A Technical Introduction to GPS for the Radio Amateur

R. Sean Anderson
KR4YO

Overview

- *SATNAV theory*
- Signals, codes and services
- Errors and accuracy

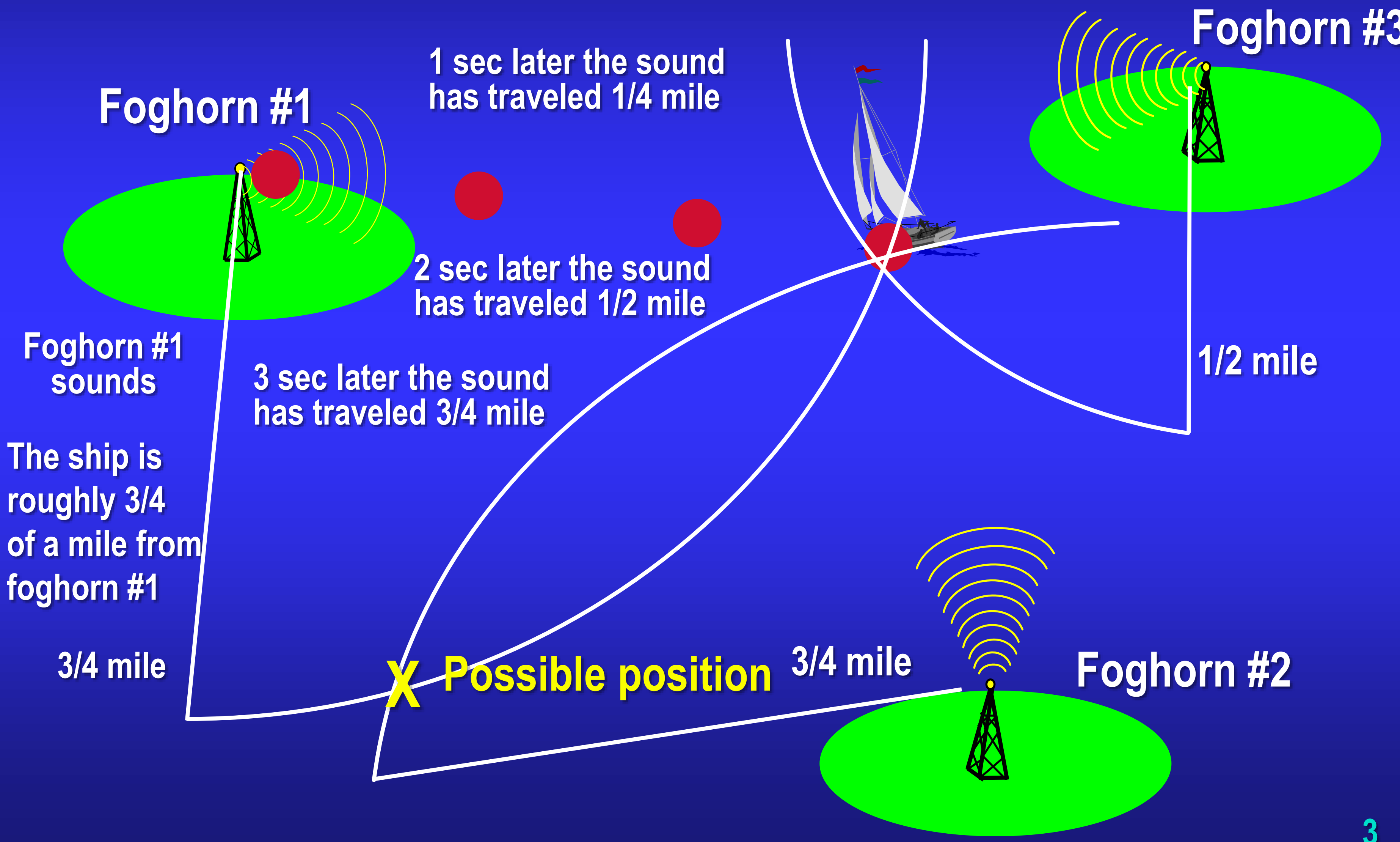
Time-of-Arrival Ranging

- SATNAV positioning uses the same *TOA* ranging concept mariners used centuries ago to navigate by foghorns
- Mariners used *maps* and ships' *clocks* to measure range to a fog horn
- Mariners knew sound travels roughly a quarter-mile per second

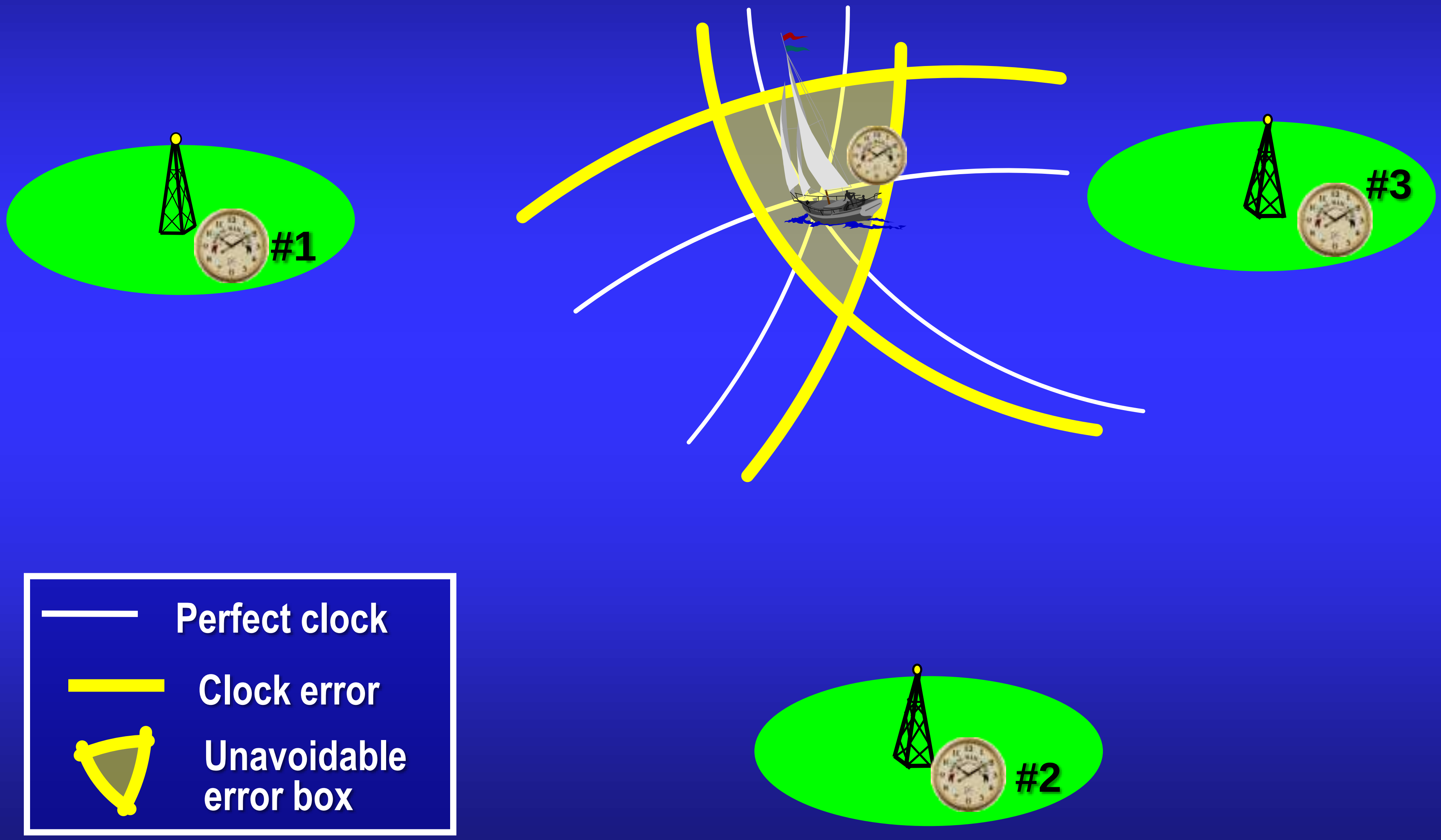
Ye ol' map

Ye ol' clock

Time-of-Arrival Example



Time-of-Arrival Example: Clock Bias



Determining Position
With Global Positioning System

To Determine Position

Three things are needed to determine position

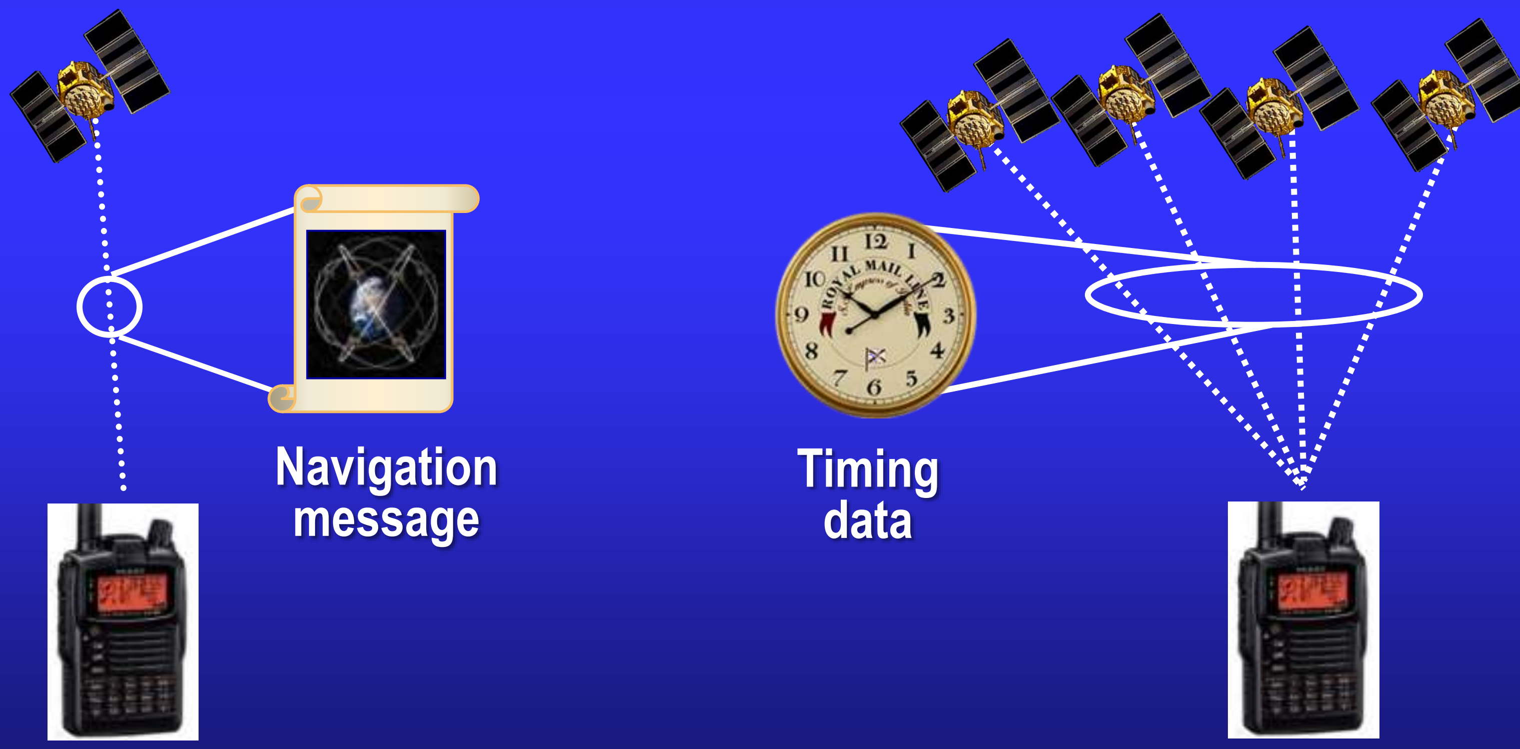
- A map showing precise location of each GPS satellite
- A signal from the GPS satellite to the receiver
- A clock to measure TOA of GPS signals at the receiver

Map

Time source

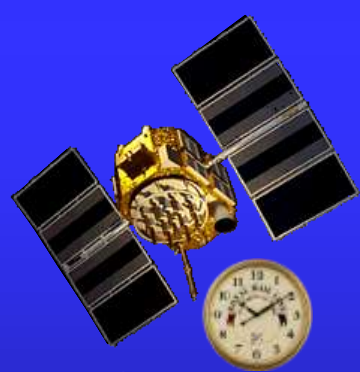
A Map and a Time Source?

