120

Table 11-53
Tower Data and Dimensions

Type	Mom. max, ft-lb	Force per ft, lb	Wt per Sec, lb	Equiv Diam, in.	No. of Secs	Length, in.	Weight, lb	v_{max}, mi/h
Rohn 25	5130	12.70	40	4.6	2	240	80	68
Rohn 45	9610	14.63	70	5.3	2	240	140	91
Rohn 55	17160	16.72	95	6.0	2	240	190	119

Mast-strength limitations lead to the application of rotating tower sections as a means of stacking multiple antennas on a single tower. The increasing popularity of this configuration suggests the consideration of rotating towers for future projects at my site. A first question is, which tower design should be used as a basis? As an example, let us postulate a moderate-size stacked installation with a 20-meter monobander at the top, a 15-meter monobander in the middle, and a 10-meter monobander at the guy ring. Using the spreadsheet methods of Chapter 7, we derive the survival speeds for the cantilever section above the highest guy ring. These survival speeds are calculated by the method of EIA RS-222-C, and are shown in Table 11-53 for the dimensions shown in Table 11-54.

We can see that, for this type of cantilever condition, the strongest tower sections offer a substantial advantage, and are the only installation that will survive at my particular site. I have treated here only the question of survival of the tower sections cantilevered above the upper

Table 11-54
Antenna Dimensions and Area Moments

Ant No.	Antenna	EI A_{eff}, ft^2	Boom A_{eff},ft^2	Height, in.	EI moment, in.^{3}	Boom moment,in.^{3}	Height, ft
1	205BA	8.1	3.9	222	388411	187013	18.5
2	155BA	4.3	3.0	96	89165	62208	8
3	105BA	2.8	2.8	6	3629	3629	0.5
Tower		6.7	6.7	240	173353	173353	
Total		21.9	16.4		654558	426203	

guy ring; a thorough analysis of the entire installation is strongly recommended and will typically be required for permit application.

Under the same conditions, replacing the upper tower section with a 2×0.25-in.-wall 1026 DOM mast will increase v_{max} for the Rohn 45 case from 91 mi/h to 98 mi/h. The survival-speed increase permits Zone B application of this configuration, but is not enough to qualify the tower section/mast combination for extreme sites or for antenna configurations with heavier wind loads. The conclusion for my own site is clearly to use only the heaviest available tower sections, so I will use Rohn 55 for future installations.

Sec 11.7 Conclusions and Suggestions for the Future

I've built and installed most of the commercial antenna examples on the masts shown in this chapter. Initial results are favorable—we're enjoying multimillion-point contest results and nothing has broken yet! In a recent contest I had the satisfaction of watching my visiting friends run contacts at 100 per hour while a 100-mi/h storm raged outside. I'm sure there is much more to learn, and the examples in this chapter are not meant to be the last word, but rather to show how to use the techniques developed in this book.

You should now be able to calculate the expected wind speed for your site, and you can use that information to analyze and design elements, clamps, booms and masts for your own use. You should now be conversant with the proper alloys, so you can order your own material and evaluate the claims of antenna and mast suppliers. If you would enjoy further reading on the subject of strength of structures, I recommend a book subtitled "Why Things Don't Fall Down."[6]

I have found that torque balance is key to rotator survival, and have applied the rubber "flexdisk" torque isolator. It would make sense to increase the diameter of directors, particularly in a tribander, to create torque balance without the need for a compensating element (a friend even suggests mounting a 2-meter Yagi at the director end to provide some balance).

If you use a computer to analyze electrical performance, you can include the Jaggard equivalent diameter algorithm and the Schelkunoff taper algorithm. You should now be able to understand the impact of lead inductance and to determine the proper driven-element length for beta match application. The information on site selection for antennas can help provide a competitive signal for those able to take advantage of sloping foreground terrain.

This concludes a weekend project that has grown to take more than two years! This book was written in the hope that it would prove useful to individuals who wish to develop better antennas, as well as to stimulate consumers who can be educated to demand even better quality from manufacturers.

