ANALYSIS OF SOME END FED ANTENNÆ

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Introduction

Some end fed antennæ is well known in the hamradio community. One of them, known as J-Pole and normally used on VHF and above, is a radiating vertical ½ wavelength element, coupled to its end through a short-circuited ¼ wavelength transmission line.
Another one, equally well-known, is the so-called Zepp[1] antenna, which is a horizontal radiating element, normally ½ wavelength long and fed at one end through a parallel line. When this line is ¼ wavelength long and also short-circuited at the lower end, the antenna is equivalent to a J-Pole one, but with horizontal radiating element.
Both are shown in Figure 1.

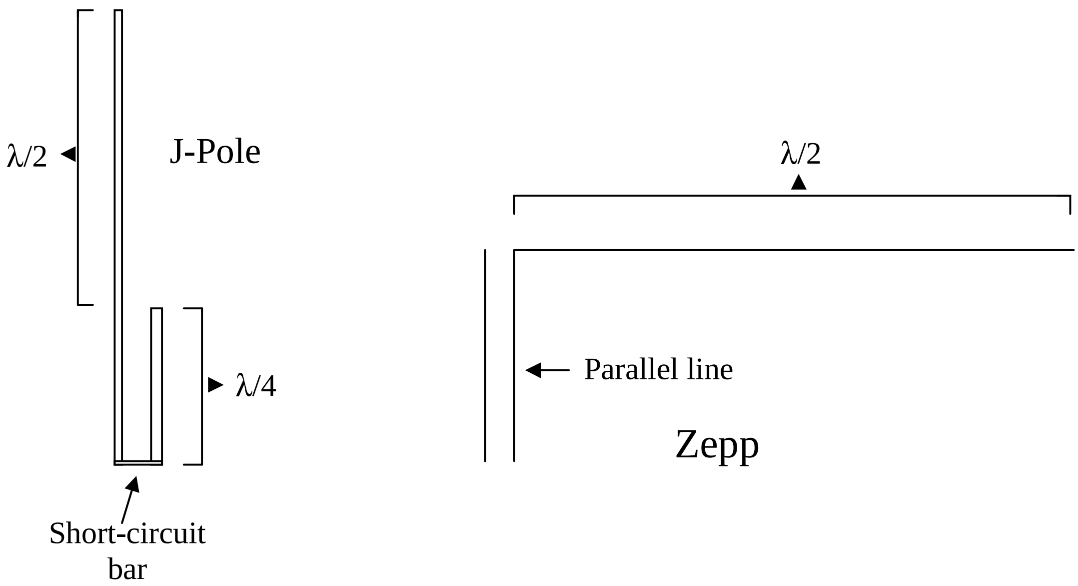

Figure 1

In the J-Pole, the feeder to the radio, normally a coaxial cable, is connected to the ¼ wavelength line at the distance from the short-circuit that correspond to the match with VSWR 1:1 at the desired frequency, permitting any length for the referred cable.
In the Zepp case, in principle the parallel line may have any length, being connected to the radio through a suitable coupler of any type, L-C or even other cables. Here we will deal only to the case where that length is ¼ wavelength with a short-circuit in the lower end, being similar to a J-Pole and being fed like this. The advantage of this method is that, below the short-circuit point the line may be connected to the ground without affecting the operation, but guaranteeing a DC path between the antenna and ground for protection against electrical discharges.

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[1]**This antenna was so called because it was firstly used in zeppelins and is also called End Fed Zepp to distinguish it from the Center Fed Zepp, that is not indeed an original Zepp, but only a dipole center fed with a high impedance line.**

Analysis

We will start our theoretical analysis by the model above referred. We will search for the point of the line that corresponds to the radio feeder cable impedance, getting the best match as possible.
In Figure 2 we have the diagram of such a system, where the ¼ wavelength line has a physical length (taking into account the velocity factor) **L = λ/4**, its impedance is **Zo**, the resistive and reactive components of the antenna end point impedance **Za** are respectively **Ra** and **Xa** and the impedance of the radio cable is **Zo'**.

