I D E A L   I N D U C T O R S   F O R   L - C   P A S S I V E   F I L T E R S

By Luiz Amaral
PY1LL/PY4LC

In the design of precise LC passive filters, some problems arise due the use of non-ideal components – dissipative capacitors and inductors. The latter are much more important because modern capacitors present neglectable losses and, in the real inductors it is very difficult to avoid windings and/or core losses.
In the case of analog multiplexers, the filters present their attenuation curves with rounded off corners in the transition region between rejection and pass bands, affecting the deepness of the resonances on the poles (Figure 1). This occurs because the design involves coefficients that are calculated in the basis of a lossless transfer functions.
There are many suggestions in the literature to solve this problem. One method employs pre-distorted coefficient tables that produce the required attenuation characteristic, but generating a grate insertion loss in the pass-band (Figure 2).
Another solution uses crystal resonators to get the correct slope of the curves.

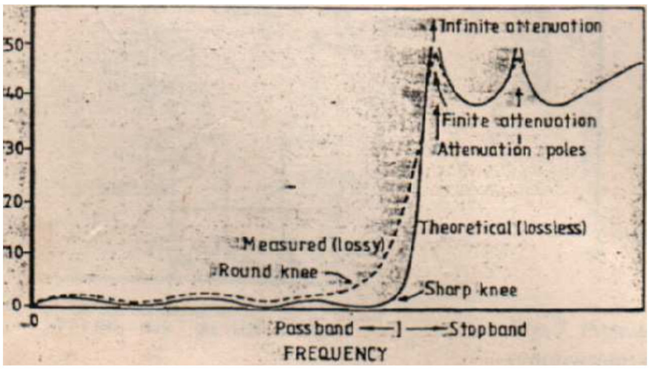

Figure 1: Example of low-pass filter, showing the inductances losses effect. There is a slight change on the pass-band, but on the transition region roundnesses occur.

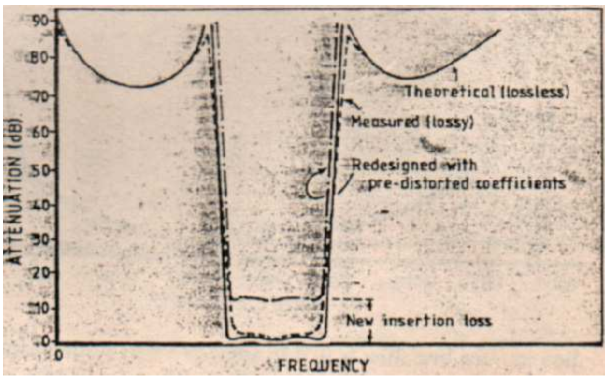

Figure 2: band-pass filter showing the theoretical and measured curves and the result of using pre-distorted coefficients that recovers the curve shape, but introduces an insertion loss.

PROPOSED SOLUTION

In the design of ladder filters, there is a minimum **Q** that satisfies the practical results. In practice, it is easier to get these **Q** values for the series inductors than for the shunt ones, meaning that, if we get the correct **Q** values for the series inductors, the theoretical ideal conditions can be easily approximated.
The proposed solution is really very simple. In the electron tubes era, **Q** multipliers were used to increase the **Q** of the inductors when those values were not obtainable with passive components; the same procedure may be used here in the present case.
Let's **Rp** be the parallel equivalent loss resistance of the inductor **L** (figure 3a). Another (negative) resistance –**Rp** put in parallel with the first one, will result in a ∞[1] value for the total resistance in parallel with the inductor, eliminating its losses. The current on a negative resistance flows in the opposite direction of a positive resistance, that is, it flows in the direction of that of a generator. Moreover, a negative resistance produces energy and thus corresponds to a generator.

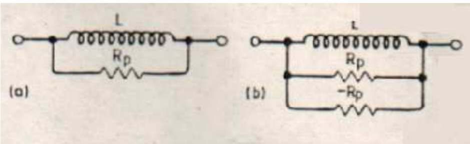

Figure 3a: equivalent loss parallel resistance in (a) can be compensated by another one with the same value but negative (b).

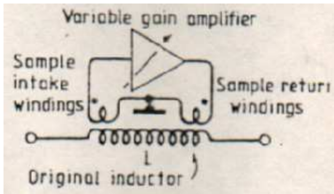

Figure 4: A sample of the signal on the inductor is amplified and returns as positive feedback to cancel losses.