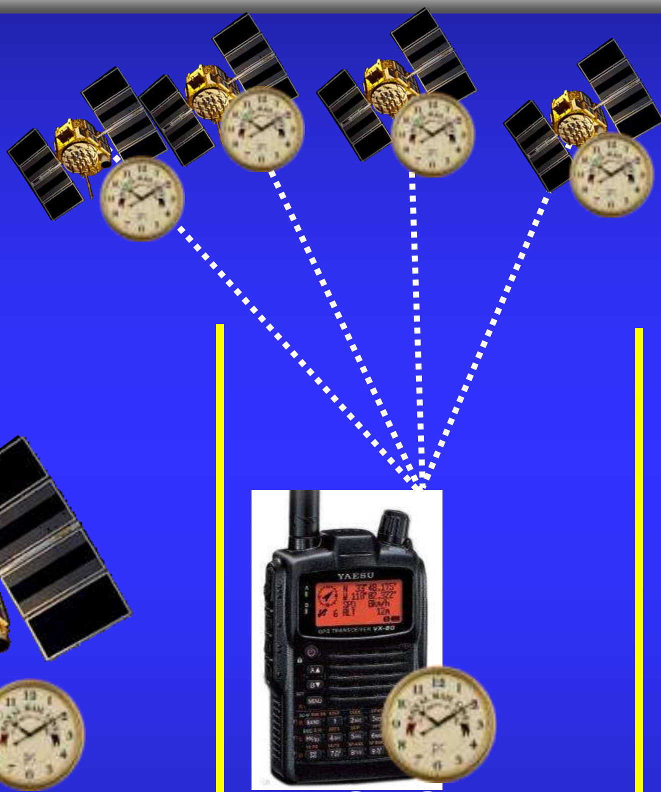
These are automatically downloaded to GPS receivers from the satellites during initialization (after turn-on; takes 12.5 min)



Time in the Global Positioning System

Four types of time:





Receiver time	Space vehicle time	GPS time	Universal time coordinated
1 second per day	8.2 nanosecond per day	UTC + 16 sec	–16 sec in navigation message

Navigation Message Format

ONE SUBFRAME = 300 bits, 6 sec

SUBFRAME #			
1	TLM	HOW	SV CLOCK COLLECTION DATA
2	TLM	HOW	SV EPHEMERIS DATA (I)
3	TLM	HOW	SV EPHEMERIS DATA (II)
	25 PAGES OF SUBFRAME 4 AND 5 = 12.5 min		
4	TLM	HOW	OTHER DATA (IONO, UTC, CRYPTO)
5	TLM	HOW	ALMANAC DATA FOR ALL SV

One data frame = 1500 bits 30 sec

TLM TELEMETRY WORD

HOW HANDOVER WORD

ONE WORD = 30 BITS, 24 DATA, 6 PARITY

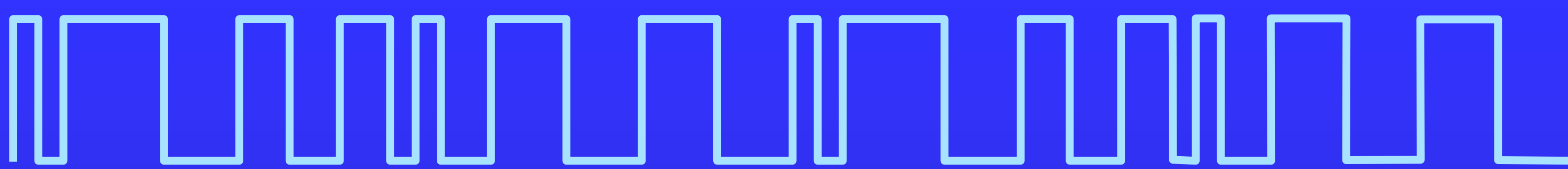
8-BIT PREAMBLE	DATA	PARITY
17-BIT TIME OF WEEK	DATA	PARITY

Pseudorange to Satellites?

A GPS satellite and a user's receiver set will generate the same satellite unique PRN at the exact same time

– The satellites continuously transmit their unique codes

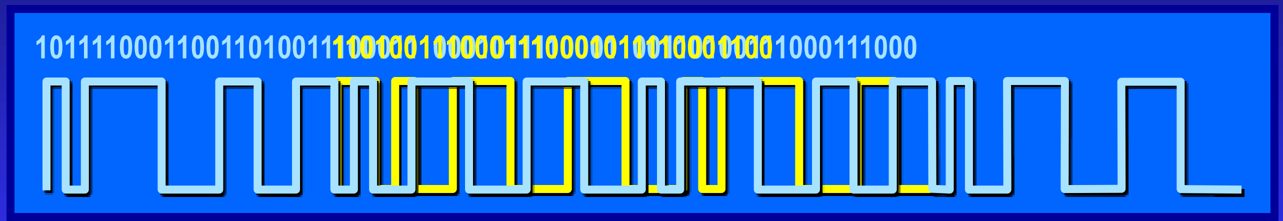
10111100011001101001110001110001011110001100110100111000111000



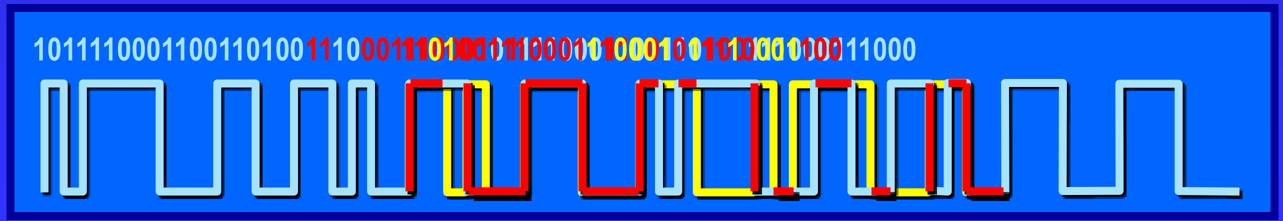
PRN 24: A short repeating PRN code sample

– The receiver set will compare the time of receipt of the satellites transmission of the code with the receiver's own generation

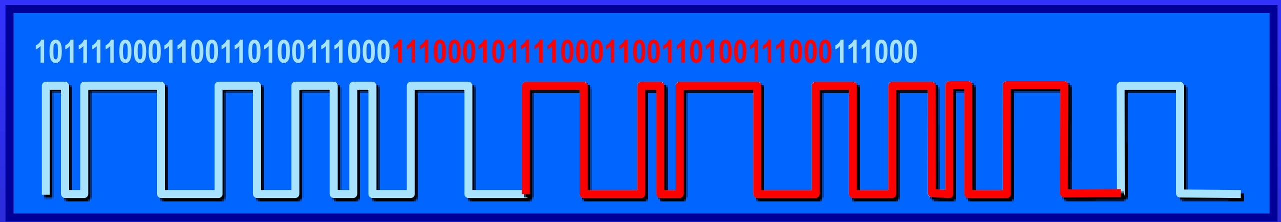
PRN Code Correlation



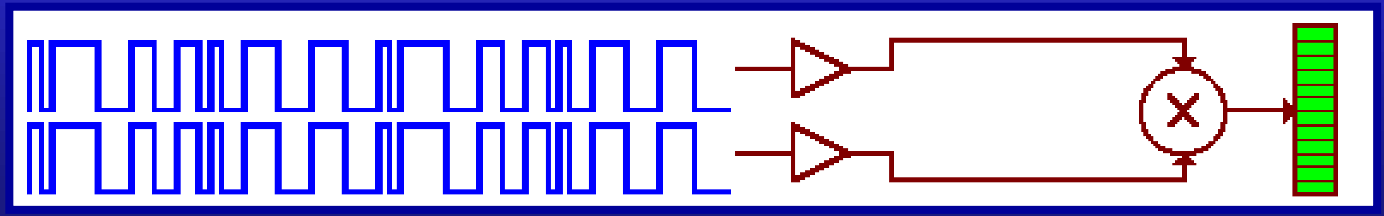
No correlation with a different PRN code



Partial correlation of identical receiver and satellite PRN code



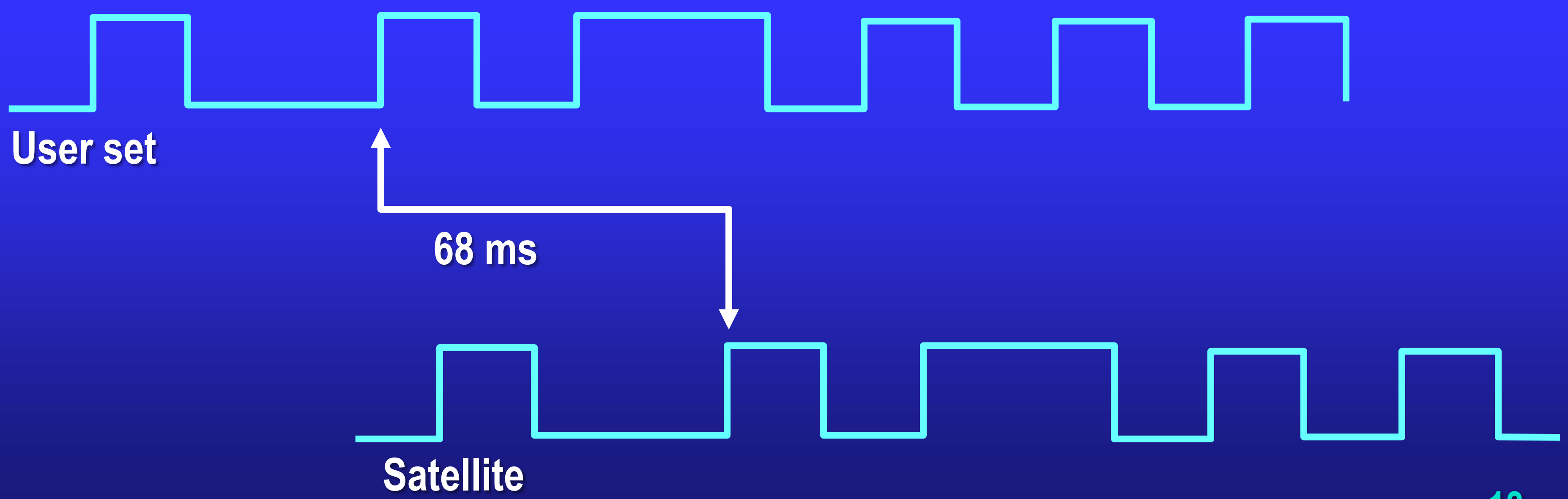
Full correlation (code phase lock) of receiver and satellite PRN code



Pseudoranging From Global Positioning System

The difference between these codes

- Corresponds directly to the time delay necessary for the signal to reach the user's set
- Simple formula yields the distance from the satellite



Global Positioning System Pseudoranging Formula

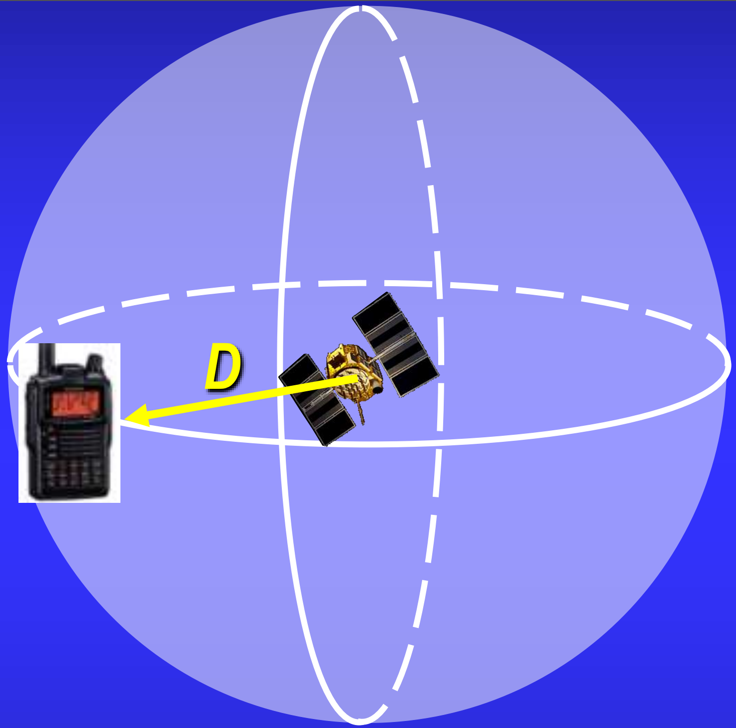
$$\text{Distance} = \text{time} \times \text{speed}$$

$$\text{Distance} = \underbrace{68\ \text{ms}}_{\textit{Time delay}} \times \underbrace{300\,000\,000\ \text{m/s}}_{\textit{Speed-of-light}}$$

$$\textit{Range} = \textit{time delay} \times \textit{speed-of-light}$$

Position by Trilateration

- A distance measurement from just one satellite will determine a receiver's position somewhere on a sphere around that satellite
- The location of the satellite is known by the receiver set because the location is transmitted as part of the navigation message

