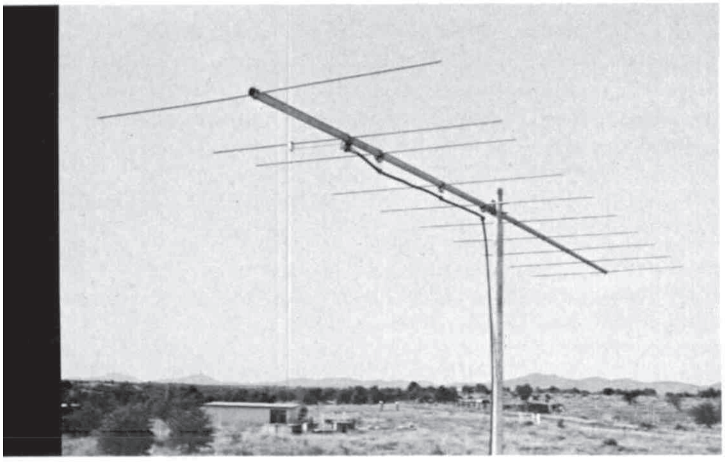

# how to design gamma-matching networks

A complete procedure for designing gamma and tee antenna-matching networks

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**When the driving-point impedance** of thin linear antenna systems is known, these systems can generally be gamma- or T-matched to a transmission line with excellent results. The gamma-match is used to match an unbalanced coaxial line to either a monopole or a dipole driven element, and the T-match is used to match a two-wire balanced line to a dipole driven element.

I will first discuss the gamma matching procedure, and will follow that with the T-match, showing that it is simply an extension of the technique used to design a gamma match.

## gamma match

The layout of the gamma-match, minus the capacitor, is one-half that of a T-match, as shown in **fig. 1**. In this illustration L is the length of one-half of the driven dipole (total length of the driven monopole) and is often near one-quarter wavelength long. The length of the gamma rod, which is parallel to the driven element, is labeled $l_\Gamma$. The center-to-center spacing between the gamma rod and driven element is S, and $Z_i$ is the input impedance to the gamma network, including the driving-point impedance.

When the element and gamma-rod diameters and spacings are *much* less than the carrier wavelength at which the system will be used, the geometry of **fig. 1** is analogous to the electrical circuit shown in **fig. 2.**[1,2,3] In **fig. 2** the input impedance, $Z_i$, is equal to

$$Z_i = \frac{(H_z Z_a)(X_s)}{H_z Z_a + X_s} \text{ ohms} \quad \textbf{(1)}$$

where $Z_a$ is one-half the dipole driving-point impedance (or monopole driving impedance), $X_s$ is the reactance of the gamma rod and $H_z$ is the antenna impedance step-up ratio.

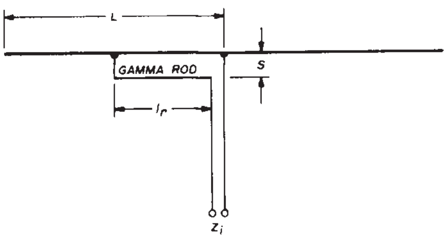



**fig. 1. The basic gamma match. L is one-half the length of the driven dipole.**

The reactance of the gamma rod $X_s$, is given by

$$X_s = jZ_o \tan kl_\Gamma \text{ ohms} \quad \textbf{(2)}$$

$$\text{where } Z_o = 60 \cosh^{-1}\left(\frac{4S^2 - D^2 - d^2}{2Dd}\right) \text{ohms}$$

in air* and $k = 2\pi/\lambda$ radians per meter. The reactance, $X_s$, is positive when the gamma rod electrical length, given by $kl_\Gamma$, is less than $\pi/2$ radians, as will be assumed here.

*I derived the characteristic impedance $\sqrt{L/C}$ for the high-frequency lossless line in air from the farads/meter length given on page 78 of *Static and Dynamic Electricity*, W. Smythe, McGraw-Hill Book Company, Inc., 1968, along with an approach similar to that given on page 210 of reference 13. The same solution appears on page 22-23 of *Reference Data for Radio Engineers*, Fifth Edition, ITT Staff, Howard W. Sams & Company, 1969.

