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Supplementary Notes

for

PHASELOCK UP TO DATE

by

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Part I

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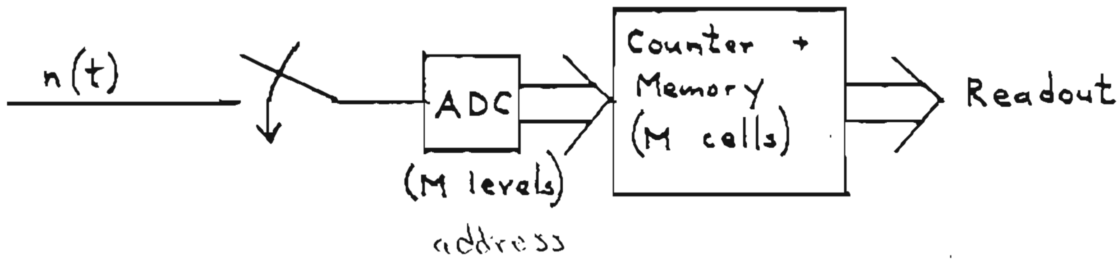
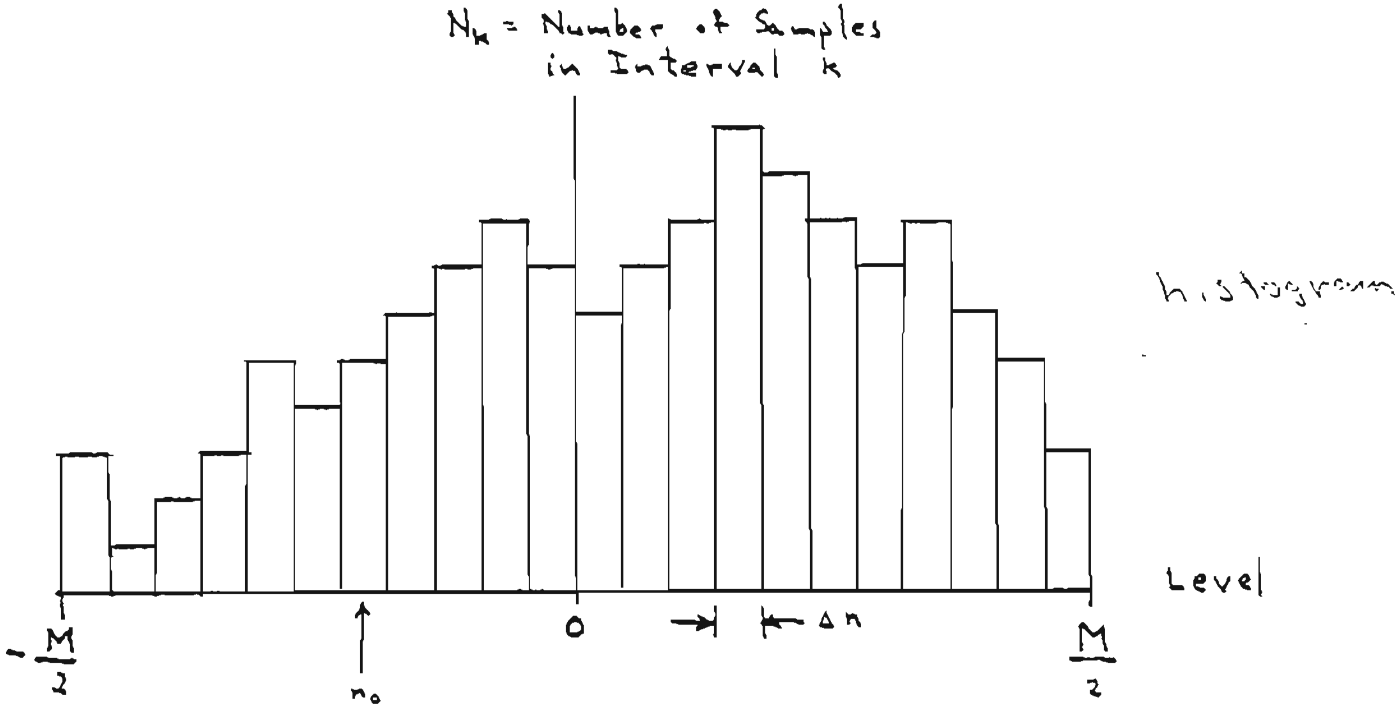
APPENDIX A





Total samples $= N = \sum_k N_k$

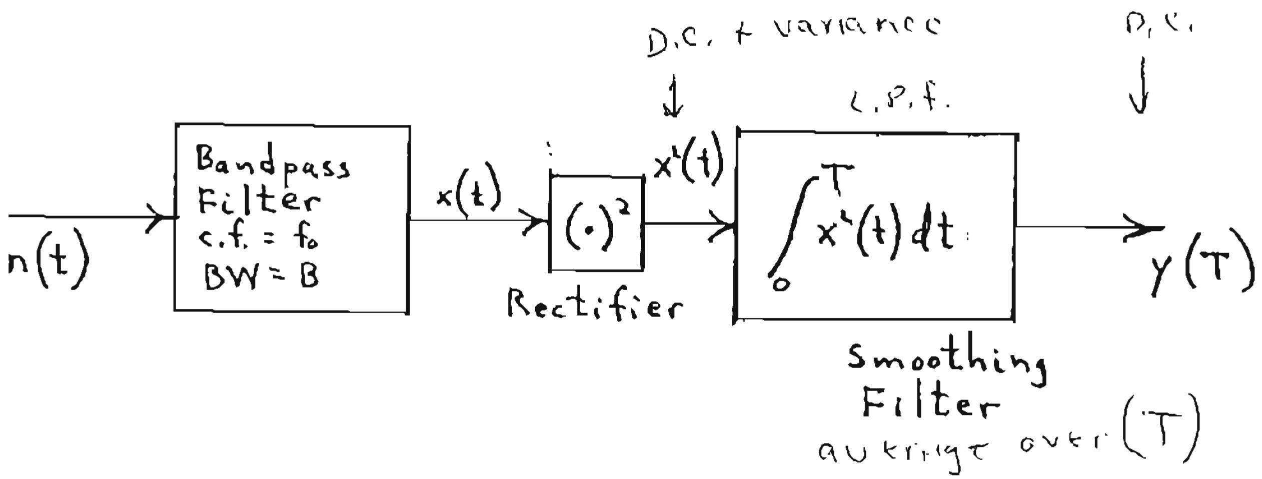
n_0 falls in $k = k_0$ th bin

$$p(n_0) \approx \lim_{\substack{N \to \infty \\ \Delta n \to 0 \\ M \to \infty}} \frac{N_{k_0}}{N}$$

equivalent to
Spectrum analyzer

$\Phi_n(f_0)$

Spectral Density.



$$\Phi_n(f_0) \propto \lim_{\substack{B\to 0\\ T\to\infty\\ BT\to\infty}} \frac{Y(T)}{B}$$

$n_i(t)$ $\qquad\qquad\qquad\qquad$ $n_o(t)$

$$\Phi_i(f) \longrightarrow \boxed{H(f)} \longrightarrow \Phi_o(f)$$

$$\Phi_o(f) = \Phi_i(f)\ |H(f)|^2$$

$$\overline{n_o^2} = \int_0^\infty \Phi_o(f)\, df$$

power transfer function

$$= \int_0^\infty \Phi_i(f)\ |H(f)|^2\, df$$

magnitude only
(no phase)