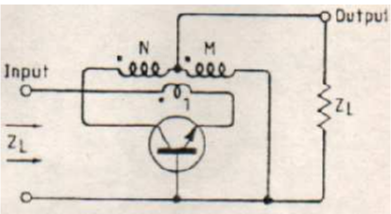

Figure 5: Norton Amplifier, with load $\mathbf{Z_L}$, $\mathbf{N = M^2 - M - 1}$ and transistors with high beta, the input impedance = $\mathbf{Z_L}$ and voltage gain = **M**.

To compensate for the losses, a signal correspondent to the total losses[2] is used. As Figure 4 shows, a signal sample is amplified and returns as a feedback to the inductor in such a manner that the feedback is positive, with correct phase and amplitude to compensate for all the losses.
If all inductors are so modified, the gain adjustment for exactly compensating for the losses produces ideal inductors. In this form, the filter can be precisely described in terms of its lossless transfer function: the attenuation curve recovers its original shape with its sharp corners.
We must note that, although using active devices, the filter continues to be passive: the local activities are used only to compensate for the losses. Thus the filter is now passive with ideal components.
Amplifiers are unidirectional devices. As the main signal flow doesn't pass through the amplifier itself, the original filter bi-directionality is preserved. This may be sometimes an important point.
The same characteristic keeps the circuit reliability, particularly in multi-channel communications, as amplifier failures degrades only the channels close to the curve corners, without total loss of the communications.
This happens to systems that have the amplifier at the main signal path.

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[1] **The value is really -∞ because it is the result Rr of two resistances Rp and -Rp in parallel:**
Rr = Rp x (-Rp) / [(Rp + (-Rp)] → -∞
[2] **The loss compensation system itself introduces its own losses that also must be compensated for.**

This method doesn't introduce any extra insertion loss and, with good amplifiers, it is possible to avoid signal-to-noise ratio degradation. The methods also preserve the group delay of the filter. The inductors are aligned separately and this makes the filter alignment very simple.

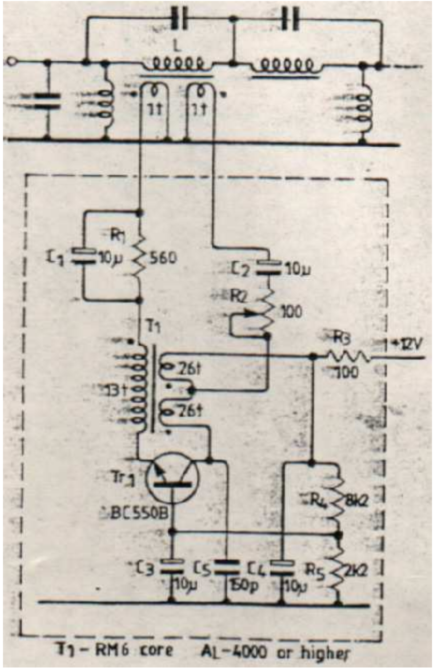

Figure 6: Band-pass filter with a series inductor compensated for losses; all series inductors must be so treated.

CIRCUIT OPERATION

To compensate for losses, the amplifier must be broadband as to amplitude and phase to keep the phase relationship between input and output. It also must present low noise to avoid degradation of the SNR and enough gain to compensate for the losses. Good stability is also important because, without losses and with positive feedback, the circuit shows a trend to oscillate.

One topology that satisfies all those conditions is the Norton amplifier Figure 5, that is almost an ideal amplifier except by two factors: is a very poor isolator because the input impedance is strongly dependent on the load impedance at its output; and its gain is fixed, with no adjustment provision. These factors, however, are not important to our purpose here.