Additionally, I hope that authors of existing antenna design programs can incorporate much of this work into their software,[7] and that this book will act as inspiration for further work in areas of antenna design and siting. Analysis is easy while synthesis is difficult, so I would expect that the best ideas are yet to come. An idea that interests me is the application of asymmetrical-diameter boom design to provide for element offset to balance element moment while maintaining boom moment balance. Also, I believe that the terraced ground model used in MININEC and its derived design programs does not adequately represent the effects of sloping foreground, and that this concept should also be developed further.

I would also enjoy seeing the development of multiband antennas pursued further. It seems that the presence of lower frequency directors compromises the performance of the higher band elements of interlaced monobanders[8] which might have inherent torque balance, while the losses in traps reduce the performance of trap multibanders. It should be possible to include trap capacitors inside the elements for electrical designs such as the KT34, and I am curious whether distributed or coiled inductors have higher Q. Perhaps open sleeve construction[9] of directors can solve the problems of interlacing. A final interest is the application

of broadband matching techniques to Yagi antennas, in a similar way to those used for broadbanding 80-meter dipoles.[10]

You can see that there is much more left to be done! I hope this has been useful to you. I'll see you in the pileups!

References

[1] L. A. Moxon, *HF Antennas for All Locations* (Potters Bar, Herts: RSGB, 1982), p 28, Fig 3-12.

[2] D. Craig, "Modifications for the Hy-Gain 105BA," *National Contest Journal*, Nov/Dec 1989, pp 11-12.

[3] Discussed in D. Pruett, "Contest Aerials," *National Contest Journal*, Sep/Oct 1989, pp 23-26.

[4] See reference 3.

[5] B. Beezley, *YO* (Yagi optimizer computer program), 507½ Taylor St, Vista, CA 92084, 1989.

[6] J. E. Gordon, *Structures, or Why Things Don't Fall Down* (New York: Da Capo Press, 1978).

[7] R. D. Straw, "Computers and Ham Radio Contesting, Part 3—Antenna Design and Optimization by Computer," *National Contest Journal*, Nov/Dec 1990, p 9.

[8] D. Rodman, "A High Performance Duoband Yagi for 15 and 20 Meters," *Ham Radio*, Aug 1989, pp 41, 43, 44.

[9] R. A. Cox, "The Open-Sleeve Antenna," in G. Hall, ed, *The ARRL Antenna Book*, 16th Ed (Newington: ARRL, 1991), p 7-4ff.

[10] F. Witt, "The Coaxial Resonator Match," in G. Hall, ed, *The ARRL Antenna Compendium, Volume 2* (Newington: ARRL, 1989), p 110.

Index

A

Antenna:
- CD-78: 7-15, 7-18
- KT34 XA: 7-15, 7-18
- QHS N2FB420: 11-55ff
- QHS N6ND510: 11-41ff
- QHS N6ND520: 11-47ff
- TH5: 11-11ff
- TH7: 6-4, 7-19, 11-19ff
- 40-2: 7-15, 7-18
- 40-2CD: 11-5
- 105BA: 6-9, 7-12, 11-24ff
- 155BA: 6-9, 7-12, 11-28ff
- 204BA: 6-9, 7-12, 9-6ff, 11-32ff
- 205BA: 6-5, 6-9, 11-36ff

Antenna siting: 10-1
Area: 6-1
- Boom: 6-21
- Effective: 6-3
- Element: 6-1ff
- 205BA element and boom: 6-5

B

Beacons: 10-9
Beam-column: 6-33ff
Bending stress: 3-6
Beta match: 9-9ff
- Beta-balun: 6-16
- QHS N6ND510: 11-42
- QHS N6ND520: 11-47
- With series inductance: 9-13ff

Boom:
- Asymmetrical: 11-45
- Attachment to mast: 7-21ff
- Center reinforcement: 6-26, 11-20, 11-25
- Center strength: 6-26
- Guyed survival calculation: 6-34ff
- Guying: 6-28ff
- Joining sections: 6-41
- Pipe construction: 11-54
- QHS N6ND520: 11-50
- Sleeve moment compensator: 11-28
- 36 ft: 11-62
- 48 ft: 11-62