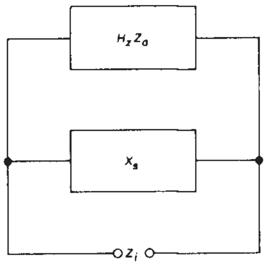



**fig. 2. Equivalent circuit to the gamma-matching system shown in fig. 1. Za is one-half the dipole input impedance, Hz is the impedance step-up ratio and Xs is the reactance of the gamma rod.**

The formula for the numeric antenna impedance step-up ratio, $H_z$, is

$$H_z = \left[1 + \frac{\cosh^{-1}\left(\frac{4S^2 - D^2 + d^2}{4Sd}\right)}{\cosh^{-1}\left(\frac{4S^2 + D^2 - d^2}{4SD}\right)}\right]^2 \quad \textbf{(3)}$$

where D is the diameter of the driven element, d is the diameter of the gamma rod and S is the spacing between the two. This factor is plotted in normalized form in **fig. 3** for use in gamma-match designs. In plotting this graph it was assumed that $H_z$ is at least 4:1, a realistic assumption in most amateur applications.

When the antenna is a self-resonant folded monopole the length of the gamma rod approaches $\pi/2$ electrical radians and $X_s$ becomes much larger than $H_z Z_a$. In this case **eq. 1** is reduced to

$$Z_i = H_z Z_a \text{ ohms} \quad \textbf{(4)}$$

In this case $Z_a$ is not highly resistive and a low vswr at the junction of the folding element and the coaxial line may not be possible.

The complex input impedance, $Z_i$, of **eq. 1** may be written as a magnitude M and phase angle as given here

$$Z_i = M \angle \psi \quad \textbf{(5)}$$

where

$$M = \frac{X_s H_z \sqrt{R_a^2 X_a^2}}{\sqrt{(H_z R_a)^2 + (X_s + H_z X_a)^2}} \text{ ohms}$$

$$\psi = \tan^{-1}\left[\frac{H_z (R_a^2 + X_a^2)}{X_s R_a} + \frac{X_a}{R_a}\right] \text{degrees}$$

To match a lossless high-frequency transmission-line characteristic resistance, $R_o$,

$$M \cos \psi = R_o \text{ ohms}$$

The lumped reactance, $X_\Gamma$, added in series with $Z_i$ to tune out or cancel the reactive component of $Z_i$ can be determined from

$$M \sin \psi = -X_\Gamma \text{ ohms}$$

For **eq. 5** to have the correct magnitude and phase angle,

$$X_s = H_z \left\{\frac{R_o X_a + \sqrt{(R_o X_a)^2 + R_o [H_z R_a - R_o] [(R_a)^2 + (X_a)^2]}}{H_z R_a - R_o}\right\} \text{ohms} \qquad \textbf{(6)}$$

From which

$$X_\Gamma = -R_o \tan \psi$$

$$= -R_o \left\{\frac{H_z [(R_a)^2 + (X_a)^2]}{X_s R_a} + \frac{X_a}{R_a}\right\} \qquad \textbf{(7)}$$

The electrical length of the gamma rod (see **eq. 2**), $kl_\Gamma$, is equal to

$$kl_\Gamma = \frac{2\pi l_\Gamma}{\lambda} \cong \frac{2\pi \, f_{MHz} \, l_\Gamma \, (360)}{(0.956)\,(300)\,(39.37)\,(2\pi)}$$

$$\cong 0.03188 \, f_{MHz} l_\Gamma \text{ degrees} \qquad \textbf{(8)}$$

The quantity $l_\Gamma$ is the length of the gamma rod in inches and the 0.956 coefficient is an average relative velocity along the gamma rod and radiating elements.[4]

As I was developing **eqs. 6** and **7** two ratios often appeared in my calculations. I have called these T and Q:

$$T = \frac{H_z}{Z_o} \text{ ohms}^{-1} \qquad \textbf{(9)}$$

$$Q = \frac{X_s}{H_z} = \frac{\tan kl_\Gamma}{T} \text{ ohms} \qquad \textbf{(10)}$$

These two ratios are useful observations because $H_z$ and $Z_o$ are a function of S/D D/d. That is, when $H_z$ is determined, T can be found, and a solution for Q eliminates a separate calculation for $Z_o$. **Fig. 4** is a normalized graph of T vs S/D and D/d, and **fig. 5** is a plot of $kl_\Gamma$ in degrees vs Q and T. These graphs are extremely helpful when designing a practical gamma matching system.

