INDUCTORS COUPLING COEFFICIENTS

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As in Figure 1, suppose we have two inductors with the same number of turns per unit of length **N**, wound on the same tube with radius **R**. The inductors have inductances and lengths respectively **L1**, **C1** and **L2**, **C2** and the distance between them on the tube is **C3**.

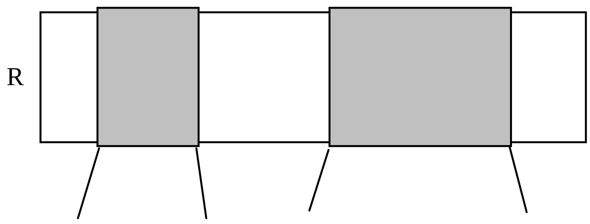

Figure 1

The expression of the inductance **L** of an inductor with **N** turns, length **C** and radius **R** is given by[1]:

$L=R^2.N^2/(9.R+10.C)$ (1)

By applying **(1)** to **L1** and **L2**, one has:

$L1= R^2.N1^2/(9.R+10.C1)$ (2)

$L2= R^2.N2^2/(9.R+10.C2)$ (3)

Besides, if we put both inductors in series, the total inductance will be:

L12=L1+L2+2.M12 (3), with **M12** being the mutual inductance between the two inductors.

M12 may be written as:

M12=K12.√(L1.L2) (4), where **K12** is the so-called coupling coefficient between the inductors and may vary from **0** to **1**.

L12=L1+L2+2.K12.√(L1.L2) (5)

In **(5)**, **L12** as **K12** are unknown due the finite distance **C3** between the inductors.

This article tries to show an expression for this coupling coefficient.

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[1] **This expression is valid only for values in inches. For centimeters a suitable conversion is necessary.**

For this let's see the case in which **C3** is zero, that is, both inductors form a continuous coil, as in Figure 2.

As both inductors are contiguous, the expression **(1)** is applicable, and we have then the total inductance value.

The inductors have now inductances, lengths and number of turns respectively **La**, **Ca**, **Na** and **Lb**, **Cb**, **Nb.**

The total inductor has inductance **Lab,** length **Cab=Ca+Cb** and number of turns **Nab=Na+Nb**.

We may write:

$Lab=La+Lb+2.Kab.\sqrt{(La.Lb)}$ **(6)**

As the expression **(1)** is valid for the total inductor as to all the individual inductors, we may use it on **(6)**:

$R^2.Nab^2/(9.R+10.Cab)=R^2.Na^2/(9.R+10.Ca)+R^2.Nb^2/(9.R+10.Cb)+2.Kab.R^2.\sqrt{[Na^2.Nb^2/(9.R+10.Ca).(9.R+10.Cb)]}$ or

$Kab=\{(Na+Nb)^2/[9.R+10.(Ca+Cb)]-Na^2/(9.R+10.Ca)-Nb^2/(9.R+10.Cb)\}/(2.\,.\sqrt{[Na^2.Nb^2/(9.R+10.Ca).(9.R+10.Cb)]}$ **(7)**

In **(7)** all elements of the second member are given and, therefore, **Kab** is determined.

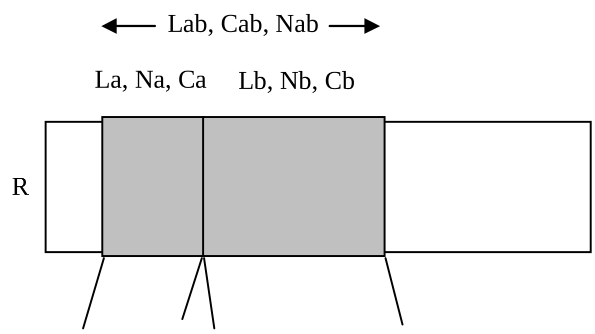

Figura 2

So, this shows that it is possible to determine the coupling coefficient in the case of contiguous inductors.

In the case of Figure 1, we can use a trick that is to fulfill the space between the inductors with an imaginary one with inductance, length and number of turns respectively **L3**, **C3** e **N3**, as in Figure 3.

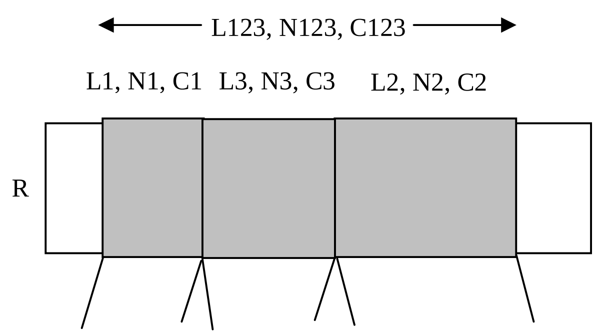

Figure 3

In this case the total inductance is **L123**, its length **C123** and number of turns **N123**.

As **L1**, **L2** and **L3** are contiguous, we can apply **(6)** to the pairs **L1-L3** and **L2-L3**, with the coupling coefficient **L12** being the unknown element to be calculated in the case **L1-L2**.

Making these applications, we have:

L13=L1+L3+2.K13.√(L1.L3) (8)

L23=L2+L3+2.K23.√(L2.L3) (9)

L12=L1+L2+2.K12.√(L1.L2) (5)

Using **(7)** in **(8)** and **(9)**, we get the values of **L13** and **L23**.

Writing the expression of the total inductance, we have:

L123=L1+L2+L3+2K12.√(L1.L2)+2.K13.√(L1+L3)+2.K23.√(L2.L3) (11)

As they are contiguous inductors, we can apply **(1)** in **L123**, having **K12** as unknown:

K12=L123-L1-L2-L3-2.K13.√(L1+L3)+2.K23.√(L2.L3) (12)

We see that all the second member is known by the use of the expressions **(6)** e **(1)**, where, inspire the fact that **L3** not be a real inductor, it is used to solve the problem presented in Figure 1, where both inductors are physically separated.

The fundamental used condition is that the tube radius is the same for all inductors and the number of turns per unit of length is the same for all too.

It is more efficient to use a computer program to solve the long equation for **K12** when all values of the second member of **(12)** are replaced **(12)**.