$$= N_o \int_0^\infty |H(f)|^2\, df$$

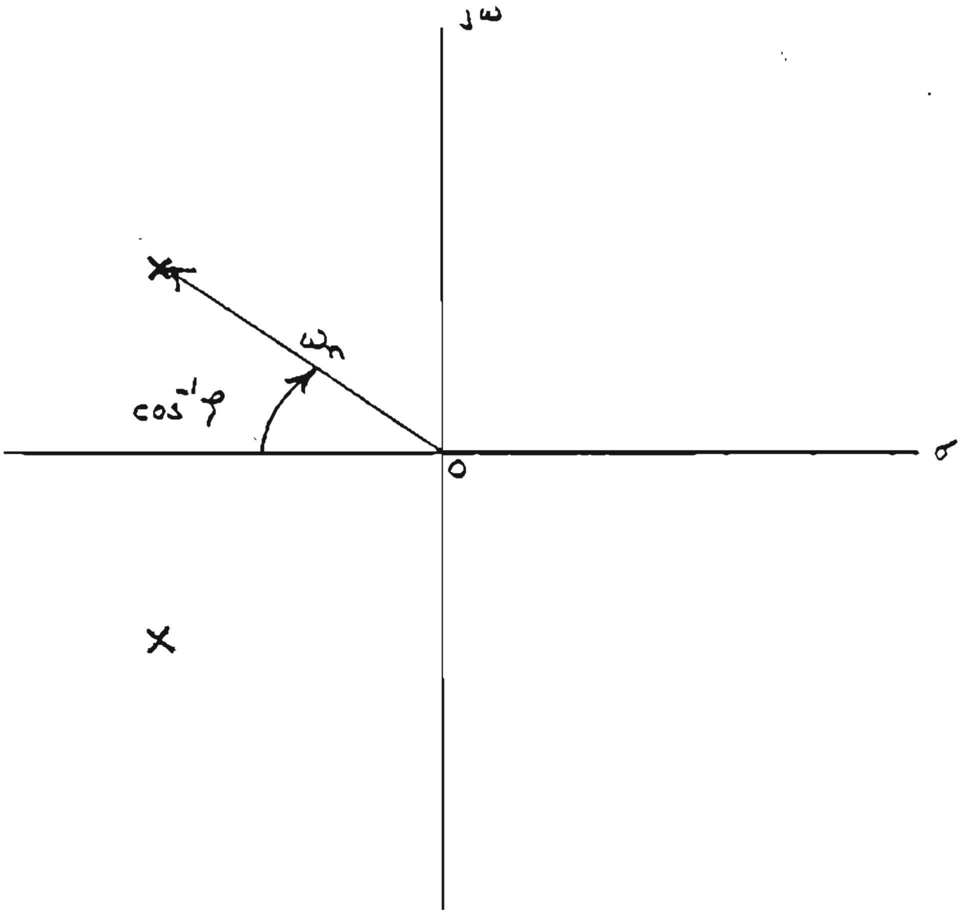
(For white noise)
N_o

$$\overline{n_o^2} = N_o \; B_N \; |H(j\omega_r)|^2$$

$$= N_o \; B_N$$

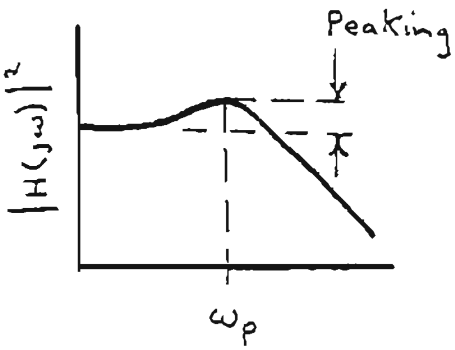
$$\text{if} \quad |H(j\omega_r)| = 1$$

CHAPTER 2

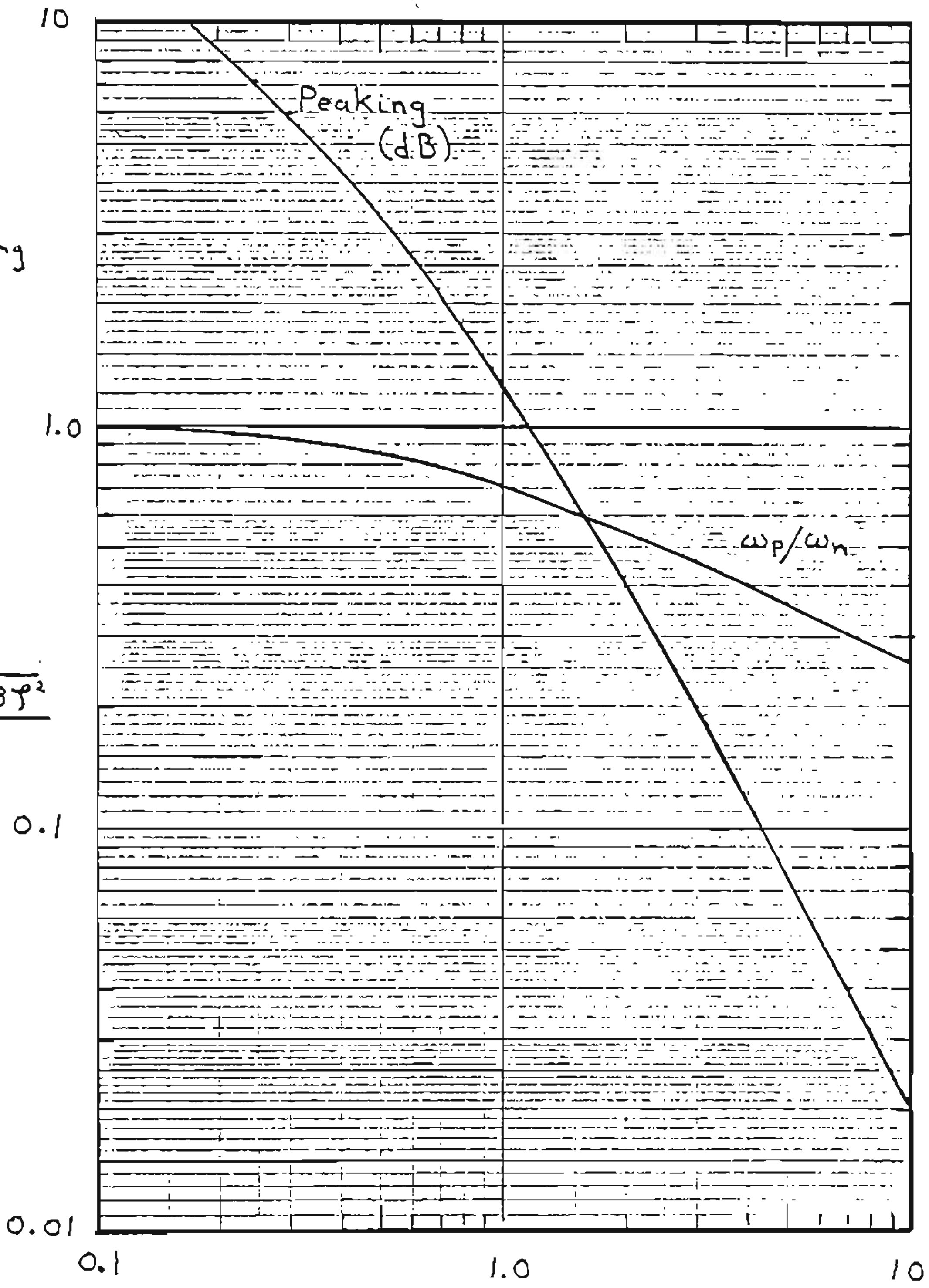


jω
ω_n
$\cos^{-1}\zeta$
0
σ

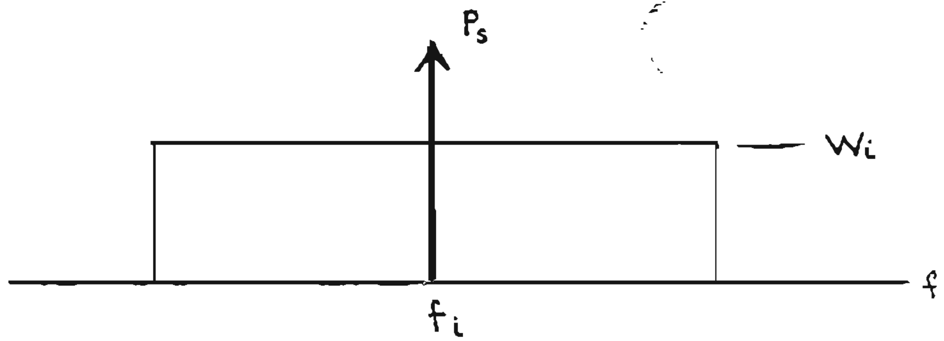
RESPONSE PEAKING IN SECOND-ORDER LOOP

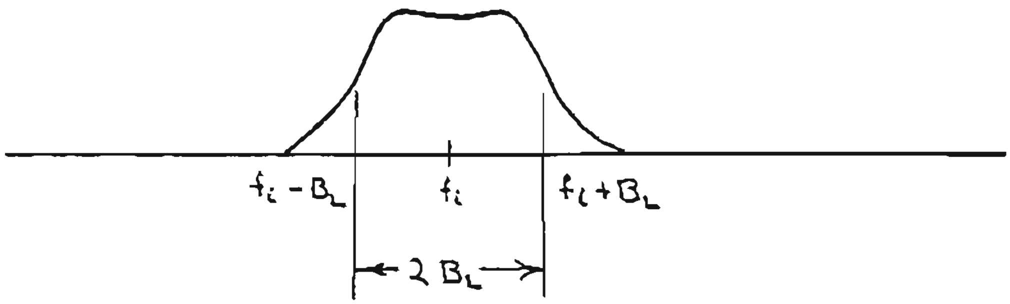


$$\omega_p^2 = \omega_n^2 \frac{-1+\sqrt{1+8\zeta^2}}{4\zeta^2}$$



CHAPTER 3





Signal power accepted by loop = $P_s \propto V_s^2/2$

Noise power accepted by loop = $2B_L W_i \propto 2B_L N_0$

$$SNR_L \triangleq \frac{P_s}{2B_L W_i}$$

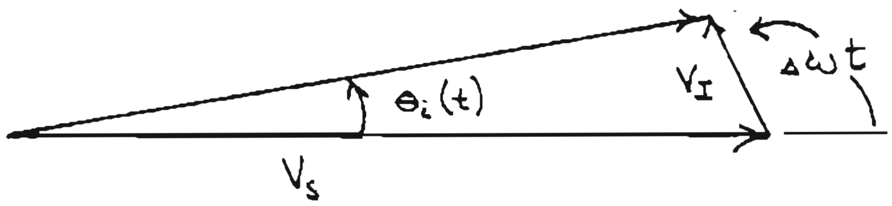
$$= \frac{V_s^2/2}{2B_L N_0}$$

$$\overline{\theta_{no}^2} = \frac{1}{2\, SNR_L}$$

Interference

$$v_{in}(t) = V_s \sin \omega_s t + V_I \sin \omega_I t$$

$$\Delta\omega = \omega_I - \omega_s$$



If $V_I \ll V_s$

$$\theta_i(t) \approx \frac{V_I}{V_s} \sin \Delta\omega t$$

Apply to PLL with phase transfer function $\dfrac{\theta_o(s)}{\theta_i(s)} = H(s)$.

$$\theta_o(t) \approx \frac{V_I}{V_s} \left|H(j\Delta\omega)\right| \sin\left(\Delta\omega t + \angle H(j\Delta\omega)\right)$$

If interference is not small compared to the signal, then nonlinear analysis is needed.

If interference is large enough and close enough in frequency, then the interference can capture the loop. Highly nonlinear problem.

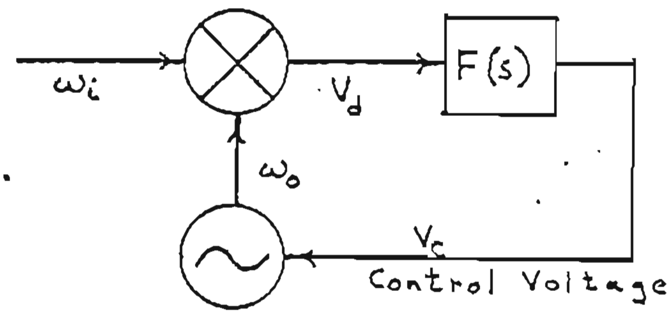
References:

Britt and Palmer, IEEE Trans., AES-3, pp. 123-135, Jan. 1967.
Blanchard, IEEE Trans., AES-10, pp. 686-697, Sept. 1974.

CHAPTER 4

Frequency Discrepancy

Free-running frequency of VCO (corresponding to zero control voltage) never coincides exactly with signal frequency. Locked operation requires a non-zero control voltage.



If $\Delta\omega = \omega_i - \omega_o$, then $V_c = \Delta\omega/K_o$ is required for lock.

Static Phase Error

Control voltage originates from phase detector output V_d coming through the loop filter.

$V_c = V_d F(0)$

Phase detector output arises from phase error: $V_d = K_d \theta_v$.

Static phase error needed to compensate for inevitable frequency offset

$$\boxed{\theta_v = \Delta\omega / K_o K_d F(0)}$$

$K_v = K_o K_d F'0)$ is velocity-error coefficient, in rad/sec.

Dynamic Lag (accelleration)

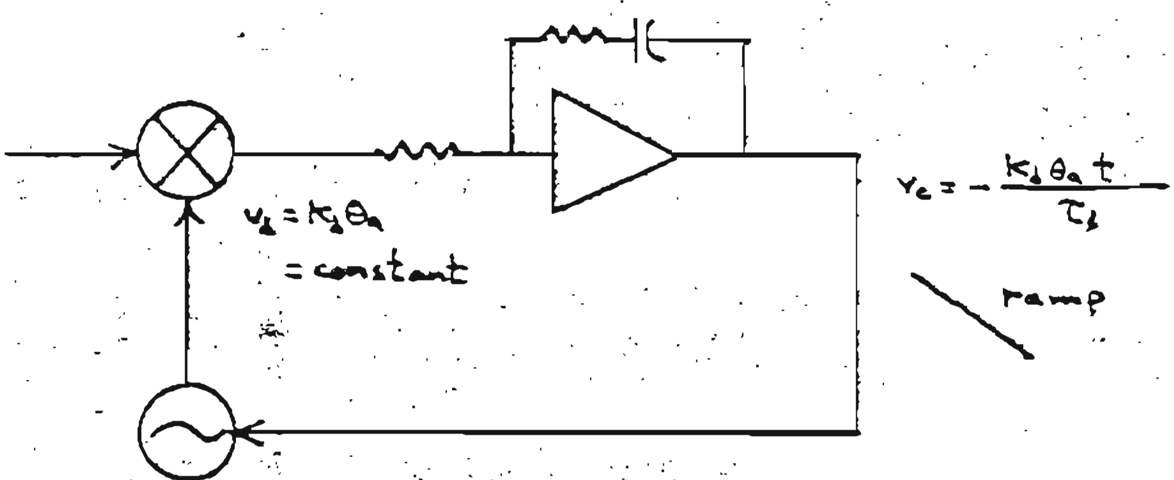
Suppose signal frequency is changing linearly at constant rate $\dot{\omega}$. VCO frequency must do the same if tracking is to be held.

VCO control voltage must be a ramp with slope $\dot{\omega}/K_o$ V/sec.

Ramp is generated by applying a constant error voltage $V_d = K_d\theta_a$ volts to integrator of loop filter.

Integrator output is $V_c = K_d\theta_a t/\tau_1 = \dot{\omega}t/K_o$ (τ_1 is integrator time constant) whereby the dynamic lag θ_a of a 2nd-order PLL, due to a frequency ramp $\dot{\omega}$ is

$$\boxed{\theta_a = \dot{\omega}\tau_1/K_oK_d = \dot{\omega}/\omega_n^2 \qquad \text{(rad)}}$$

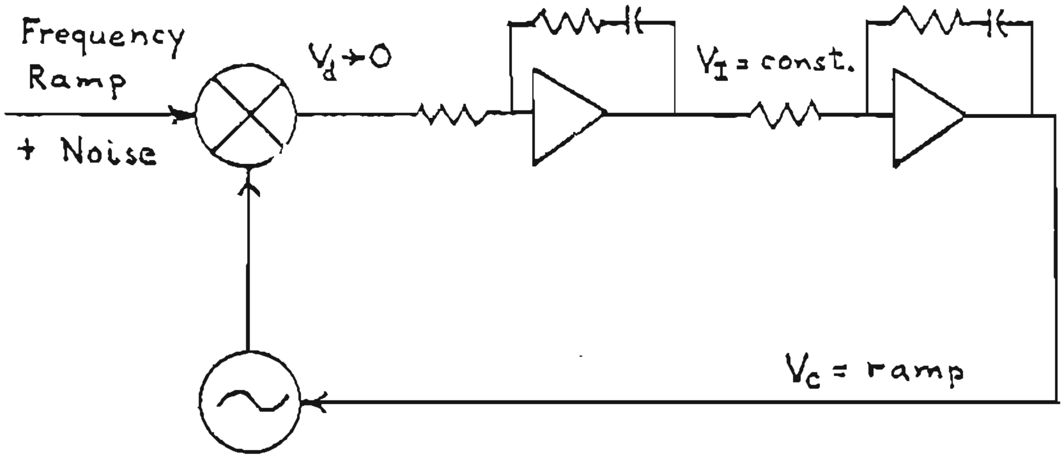


Third-Order PLL

A large ω_n is needed in a 2nd-order loop to achieve small dynamic lag in the presence of a large rate of frequency change. But large ω_n is incompatible with small noise bandwidth.

Third-order PLL has two integrators in loop filter.

Permits near-zero dynamic lag error together with small noise bandwidth.



Second-order PLL suffices for most applications but signal dynamics sometimes require 3rd-order.

Error Limits

If phase detector has sinusoidal characteristic (common, but not universal) then error voltage is $V_d = K_d \sin \theta_e$.

Error voltage increases for increasing phase error, up to $\theta_e = 90^0$.

If θ_e exceeds 90^0, the phase detector cannot generate the corresponding increase of V_d.

Therefore, lock fails for conditions requiring $\sin \theta_e > 1$.

Frequency-tracking limit (hold-in limit)

$\Delta\omega_H = K_v$ (But see next page.)

Frequency-rate limit

$\Delta\dot{\omega} = \omega_n^2$ (Second-order PLL)

Op-Amp Saturation

Foregoing hold-in limit based upon nonlinearity of phase detector.

If K_v is very large, then op amp in active filter saturates at $\pm V_m$--long before V_d reaches maximum level.

Actual hold-in range is $V_m K_o$ rps, rather than the much-larger K_v.

CHAPTER 5

Discriminator - Aided PLL

For Fig. 5.13 $(\tau_f = R_f C)$

$$H(s) = \frac{sK_o\left(\frac{K_d \tau_2}{\tau_1} + \frac{K_f}{\tau_f}\right) + \frac{K_o K_d}{\tau_1}}{s^2 + sK_o\left(\frac{K_d \tau_2}{\tau_1} + \frac{K_f}{\tau_f}\right) + \frac{K_o K_d}{\tau_1}}$$

$$= \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{K_o K_d}{\tau_1}$$

$$\zeta = \frac{K_o}{2\omega_n}\left(\frac{K_d \tau_2}{\tau_1} + \frac{K_f}{\tau_f}\right)$$

$$K = 2\zeta\omega_n = K_o\left(\frac{K_d \tau_2}{\tau_1} + \frac{K_f}{\tau_f}\right)$$

$$a_2 = \frac{K_d}{\tau_1\left(\frac{K_d \tau_2}{\tau_1} + \frac{K_f}{\tau_f}\right)}$$

K_f is discriminator gain in volts per rad/s

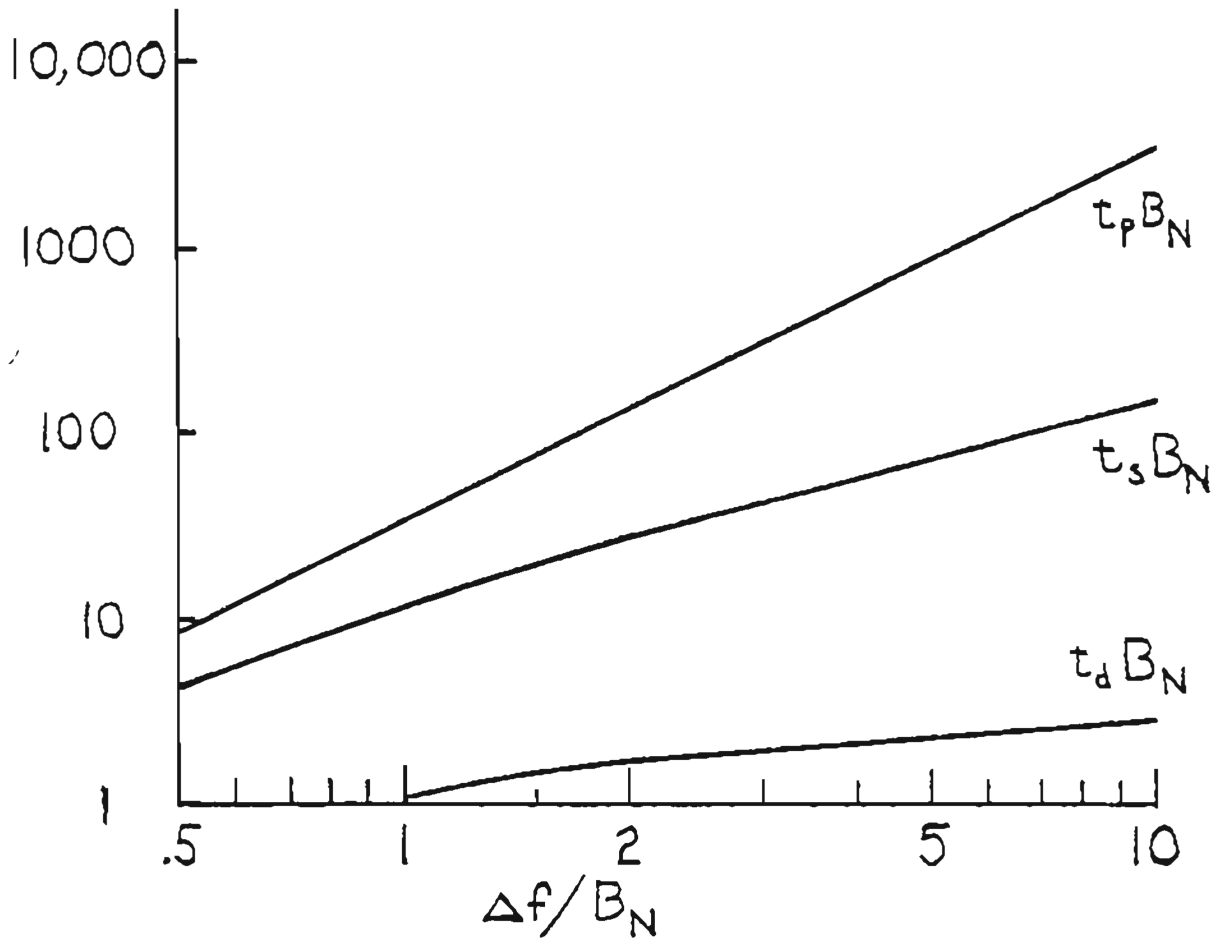
COMPARISON OF FREQUENCY-ACQUISITION TIMES

t_p = Pull-in time (proportional to Δf^2)

t_s = Sweep time (proportional to Δf)

t_d = Discriminator-aided time (proportional to ln Δf)

Curves are for second-order loop with damping factor of 0.707.
B_N is noise bandwidth.