Boom strength calculation: 6-18ff
Broadband matching: 10-16

C

Capacitance:
- Insulated element clamp: 9-8ff

Catenary: 10-21
Centroid: 6-18
Chain drive: 7-27
Clamp:
- Channel: 5-18ff
- Element-to-boom: 5-1ff
- Equivalent diameter: 9-1ff
- Hy-Gain: 5-9ff
- Hy-Gain insulated: 9-8
- Hy-Gain insulated reactance table: 9-9
- Insulated: 5-20ff
- Pipe railing: 5-19
- Plate: 5-14ff
- QHS N6ND520 element: 11-52
- Section modulus: 5-3
- Wind angle: 5-4

Coaxial cable: 10-16ff
Column:
- Compression: 6-32

Compensating element: 6-22ff
Computer Program:
- Further studies: 10-15
- MININEC: 1-8
- MN: 8-27
- YO: 11-59

Concentric tubes: 4-23ff
Concrete: 10-23
Contest tips: 10-18ff
Corrosion: 3-12
Cylindrical Yagi designs: 11-59ff
- 10M5: 11-60
- 10M6: 11-60
- 15M5: 11-60
- 15M5L: 11-60
- 15M6S: 11-60
- 20M5: 11-60
- 20M5M: 11-60
- 20M5S: 11-60

D

Dimple: 4-38
Dipole:
- Characteristic impedance: 8-7ff
- Conical: 8-8
- Cylindrical: 8-11
- Cylindrical equivalent to tapered: 8-23
- Effect of shape: 8-7ff
- Equivalent shapes: 8-4ff
- Reactance: 8-1ff
- Spheroidal: 8-10
- Step tapered: 8-18

E

Effective area: 6-1ff
- Wind angle: 6-9

Electrolytic potential: 3-12
Element:
- Area: 5-6
- Clamp to boom: 5-1ff
- Cylindrical: 4-3ff
- Effective area: 6-3
- Guyed: 6-45ff
- Moments: 5-6ff
- Multisection tapered: 4-12ff
- Projected area: 4-1
- Reinforcement: 4-18ff
- Triband: 4-33ff
- Wind Force: 4-1ff
- 2-section tapered: 4-8
- 10 meter: 4-32ff
- 15 meter: 4-28ff
- 20 meter: 4-20ff
- 40 meter: 4-36ff

Equivalent diameter: 9-1ff
Equivalent radius: 9-4

F

Fasteners: 3-8ff
Fatigue: 3-7, 4-38
- Hole location to avoid: 4-38

Force:
- Cantilever structures: 3-3

Foreground shadowing: 10-6

G

Gain:
- Comparison using fading statistics: 10-8
- Ground reflection: 10-3

Ground reflection: 10-1ff
- Brewster angle: 10-2
- Effect on resonance: 10-11ff
- Foreground shadowing: 10-6
- Fresnel reflection zone: 10-7
- Gain: 10-2ff
- Sloping foreground: 10-2ff
- Smoothness criterion: 10-2
- Stacking gain: 10-14

Guy:
- Attachment: 6-39
- Cable strength: 6-48
- Element: 6-45
- Guy support offset: 11-38
- Side guy support: 6-40, 11-38
- Support strength: 6-37

Guyed Boom: 6-28ff

H

Hole:
- Drilling in round tube: 6-42ff
- Light-drive fit: 6-43
- Stress riser: 4-38

Hose clamps: 3-9

I

Ice loading: 2-21, 4-5
Installation safety: 10-20
Isolator:
- Torque: 7-28

L

Lawson algorithm spreadsheet: 9-7

M

Manufacturer's antenna parts: 11-65
Mast:
- Attachment of boom: 7-21ff
- Hy-Gain clamp: 7-23
- Reinforced: 7-20, 11-69
- Section modulus table: 7-19
- Stacked antennas: 7-10ff, 11-64ff

Mast moment: 7-6ff
Moment: 3-2
- About mast: 6-9ff
- Boom: 6-21
- Boom bending: 6-16ff
- Cantilever beam: 6-17
- End-supported beam: 6-30
- Intermediate-supported beam: 6-30
- Mast bending: 7-6ff

