S I D E B A N D S, H A R M O N I C S, E T C

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Introduction

Have you thought about what modulation sidebands and harmonics are really? Have you *actually* understood the involved concepts?
Let's remember again some important basic concepts.
First we consider here only time independent circuits, that is, whose parameters are constant (the components don't vary with time).
A linear circuit is not able to produce, in its output, signals that contain frequencies different from those of its input. This is easy to understand: if a circuit is linear, its output is (by definition) *proportional* to its input, that means, if an input is sinusoidal with certain frequency, being proportional, its output will be too[1].
Na AM modulator circuit, for instance, receive at it input the modulating signal of frequency $\boldsymbol{\omega}$ and the carrier to be modulated of frequency $\boldsymbol{\Omega}$. But, it has at its output, besides, eventually, the input frequencies, other ones, like harmonics (integer multiples) of $\boldsymbol{\omega}$ and of $\boldsymbol{\Omega}$ and still other like $\boldsymbol{\Omega-\omega}$ and $\boldsymbol{\Omega+\omega}$, that are the so called sidebands.
A frequency multiplier circuit (doubler, Tripler, etc) has a frequency at its input and one of its multiple at its output.
So, the modulator circuit as much as the frequency multiplier one, have, at their outputs, frequencies *different* from those at their inputs and, therefore, **cannot be linear circuits**[2].
This is a fundamental concept.
Another important one is that, every not sinusoidal periodic signal is composed by a sum of sinusoids of frequencies multiple of $\boldsymbol{\omega}$, called fundamental frequency (corresponding to the periodic signal period), as seen in the example of Figure 1. In the case of this figure, a sinusoid with frequency **f** is summed with another one with frequency **3f** and amplitude 1/3 of the first one. In blue, we see (approximately) a sketch of the sum of those sinusoids. We see that a square wave is beginning to be formed. If we sum a sinusoid with frequency **5f** and amplitude 1/5 of the first one, the square wave will be more precise. If we use all odd order harmonics since **1** to **n**, the square wave becomes perfect. We may say that a square wave has energies in all odd harmonics. This means that, if we pass that wave shape through bandpass filters centered at those odd harmonics, all of them will produce output, but if we excite the filters with the fundamental sinusoid, no harmonic filter (for $\mathbf{n \neq 1}$, that is the fundamental itself) will produce output.

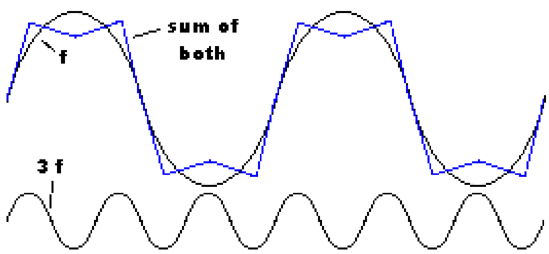

Figure 1

. A time dependent periodic signal can be observed with an oscilloscope, where is seen its waveform in the so-called time domain. But the same signal can be observed in a different manner, with a spectrum analyzer,

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[1]**An introduction of a phase delay that generally depends on the frequency may occur, but it doesn't alter the latter.**
[2] **Indeed, both must be, ideally, if time independent, quadratic circuits, that is, of the second degree.**

where one can see their several sinusoidal components, being called the frequency domain. Both forms are equivalent and describe the same signal.

The New Frequencies Generation

A signal with only one frequency **Ω** is a pure sinusoid (the phase doesn't matter). When we modulate in AM this sinusoid, the signal shape in the time domain is not sinusoidal any more[3]. This means that the modulator itself created other frequencies. Mathematically it is easy to demonstrate that it were created just the new frequencies **Ω-ω** e **Ω+ω**, where **ω** are the several frequencies components of the modulating signal (not necessarily sinusoidal). They are the so-called *sidebands*.

Note that, in the time domain, we don't observe the sidebands and in the frequency domain, we don't observe the signal shape varying with time (modulated). In the frequency domain, as we cannot see amplitude variations, the carrier amplitude is constant. In the time domain, only the temporal composition of constant amplitude sinusoids generates a non-sinusoidal signal and, therefore, with variable amplitude.