CHAPTER 6

Phase Noise Relation to RF Spectrum

Phase-modulated signal

$$s(t) = A \cos[\omega_c t + \theta(t)]$$

Consider sinusoidal phase modulation:

$$\theta(t) = \theta_m \sin \omega_m t$$

so phase variance $\overline{\Delta\theta^2} = \frac{\theta_m^2}{2}$.

For $\theta_m \ll 1$ rad

$$\begin{aligned} s(t) \approx A \{ & J_0(\theta_m) \cos \omega_c t && \leftarrow \text{(carrier)} \\ & + J_1(\theta_m) \cos(\omega_c + \omega_m)t && \leftarrow \text{Upper Sideband} \\ & - J_1(\theta_m) \cos(\omega_c - \omega_m)t \} && \leftarrow \text{Lower Sideband} \end{aligned}$$

For $\theta_m \ll 1$; $J_0(\theta_m) \approx 1$, $J_1(\theta_m) \approx \frac{\theta_m}{2}$

$$s(t) \approx A\left\{ \cos \omega_c t + \frac{\theta_m}{2} \cos(\omega_c + \omega_m)t - \frac{\theta_m}{2} \cos(\omega_c - \omega_m)t \right\}$$

$$\text{Ratio} \left\{ \frac{\text{Amplitude of one sideband}}{\text{Amplitude of carrier}} \right\} = \frac{\theta_m}{2}$$

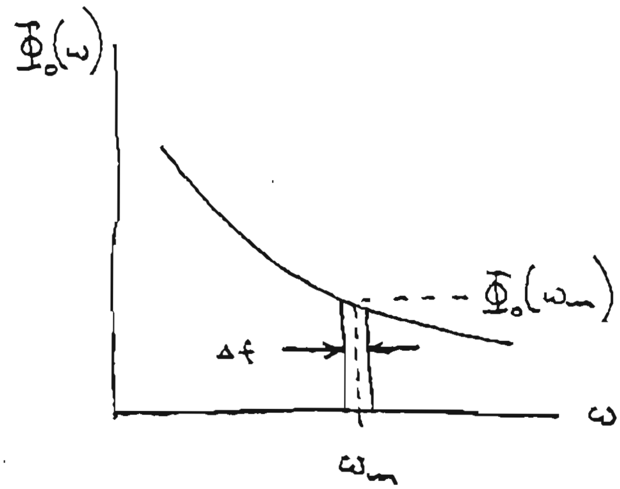
In dB: $\quad 20 \log_{10} \frac{\theta_m}{2} = 10 \log_{10} \frac{\overline{\Delta\theta^2}}{2} \quad$ dBc

Now consider random phase modulation $\theta(t) = \theta_n(t)$ which has spectral density $\Phi_0(\omega)$ rad^2/Hz.

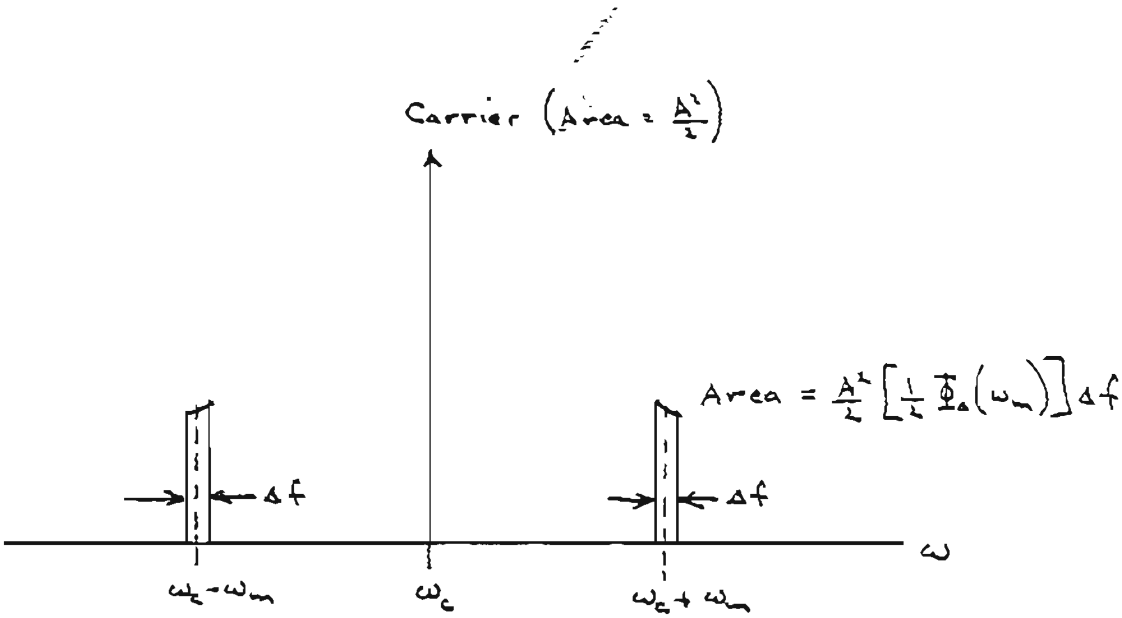
In narrow slot Δf Hz wide at frequency ω_m, the phase noise has variance

$$\overline{\Delta\theta^2} = \Phi_0(\omega_m)\Delta f \quad \text{rad}^2$$

and has near-sinusoidal character.



Contribution to RF Spectrum



$$\Phi_{RF}(\omega_c \pm \omega_m) \approx \frac{A^2}{2}\left[\frac{1}{2}\Phi_0(\omega_m)\right] \quad V^2/Hz$$

$(\omega_m \gg 0)$

Single-sideband noise ratio $\approx$

$$\boxed{10 \log_{10} \frac{\Phi_0(\omega_m)\,\Delta f}{2} \quad dBc}$$

(Power ratio in bandwidth Δf Hz at frequency $f_m = \omega_m / 2\pi$ away from carrier.)

Conditions:

— $\overline{\theta_n^2} << 1 \text{ rad}^2$
(Never true for small-enough ω_m)

— In-phase (AM) noise small compared to phase noise.

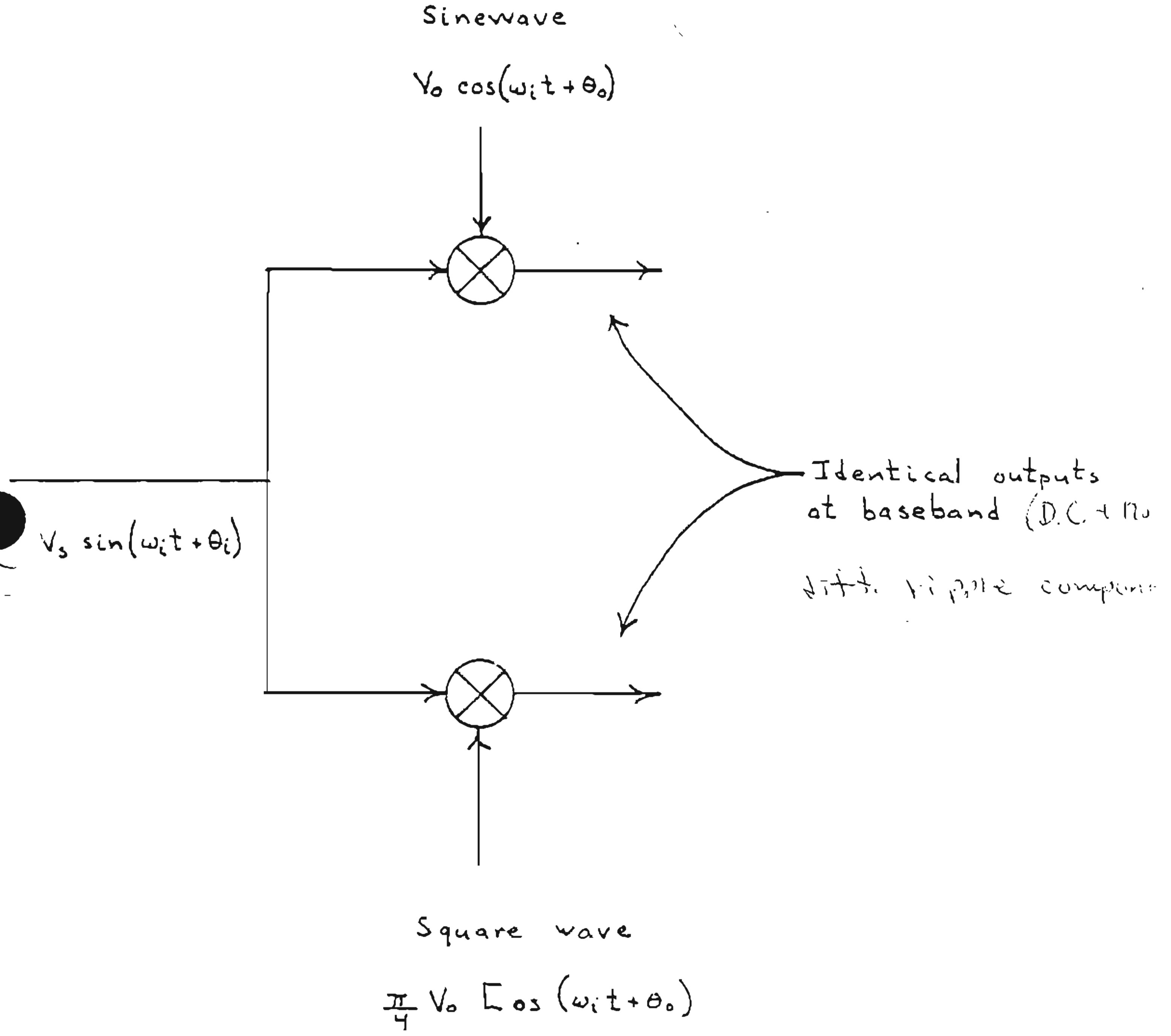
(Good approximation close-in to carrier but AM and PM are likely near-equal for large ω_m.)

Notation Convention:

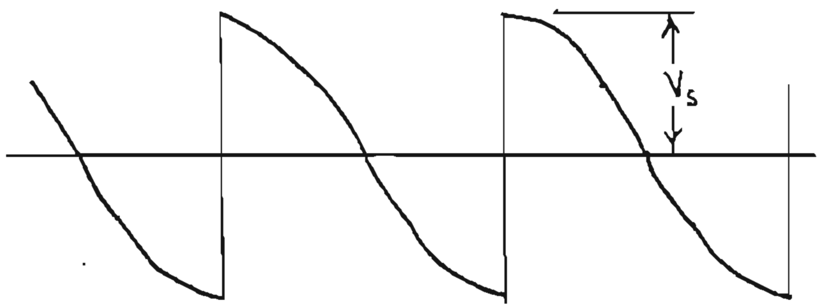
$$\mathcal{L}(f_m) \triangleq \frac{1}{2}\,\Phi_o(\omega_m)$$

$$= \text{Ratio}\left[\frac{\text{Single-sideband power in 1-Hz BW at } f_m \text{ Hz away from carrier}}{\text{Carrier power}}\right]$$

$$f_m = \omega_m / 2\pi$$


Sinewave
$V_o \cos(\omega_i t + \theta_o)$
$V_s \sin(\omega_i t + \theta_i)$
Identical outputs
at baseband (D.C. +
Square wave
$\frac{\pi}{4} V_o \,\mathrm{Cos}\,(\omega_i t + \theta_o)$

RIPPLE IN SWITCHING PD



$$K_d = \frac{2}{\pi} V_s$$

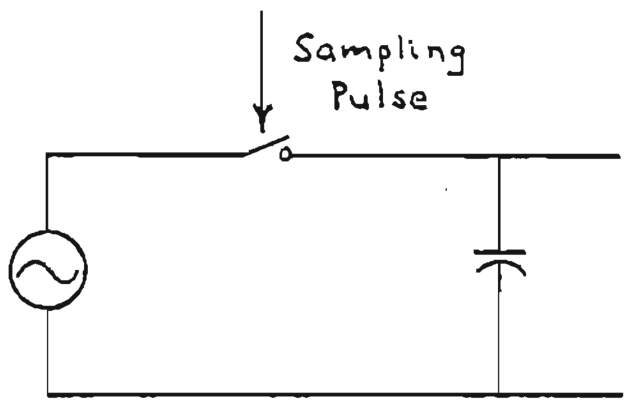
From Fourier series analysis, amplitude of nth-harmonic of the input frequency is

$$= \frac{4nK_d(-1)^{1+n/2}}{n^2 - 1} \qquad \text{for n an even harmonic}$$

$$= 0 \qquad \text{for n an odd harmonic}$$

n	Harmonic Amplitude
2	$8K_d/3$
4	$16K_d/15$
6	$24K_d/35$
8	$32K_d/63$

Sample and Hold Phase Detector



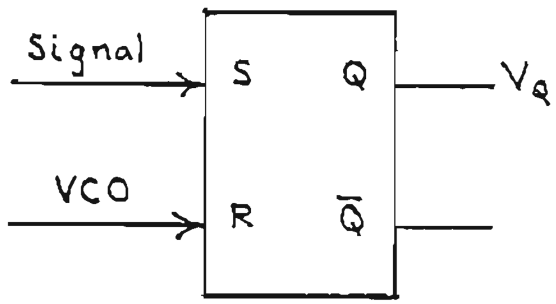
DC output vs. phase has same shape as input signal (when above noise).

Output ripple is negligible.