When **eq. 10** is substituted into **eq. 6**, a solution for Q is a positive discriminant root of a quadratic equation as follows,

$$Q = A + \sqrt{A^2 + B} \text{ ohms} \qquad \textbf{(11)}$$

Where

$$A = \frac{R_o X_a}{H_z R_a - R_o} \text{ ohms}$$

$$B = \frac{R_o [(R_a)^2 + (X_a)^2]}{H_z R_a - R_o} \text{ ohms}^2$$

When **eq. 10** is used with **eq. 7** a solution for the reactance of the gamma capacitor can be obtained

$$X_\Gamma = -\frac{1}{2\pi f C_\Gamma} = -[E + F] \text{ ohms} \qquad \textbf{(12)}$$

This equation may be rearranged to provide the value of the gamma capacitor in pF

$$C_\Gamma = \frac{10^6}{2\pi \, [E + F] \, f_{MHz}} \text{ pF} \qquad \textbf{(13)}$$

Where

$$E = \frac{R_o}{R_a}\left[\frac{(R_a)^2 + (X_a)^2}{Q}\right] \text{ohms}$$

$$F = \frac{R_o}{R_a} X_a \text{ ohms}$$

In **eqs. 12** and **13** the sum of E and F is positive. Otherwise, the phase angle is in the fourth quadrant and the gamma-matching capacitor $C_\Gamma$ cannot be used. For the reactance of the gamma match, $X_s$, to remain positive, the electrical length of the gamma rod, $kl_\Gamma$, must be less than 90 electrical degrees and **eqs. 6** and **11** have the restriction

$$H_z > \frac{R_o}{R_a} \qquad \textbf{(14)}$$

This equation is very important to broadband array operation. That is, $R_a$ may change considerably with frequency changes and $H_z$ should be large enough to assure the restriction given by **eq. 14.** Furthermore, vswr readings over a broad band reflect the combined characteristics than 20 to 30 may not be practical.

When the driven element or parasitic elements of an array are arbitrarily pruned, element length and/or spacing, to obtain a resistive driving-point impedance, the forward gain may be reduced. There are many combinations of $R_a$, $X_a$, $H_z$, $Z_o$

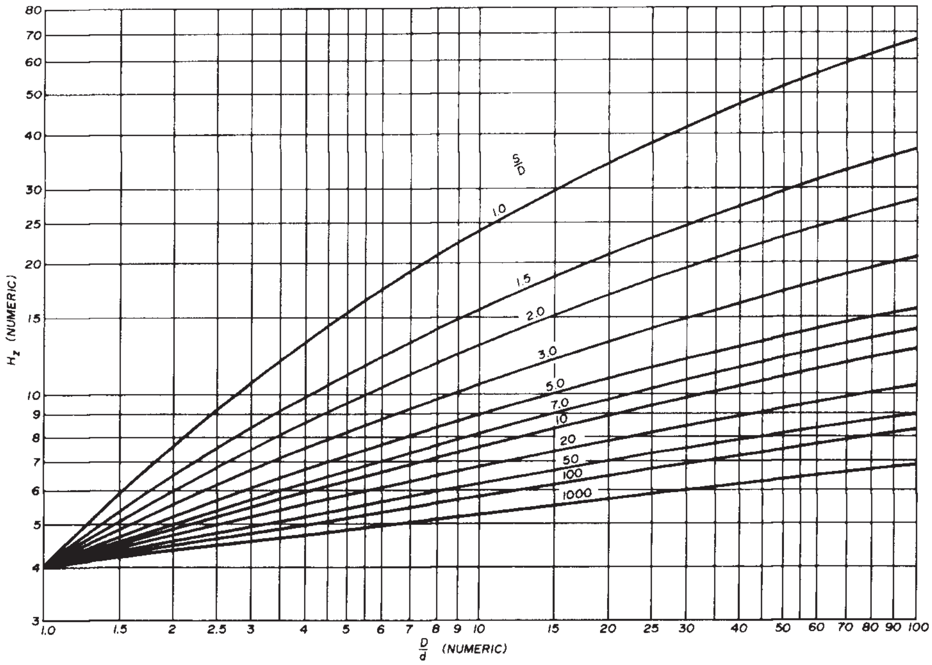



**fig. 3. Antenna impedance step-up ratio, Hz, as a function of element diameters and spacing.**

of the antenna and the matching network, and a well designed gamma- or T-network will follow the inverse behavior of the antenna driving-point impedance.

Large values of $H_z$ imply small values of S together with large wire diameter ratios, D/d. At the same time, the electrical length of the gamma rod, $kl_\Gamma$, should be greater than about 15 electrical degrees to minimize ohmic circuit losses as well as to maximize antenna system bandwidth. However, large values of $H_z$ along with large design values of $kl_\Gamma$ can lead to construction errors and/or instability in $H_z$, $Z_o$, T and Q. When the driven element diameter is not very large an $H_z$ of more and $l_\Gamma$ which satisfy **eq. 6**. Also, many *combinations* of D, d, S and C which are correct solutions, but not necessarily best.