It is fundamental to understand that, with the perfect equivalence of both domains, the sidebands are not **due to** the modulation, but they are the modulation itself, only observed in another domain (frequency domain).

So, when we say that an AM modulated signal AM[4] contains sidebands, it means that we are characterizing it in the frequency domain. If characterized in the time domain, we may only talk about its amplitude variation.

The frequency multiplication process is *identical* to the AM modulation. In the doubler, for example, we modulate a signal of frequency **Ω** by another also with the same frequency **Ω**. The sidebands are **Ω+Ω**, that results in **2×Ω** (frequency double) and **Ω-Ω** that results in **0** (DC). For the tripler, the modulating signal would be **2×Ω**[5] and so forth[6].

Have you understood all of this in an enough deep manner? Yes? Then answer this question: a transmitter is generating only one frequency, that of the not modulated carrier, therefore, a pure sinusoid. A receiver in the line of sight (not to be disturbed by propagation effects) is receiving, so, a constant signal. But suppose that the transmission antenna is a turning directional one and can turn freely (hypothetically we don't have any problems with cables) and it is turning with a speed of 1000 turns/second (also hypothetically!). The transmitter goes on generating a pure sinusoid, but in the receiver side the signal is varying in amplitude 1000 times per second. The question is: in the receiver do we observe sidebands? If positive, who generated them and where does come the energy of the sidebands?

The answer is yes; the receiver 'sees' a modulated signal, therefore, observing, in the frequency domain, the presence of sidebands, with its amplitude less than the static case. The energy of those sidebands comes just from the carrier amplitude decrease, that is, is taken from the latter. The transmitter doesn't see any sideband, but the receiver does.

Clearly a question arises here: but the modulator here, obviously, is the rotating antenna that is a linear element. How did a modulation occur? That's a good question!

The answer is in the beginning if the **Introduction**, where we said that we would consider only time independent circuits. The antenna here has a time dependent parameter that is its gain in the receiver direction. This time dependence of circuits, even linear ones, can lead to the creation of new frequencies too[7].

When the modulating frequency is of the order of the carrier one and sinusoidal, the AM modulation phenomenon is used for frequencies conversion (that happens at the converter stage of transmitters and receivers), but the process is exactly the same of the conventional AM modulation.

When the modulation is not AM, but, for instance, in frequency (FM), the amplitude of the modulated signal stays constant in time (time domain), but the shape any more sinusoidal, is the result of the creation of

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[3]A signal is pure sinusoidal if its amplitude is constant along the time, from -∞ to +∞. Any deviation from this standing shape means that the signal IS NOT pure sinusoidal any more.

[4]Any modulation type transforms the signal to a non-sinusoidal one, not only the AM, but we are dealing with this more evident case.

[5]The signal 2×Ω would be the result of the non-linear generation, inner to the multiplier, of the frequency double.

[6]Normally, due the fat that is difficult to control the exact degree of non-linearity, the multiplier generates undesirable frequencies that are eliminated by resonant filtering.

[7]There are mixers/modulators circuits that work just by parametric variation.

sidebands[8] and generally is generated by parametric modulation (time dependence) and not by non-linear modulation.

Conclusion

This article only tries to present the correct concepts about the shown subject that is, many times, misunderstood by many people. A rather deep and reaching presentation was tried, but with an accessible manner to the technician or common radio amateur.

Specially, it is called the attention to the total equivalence between AM modulation and frequency multiplication and conversion. Besides, it is emphasized that 'well-behaved' circuits, that is, linear and time independent ones, cannot be used as modulators and frequency multipliers or converters. It is also presented the concept of time and frequency domains, reinforcing the fact that to observe a modulated signal on an oscilloscope, gives the same information on the signal that to observe it on a spectrum analyzer, being equivalent observations, each with its own domain vision.

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[8] **Here the sidebands are not simply of the form $\Omega+\omega$ e $\Omega-\omega$, but much more complicated, depending upon mathematic functions very transcendent.**