Sometimes useful for phaselocking to harmonics.

Flip-Flop Phase Detectors

The simplest sequential PD is an RS Flip Flop.

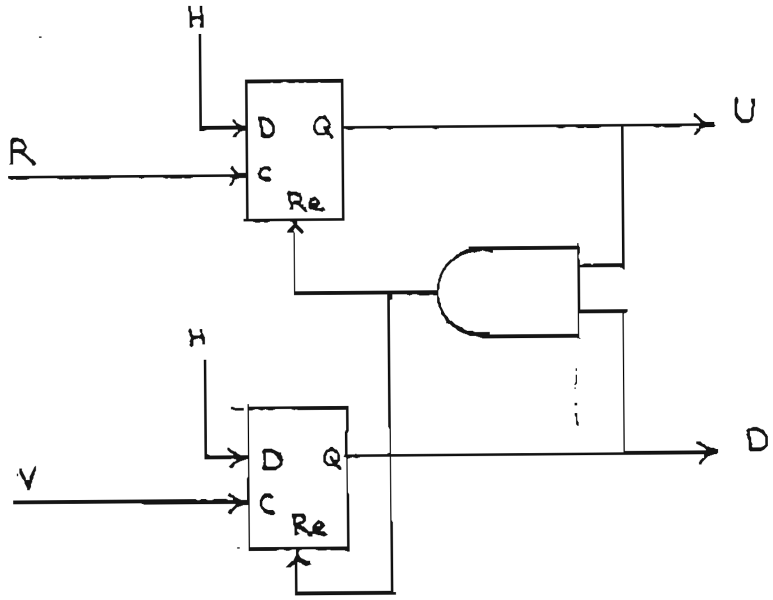


Typical operation:

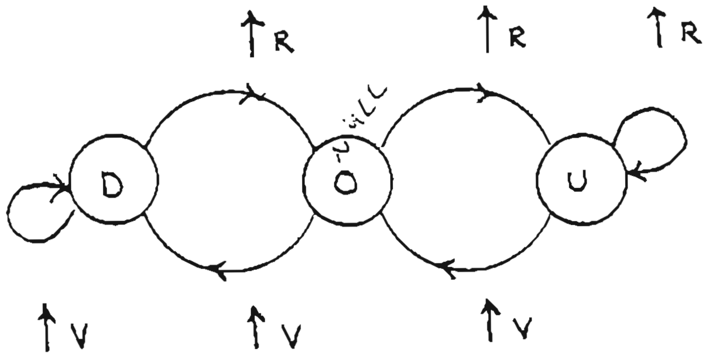
Q is set true by negative edges of signal.

Q is reset by negative edges of VCO.

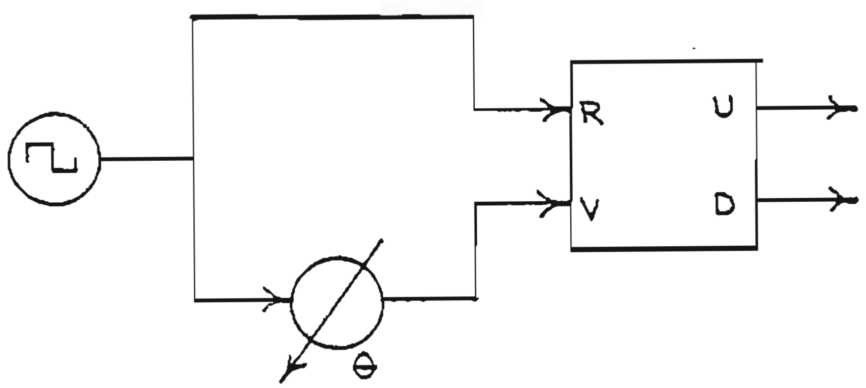
Widely used in laboratory phase meters.



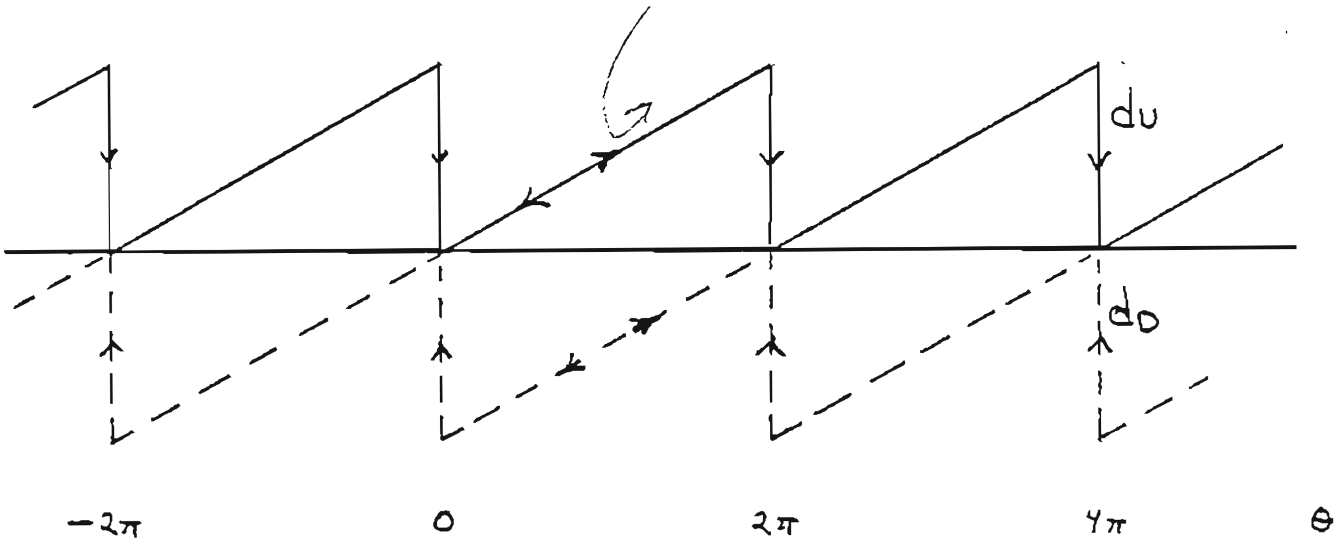
(Output Logic: High = True)



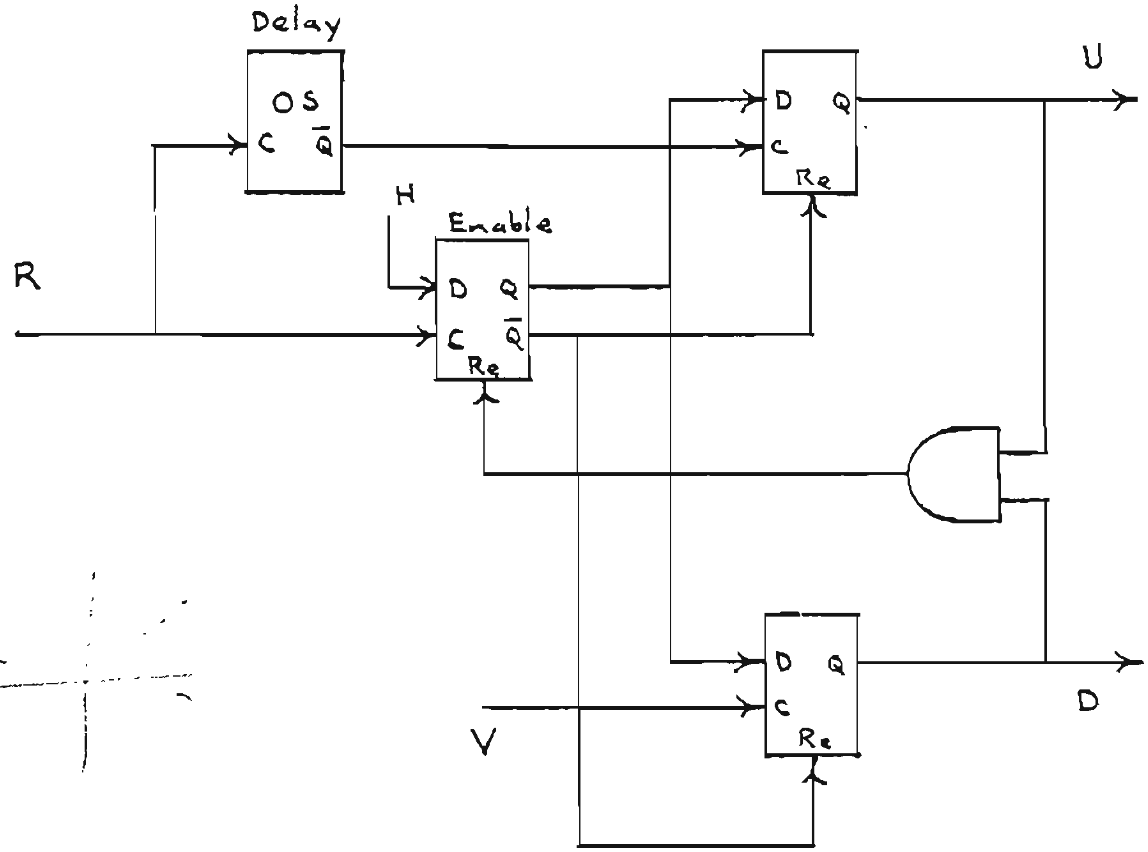
Refs: J. Tal and R.K. Whitaker, IEEE Trans. AES-15, March 1979
C.A. Sharpe, EDN, Sept. 20, 1976



$f_R > f_V$ rate $= (f_R - f_V)$



$f_R < f_V$
rate $= (f_V - f_R)$



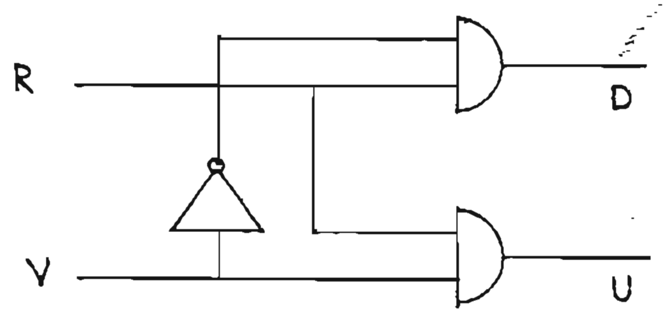
(Output Logic: High = True)

used if R is missing on occasion to set ~~start~~ outputs to 0

Sequential PD for

Bit Synchronizer

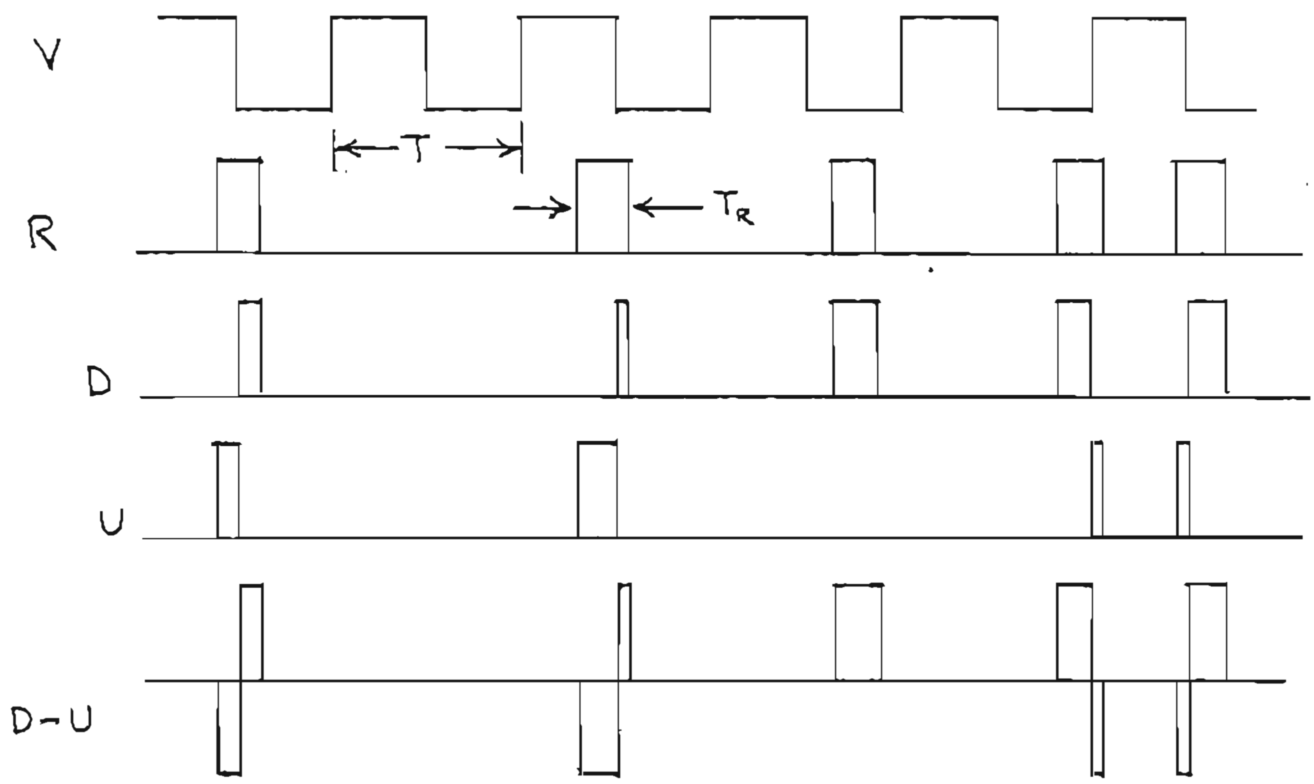
narrow freq range of operation due to O.S.