For example, if it is desirable to reduce the length of the gamma rod, **eq. 10** shows that Q will be reduced (T is inversely related). Looking further, **eq. 6** shows that a reduction in Q requires an increase in $H_z$, and **eq. 2** shows that an increase in $H_z$ requires an increase in D/d or a decrease in S/D (or combination of both). From **eq. 3** it can be seen that an increase in D/d increases $Z_o$, while a decrease in S/D decreases $Z_o$. The ambiguity here is eliminated by plotting T, given in **eq. 9** and plotted in **fig. 4**.

## practical gamma-match

Some time ago I obtained a homebrew 10-element, two-meter Yagi-Uda type in-line end-fire array with somewhat unusual spacings and element lengths. Measurements with the antenna's dissimilar diameter folded-dipole driven element connected to 300-ohm open-wire line indicated that its best overall performance was near lower end of the 144-MHz band. An analysis of element lengths and spacing confirmed this observation, and calculations indicated that best performance was at about 144.6 MHz.

Since I wanted to use the antenna at a somewhat higher frequency, 145.4 MHz, I discarded the folding element (which transfers *any* antenna driving-point impedance, real or complex, to the transmission line) and shortened all the elements. I did not change element spacing or boom length, so performance was compromised somewhat. However, calculations indicated that the array's performance was much improved over the 145.35- to 146.25-MHz segment of the band. The free-space driving-point impedance, $Z_a'$, is slightly inductive, about 14 + j3 ohms at 145.4 MHz where I'll be using the antenna most of the time.

To use the previously discussed gamma-match design equations, the free-space input impedance, $Z_a'$, must be divided by 2. Therefore, $Z_a = 7 + j1.5$ ohms. Since I planned to use 50-ohm polyfoam RG-8/U with the antenna, $R_o = 50$ ohms. The diameter of the driven element is 0.375 inch and the diameter of the gamma rod, which is made from number-12 wire, is 0.0808 inch. The spacing between the elements, S, is approximately 0.6029 inch (0.375-inch long insulator plus one-half the diameter of the driven element, plus one-half the diameter of the gamma rod).

With these figures, the gamma-match design procedure can begin by finding two simple ratios:

$$\frac{D}{d} = 4.64$$

$$\frac{S}{D} = 1.61$$

When these two ratios have been found, the antenna impedance step-up ratio, $H_z$, can be found from **fig. 3**. In this case, $H_z$ is equal to approximately 11.0. The quantity T can be determined with the help of **fig. 4**, approximately 0.046 $\text{ohm}^{-1}$ in this case. Now, calculate A and B and use those quantities to find Q:

$$A = \frac{R_o X_a}{H_z R_a - R_o} \cong \frac{+75}{27} \cong +2.77 \text{ ohms}$$

$$B = \frac{R_o\left[(R_a)^2 + (X_a)^2\right]}{H_z R_a - R_o}$$

$$\cong \frac{2562.5}{27} \cong 94.9 \text{ ohms}^2$$

$$Q = +A + \sqrt{A^2 + B}$$

$$\cong +2.77 + \sqrt{102.57} \cong 12.9 \text{ ohms}$$

With the quantities Q and T known, the electrical length of the gamma rod, $kl_\Gamma$, in degrees can be determined with the help of **fig. 5**. In this case the gamma rod is approximately 31.0 electrical degrees long. This can be converted into inches by rearranging **eq. 8**.

$$l_\Gamma \cong \frac{kl_\Gamma}{0.03188 f_{MHz}} \cong \frac{31}{0.03188(145.4)}$$

$$\cong 6.69 \text{ inches}$$

To find the required gamma capacitance, calculate the quantities E and F from **eq. 13**.

$$E = \frac{R_o}{R_a}\left[\frac{(R_a)^2 + (X_a)^2}{Q}\right]$$

$$\cong \frac{2562.5}{(7)\ (12.9)} \cong 28.38 \text{ ohms}$$

$$F = \frac{R_o}{R_a} X_a \cong \frac{50\ (+1.5)}{7} \cong 10.7 \text{ ohms}$$

$$C_\Gamma = \frac{1{,}000{,}000}{2\pi\ [E + F]\ f_{MHz}}$$

$$\cong \frac{1{,}000{,}000}{35{,}700} \cong 28 \text{ pF}$$

Thus, the gamma-matching network consists of a gamma rod made from number-12 wire, approximately 6.69 inches long with an axial center-to-center spacing of 0.6029 inch from the *parallel* driven element. The gamma capacitor is about 28 pF, which can be provided by the 7- to 45-pF variable I happen to have in my junkbox.