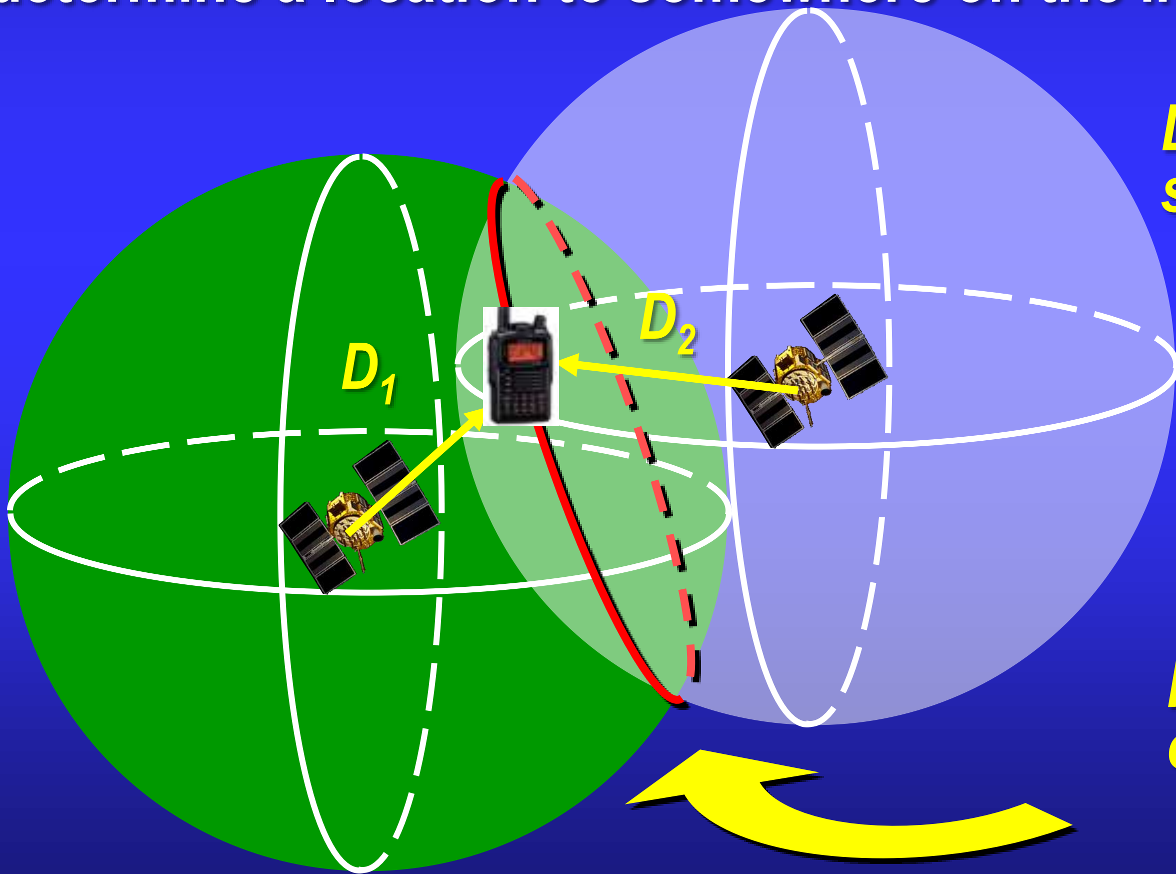


D: Pseudorange from satellite to receiver

Surface-of-sphere: All possible solutions

Add a Second Satellite

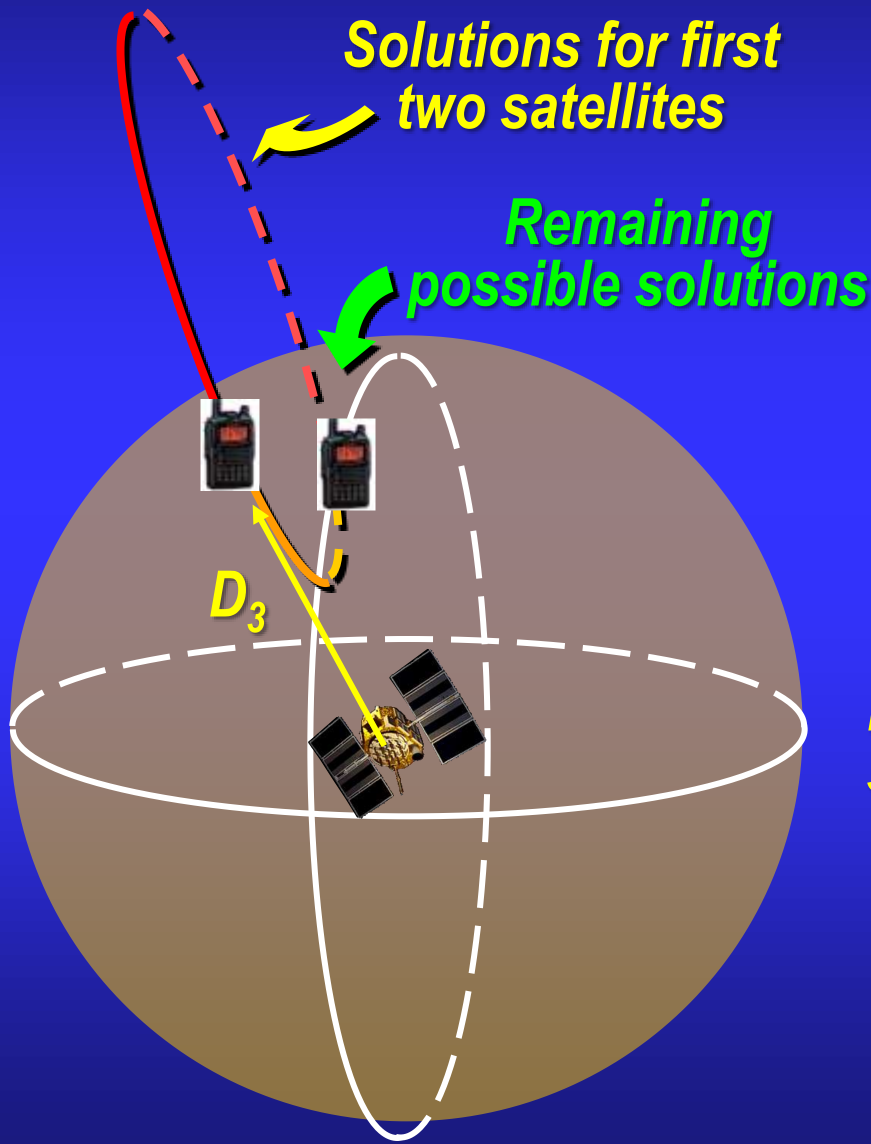
Two satellite measurements will result in two spheres that will determine a location to somewhere on the intersecting circle



D: Pseudorange from satellite to receiver

Red ring: Intersection of two spheres

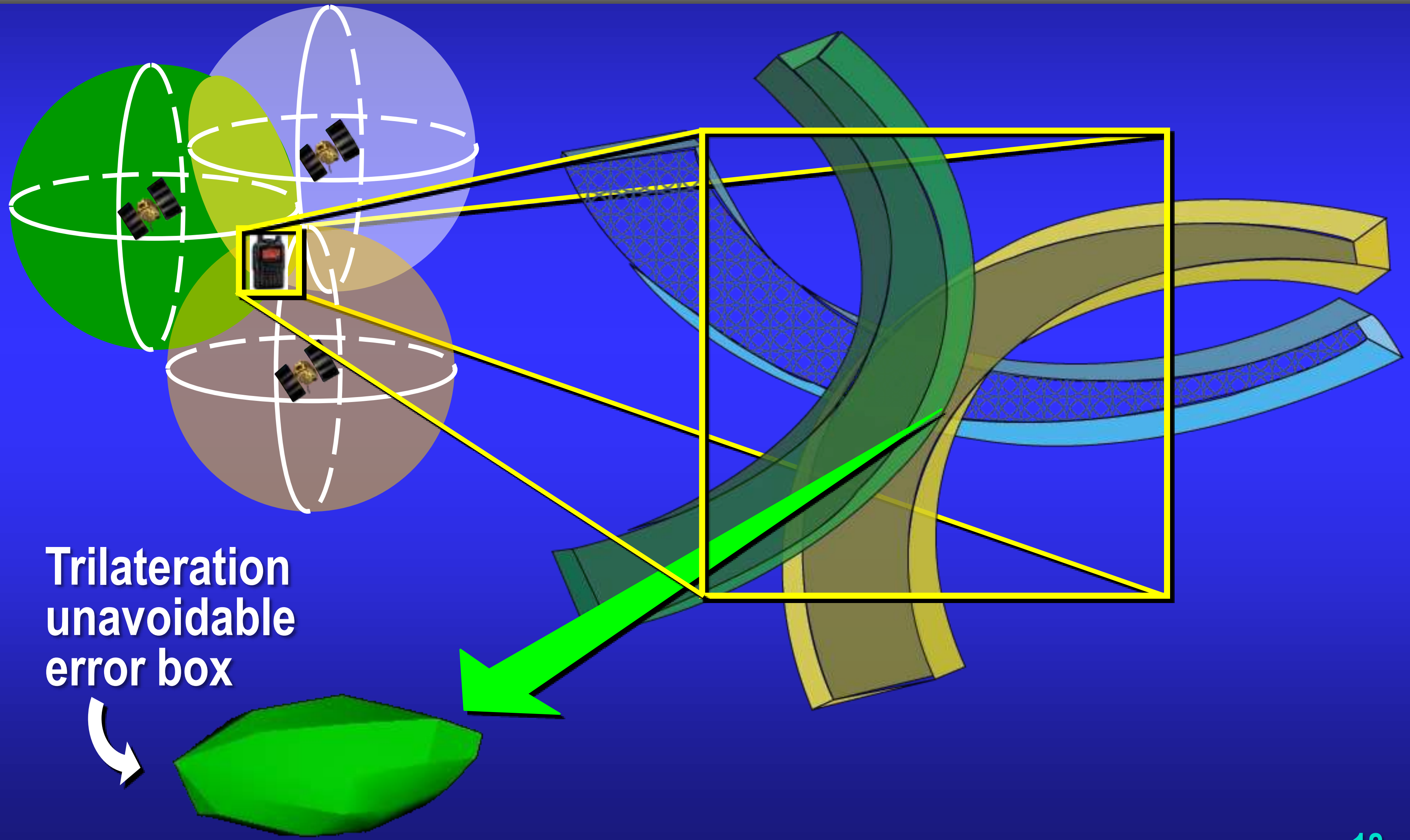
Add a Third Satellite



Three satellite measurements will determine a location to two points on that circle; only one of which will be logical

D: Pseudorange from satellite to receiver

Trilateration
Unavoidable Error Box



Add a Fourth Satellite

- Because receiver and satellite clocks are not perfect, time (t) becomes a variable in addition to the x, y and z values
- Four satellites are needed for the receiver to solve for the clock bias error

$$Position = \begin{bmatrix} GPS_1\ x, y, z, t \\ GPS_2\ x, y, z, t \\ GPS_3\ x, y, z, t \\ GPS_4\ x, y, z, t \end{bmatrix}$$