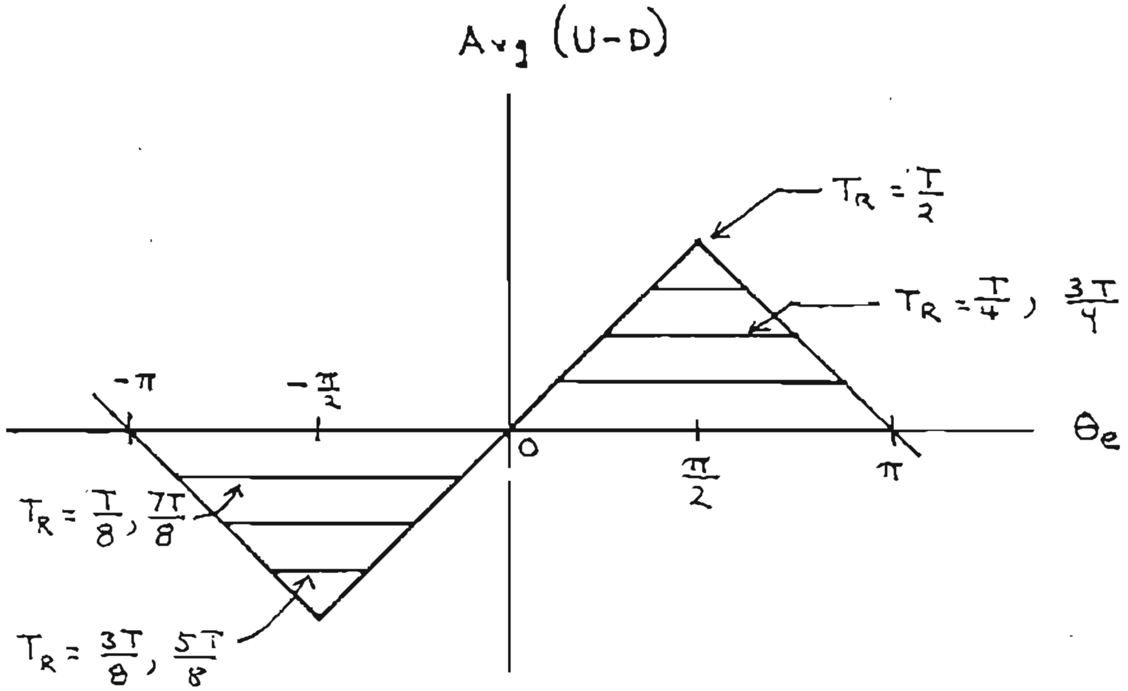


$U = RV \qquad D = R\overline{V} \qquad N = \overline{R}$

Multiplier (Combinatorial)

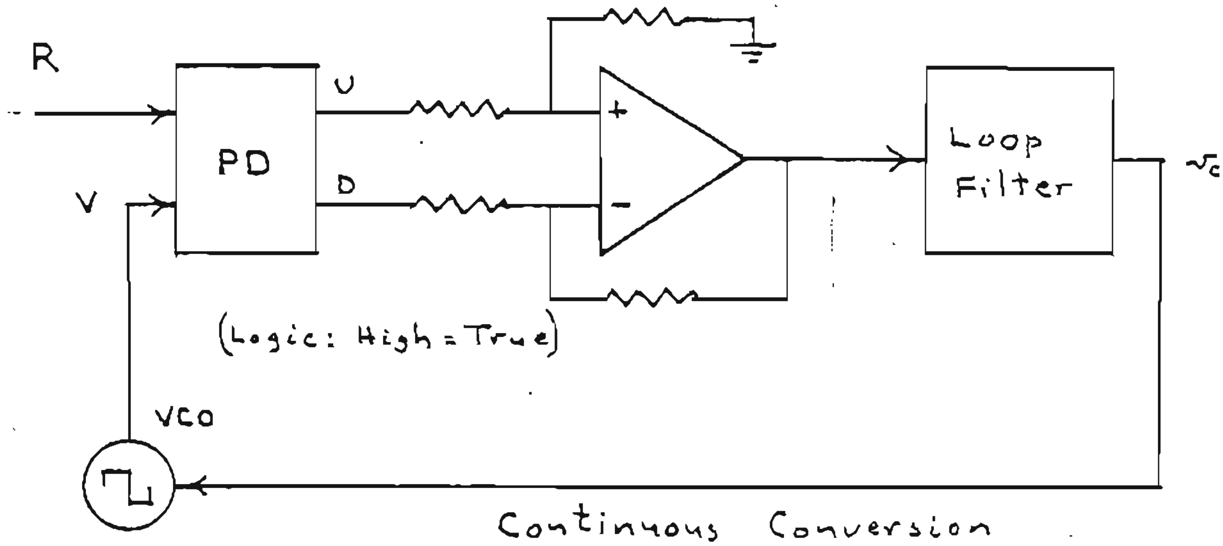
3-State Phase Detector / NO FREQ DET.



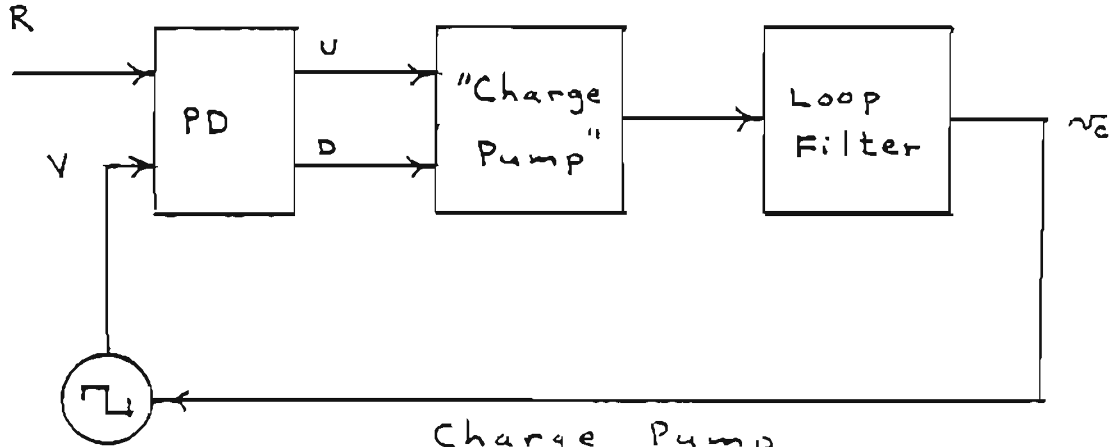


<u>PD Characteristic</u>

3-State Multiplier PD

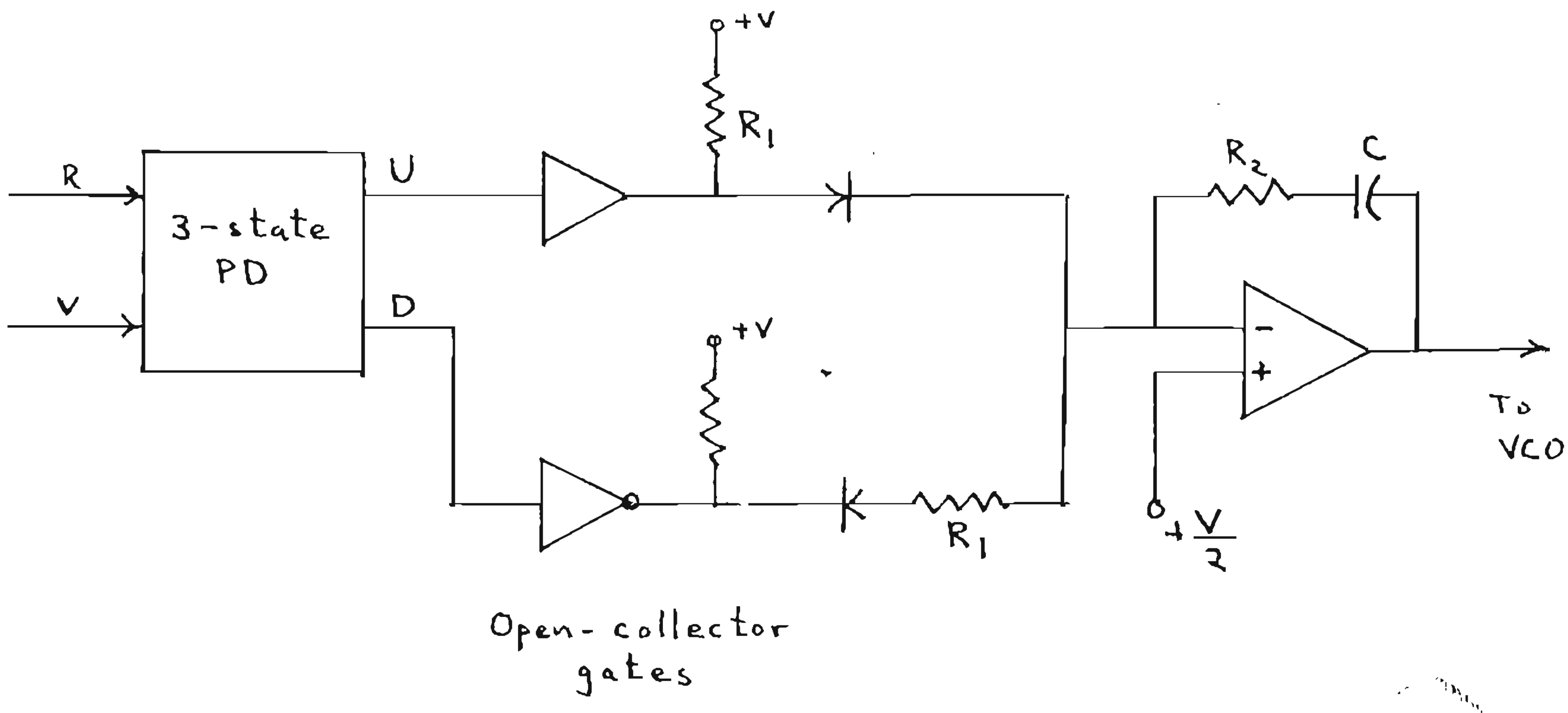


Continuous Conversion

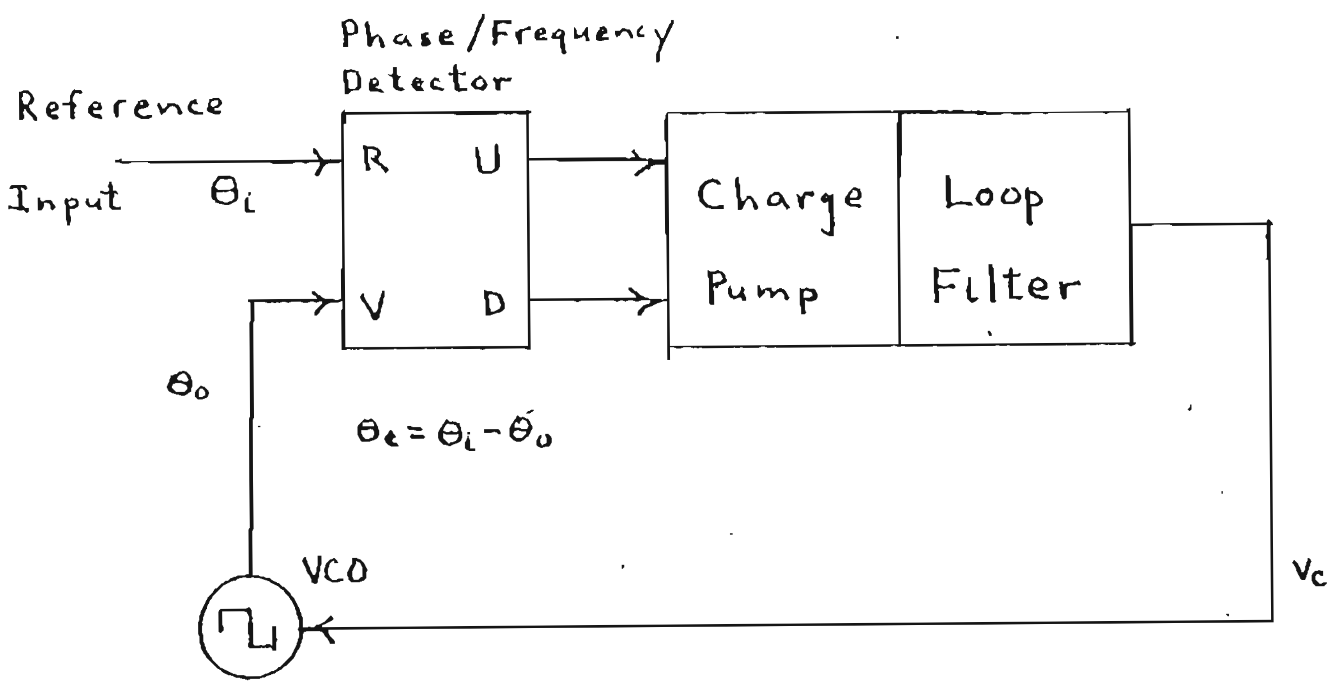


Charge Pump

Logic-to-Analog Conversion



<u>Typical Charge Pump</u>
(Widely Used)



$$\omega_o = \Omega_o + K_o V_c$$

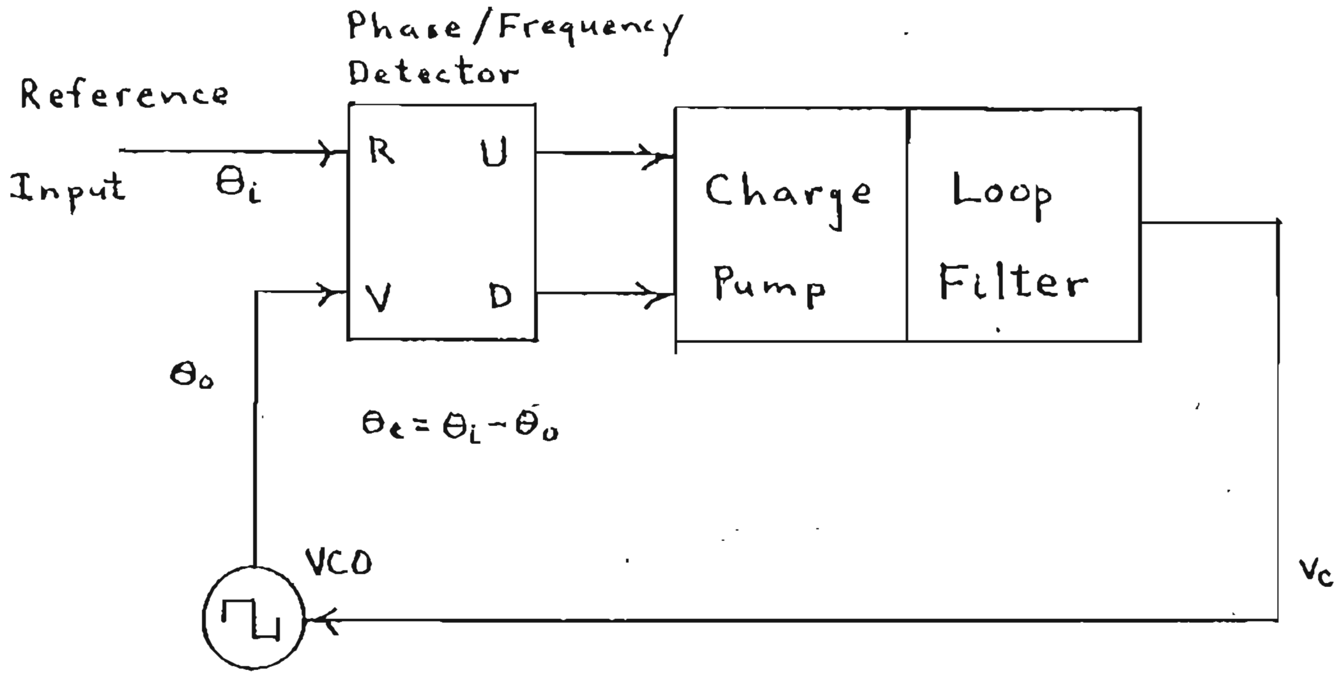
$$\theta_o(t) = \theta_o(0) + \int_0^t \omega_o(t')\,dt'$$