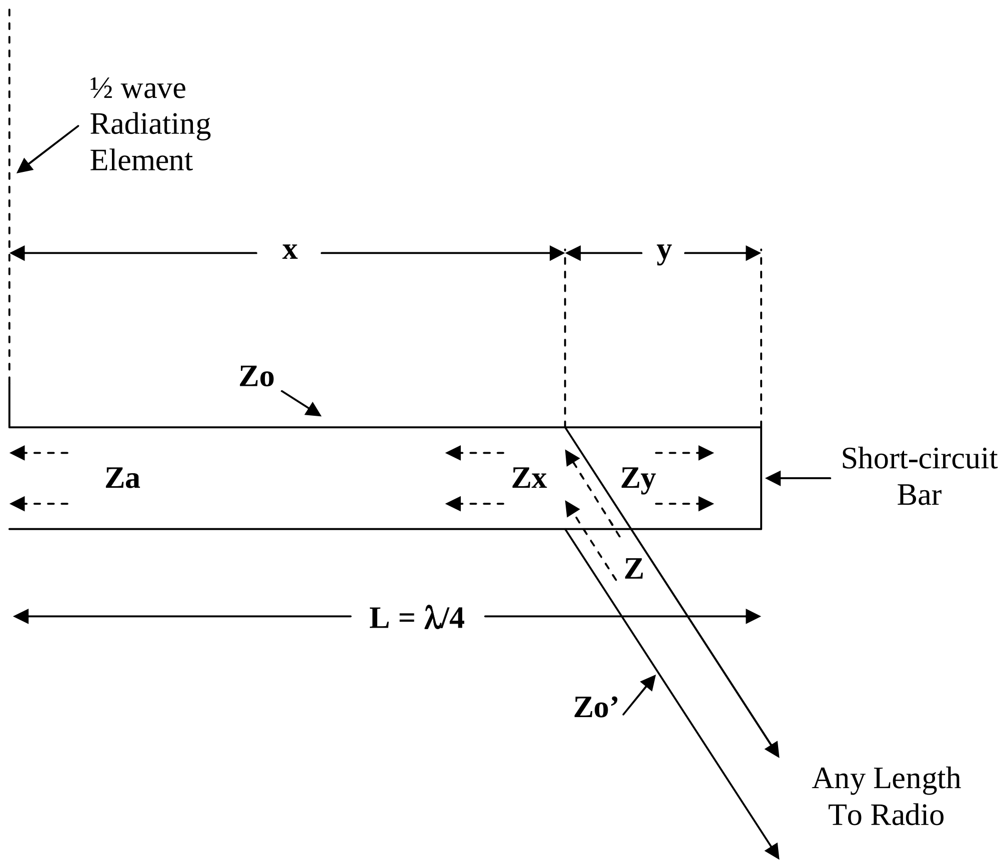

Figure 2

From Figure 2 we see that the line that goes to the radio 'sees' on the ¼ wavelength line an impedance **Z** that is the result of paralleling two impedances **Zx** e **Zy**. The first one is the reflected impedance by the antenna load **Za** over the line of length **x**; the second is the reflected impedance by the null load (short-circuit) over the line of length **y**.
The expression of the impedance **Z** reflected over an ideal line[2] of impedance **Zo**, length **l** and load **Zc** is given by:

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[2]**Here we are considering all lines as ideal, that is, lossless, because normally we choose low loss lines in the practical systems: by taking into account the losses, we would only be led to more complications on the involved equations with no substantial results changing.**

$\mathbf{Z = Zo.(Zc + j.t.Zo)/(Zo + j.t.Zc)}$ **[I]**, where $\mathbf{t = tg\ 2.\pi.l/\lambda}$

Here λ is the wavelength on the line, that means, taking into account its velocity factor and **tg** the trigonometric function **tangent**.
We can use **[I]** to calculate the impedances **Zx** and **Zy.**
For **Zx**, $\mathbf{l = x}$, $\mathbf{Zc = Za}$ and $\mathbf{t = tx = tg\ 2.\pi.x/\lambda}$.
For **Zy**, $\mathbf{l = y = L - x = \lambda/4 - x}$, $\mathbf{Zc = 0}$ and $\mathbf{t = ty = tg\ 2.\pi.(L - x)/\lambda}$.
So, we have:

$$\mathbf{Zx = Zo.(Za + j.tx.Zo)/(Zo + j.tx.Za)}$$

As $\mathbf{Za = Ra + j.Xa}$,

$$\mathbf{x = Zo.[Ra + j.(Xa + tx.Zo)]/[(Zo - tx.Xa) + j.tx.Ra]} \quad \mathbf{[II]}$$

$$\mathbf{Zy = Zo.\ j.ty.Zo/Zo = j.ty.Zo}$$

But $\mathbf{ty = tg\ 2.\pi.(\lambda/4 - x)/\lambda = tg\ (\pi/2 - 2.\pi.x/\lambda) = 1/tg\ x/\lambda = 1/tx}$. Therefore **Zy** may be written as:

$$\mathbf{Zy = Zo.j.Zo/tx} \quad \mathbf{[III]}$$

The impedance **Z** seen by the cable **Zo' is the** result of **Zx** paralleled with **Zy**:

$$\mathbf{Z = Zx.Zy/(Zx + Zy)} \quad \mathbf{[IV]}$$

After some manipulation, after using **[II]** and **[III]** in **[IV]**:

$$\mathbf{Z = [Ra + j.(Zo.tx + Xa)]/(1 + tx^2)}$$

Note that, as **tx**, **Zo**, **Ra** and **Xa** are real, **Z** results to be a complex number. But, as we have to couple a real impedance cable to the ¼ wavelength line, **Z** must be real and, therefore, its imaginary component must be zero. So:

$\mathbf{Zo.tx + Xa = 0}$ or

$$\mathbf{Xa = -Zo.tx} \quad \mathbf{[V]}$$

[V] shows that the reactance of the antenna input point must be negative (or capacitive), as **Zo** and **tx** are both positive. This means that, for these so coupled antennæ, the radiating element must be shortened with regard to the resonant length $\mathbf{\lambda/2}$.
As **Z** must be equal to the impedance **Zo'** of the radio cable, we have:

$\mathbf{Z = Zo' = Ra/(1 + tx^2)}$ or

$$\mathbf{Ra = (1 + tx^2).Zo'} \quad \mathbf{[VI]}$$

As $\mathbf{tx = tg\ 2.\pi.x/\lambda}$ **[VIIa]**, we have for **x**:

$\mathbf{x = [\lambda/(2.\pi)].tg^{-1}\ tx}$ **[VII]**, where $\mathbf{tg^{-1}}$ is the inverse function of **tg**, that is, **arctg**.

Eliminating **tx** between **[VI]** and **[VII]**, one has:

$\mathbf{Ra = Zo'.(1 + tg^2\ 2.\pi.x/\lambda)}$ or $\mathbf{Zo' = Ra.cos^2\ 2.\pi.x/\lambda = Ra.cos^2\ tx}$, where **cos** is the function **cosine**.

$\mathbf{x = [\lambda/(2.\pi)].cos^{-1}\ \sqrt{(Zo'/Ra)}}$ **[VIII]**, where $\mathbf{cos^{-1}}$ is the inverse function of **cos**, that is, **arccos.**

Or in another way:

$$\mathbf{x = [\lambda/(2.\pi)].tg^{-1} \sqrt{(Ra - Zo')/Zo'}} \quad \mathbf{[VIIIa]}$$

The distance **y** from the short-circuit bar to the connection point of the two lines (more convenient than **x**) is:

$$\mathbf{y = [\lambda/(4.\pi)].[\pi - 2. \cos^{-1} \sqrt{(Zo'/Ra)}]} \quad \mathbf{[IX]}$$

Or in another way:

$$\mathbf{y = [\lambda/(2.\pi)].tg^{-1} \sqrt{Zo'/(Ra - Zo')}} \quad \mathbf{[IXa]}$$