Some Need to See the Math

GPS Pseudorange Navigation Example - Peter H. Dana - 4/24/96

Satellite (SV) coordinates in ECEF XYZ from Ephemeris Parameters and SV Time

$SVx_0 := 15524471.175$ $SVy_0 := -16649826.222$ $SVz_0 := 13512272.387$ SV 15

$SVx_1 := -2304058.534$ $SVy_1 := -23287906.465$ $SVz_1 := 11917038.105$ SV 27

$SVx_2 := 16680243.357$ $SVy_2 := -3069625.561$ $SVz_2 := 20378551.047$ SV 31

$SVx_3 := -14799931.395$ $SVy_3 := -21425358.24$ $SVz_3 := 6069947.224$ SV 7

Satellite Pseudoranges in meters (from C/A code epochs in milliseconds)

$P_0 := 89491.971$ $P_1 := 133930.500$ $P_2 := 283098.754$ $P_3 := 205961.742$ Range + Receiver Clock Bias

Receiver Position Estimate in ECEF XYZ

$Rx := -730000$ $Ry := -5440000$ $Rz := 3230000$

For Each of 4 SVs $i := 0..3$

Ranges from Receiver Position Estimate to SVs (R) and Array of Observed - Predicted Ranges

$$R_i := \sqrt{\left(SVx_i - Rx\right)^2 + \left(SVy_i - Ry\right)^2 + \left(SVz_i - Rz\right)^2} \qquad L_i := \mathrm{mod}\left[\left(R_i\right), 299792.458\right] - P_i$$

Compute Directional Derivatives for XYZ and Time

$$Dx_i := \frac{SVx_i - Rx}{R_i} \qquad Dy_i := \frac{SVy_i - Ry}{R_i} \qquad Dz_i := \frac{SVz_i - Rz}{R_i} \qquad Dt_i := -1$$

Solve for Correction to Receiver Position Estimate

$$A := \begin{bmatrix} Dx_0 & Dy_0 & Dz_0 & Dt_0 \\ Dx_1 & Dy_1 & Dz_1 & Dt_1 \\ Dx_2 & Dy_2 & Dz_2 & Dt_2 \\ Dx_3 & Dy_3 & Dz_3 & Dt_3 \end{bmatrix} \qquad dR := \left(A^T \cdot A\right)^{-1} \cdot A^T \cdot L \qquad dR = \begin{bmatrix} -3186.496 \\ -3791.932 \\ 1193.286 \\ 12345.997 \end{bmatrix}$$

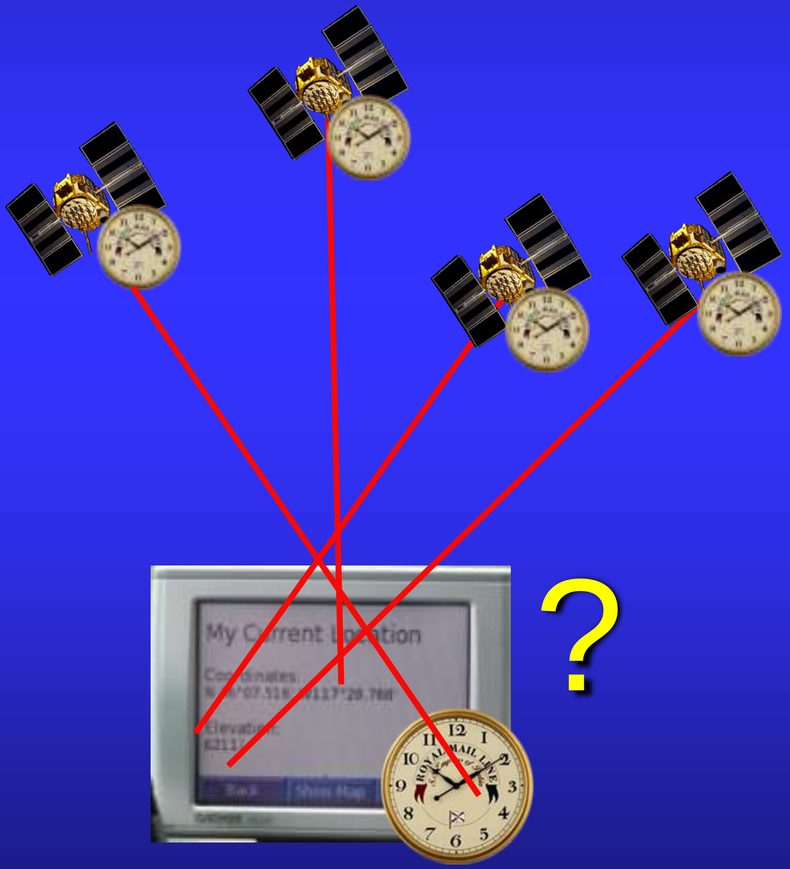
Apply Corrections to Receiver XYZ and Compute Receiver Clock Bias Estimate

$Rx := Rx + dR_0$ $Ry := Ry + dR_1$ $Rz := Rz + dR_2$ $Time := dR_3$

$Rx = -733186.496$ $Ry = -5443791.932$ $Rz = 3231193.286$ $Time = 12345.997$

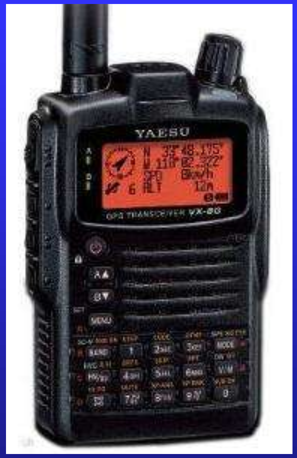
Some Need to Hear This in English

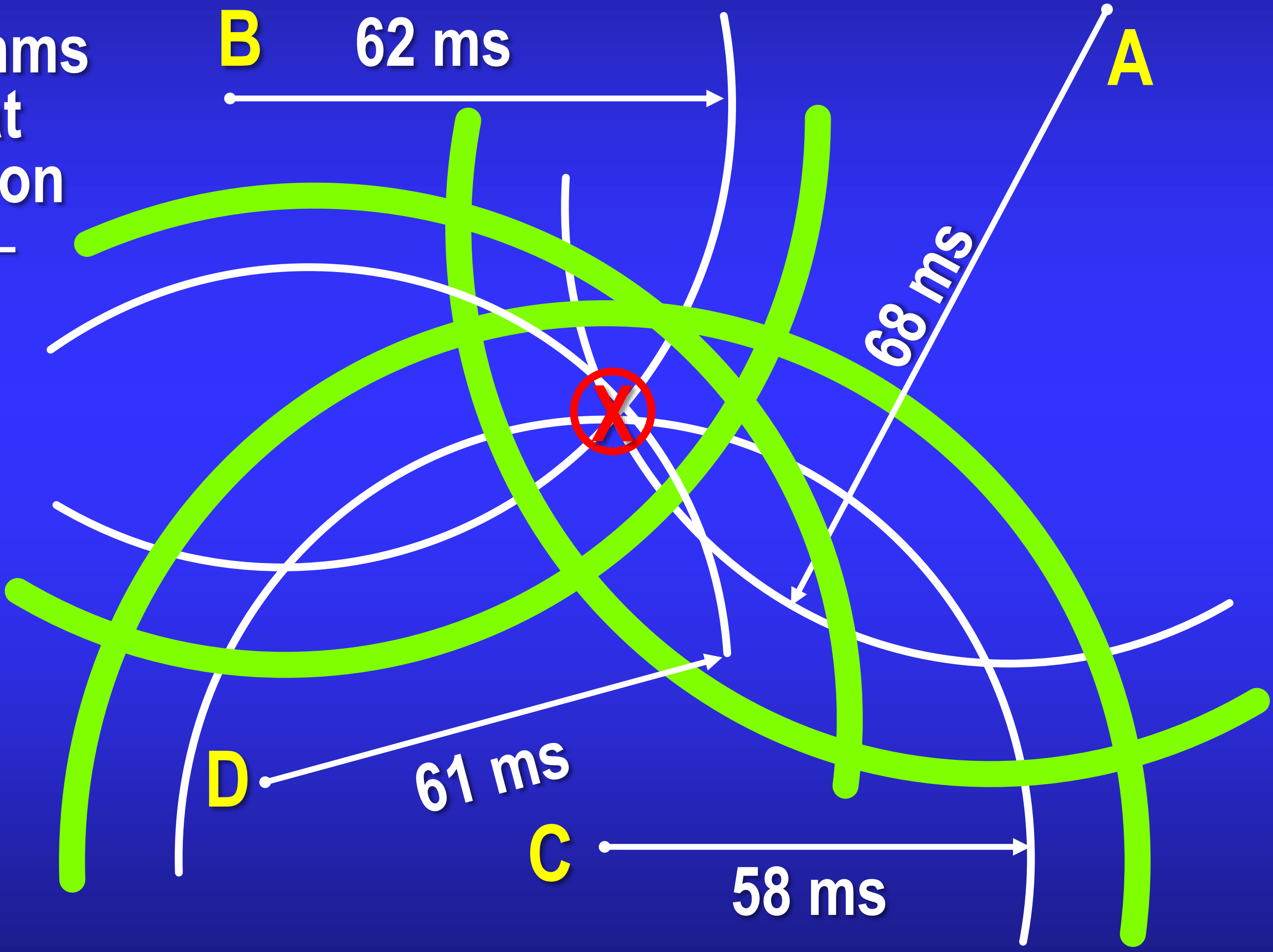
Because the receiver and satellites clocks are not precise, a fourth satellite is needed to further refine positioning to result in a relatively accurate position