This same basic design procedure can also be used if the driving-point impedance is slightly capacitive. For example, if my two-meter array driving-point impedance had been 14 − j3 ohms at 145.4 $MH_z$, $Z_a$ for design purposes would be 7 − j1.5 ohms. The values for D/d, S/D, $H_z$, T, A and B would be the same as for the previous case, although the A quantity would have a negative sign. This changes the magnitude of Q to 7.36 ohms. Referring to **fig. 5**, the length of the gamma rod is approximately 20.0 electrical degrees, and its length is

$$l_\Gamma \cong \frac{k l_\Gamma}{0.03188 f_{MHz}}$$

$$\cong \frac{20}{0.03188(145.4)} \cong 4.31 \text{ inches}$$

Now, calculate the quantities E and F and determine the value of the gamma capacitor:

$$E = \frac{R_o}{R_a}\left[\frac{(R_a)^2 + (X_a)^2}{Q}\right]$$

$$\cong \frac{2562.5}{(7)\,(7.36)} \cong 49.73 \text{ ohms}$$

$$F = \frac{R_o}{R_a} X_a \cong \frac{50(-1.5)}{7} = +\,10.7 \text{ ohms}$$

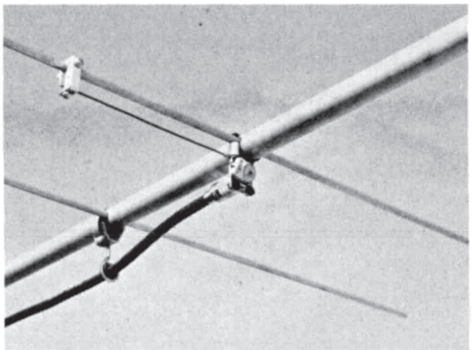
$$C_\Gamma = \frac{1{,}000{,}000}{2\pi\,[E + F]\, f_{MHz}}$$

$$\cong \frac{1{,}000{,}000}{35{,}700} \cong 28 \text{ pF}$$

Thus, when the gamma rod is shortened from 6.69 inches to about 4.31 inches, and no other changes are made, the gamma network will properly terminate the 50-ohm coaxial feedline into an antenna impedance of 14 − j3 ohms.

## testing the design

Using an adjustable gamma rod 7.5 inches long and a 7- to 45-pF variable gamma capacitor, I assembled the gamma match to the antenna. When the antenna was erected horizontally, 18-feet above the ground, with the gamma rod adjusted to 6.75-inches long and the gamma capacitor set to approximately 25 pF, the vswr was as follows:



**Gamma-matching system for the 144-HMz Yagi beam antenna.**

146.3 MHz 1.38:1 vswr
145.4 MHz 1.03:1 vswr
144.9 MHz 1.16:1 vswr

As can be seen, at the design frequency of 145.4 MHz the standing-wave ratio is extremely small.

Calculations indicated that the driving-point impedance of this modified home-brew array becomes slightly capacitive at both 144.9 and 146.3 MHz. Therefore, I began to shorten the length of the gamma rod and increase the gamma capacitance to see if I could obtain a *constant* vswr. With a gamma rod length of 5.5625 inches and the gamma capacitor set to 35 pF I obtained what I was looking for:

146.3 MHz 1.27:1 vswr
145.4 MHz 1.28:1 vswr
144.9 MHz 1.27:1 vswr

I then twisted the antenna boom 90° so that the antenna was vertically polarized on the *metal* mast. Without touching the gamma adjustments, I slipped the boom back and forth until I obtained a minimum vswr of 1.3:1 at 145.4 MHz.

The boom was locked at this point, and both the gamma rod and gamma capacitor were adjusted (in that order) to obtain a minimum vswr of 1.09:1.

I then started to raise the height of the array by increasing the height of the mast. When the array was about two feet higher (13 quarter-wavelengths above ground) the vswr was maximum at 1.6:1. Above this height the vswr began to decrease. I expected maximum mast coupling to occur at odd quarter-wavelengths of mast, and this behavior confirms it.

I locked the antenna at 21 feet above the ground (I wasn't able to reach the gamma capacitor at greater heights) and adjusted both the gamma capacitor and the gamma rod. I then pushed the antenna up to slightly more than 25 feet above ground, the maximum height of the mast, and made the vswr measurements. After several adjustments at the 21-foot level, at the 25-foot height, with a gamma capacitance of 30 pF and a gamma rod length of 5.875 inches, I obtained the following vswr measurements:

146.3 MHz 1.10:1 vswr
145.4 MHz 1.02:1 vswr
144.9 MHz 1.04:1 vswr

## performance

Essentially all of the two-meter stations in Tucson, about 35 miles from my station, are vertically polarized, and my propagation is by diffraction over the north end of the Santa Rita mountain range. In running tests with K7RMH in Tucson, I was delighted to find that my British dual-eight skeleton slot antenna, which is 3 to 4 feet higher than the homebrew array, was only about 3 dB better with the same power input. That is, with one-watt rf input, at K7RMH's station I was S9 +30 dB with the homebrew array and S9 +33 dB with the skeleton slot.

