R M S P O W E R...WHAT IS THAT?

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Many articles have referred to *RMS power* in their texts. It is, indeed, a serious conceptual mistake, unacceptable in official texts.

The power with a clear physical meaning is the *average power* that is the result of an RMS voltage or current.

On a resistance **R** one applies a periodic voltage **V(t)** with period **T** (we can do the same for a periodic current **I(t)** on **R**). We ask: which is the *continuous* voltage $\mathbf{V_{eff}}$ applied during the same time **T** on the same resistance **R** that produces the same dissipated energy as **V(t)**? $\mathbf{V_{eff}}$, called efficacious (or efficacious current, in the case of $\mathbf{I_{eff}}$). But the calculi show that (see Appendix) that the efficacious voltage is equal to the RMS (Root Mean Square) of the voltage **V(t)**, that is, $\mathbf{V_{RMS}}$. The RMS of a time function is only a special average taken on all values of that function. So, the RMS voltage has a clear and useful physical meaning, as it corresponds to de DC voltage that, during the same time **T** produces the same energy dissipation of the variable voltage **V(t)** (that may have both components DC and AC). The constant power that produces such dissipation is the average power $\mathbf{P_m}$, as seen in the appendix. For the sinusoidal case, that is, when the voltage is given by $\mathbf{V(t)} = \mathbf{V_0} \times \mathbf{\sin \omega t}$ (or a current $\mathbf{I(t)} = \mathbf{I_0} \times \mathbf{\sin \omega t}$), then $\mathbf{V_{RMS}} = \mathbf{V_0} \div \sqrt{\mathbf{2}}$ or $\mathbf{I_{RMS}} = \mathbf{I_0} \div \sqrt{\mathbf{2}}$ and the average power is $\mathbf{P_{av} = ½\ (V_0\ .\ I_{0)} = ½ \times V_0^2 \div R = ½ \times I_0^2 \times R = ½ \times P_0}$, where $\mathbf{P_0}$ is the peak power.

We must call the reader attention to the fact that the efficacious value is a something physical, but the RMS one is purely mathematical, only a special quadratic average.

It is clear that the power has its own RMS value, $\mathbf{P_{RMS}}$, as this is only the result of an average calculus. But which is its physical interpretation? Which is its use? Indeed there is no special physical interpretation or use for it. So, it **cannot** be used with the meaning of the average power or in its place. Therefore the product of a RMS voltage by an RMS current **is not** an RMS power, **but** an average one. This is conceptual, not only a matter of definition.

The value of the RMS power for the sinusoidal case, as shown in the appendix, results to be:

$\mathbf{P_{RMS} = (\sqrt{3/8}) \times P_0}$ or $\mathbf{P_{RMS} = (\sqrt{3/2}) \times P_{av}}$.

MATHEMATICAL APPENDIX

The instant power corresponding to a periodic voltage **V(t)** with period **T** applied on a constant resistance **R** is given by:

$\mathbf{P(t)} = \mathbf{V(t)^2 \div R}$

The dissipated energy on the resistance during the period **T** is:

$$\mathbf{E} = \int_0^T \mathbf{P(t) \times dt = (1/ R) \times} \int_0^T \mathbf{V(t)^2 \times dt}$$

The average power in this period is given by the energy dissipated in the period divided by this latter:

$$\mathbf{P_{av} = (1/ R) \times (1/T) \times} \int_0^T \mathbf{V(t)^2 \times dt}$$

As a power may be written as a voltage squared divided by the resistance, we may write the average power as:

$$\mathbf{P_{av} = V_{eff}^2 / R}, \text{ where } \mathbf{V_{eff} = [\ (1/T) \times} \int_0^T \mathbf{V(t)^2 \times dt\ ]^{1/2}}$$

By definition, the square root above, that contains the inverse of the period and the time integral, is called the root mean square, or simply RMS of the function **V(t)** in the period **T**, called here $\mathbf{V_{RMS}}$.

As $\mathbf{V_{eff}}$ a constant voltage, it corresponds to the DC voltage value that dissipates in **R** and during the period **T** the same energy dissipated in the same time by the variable voltage **V(t)**. So, we conclude that $\mathbf{V_{eff}} = \mathbf{V_{RMS}}$.

In the case of a pure sinusoidal voltage **V(t)**, that is, $\mathbf{V(t)} = \mathbf{V_0 \times \sin \omega t}$, :

$$\mathbf{V_{RMS} = V_0 \times [\ (1/T) \times} \int_0^T \mathbf{\sin^2 \omega t \times dt\ ]^{1/2}}, \text{ resulting in } \mathbf{V_{RMS} = V_0 \div \sqrt{2}}, \text{ as the integral is } \mathbf{1/2}.$$

As $\mathbf{P_{av}} = \mathbf{V_{eff}^2} / \mathbf{R}$, $\mathbf{P_m} = \mathbf{V_0^2 \div (2 \times R)}$ and as $\mathbf{V_0 = I_0 \times R}$, $\mathbf{P_{av} = V_0 \times I_0 \div 2 = I_0^2 \times R \div 2}$.

The RMS power would be given by a similar expression:

$$\mathbf{P_{RMS} = [\ (1/T) \times} \int_0^T \mathbf{P(t)^2 \times dt\ ]^{1/2}}$$

There is no special meaning for this value.

In the sinusoidal case, we have:

$$\mathbf{P_{RMS} = (V_0^2 \div R) \times [\ (1/T) \times \int_0^T \sin\ \omega t \times dt\ ]^{1/2}}$$

As the integral is $\mathbf{T \times 3/8}$ and $\mathbf{V_0^2 \div R}$ is the peak power $\mathbf{P_0 = 2 \times P_{av}}$, we have:

$$\mathbf{P_{RMS}} = \mathbf{(\sqrt{3/8}) \times P_0 = (\sqrt{3/2}) \times P_{av}}$$

All calculi may be done with $\mathbf{I(t)}$ rather than $\mathbf{V(t)}$, by putting the power in the form $\mathbf{P = R \times I^2}$.