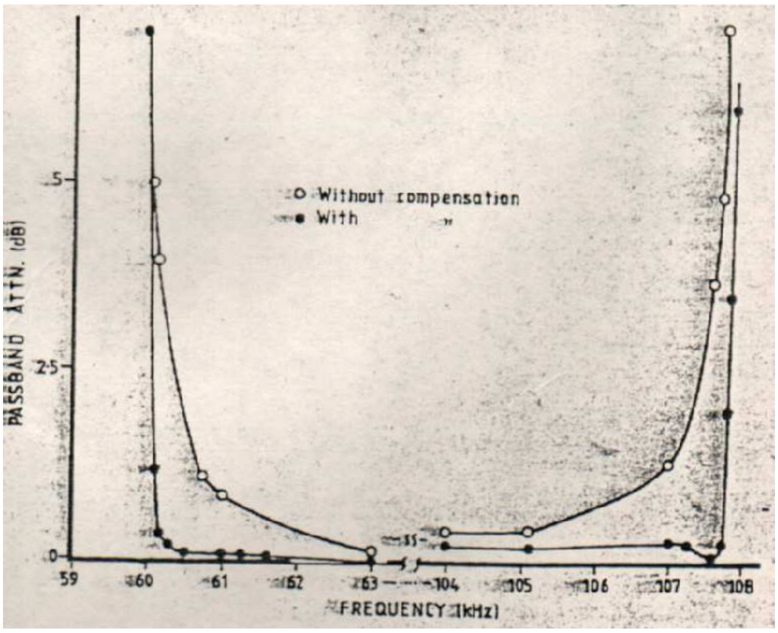

Figure 7: Attenuation curve of the filter of Figure 6 with and without inductors compensation. Note the difference at the corners, rounded by the losses.

In the text:

Rp = equivalent parallel loss resistance of the inductor
Po = amplifier output power
Pi = the power at its input
Pg = lost power on **Rp**
Vo = voltage on the **No** turns
Vi = voltage on the **Ni** turns
V$_L$ = amplifier output voltage
No = number of turns of the return winding
Ni = number of turns of the signal sample winding
Np = number of turns of the inductor **L**
Zo = input impedance of the **No** turns; **Zi** = impedance seen by the **Ni** turns
Z$_L$ = amplifier output impedance; **Q** = quality factor of the inductor **L**
f = frequency of the defined **Q**; **A**$_L$ = inductance / (turns)^2 for the inductor core
M = voltage gain of the amplifier
N = load number of turns of the Norton amplifier

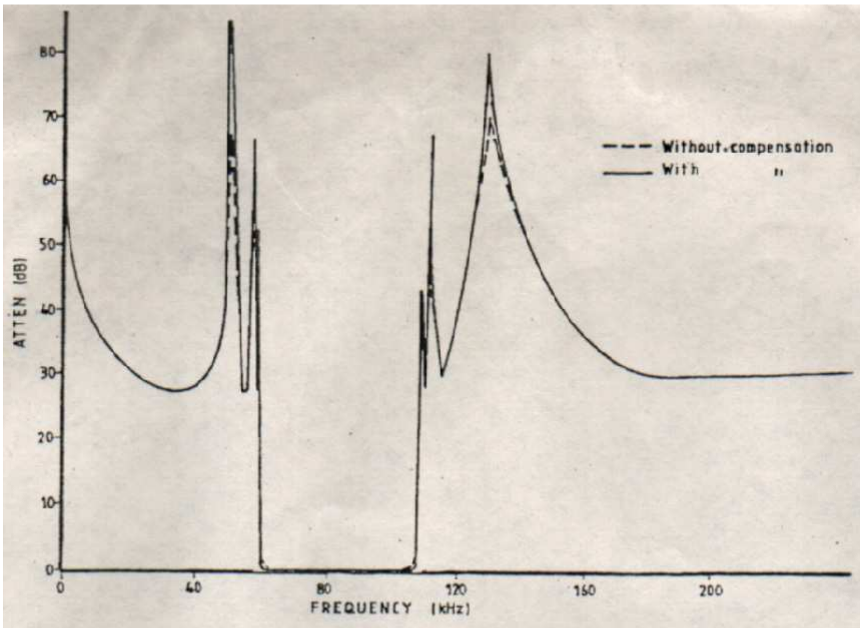

Figure 8: General attenuation curve of the assembled unit. Note the deeper due the greater **Q** values.

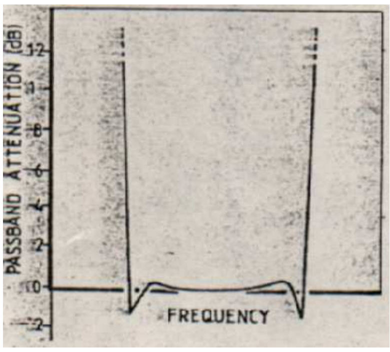

Figure 9: Pass-band attenuation curve due the over-compensation of inductors. At the peaks, the attenuation is negative, that is, an undesirable real gain occurs, being possible for the amplifier to oscillate.