Upon further testing with a two-position coaxial switch in the feedline for rapid switching between the line tuner and the RG-8/U coaxial cable, I discovered that the coaxial switch generates a vswr of 1.5:1 at the transmitter. This results in

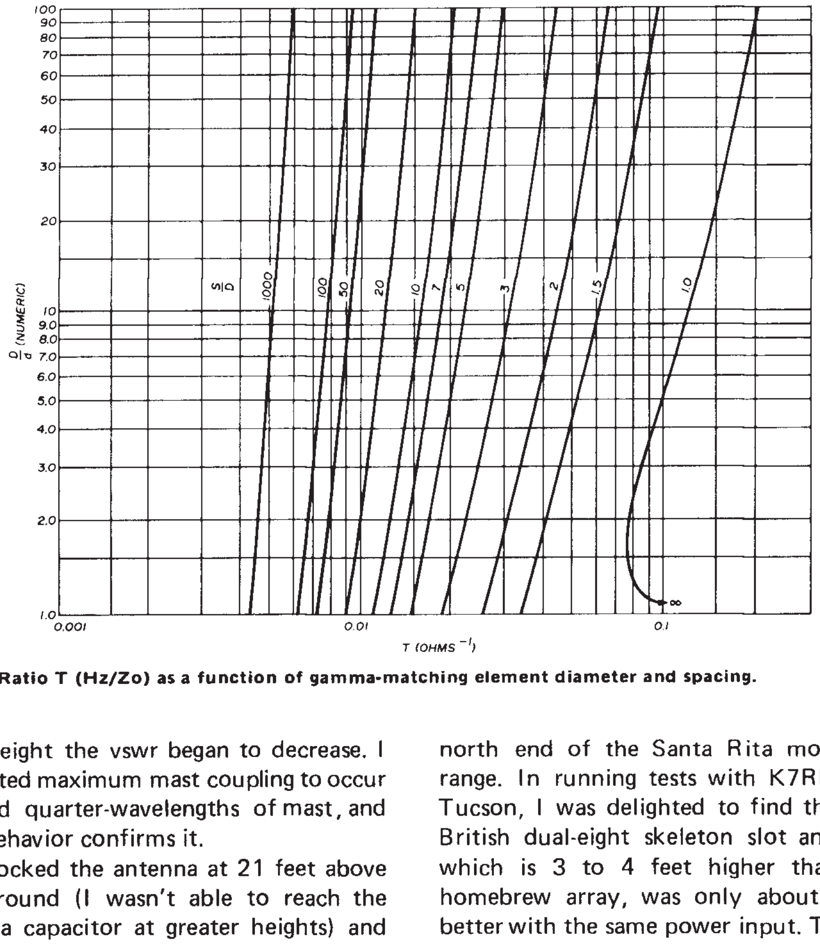



**fig. 4. Ratio T (Hz/Zo) as a function of gamma-matching element diameter and spacing.**

about a 1.0 dB loss at K7RMH's station which suggests that the K7RMH signal readings are intensity, *not* power, sensitive. This indicates that a change of 3 dB at K7RMH is a *power* change of 6 dB.

Calculations indicate that the gain of the skeleton slot is about 4 dB better than that of the homebrew array. I am using *Propagation* calling for papers on antenna measurements.

It may be that the recent article by W3TQM which described how to determine antenna impedance by direct swr measurements may prove popular with amateurs.[5] I hope to try this procedure sometime to confirm my calculations on