Fig: 1

Phase lock Loop with

Phase/Frequency Detector

and Charge Pump



$$\omega_o = \Omega_o + K_o v_c$$

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Fig: 1

Phaselock Loop with Phase/Frequency Detector and Charge Pump

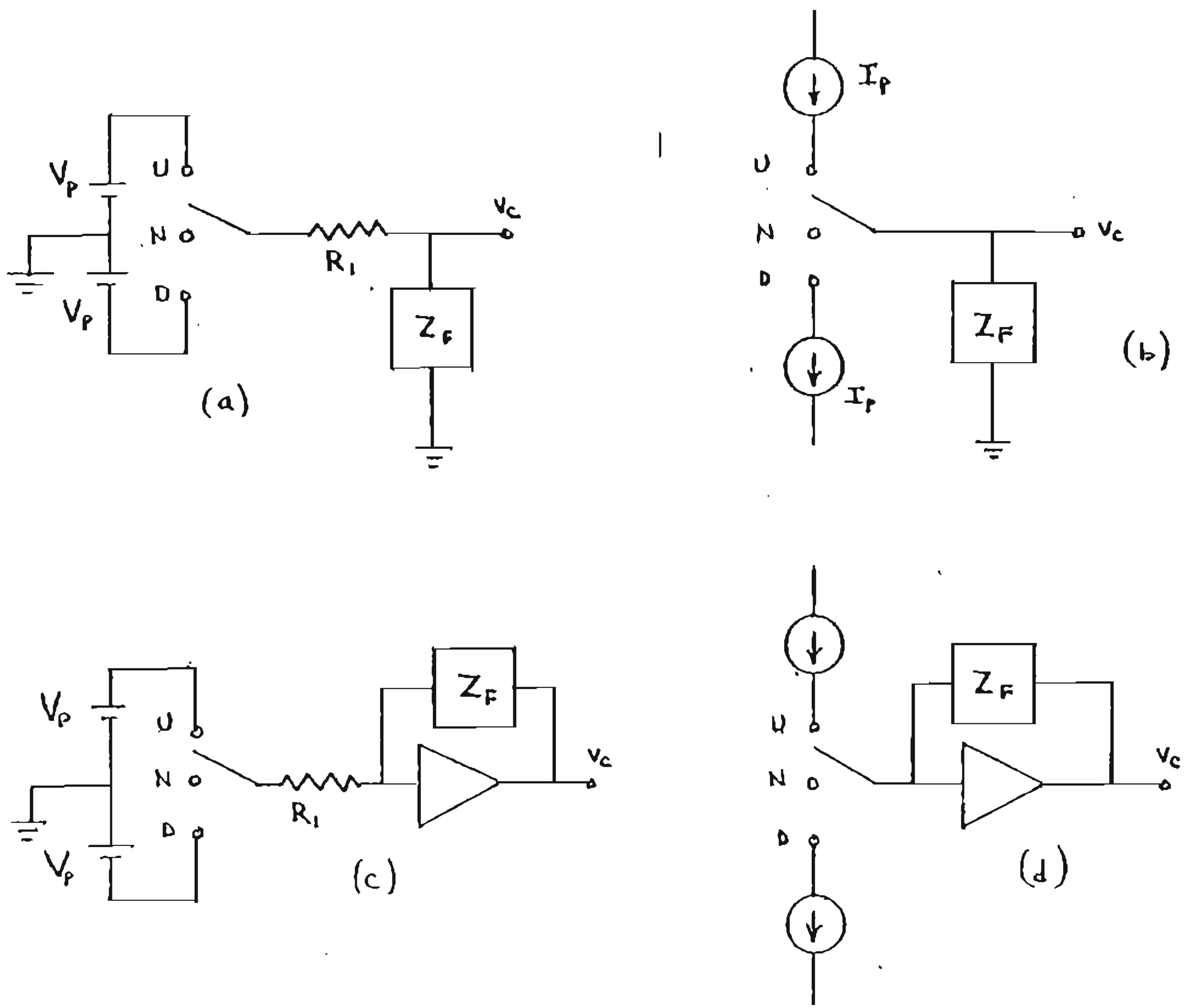
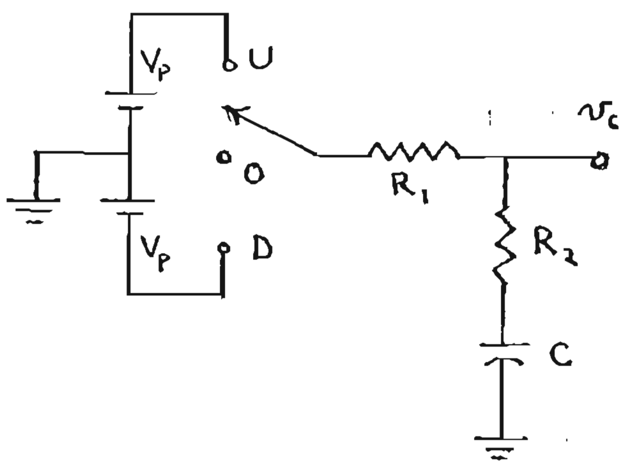
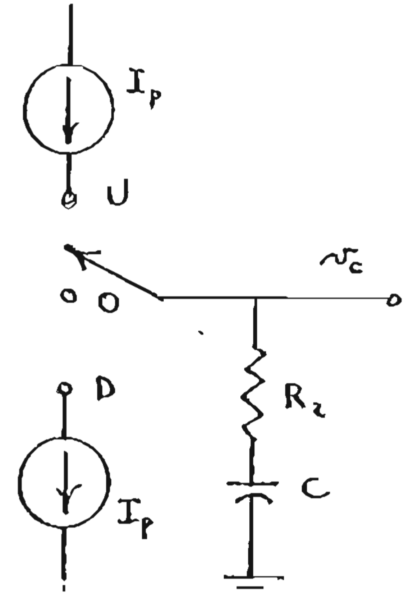
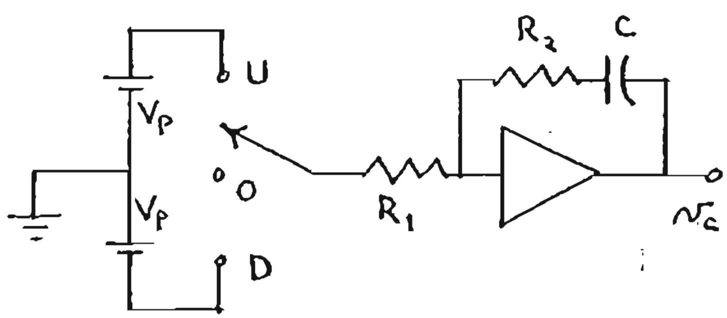


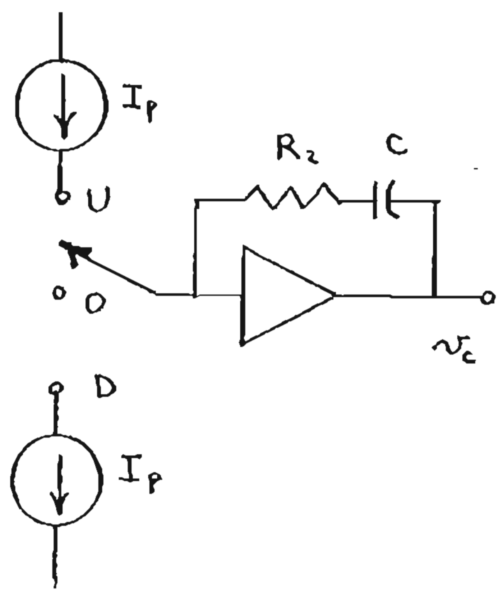
Fig. 2

Charge Pumps and Loop Filters









Charge Pumps

Current-Switch Charge-Pump

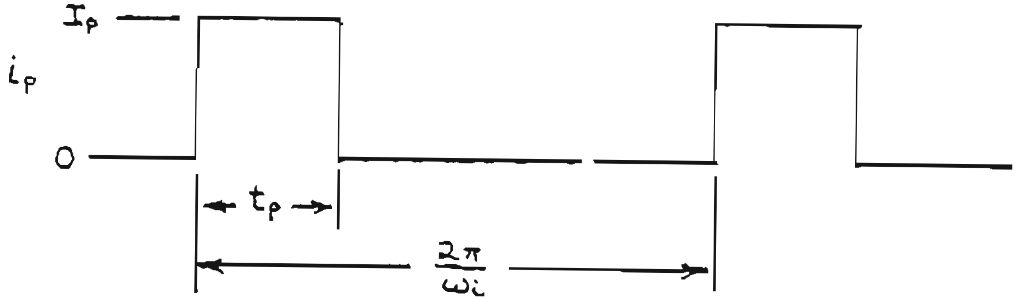
Averaging Approximation

Input frequency = ω_i rad/s

Pump current = $\pm I_p$ amps

(+ for $\theta_e > 0$: U) (- for $\theta_e < 0$: D)

Pump duration = $t_p = |\theta_e| / \omega_i$ sec



Average pump current = $i_d = \dfrac{\pm I_p t_p \omega_i}{2\pi}$

$= \dfrac{|I_p| \theta_e}{2\pi}$ amps

PD gain = $K_d = \dfrac{|I_p|}{2\pi}$ amp/rad

Loop Transfer Function

For any one-port loop filter $Z_F(s)$

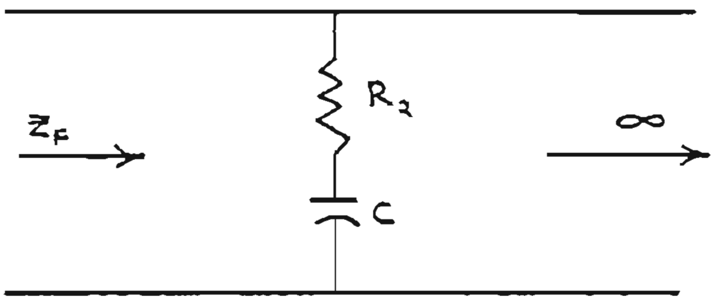
$$V_c(s) = I_d(s)\, Z_F(s) = K_d\, \theta_e(s)\, Z_F(s)$$

$$\theta_o(s) = \frac{K_o V_c(s)}{s}$$

$$\theta_e(s) = \theta_i(s) - \theta_o(s)$$

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_o |I_p|\, Z_F(s)}{2\pi s + K_o |I_p|\, Z_F(s)}$$

For second-order loop:



$$Z_F = R_2 + \frac{1}{sC} = R_2\left(1 + \frac{1}{s\tau_2}\right)$$

$$H(s) = \frac{\dfrac{K_0 |I_p| R_2}{2\pi}\left(s + \dfrac{1}{\tau_2}\right)}{s^2 + \dfrac{K_0 |I_p| R_2}{2\pi} s + \dfrac{K_0 |I_p| R_2}{2\pi \tau_2}}$$

$$= \frac{K\left(s + \frac{1}{\tau_2}\right)}{s^2 + Ks + \frac{K}{\tau_2}}$$

$$= \frac{2\zeta\omega_n\left(s + \frac{1}{\tau_2}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$K = \frac{K_o |I_p| R_2}{2\pi}$$

$$\omega_n = \sqrt{\frac{K_o |I_p| R_2}{2\pi \tau_2}} = \sqrt{\frac{K}{\tau_2}}$$

$$\zeta = \frac{1}{2}\sqrt{\frac{K_o |I_p| R_2 \tau_2}{2\pi}} = \frac{1}{2}\sqrt{K \tau_2}$$

Static Phase Error:

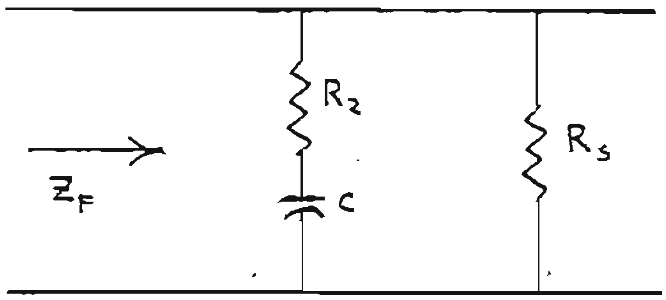
$$\theta_v = \frac{\Delta\omega}{K_v}$$

$$K_v = K_d K_o Z_F(0)$$

$$Z_F(0) = \infty$$

$$\boxed{\therefore \quad \theta_v = 0}$$

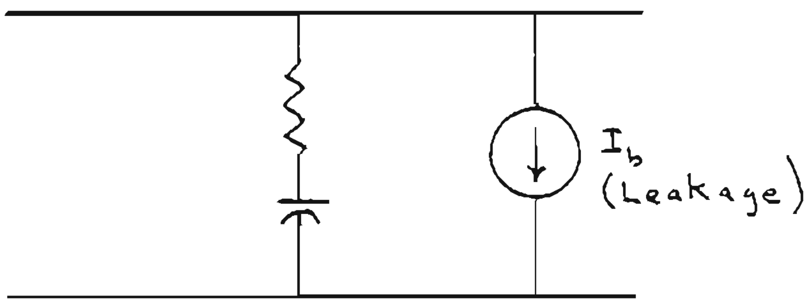
Effect of loading:



$$Z_F(0) = R_s \quad ; \quad K_v = \frac{|I_P|}{2\pi} K_o R_s$$

$$\theta_v = \frac{2\pi \Delta\omega}{|I_P| K_o R_s}$$

Effect of leakage:



$$\theta_b = \frac{2\pi I_b}{|I_P|}$$

accurate tracking:

1) charge pump
2) fast switching PFD

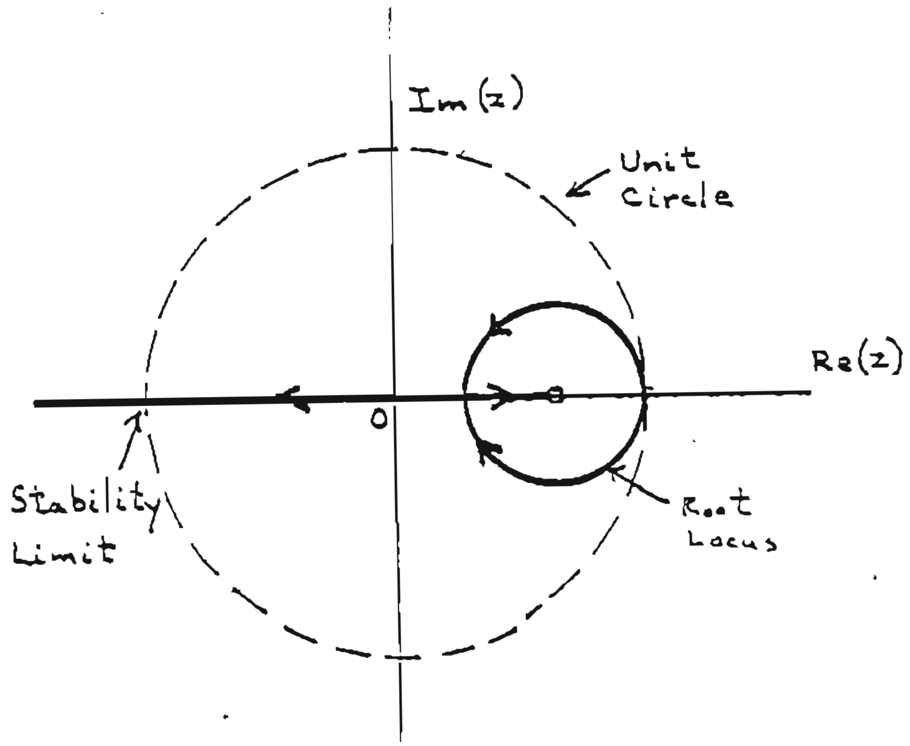


Fig. 3

Root Locus Plot of Second-Order Loop

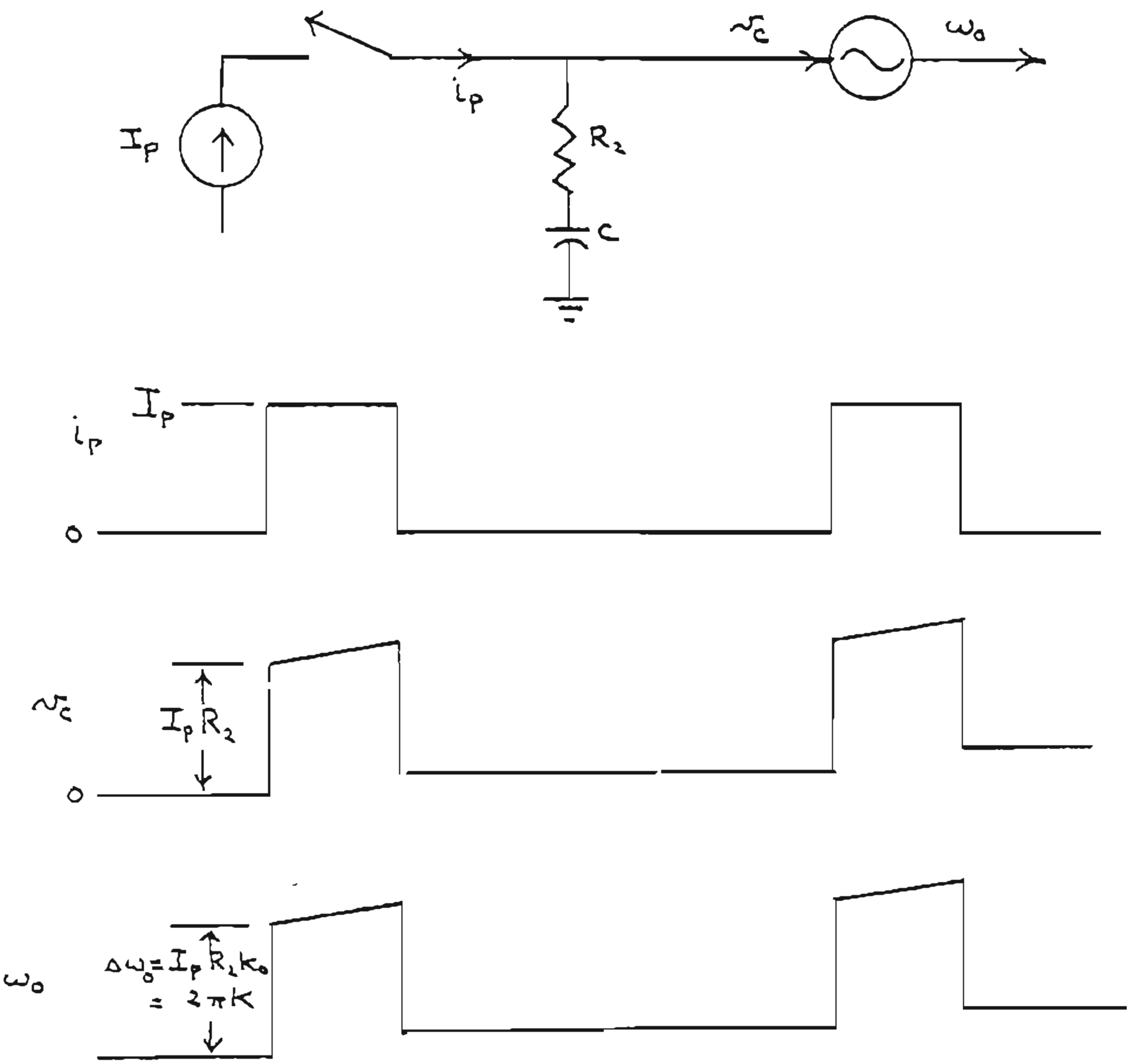
(in z-plane)

Stability Limit: Second-Order Loop

(from sampling analysis)

$$K\tau_2 < \frac{4\left(\frac{\omega_i \tau_2}{2\pi}\right)}{2 + \frac{2\pi}{\omega_i \tau_2}}$$

VCO Overload



Frequency jump $= 2\pi K$ rad/s at each cycle.

Control voltage must remain within dynamic limits of VCO control range.

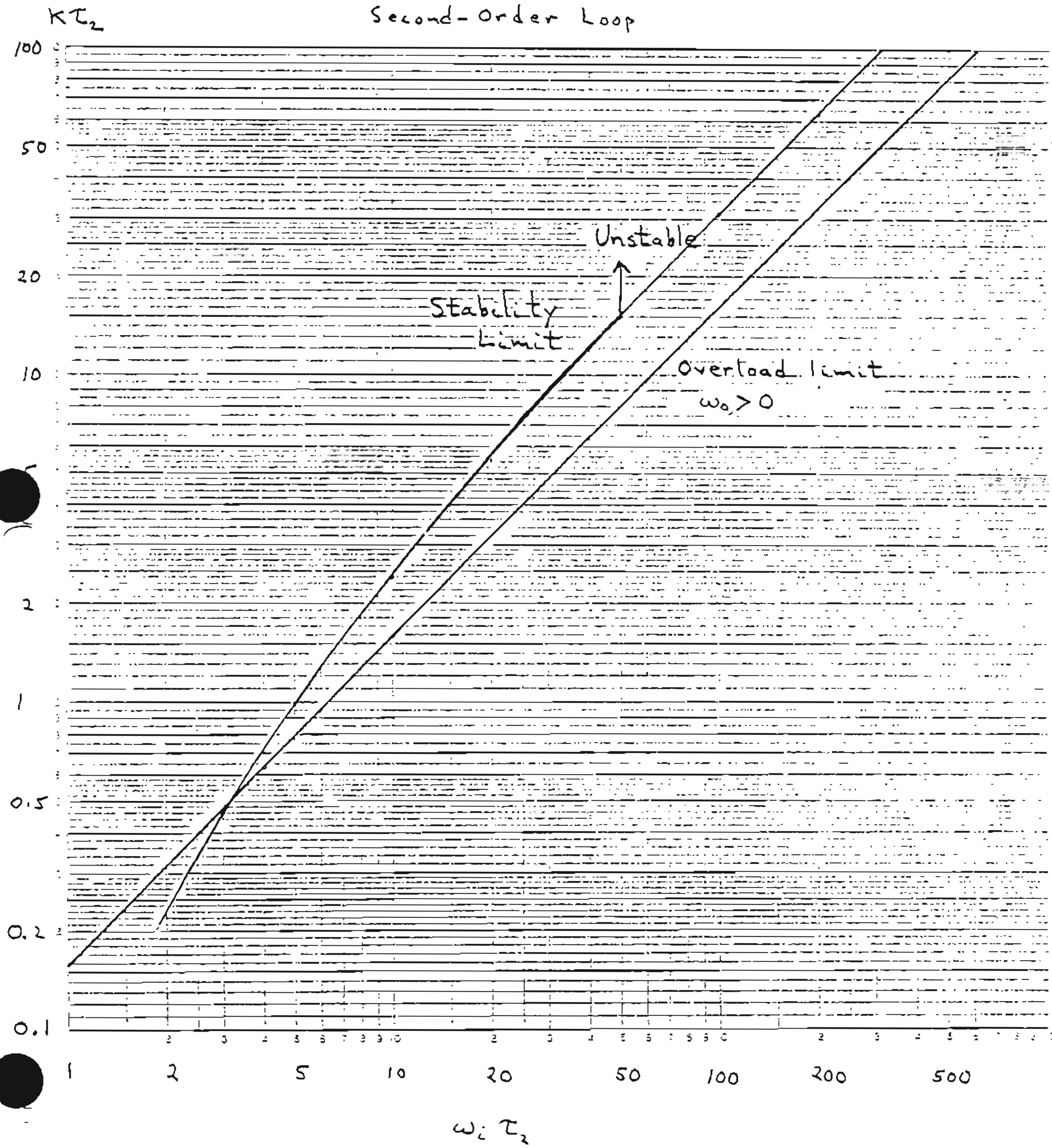
Extreme limit: $\omega_o > 0$
(Cannot have negative frequency)

$$\therefore \quad 2\pi K < \omega_i$$

$$K\tau_2 < \frac{\omega_i \tau_2}{2\pi}$$

Stability and Overload Limits

Second-Order Loop



Other Charge Pumps

Voltage switch, passive filter:

Let $I_p = V_p / R_1$; use current-switch equations.

Valid only if $v_c << V_p$.

Otherwise, pump current depends upon v_c; operation becomes nonlinear in peculiar manner.

Active filters:

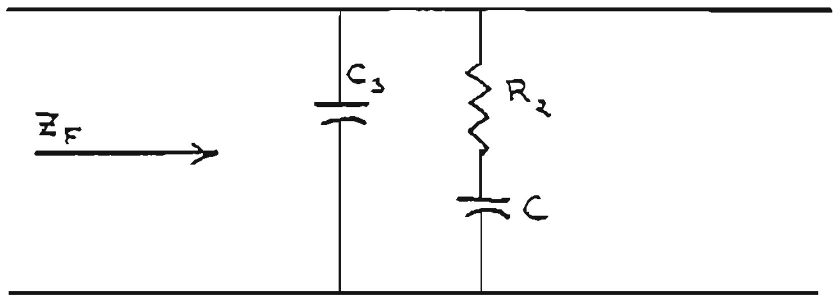
Use equations for current switch and passive filter, taking polarity reversal into account. For voltage switch, use $I_p = V_p / R_1$.

Amplifiers isolate any loading by VCO control port.

Amplifier must be capable of delivering I_p amps on each cycle and jumping $I_p R_2$ volts without slew limiting or other overload.

Jump Suppression

To convert jump to smaller-amplitude ramp, add capacitor across filter.



$$Z_F = \frac{sCR_2 + 1}{sC\left(sC_3R_2 + 1 + \frac{C_3}{C}\right)}$$

Define:

$$\tau_2 = R_2 C$$

$$b = 1 + \frac{C}{C_3}$$

$$Z_F = \left(\frac{b-1}{b}\right)\frac{s\tau_2 + 1}{sC\left(\frac{s\tau_2}{b} + 1\right)}$$