Installation

The best process to installing an antenna with that coupling method is start by the radiating element.
We install this element with its resonant length, as the resistive component does not vary too much around resonance[3]. We measure the value of the component **Ra** and we apply it in the expression **[IX]** together the values of **Zo'**, the impedance of the radio cable (normally 50Ω), and **λ** (remembering again that this is calculated on the line, that is, taking into account the line velocity factor). The value **y** got (in the same unit as **λ**) is the distance (from the short-circuit of the ¼ wavelength line) where we must connect the radio cable. Note that that distance doesn't depend on **Zo**, the ¼ wavelength line impedance. We calculate **x** using **[VIII]** or **[VIIIa]** and **y** using **[IX]** or **[IXa]**. This value of **x** is applied to the expression **[VIIa]**, getting **tx**. The latter enters the expression **[V]** and we get **Xa**, after have chosen the desired value for the impedance **Zo**[4]. Then we install the ¼ wavelength line. Measuring the antenna input impedance[5], we adjust its length to get the reactance **Xa** value. Now we may close the short-circuit on the ¼ wavelength line and connect the radio cable on the correct position and this may have any length. Due small variations when redimensioning the antenna radiating element to adjust Xa, it is clear that we can redo all the process from **Ra** measuring how many times we want to get the best coupling as possible.
The antenna physical length misadjusting, that is, the use of a not appropriate **Xa** value, will lead to the impossibility of the match of the radio cable with the ¼ wavelength line, resulting on that cable a VSWR always greater than 1:1.
In the specific case of the J-Pole, the ¼ wavelength line is built together the antenna itself, but we may use a commercial parallel line, with some loss increase (remembering that this antenna is used in higher frequencies).
There are some great differences between a J-Pole and a Zepp. The first, working in a normally higher frequency, it is always put far (in wavelengths) from objects and metallic conductors and it is more balanced in the sense that the ¼ wavelength doesn't radiate too much.
The second suffers almost always the interference from elements of the surroundings, is normally close to the ground and, due the relative position between the radiating element and the ¼ wavelength line, the latter always leads common mode currents ("antenna currents"), producing radiation. These currents go to the ground and, so, the connection to ground of the lower end of the line modifies the global antenna characteristics, being able to alter its performance, depending on the ground quality itself. So, for the Zepp antennæ, it is advisable that we attenuate those common mode currents to a minimum by the use of baluns

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[3]**For this type of operation to be correctly performed, we must always use an antenna impedance meter. This has to be able to measure resistance and reactance values up to the order of 10KΩ. (a good example is the AIM-4170 from Array Solutions) to be possible to deal with antennæ end point impedances.**
[4]**Normally we must choose the line with the highest impedance as possible because we get the smallest VSWR on it, decreasing losses and limiting less the transmitted power. It seems not to exist commercial lines with more than 600Ω and, using the expression of the parallel line impedance Zo = 276 log 2S/d, for practical values of the distance S between conductors, around 70cm, and minimum conductors diameter, say 1mm, we verify that the maximum obtainable impedance is around 870Ω, still much smaller than the input impedance of a wire antenna on its end. With a distance between conductors of 45cm and using 1,5mm wires, much more practical, we get an impedance of about 767Ω.**
[5]**Meters, as the commercial example of the note 3, presents the possibility of measuring the antenna input impedance through the line itself, therefore without the need of measuring directly on that input point, what would be very inconvenient.**
There are free computer programs in the internet that perform the same function. The used line may be the ¼ wavelength line without the short-circuit.

(normally of current type). This need can be easily noted by connecting and disconnecting the ground from the lower end of the ¼ wavelength line and observing the VSWR change and/or the signal reported by another station. The greater those changes, the greater misbalance of the line.

An Alternative

For the Zepp antennæ that, as we saw before, are rather hard to adjust because it needs many antenna lowering-adjust-rising cycles, one solution can be obtained by replacing the short-circuit bar by a reactance. This, fortunately, is positive or inductive and, therefore it may be got with an inductance that guarantees the DC path between both line wires, keeping the protection against electric discharges. The Figure 2 may be used here, but with the short-circuit bar replaced by a positive reactance **Xh**.
The expression **[II]** that defines **Zx** is obviously maintained, but the definition of **Zy** is altered and the expression **[III]** doesn't hold any more. Using the general equation **[I]**, one can obtain the expression for the new **Zy**, with **Zh = j.Xh**:

Zy = Zo.(j.Xh + j.ty.Zo)/(Zo – ty.Xh) that results in:

Zy = j.Zo.(Xh + ty.Zo)/(Zo + ty.Xh)   [III']