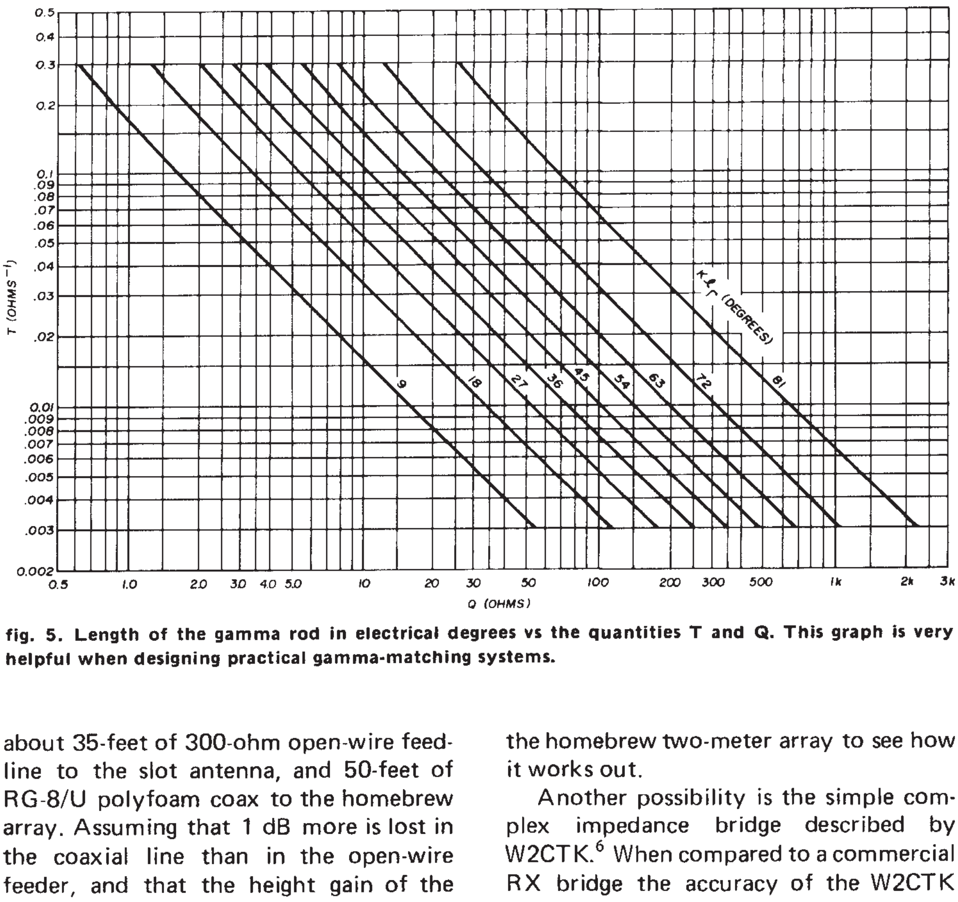



**fig. 5. Length of the gamma rod in electrical degrees vs the quantities T and Q. This graph is very helpful when designing practical gamma-matching systems.**

about 35-feet of 300-ohm open-wire feedline to the slot antenna, and 50-feet of RG-8/U polyfoam coax to the homebrew array. Assuming that 1 dB more is lost in the coaxial line than in the open-wire feeder, and that the height gain of the slot antenna is 1 dB better, the test results check out very well.

## impedance measurements

A problem many amateurs face when designing their own antenna is measuring the driving-point impedance. Often, they do not have the necessary equipment or the technique is long and drawn out, and provides erroneous results. This seems to be underscored by an article in the July, 1972, *IEEE Trans. on Antennas and* the homebrew two-meter array to see how it works out.

Another possibility is the simple complex impedance bridge described by W2CTK.[6] When compared to a commercial RX bridge the accuracy of the W2CTK bridge is quite good. A complete discussion on the use of this instrument appears elsewhere in this issue.

In the absence of impedance measurements, the amateur must generally resort to existing published material which provides the driving-point impedance for various antenna configurations. However, with different (unscaled) antenna layouts, the magnitude and phase angle of the driving-point impedance will be different.

However, a review of some of the

available material suggests that, most of the time, the resistive, $R_a$, and reactive, $X_a$, portions of the antenna driving-point impedance will fall within the following limits:

$$+3 \leqslant R_a \leqslant +175 \text{ ohms}$$

$$-70 \leqslant X_a \leqslant +55 \text{ ohms}$$

For estimates of the driving-point impedance, consult references 1, 2, 4, 7, 8, 9, 10, 11, 12, 13 and 14. Computer programs have been developed for calculating the driving-point impedance vs the layout of the antenna, but unfortunately, these programs are generally not available to amateurs.

## another design approach

When $H_z$ can have a value of 4.0, **fig. 3** suggests that the S/D ratio can be any value when D/d = 1.0. In this case a decision as to what value of S/D to use can depend upon a *desired* $kl_\Gamma$. Using this approach, Q is determined first as a function of the A and B terms on **eq. 11**. At the same time $kl_\Gamma$ should be greater than about 15 electrical degrees — I would use one of the 18- to 45-degree lines in **fig. 5** if I were not insisting upon a specific gamma-rod length. Then T can be obtained from **fig. 5** as a function of $kl_\Gamma$ and Q. With T determined, S/D lies along the abscissa of **fig. 4** where D/d = 1.0.

For example, consider a self-resonant vertical monopole antenna on a good ground plane with an assumed driving-point impedance of 34 + j17 ohms at 145.4 MHz. Therefore, $R_a$ = 34.0 ohms and $X_a$ = 17.0 ohms. With a polyfoam RG-8/U feedline, $R_o$ = 50.0 ohms. Since the antenna impedance step-up ratio, $H_z$, must be larger than the ratio $R_o/R_a$ (see **eq. 14**), if a value of 4.0 is chosen for $H_z$, in this example it will satisfy that requirement.