Moment of area: 4-2
Moment of force: 4-2
- Maximum: 5-3

Moment of inertia: 3-3

N

Northern California DX Foundation:
- Beacons: 10-9

P

Pipe: 3-11ff
- Steel: 7-2ff

Plastic insulation, testing: 5-22
Power measurement: 10-18

R

Reactance:
- Dipole near resonance: 8-1ff
- Element section length: 9-16ff

Resonance:
- Percent shortening: 8-12

Reynolds number: 2-17
Rivets: 3-9
Roller chain: 7-27
Rotator: 7-24ff
- Antenna moment of inertia: 7-24
- Torque: 7-25
- Torque table: 7-26

S

Safety: 10-20
Schelkunoff algorithm spreadsheet: 9-6
Section modulus: 3-3ff, 7-1
- Hollow cylinder: 3-5
- Mast table: 7-19

Shear flow: 4-19
Siting:
- Foreground slope: 1-10, 2-9, 10-1ff
- Ground roughness: 2-11
- Hilltop: 1-2

Spreadsheet:
- Boom: 6-18ff
- Constructing: 4-12ff
- Element strength: 4-12ff
- Element taper algorithm: 8-22, 9-6ff
- Mast: 7-10ff

Standards:
- British Standards Institution CP3: 2-7
- Comparison: 2-19ff
- EIA RS-222-C: 2-6ff
- EIA-222-D: 2-6
- Los Angeles County: 7-23
- Uniform Building Code (UBC): 2-8

Steel:
- Tubing catalog: 7-5
- Tubing manufacturing process: 7-3ff

Steel Tube and Pipe: 7-2ff
Strength, boom: 6-18
Stress: 4-2
Survival speed:
- Boom calculation: 6-24ff

T

Taper algorithm: 8-18
- Experimental verification: 10-10ff
- Lawson tapered dipole: 8-27
- Schelkunoff, for tapered dipole: 8-18ff
- Schelkunoff, MININEC compared: 8-24ff
- Spreadsheets: 9-6ff

Tapered dipole: 8-18
Tie-down: 10-22
Torque: 3-1
- Balance for coaxial cable: 11-10
- Isolator flex-disk: 7-28
- Mast: 6-11

Torque compensation: 6-13ff

Tower:
- Effective height: 10-8
- Rohn 25: 7-18
- Rohn 45: 7-17
- Rohn 55: 7-17
- Rotating: 7-17, 11-70

Tube:
- Aluminum: 3-10
- Steel: 7-2ff

V

Vibration: 4-38

W

Wind:
- Basic wind speed (defined): 2-5
- Calculating expected conditions: 11-2ff
- Fastest mile of wind: 2-5
- Force, on cylinders: 2-16ff
- Force, 205BA: 6-6
- Gradient wind: 2-2
- Gust factor: 2-10, 6-7
- Local wind statistics: 2-5
- Maximum 50-year pressure: 2-14
- Maximum wind speed v_{max}: 4-3
- Variation with height: 2-1ff
- Wind pressure: 2-12
- Wind speed: 2-3ff
- Wind speed factors: 2-7ff
- Wind zones A, B and C: 2-6ff

Y

Yagi-Uda: 1-3ff
Yield strength: 3-7ff
- Steel tube and pipe table: 7-3
- Table: 3-8

PHYSICAL DESIGN
OF
YAGI ANTENNAS

PROOF OF
PURCHASE

ARRL MEMBERS

This proof of purchase may be used as a $2.00 credit on your next ARRL purchase. Limit one coupon per new membership, renewal or publication ordered from ARRL Headquarters. No other coupon may be used with this coupon. Validate by entering your membership number—the first 7 digits on your QST label—below:

FEEDBACK

Please use this form to give us your comments on this book and what you'd like to see in future editions.

Name _____ Call sign _____

Daytime Phone ( ) _____ Age _____

Address _____

City, State, ZIP _____

If licensed, how long? _____

Other hobbies _____

Occupation _____

Edition 1 2 3 4 5 6 7 8 9 10 11 12

Printing 1 2 3 4 5 6 7 8 9 10 11 12

From _____

_____

_____

Please affix postage. Post Office will not deliver without sufficient postage.

Editor, Physical Design of Yagi Antennas
American Radio Relay League
225 Main Street
Newington, CT USA 06111

..... please fold and tape .....