Reducing Error Box Size

Solved by algorithms in the receiver that balance the solution equation +++ ––––

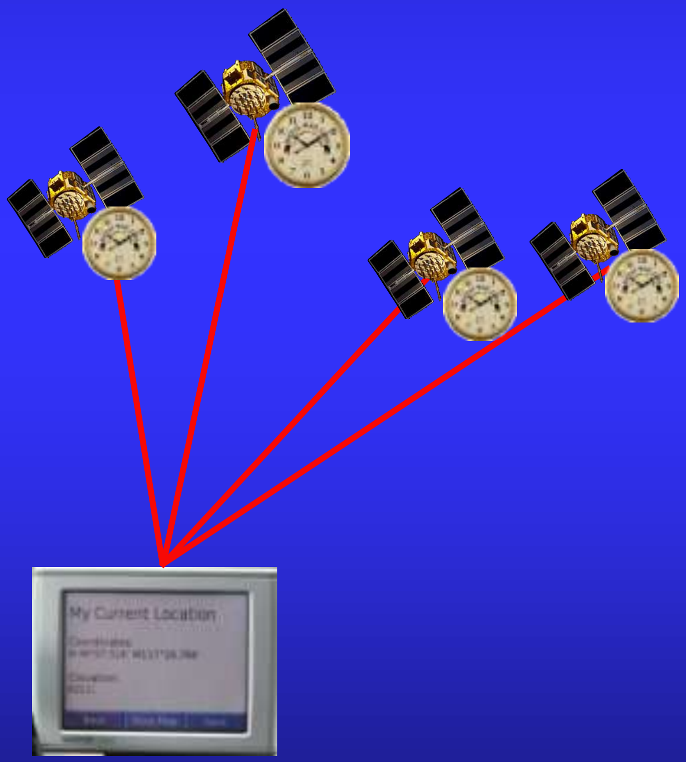




Factor Analysis

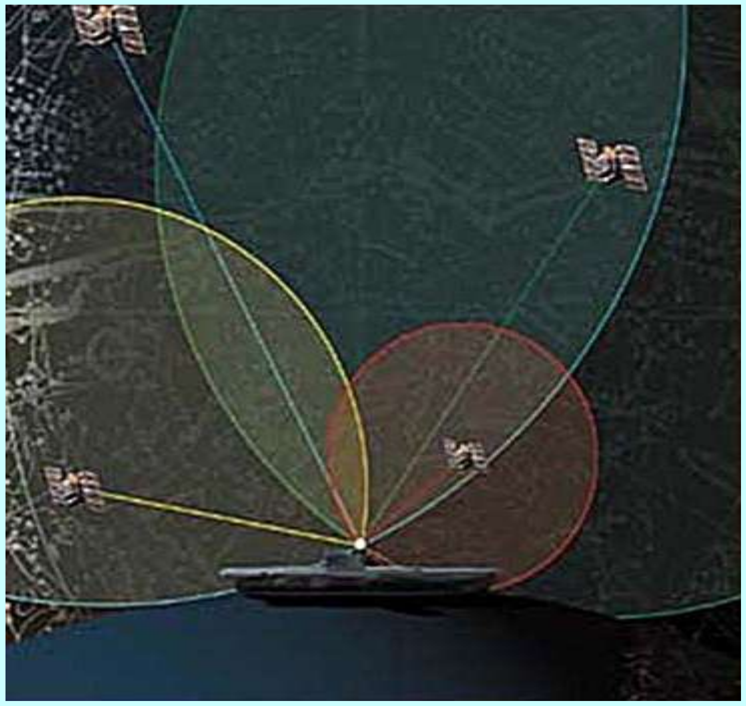
Without the time factor analysis

With the time factor analysis



Overview

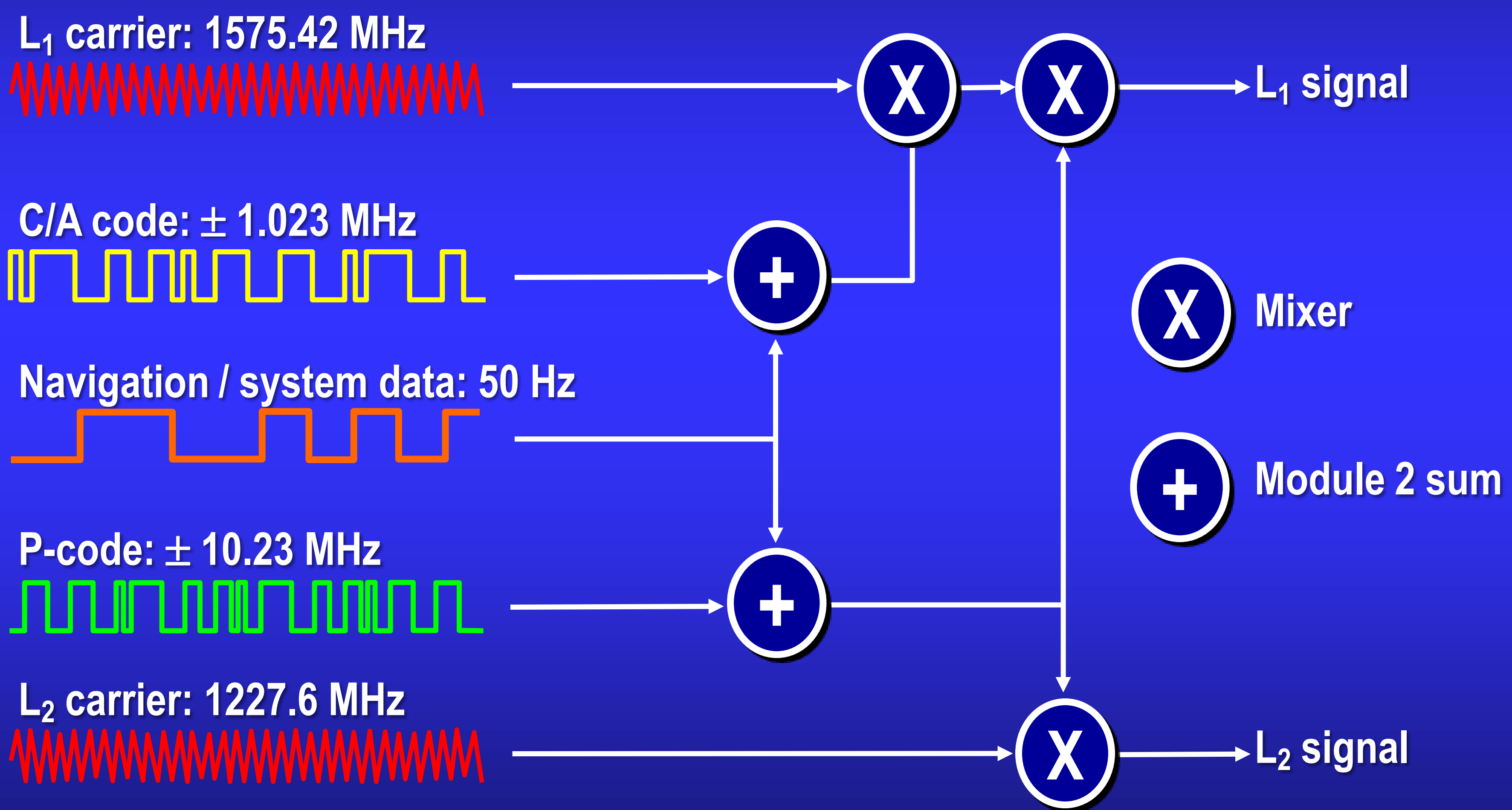
- SATNAV theory
- *Signals, codes and services*
- Errors and accuracy



Global Positioning System Signals

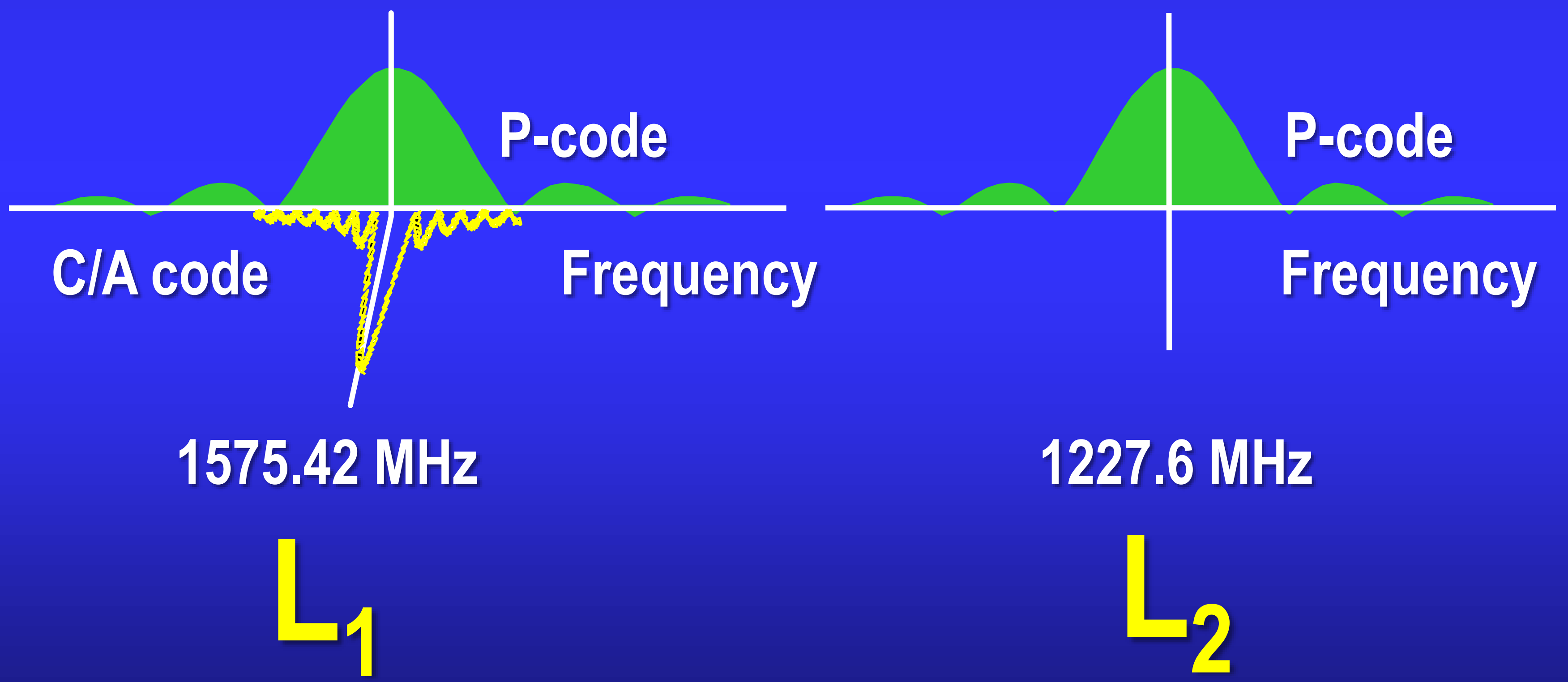
- GPS for civilians broadcasts over one center frequency
  - *L1:* 1575.42 MHz, P-code and C/A code
- Additional frequencies for mil/gov't use
  - *L2:* 1227.60 MHz, P-code only
  - *L3:* 1381.05 MHz, NUDET only
- *C/A code = course acquisition code*
- *P-code = precision code*

Global Positioning System Signals-in-Space



Global Positioning System Signal Spectrum

Each GPS satellite broadcasts continuously on two center frequencies, called L_1 and L_2

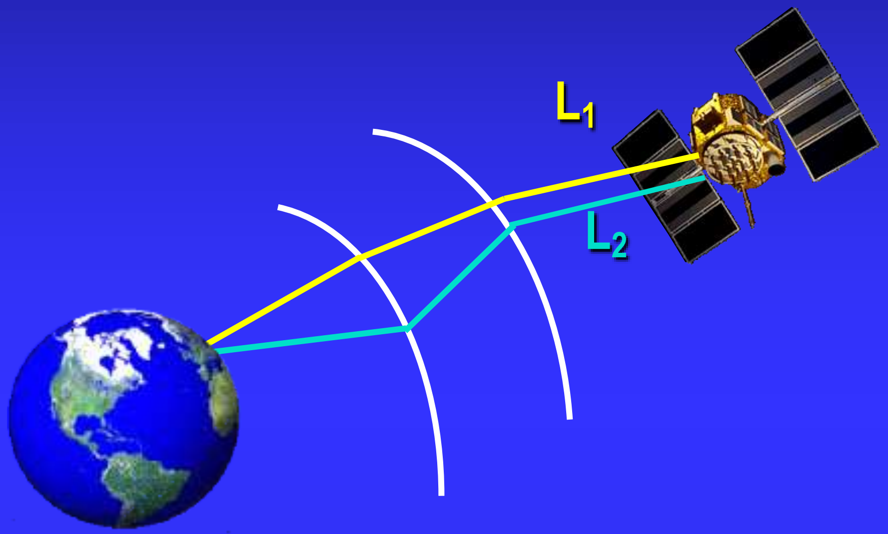


Global Positioning System Ranging Codes

- C/A code (repeats every ms)
  - Short PRN sequence: 1023 bits
  - Narrow bandwidth: ±1.046 MHz
  - Repeats every ms
  - Fast, direct acquisition
  - Easy to detect / jam
  - On L_1
  - Assists in acquiring the P-code
- P-code (repeats every week)
  - Long PRN sequence: 6.2 trillion bits
  - Broad bandwidth: ±10.46 MHz
  - Repeats every week
  - Slow, direct acquisition
  - Harder to detect / jam
  - On L_1 and L_2
  - Encryptable to form Y-code

Users Access to Signals and Codes

- Civilian / commercial receivers use only C/A on L_1
- Authorized receivers use both L_1 and L_2
- Using both frequencies reduces error
  - Allows dynamic modeling of the ionospheric delays
  - If only one frequency is used, the receiver set must use an ionospheric model in the navigation message



Because the travel distance of L_2 is greater than L_1, the TOA is slightly longer. Two-frequency receivers can, therefore, model ionospheric error

Two Global Positioning System Services

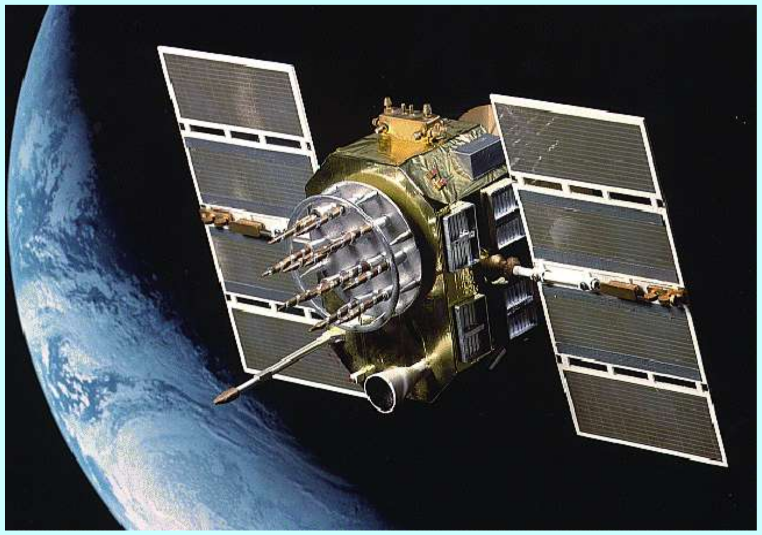
- Precise positioning service
  - Can decrypt Y-code
  - P-code based
  - PPS 95% 3-D position error: 3.76 m (95%) in 2004
  - PPS 95% NAV user time transfer error: 8.1 ns (95%) in 2004
- Standard positioning service
  - Actual error based on current DoD policy
  - Cannot decrypt Y-code or remove SA error
  - C/A code based
  - SPS position / timing accuracy not currently tracked
    - GPSOC position accuracy estimate: ~5 m
    - Represents GPSOC estimate of PPS + 30% to 40%