With $H_z$ = 4.0, the diameter ratio D/d may be found from **fig. 3** to be 1.0. Assuming driven element and gamma rod diameters of 0.375 inch, the values for A, B and Q are as follows:

$$A = \frac{R_o X_a}{H_z R_a - R_o} = \frac{+850}{86} \cong +9.884 \text{ ohms}$$

$$B = \frac{R_o\left[(R_a)^2 + (X_a)^2\right]}{H_z R_a - R_o}$$

$$= \frac{72250}{86} \cong 840.1 \text{ ohms}^2$$

$$Q = +A + \sqrt{A^2 + B}$$

$$\cong 9.884 + \sqrt{937.78} \cong 40.51 \text{ ohms}$$

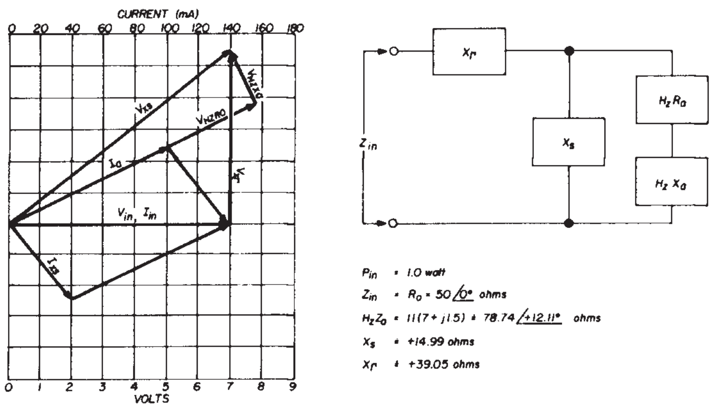



**fig. 6. Voltage and current distribution of the gamma-matching network designed to match the 50-ohm coaxial line to the 10-element beam at 145.4 MHz (Za = 7 + j1.5 ohms). Voltage and current values are based on power input of 1 watt.**

Using a selected gamma-rod length of 36.0 degrees, the length of the rod in inches is

$$l_\Gamma \cong \frac{kl_\Gamma}{0.03188 f_{MHz}}$$

$$= \frac{36}{0.03188(145.4)} = 7.766 \text{ inches}$$

When $kl_\Gamma$ is 36.0 degrees, T is 0.018 $ohms^{-1}$ (see **fig. 5**). When both T and D/d are known, the spacing-to-diameter ratio S/D can be found from **fig. 4**. In this case, S/D = 3.3. Therefore, the center-to-center spacing is

$$S = 3.3D = 3.3(0.375) \cong 1.24 \text{ inch}$$

Now, the quantities E and F may be determined, and the value of the gamma capacitor, $C_\Gamma$, calculated:

$$E = \frac{R_o}{R_a}\left[\frac{(R_a)^2 + (X_a)^2}{Q}\right]$$

$$\cong \frac{72250}{1377.3} \cong 52.46 \text{ ohms}$$

$$F = \frac{R_o}{R_a} X_a = \frac{50(+17)}{34} = +25.0 \text{ ohms}$$

$$C_\Gamma = \frac{1,000,000}{2\pi\,[E + F]\, f_{MHz}}$$

$$\cong \frac{1,000,000}{70,711} \cong 14.14 \text{ pF}$$

Therefore, the completed gamma-matching network consists of a 0.375-inch diameter gamma rod, about 7.75-inches long, with a 14pF gamma capacitor.

## tee match

Although the previous discussion pertains to the gamma match, the same design philosophy may be applied directly to the tee match. If you want to tee-match a balanced transmission line to an isolated thin linear antenna at the driving impedance point, the driving-point impedance, $Z_a'$, and the line characteristic resistance, $R_o'$, are halved and the procedure for findings S/D, D/d, $kl_\Gamma$, and $C_\Gamma$ for one arm of the tee-match follows that for the gamma match. The results are merely imaged to, or flipped over, to the other arm of the tee- or double-gamma configuration.

For example, to match a balanced feedline to the previously discussed home-brew array, which has an input impedance of 14 + j3 ohms, the gamma-match values on page 51 would be used in each arm of the tee-match to provide a match to *balanced* 100-ohm transmission line. If you want to use 300-ohm balanced line, use the same gamma-match design procedure. However, use $R_o$ = 150 ohms for *each* arm of the tee.

## acknowledgement

I want to thank W7ERU for his assistance. Without his computations, reasonable limits and graph plotting would have been a tedious task.

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