A quick analysis of the circuit in Figure 5 shows:

$Zi = Z_L$, assuming that **$N = M^2 - M - 1$**

Connecting Norton circuit as in Figure 5, one has:

$Z_L = R + Zo$

that is the load impedance of the amplifier (R is an adjustable resistor provided to be possible to adjust the amplifier gain).
Therefore:

$Zi = R + Zo$ [1]

$Vo = M \times Vi$ [2]

Suppose **$Ni = No$**, that is, the auxiliary windings have the same number of turns (in practice 1 turn is enough):

$Vo = Vi$ [3]

From the resistive attenuator formed by **R** and **Zo**, we have:

$\mathbf{Vo = V_L \times Zo / (R + Zo)}$ or $\mathbf{V_L = Vo \times (R + Zo) / Zo}$ **[4]**

The load **Zo** is the result of the loads **Rp** and **Zi** in parallel transferred to the **No** turns, that is, taking into account the turns ratio:

$\mathbf{Zo = Rp / Np^2 \,//\, Zi}$ or $\mathbf{Zo = Zi \times (Rp / Np^2) / (Zi + Rp / Np^2)}$ **[5]**

With [2] in [4]:

M x Vi = Vo x (R + Zo) / Zo or **M x Vi x Zo = Vo X R + Vo x Zo**

From we take **R**:

R = Zo x (M x Vi - Vo) / Vo that with **[3]**, we get:

R = Zo x (M -1) or Zo = R / (M - 1) [6]

With **[1]** in **[5]** we have:

$\mathbf{Zo = Rp \times (Zo + R) / [Rp + Np^2 \times (Zo + R)]}$ **[7]**

With **[6]** in **[7]** we get:

R / (M - 1) = Rp x R x M / (M - 1) or $\mathbf{Rp + Np^2 \times R \times M / (M - 1) = Rp \times M}$

Therefore $\mathbf{R = Rp \times (M - 1)^2 / (Np^2 \times M)}$ **[8]**

But, for a parallel L-R circuit, we have:

$\mathbf{Q = Rp / (2 \times \pi \times f \times L)}$ or $\mathbf{Rp = 2 \times \pi \times f \times L \times Q}$ **[9]**

and $\mathbf{L = Np^2 \times A_L}$ **[10]**

with **[10]** and **[9]** in **[8]**, we have finally:

$\mathbf{R = 2 \times \pi \times f \times A_L \times Q \times (M-1) / M}$ **[11]**

We muse remember that this result holds for the use of a Norton amplifier with a series resistance to compensate the losses; the expression would be different for another type of circuit.
Although the value of **R** in equation **[11]** gives the theoretical null loss to the inductor, it may lead to oscillations or instabilities. A variable resistor (preferentially multiturn) must be used for experimentally getting the desired curve for the filter.
So, equation **[11]** gives us an approximate value for **R**.
The Norton transformer must have the coupling among windings the most perfect as possible to avoid filter perturbations due stray capacitances. The number of turns **N** of that transformer must be the smallest as possible, **N = 1**, preferably.

PRACTICAL RESULTS

To demonstrate the real practicability of this approximation, a band-pass multisection filter from 60 to 108kHz was built, one of its section showed in Figure 6: the inductor **L** is that compensated for the losses. The sample signal is got from **L** through a one turn winding at left, entering the Norton amplifier crossing the dotted line and returning to the inductor through the right via another one turn winding. The resistor **R2** adjusts the gain and **C5** prevents HF oscillations due transformer parasitic capacitances.

Figures 7 and 8 show the attenuation curves with and without the losses compensation. Figure 7 being only an expansion of Figure 8. There is clearly a considerable improvement on both pass and attenuation bands. The compensation is not very affected by temperature variations due the great thermal stability of the Norton amplifier.

ADJUSTMENTS

For the filter alignment, we must use the formal procedure of the filter adjustment, but with the amplifier power supply turned off (with no losses compensation). When all inductors are aligned, we turn on the power supply with the gain control resistors at their maximum value (minimum compensation)
Observe the dips in the attenuation band and, for each pole, increase those dips by means of the corresponding gain controls. An increase of 10dB is reasonable. Observing the attenuation curve, one makes some little readjustments to get the sought response. This must be done with care not to overcompensate the inductors and produce oscillations or undesirable peaks at the pass-band corners, as in Figure 9.
The method generated a patent. Unfortunately, with the coming of the digital technology, filtering by frequency bands became much less used. Even so, in the cases where the passivity and bi-directionality are important factors, they still have some use[3].

REFERENCES

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2. Zverev A. I. Handbook of Filter Syntesis. John Wiley, 1967
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[3] **For example, some SSB generation/detection systems by the so-called 'third method', or even audio filters, can use this type of solution.**