Overview

- SATNAV theory
- Signals, codes and services
- *Errors and accuracy*

Errors and Accuracy

– *URE*
– DOP
– Calculating GPS accuracy



GPS Blk IIA satellite

User Range Error Defined

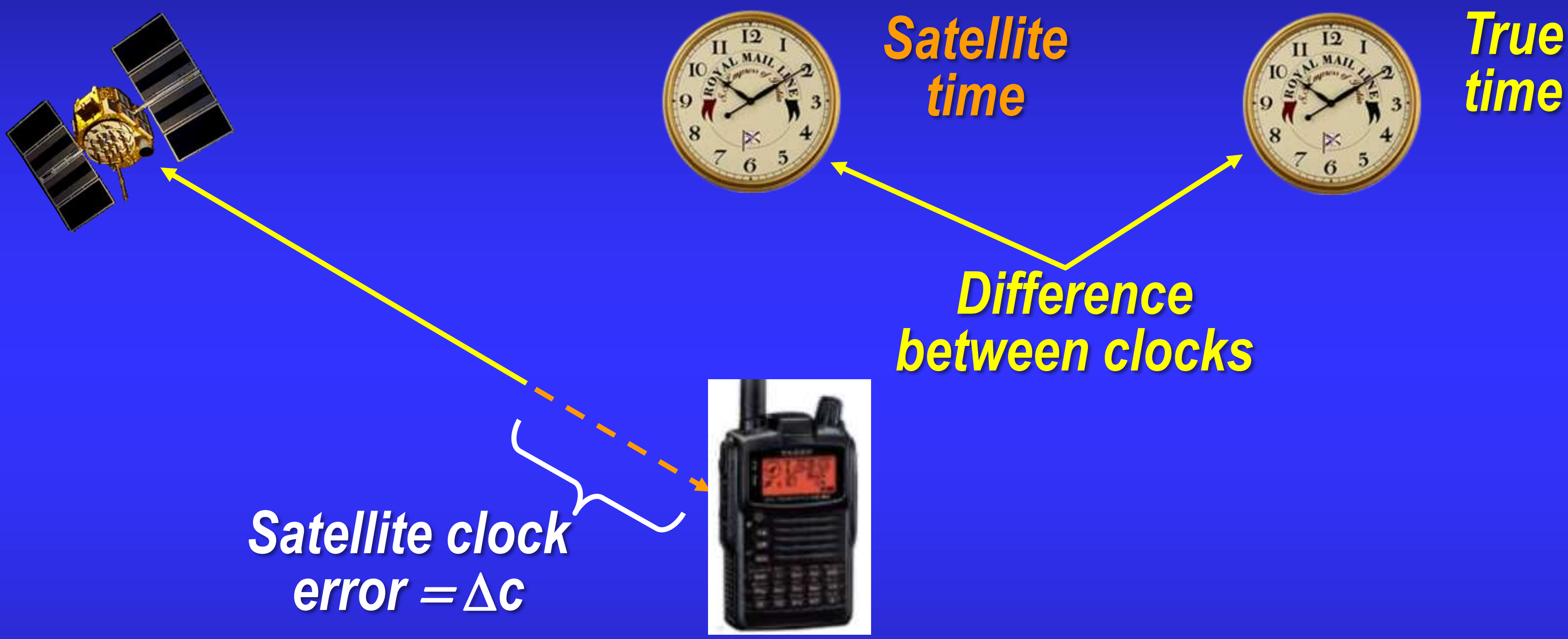
- *Error in satellite to receiver range measurement*
- URE relates a single satellite to the receiver
  - Four satellite solution = four distinct URE
  - Constantly changing with time
- Six factors: Majority of error can be corrected
  - *Uncontrolled:* Built-in corrections (models)
  - *Controlled:* Periodic satellite uploads (ephemeris and clock)

Sources of Error

Representative per satellite error budget (contractually allowed)	*SPS*	*PPS*
– Satellite clock error (Δc)	2.1	2.1
– Ephemeris error (Δp)	<19.6	8.2
– Ionosphere (Δi)	4.5	4.5
– Troposphere (Δt)	3.9	3.9
– Receiver noise (Δr)	2.9	2.9
– Multipath (Δm)	2.4	2.4

All units are meters, statistically at 95% probability

Satellite Clock Error



Error in pseudorange caused by difference (error) between true time and clock time

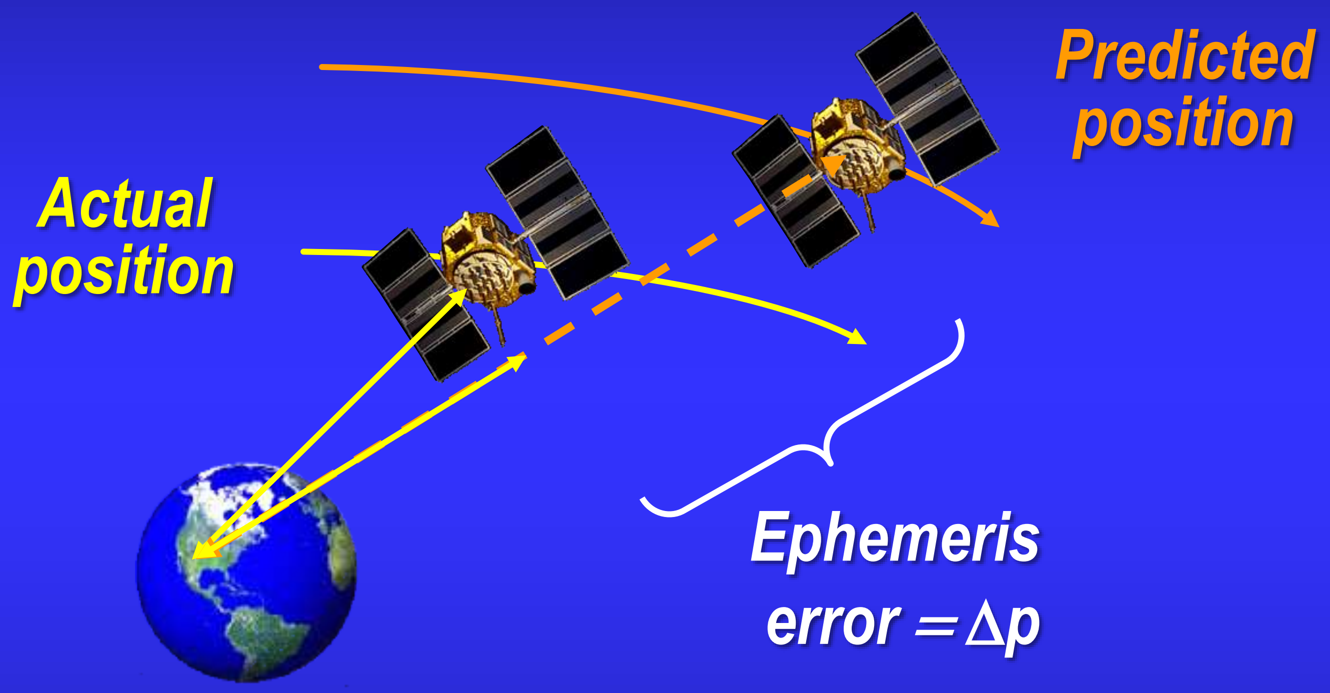
Accurate Satellite Clocks

- Uses cesium and rubidium oscillators
- Stability of approximately 1 part in 1×10^{-13} per day
- Equates to clock error of 8.6×10^{-9} second per day
- Equates to range error of 2.5 meters per day
- Error grows slowly over time

Satellite Clock Corrections

- *Correction: Periodic satellite clock uploads*
  - Satellite operations crews at MCS
  - Typically performed once a day
  - Can be increased based on requirements
- *Error contribution: Approximately 2 m to 4 m*

Ephemeris Error



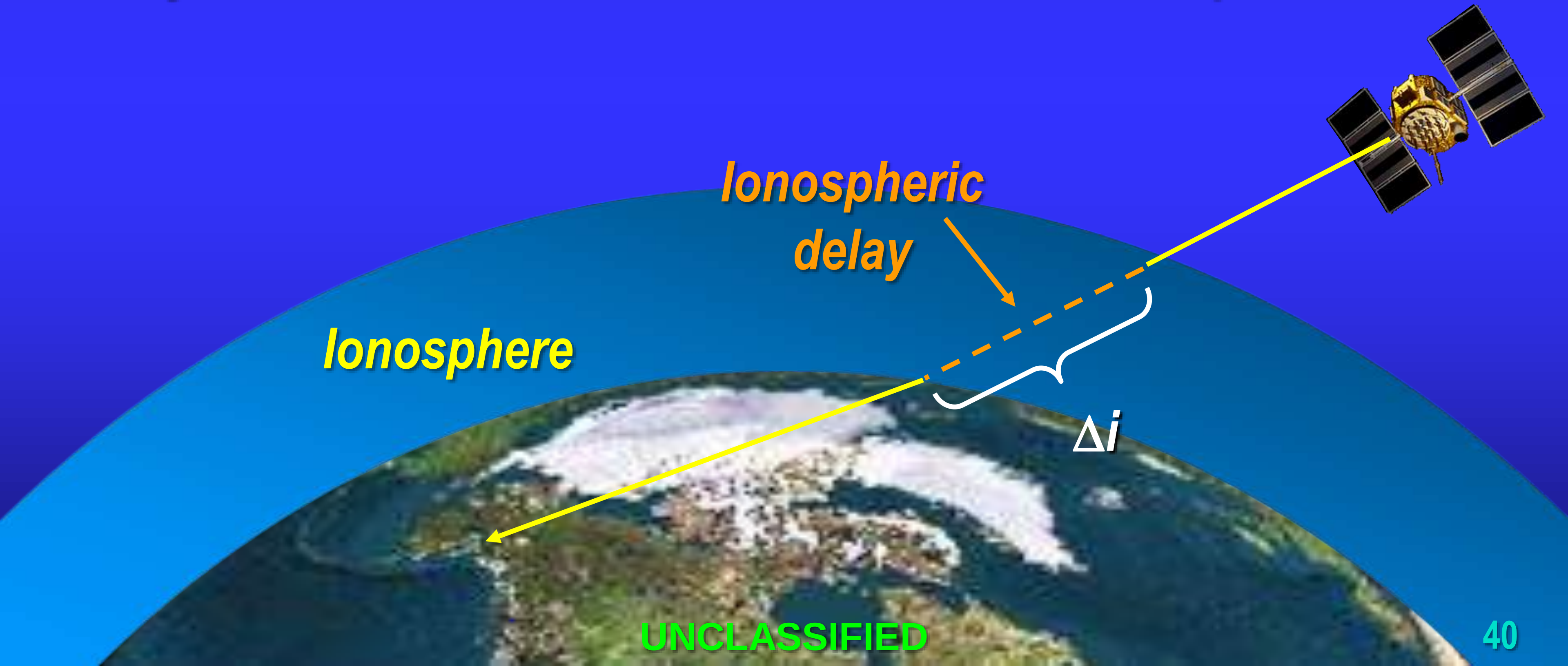
Error in pseudorange caused by difference (error) between true and predicted positions

Ephemeris Error Corrections

- Correction: Periodic satellite ephemeris uploads
  - Satellite operations crews at MCS
  - Typically performed once a day
  - Can be increased based on requirements
- Error contribution: Approximately 2 m to 3 m

Ionospheric Error

– Greatest natural source of GPS error
– Error in pseudorange due to signal delay (error) caused by interaction with free electrons in the ionosphere