The impedance of the radio cable connection point **Z** that is the parallel of **Zx** and **Zy**, after some manipulation and **ty = 1/tx** still holding, results in:

$$\mathbf{Z = Zo.[tx.Xh = +Zo)].[Rb + j.(Xb + tx.Zo)].[(Zo^2 - Xb.Xh) - j.Rb.Xh]/\{(1 + tx^2).[(Zo^2 - Xb.Xh)^2 + Rb^2.Xh^2]\}}$$

As we want **Z** real, its imaginary component must be zero. This results in:

$$\mathbf{Xb.Zo^2 - Xb^2.Xh + tx.Zo^3 - Xb.tx.Zo.Xh - Rb^2.Xh = 0}$$

Here the radiating element may be resonant and, in this case, **Xb = 0**. Thus we get:

$\mathbf{Tx.Zo^3 = Rb^2.Xh}$ or

$\mathbf{Xh = tx.Zo^3/Rb^2}$ **[V']**

[V'] shows that, if the radiating element is resonant, we need a positive reactance (inductive) for the match to the radio cable to be possible. The value of the inductance that corresponds to that reactance is given by:

$\mathbf{Lh = Xh/(2.\pi.f)}$ **[V'']**, where **f** is the operation frequency.

With its imaginary component zeroed, the impedance **Z** is written as:

$Z = (tx^2.Zo^2 + Rb^2)/[Rb.(1 + tx^2)]$ [VI']

As we want **Z = Zo'**, the radio cable impedance, using **[VI']**, we have for **tx** and **ty**:

$\mathbf{tx = \sqrt{[Rb.(Rb - Zo')/(Rb.Zo' - Zo^2)]}}$

$\mathbf{ty = \sqrt{(Rb.Zo' - Zo^2)/[Rb.(Rb - Zo')]}}$

As $\mathbf{ty = 2.\pi.y/\lambda}$, we have:

$\mathbf{y = [\lambda/(2.\pi)].tg^{-1} \sqrt{(Rb.Zo' - Zo^2)/[Rb.(Rb - Zo')]}}$ **[IX']**

The adjust process is similar to that of the former system, but the coupling adjust is performed by the value of the reactance on the line lower end. To make the process easier, as the use of only a coil is not a must,

we may regulate the reactance value by the use of a variable capacitor in parallel with an inductor (this must have a slightly higher value than that calculated by expression **[V'']**, say 10 to 20% more for the adjust to be possible). With the capacitive reactance in parallel, it is possible to obtain the reactance positive value that we want.
The wanted reactance **Xh** is the result of an inductive reactance $\mathbf{X_L}$ paralleled with the capacitive one **Xc**:

$\mathbf{Xh = X_L.Xc/(X_L + Xc)}$, and we get for **Xc**:

$\mathbf{Xc = Xh.X_L/(Xh - X_L)}$ **[X']**[6] or

$\mathbf{C = 1/(2.\pi.f.Xc)}$ **[XI']**

Here the variable capacitor **C** may be chosen to resonate with $\mathbf{X_L}$ approximately in the half of its maximum capacity, giving enough margin for the adjust in the entire operation band. Perhaps there is the need to recalculate the inductance $\mathbf{X_L}$ to adapt a variable capacitor that we already have. Obviously we can use a variable capacitor in series with a fixed one, both with maximum working voltage enough to tolerate the involved power. But, as the impedance at that line end is lower than **Zo'** (the radio cable impedance, normally 50Ω), the voltage will be lower than that of the radio cable. For a power of 2kW, for example, the voltage in the 50Ω coaxial cable is less than 320V, so there must not be big problems concerning the isolation of those capacitors.
As we said before, for the J-Pole there is no reason for using this method because it is very simple to adjust of the antenna by the former method, which has no extra reactance.

Conclusion

This article shows how it is possible to adjust end fed antennæ using two slightly different methods, both employing ¼ wavelength lines. It also shows the similarities and differences between the J-Pole and the Zepp antennæ, letting the user designs, assembles and measures his antennæ of those two types.

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[6]As the inductance is slightly greater that that calculated by [V''], in [X'] the reactance Xh is smaller than X_L, and this results in a negative Xc, that means, a capacitor.