Ionospheric Error Delays

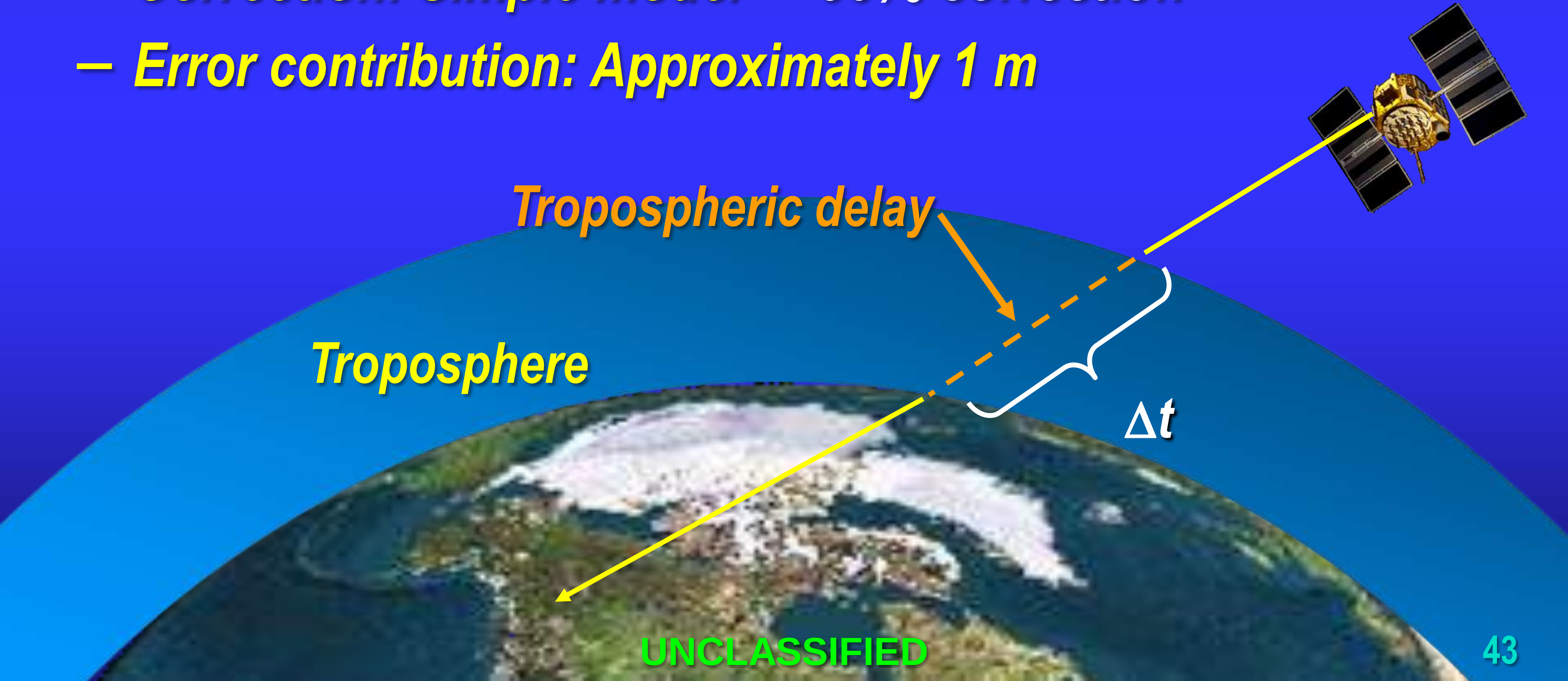
- Delay directly proportional to electron density
  - Fluctuates: Hourly, daily and monthly cycles
  - Impacted by solar activity (flares or solar max)
  - Typically relatively stable in temperate zones
  - Considerable flux in polar and equatorial zones
- Delay due to signal path: Low-elevation satellites have longer path through ionosphere

Ionospheric Error Corrections

- *SPS correction: Ionosphere modeling*
  - Very computationally complex models
    - Standard GPS receivers: Only 50% correction
    - State-of-the-art models: Only 75% correction
  - *Error contribution: 2 m to 4 m*
- *PPS correction: Modeling and dual-frequency real-time modeling*
  - Delay inversely proportional to signal frequency
  - *Error contribution: 1 m to 2 m*

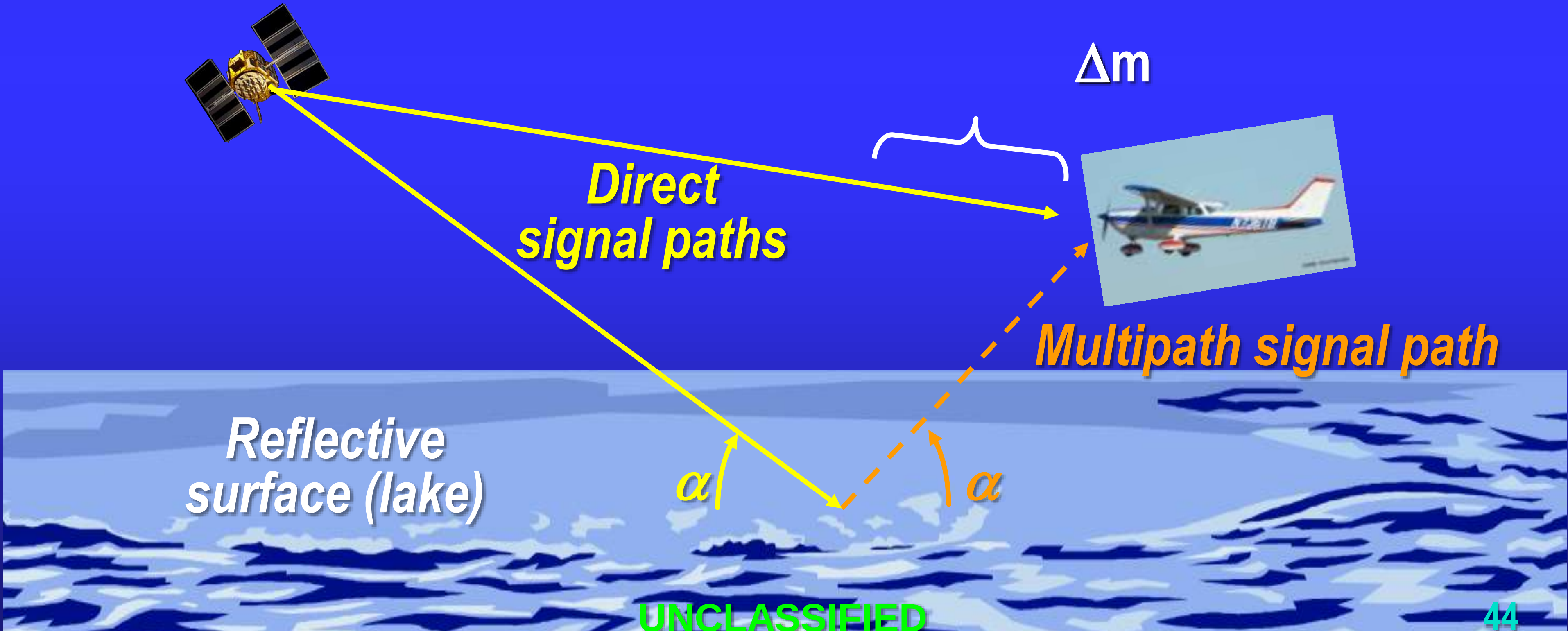
Tropospheric Error

– Error in pseudorange due to signal delay (error) caused by refraction through the troposphere
– *Correction: Simple model* — 90% correction
– *Error contribution: Approximately 1 m*



Multipath Error

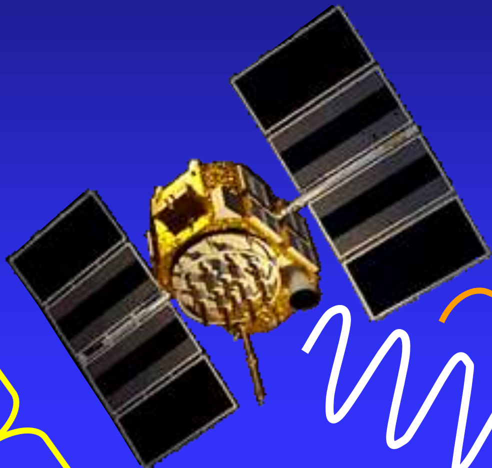
- Error in pseudorange due to increased time lag (error) caused by reflected signal
- *Correction: Masking angle and antenna design*
- *Error contribution: < 1.5 m*



Receiver Error

- Error in pseudorange due to receiver itself (error) caused by microprocessor and antenna design
- Current technology has reduced to a minimum
- ***Correction: None***
- ***Error contribution: Approximately 0.5 m***

Six User Range Error Factors



Two atmospheric errors:

1. Ionospheric error (Δi)
2. Tropospheric error (Δt)

Two satellite errors:

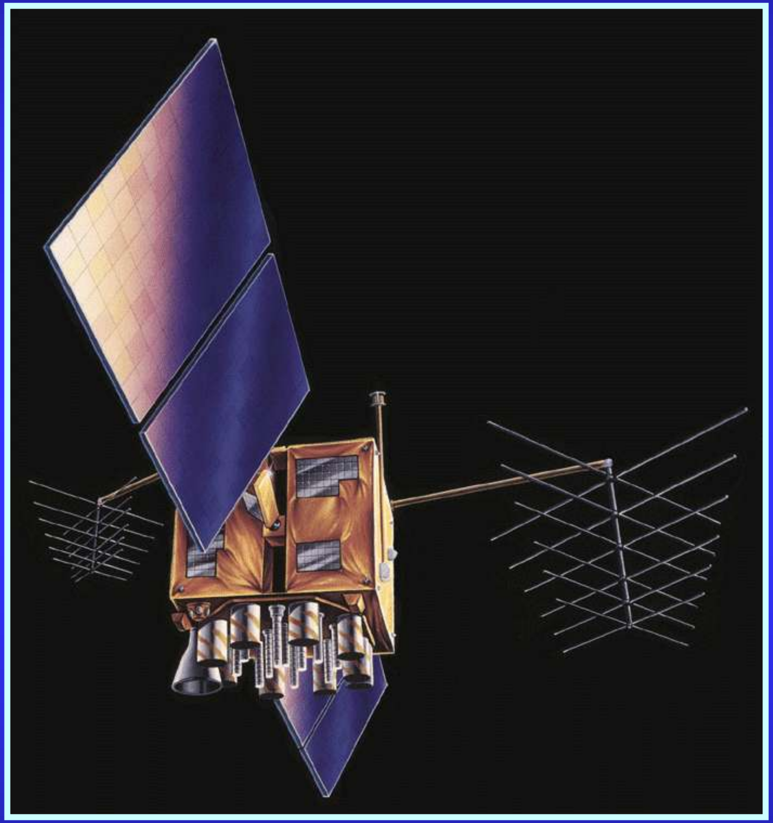
1. Clock error (Δc)
2. Ephemeris error (Δp)

Two receiver errors:

1. Multipath (Δm)
2. Receiver noise (Δr)

Errors and Accuracy

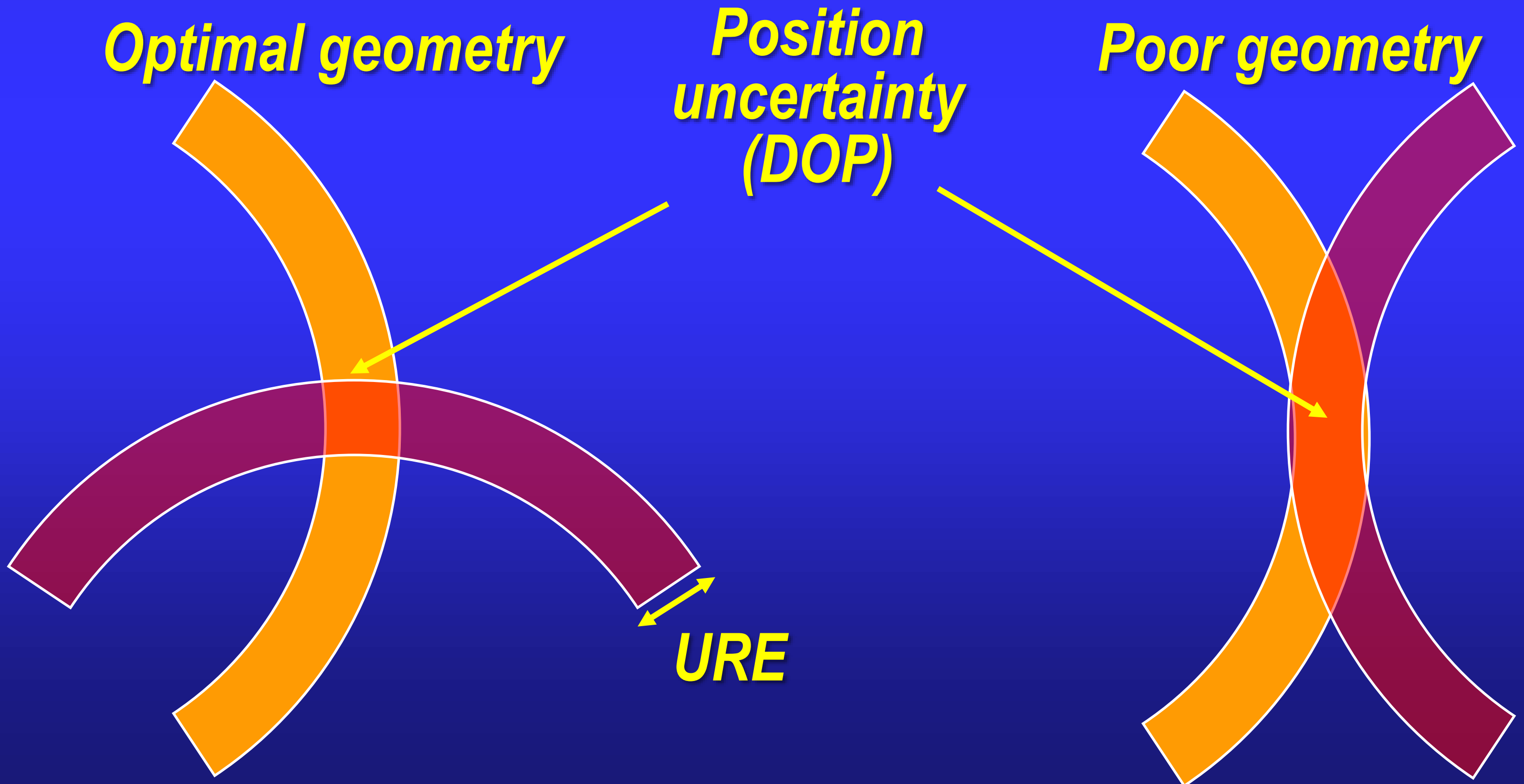
- – URE
- – *DOP*
- – Calculating GPS accuracy



GPS Blk IIR Satellite

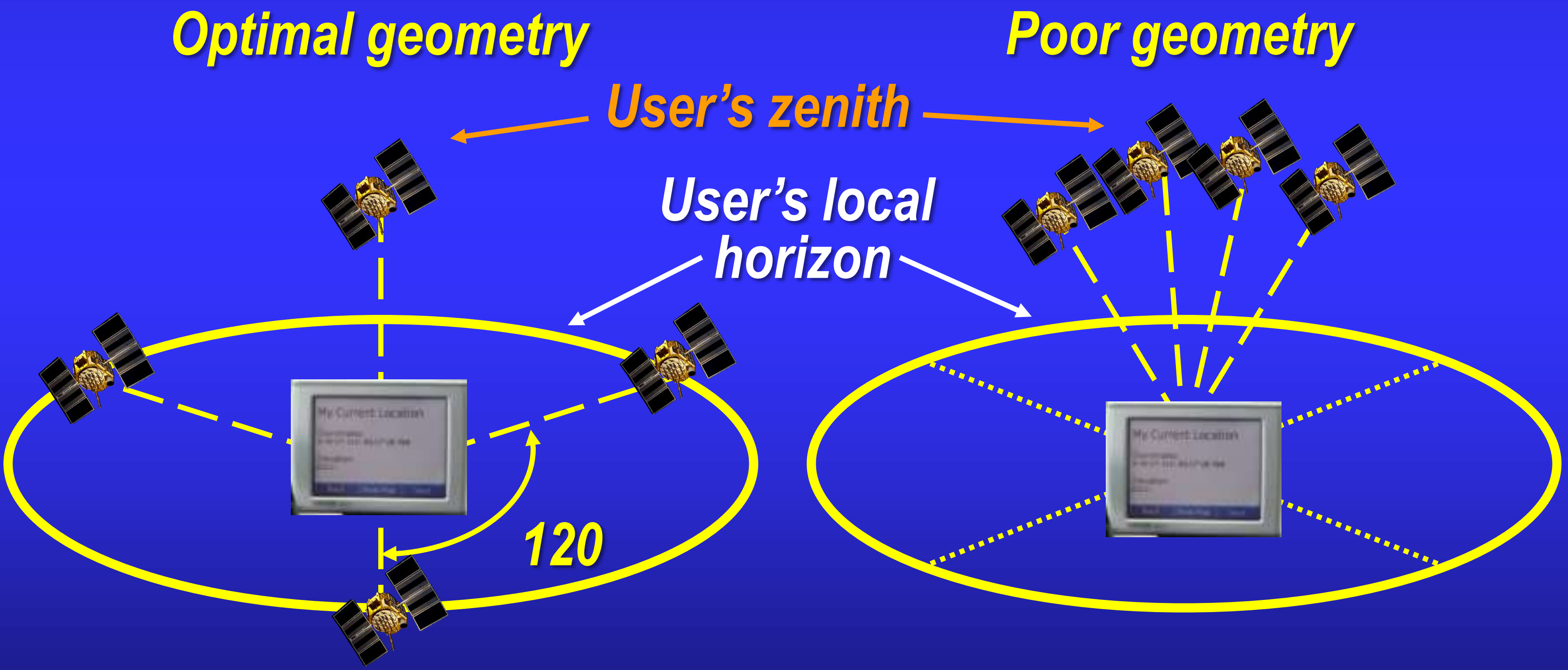
Dilution-of-Precision Defined

– *Error due to geometric relationship of the satellites and receiver* (unitless measure)
– Simple 2-D example for overlapping areas of error:



Dilution-of-Precision Defined in 3-D

3-D example (four satellites): Much more complicated



Dilution-of-Precision Factors

- HDOP: Satellite geometric effect on horizontal or latitude-longitude errors
- VDOP: Satellite geometric effect on vertical or altitude errors
- PDOP: Satellite geometric effect on combined vertical and horizontal (3-D) errors
- TDOP: Geometric effect on time error
- GDOP: Satellite geometric effect on combined vertical, horizontal and time error

How Dilution-of-Precision Is Used

- Unitless figure of merit: Low is good; high is bad
- GPS receivers continually optimize DOP
  - Calculates for all possible satellite combinations
  - Picks best combination for navigation solution
- DOP prediction software
- Can bad DOP be corrected?
  - No; strictly a function of satellite geometry
  - More satellites typically = better DOP

Dilution-of-Precision Characteristics

- Minimum of four satellites required for accurate PVT solution
- Optimal four satellite geometry: Three satellites on horizon (equally spaced in azimuth) and one overhead
- More satellites increases the *opportunity* for good DOP

Errors and Accuracy

- – URE
- – DOP
- – *Calculating GPS accuracy*

GPS Blk IIF satellite

Calculating Accuracy

- *All measurements have errors*, no matter how exact the measuring device or perfect the operator
- Take *multiple measurements* of the same thing
- GPS error due to both *predictable* (DOP) and *statistical* (URE) factors
  - Error is simple product of DOP and URE

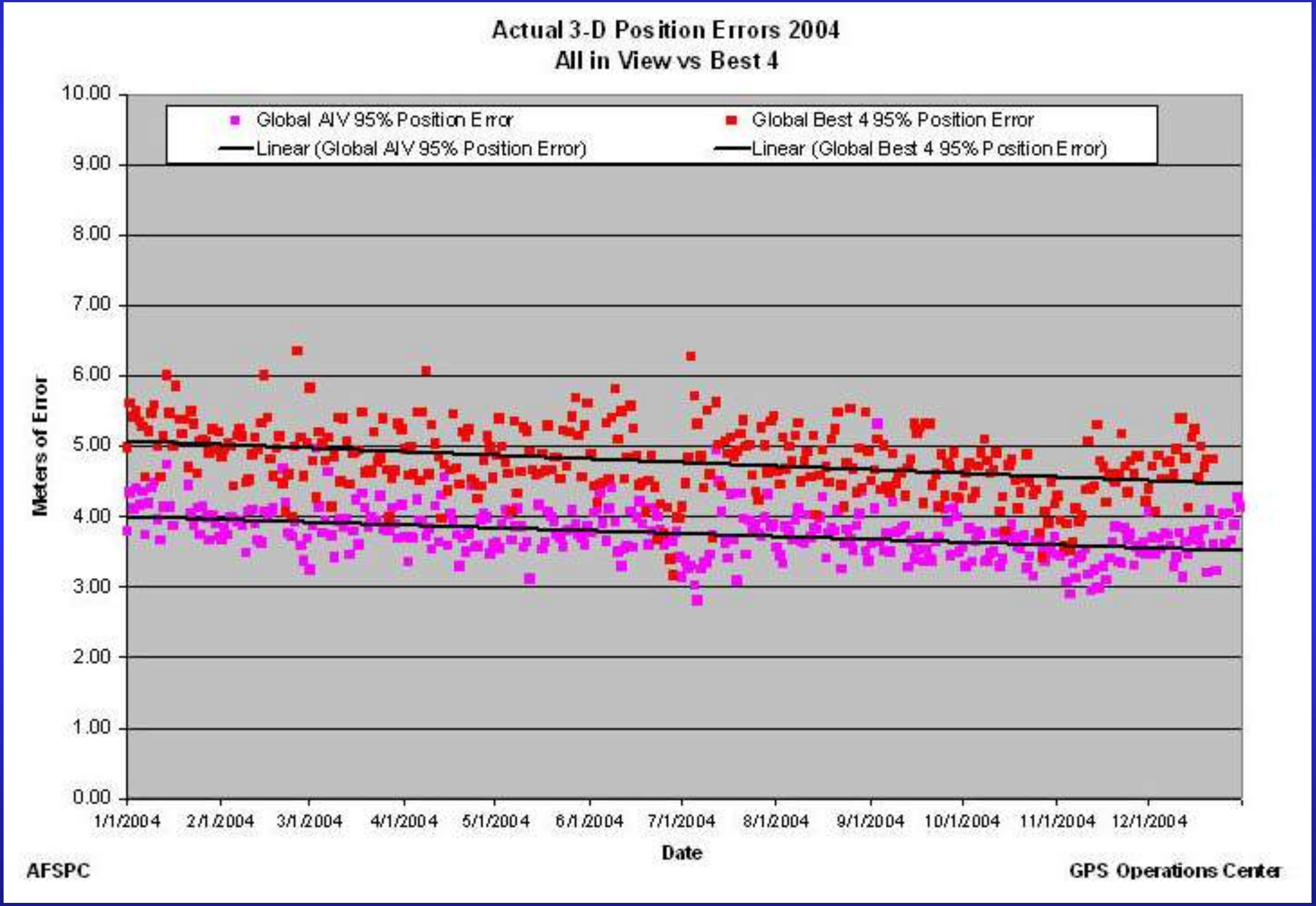
$$GPS_{ERROR} = DOP \times URE$$

  - URE is not a *simple* sum; six components are statistically *added* using root sum square
- $URE = \sqrt{(\Delta p)^2 + (\Delta c)^2 + (\Delta i)^2 + (\Delta t)^2 + (\Delta m)^2 + (\Delta r)^2}$
  - URE are statistical samples — not exact figures

Bottom Line

- GPS accuracy? Standard answer: It depends
- Many factors contribute (time, region of Earth, orbital parameters, constellation status)
  - Some correctable or minimized
  - Some predictable, but not correctable
  - Some fluctuate greatly and difficult to predict
- Despite this, many still want an actual number

2004 3-D Position Error



Summary

– SATNAV theory
– Signals, codes and services
– Errors and accuracy

Questions?

R. Sean Anderson, KR4YO

kr4yo@arrl.net

(703) 707-9025