zero at $s = -1/\tau_2$

pole at $s = -b/\tau_2$

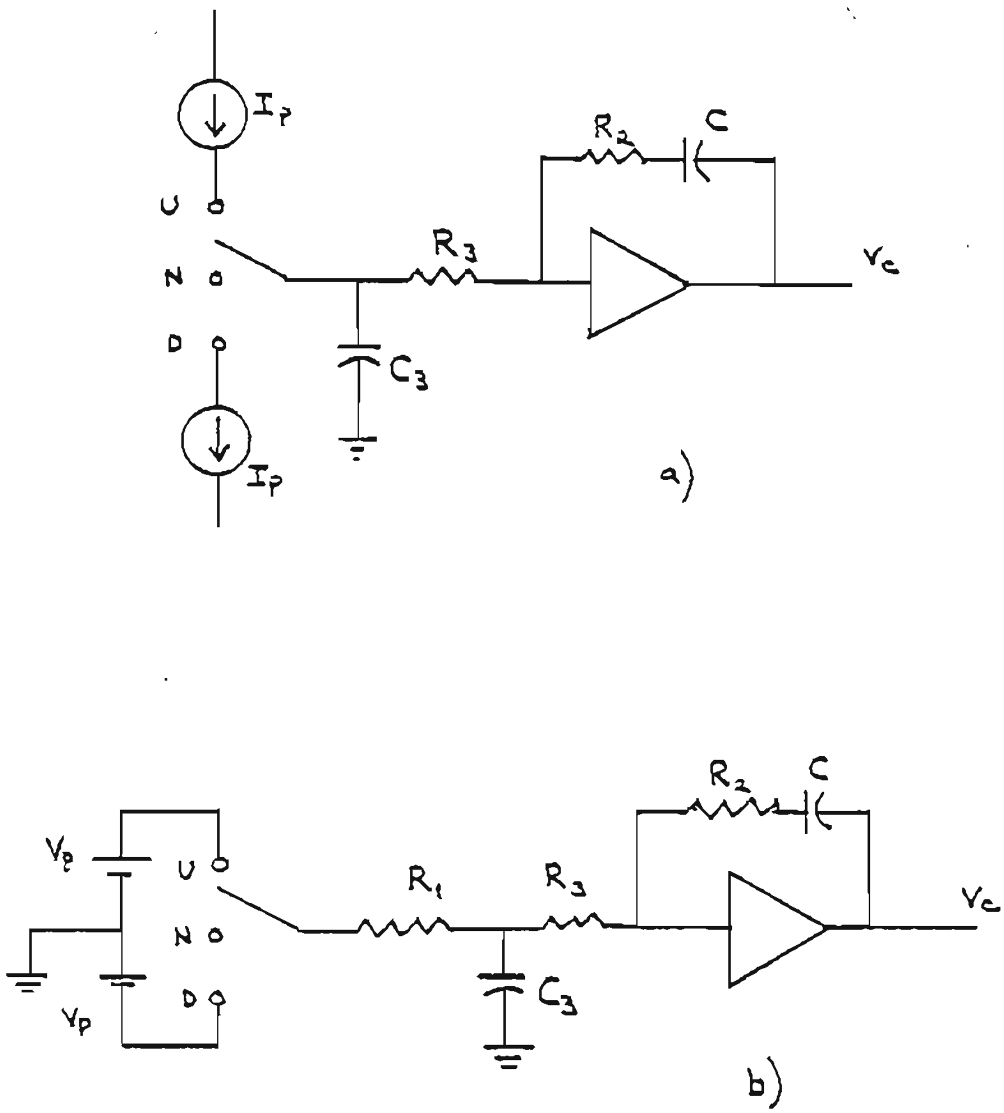


Fig. 6

Jump Suppression for Active Filters

Transfer Function: Third-Order Loop

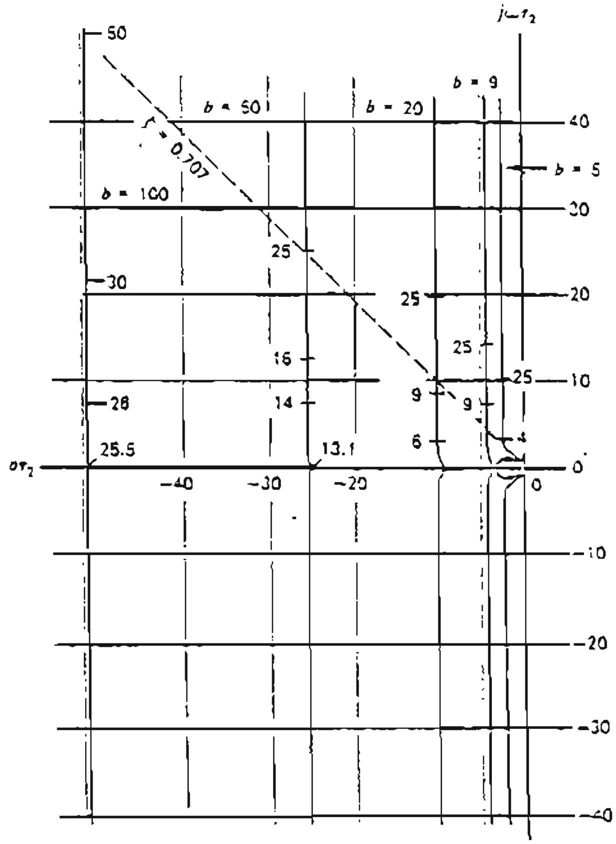
passive Filter

$$K \triangleq \frac{K_o |I_p| R_2}{2\pi}$$

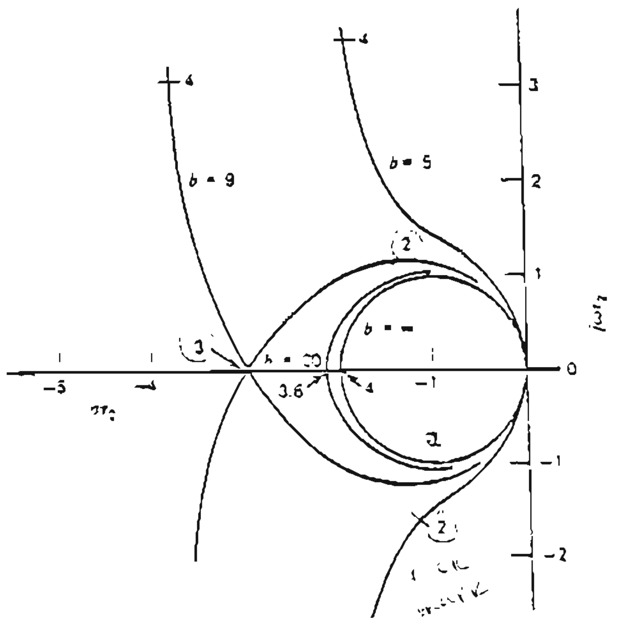
$$H(s) = \frac{K\left(\frac{b-1}{b}\right)\left(s + \frac{1}{\tau_2}\right)}{\frac{s^3 \tau_2}{b} + s^2 + K\left(\frac{b-1}{b}\right)s + \frac{K(b-1)}{b\,\tau_2}}$$

$$K' = \frac{b-1}{b} \tau_2 K$$

passive filter only



a)



b)

Fig. 7

Root Locus Plots for Third-Order, Type-II PLL

a) Large scale

b) Expanded scale

Tick marks show values for $K' \triangleq K\tau_2$

Stability Limit: Third-Order Loop

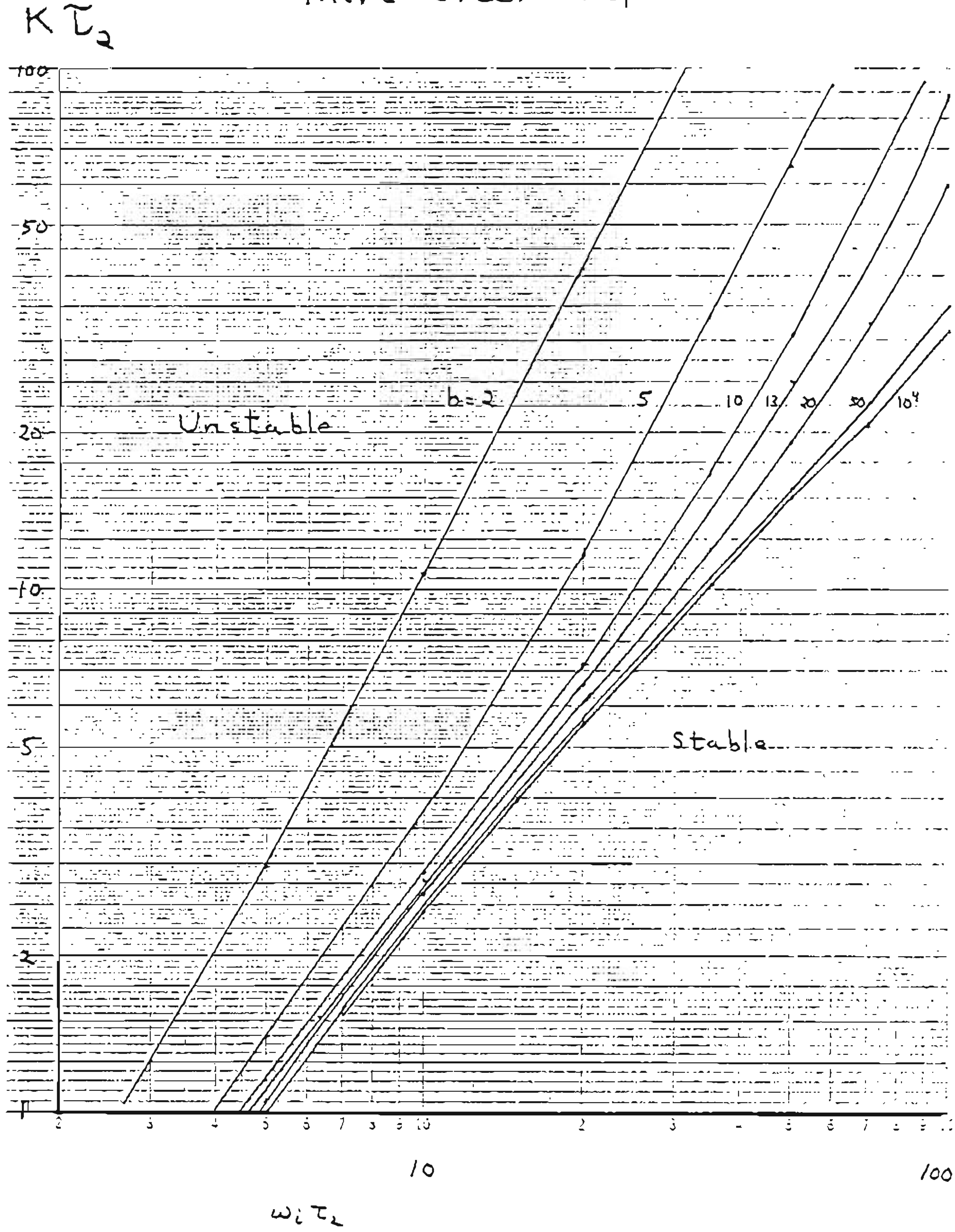
$$K\tau_2 < \frac{4(1+a)}{\dfrac{2\pi(b-1)}{b\,\omega_i\tau_2}\left[\dfrac{2\pi(1+a)}{\omega_i\tau_2} + \dfrac{2(1-a)(b-1)}{b}\right]}$$

$$a \triangleq e^{-\frac{2\pi b}{\omega_i \tau_2}}$$

(Caution: Stability does not assure satisfactory operation. Actual pole locations are needed.)

Stability Limit

Third-Order Loop



Effectiveness of Jump Suppression:

Ramp amplitude

$$|\Delta\omega_0| = 2\pi K\left(\frac{b-1}{b}\right)\left[\frac{b-1}{b}\left(1-e^{-\frac{b|\theta_e|}{\omega_i\tau_2}}\right)+\frac{|\theta_e|}{\omega_i\tau_2}\right]$$

For 2nd-order loop, $|\Delta\omega_0| = 2\pi K$

Suppression factor

$$\beta = \left(\frac{b-1}{b}\right)\left[\frac{b-1}{b}\left(1-e^{-\frac{b|\theta_e|}{\omega_i\tau_2}}\right)+\frac{|\theta_e|}{\omega_i\tau_2}\right]$$

If $\frac{b|\theta_e|}{\omega_i\tau_2} << 1$ (Not necessarily true)

then

$$\beta \approx \frac{(b-1)|\theta_e|}{\omega_i\tau_2}$$

(To be effective, time constant τ_2/b introduced by extra capacitor must exceed $1/\omega_i$ substantially.)

Supplementary Notes

for

PHASELOCK UP TO DATE

by

Dr. Floyd M. Gardner

Part II

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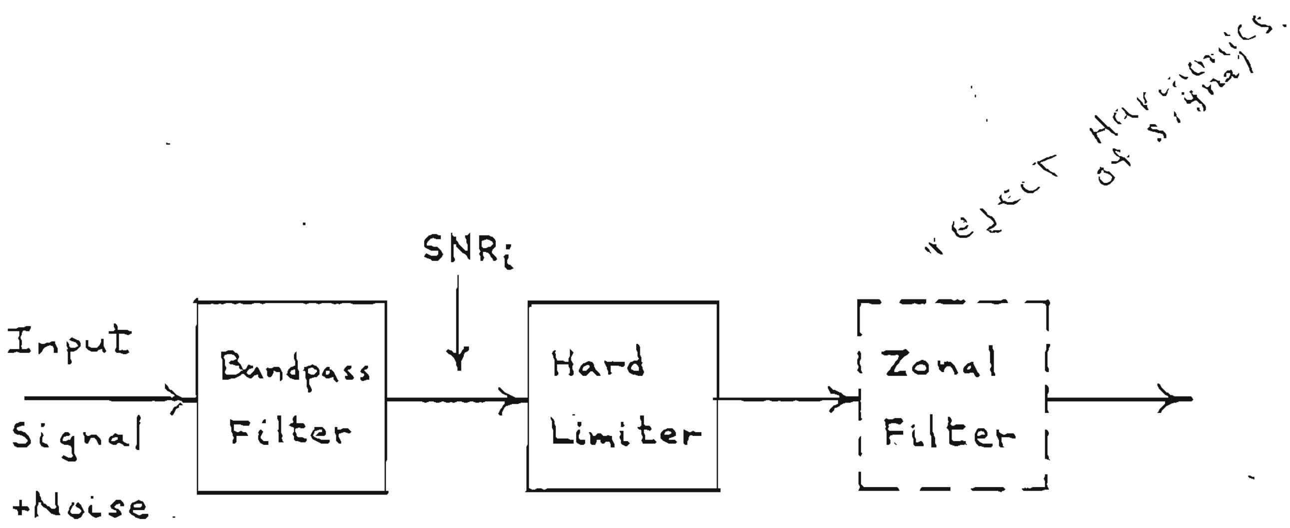
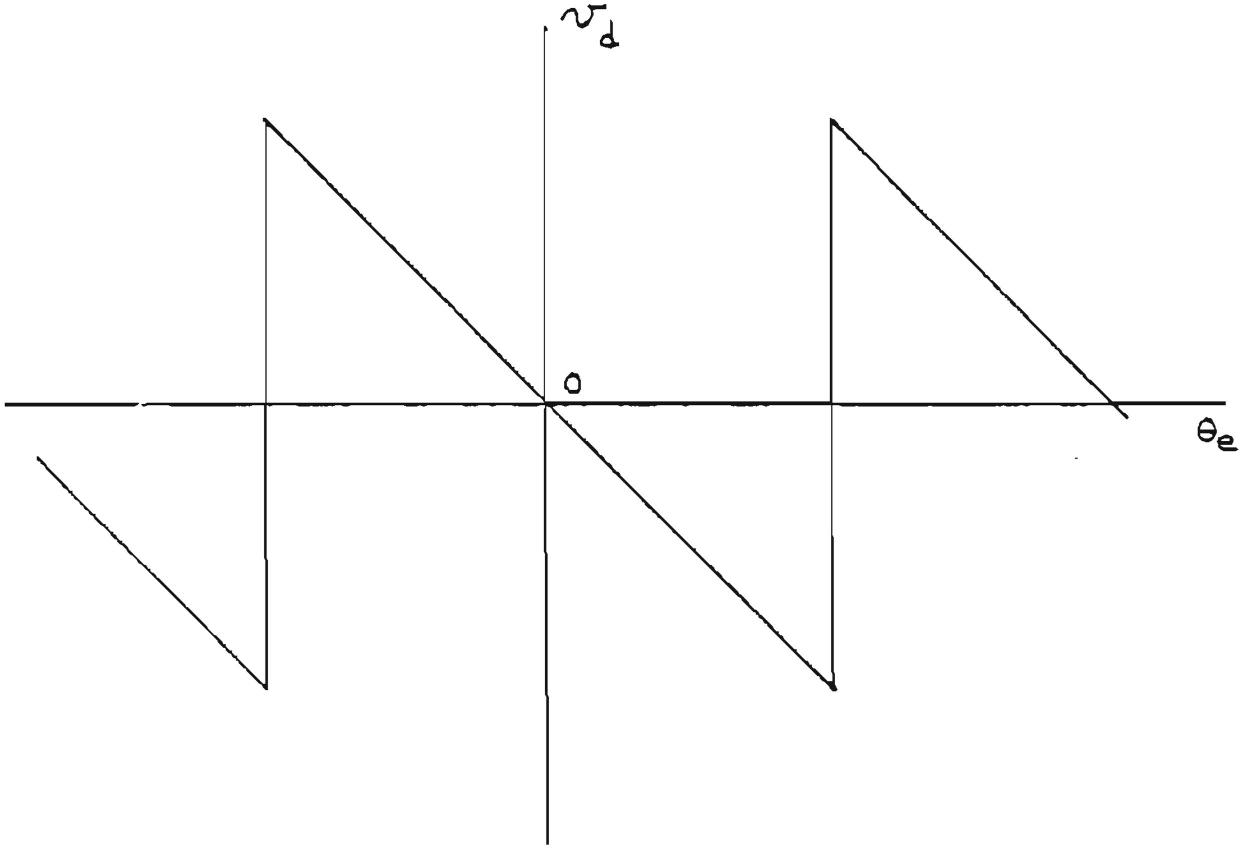
Palo Alto, CA 94301
(415) 328-8855

CHAPTER 6

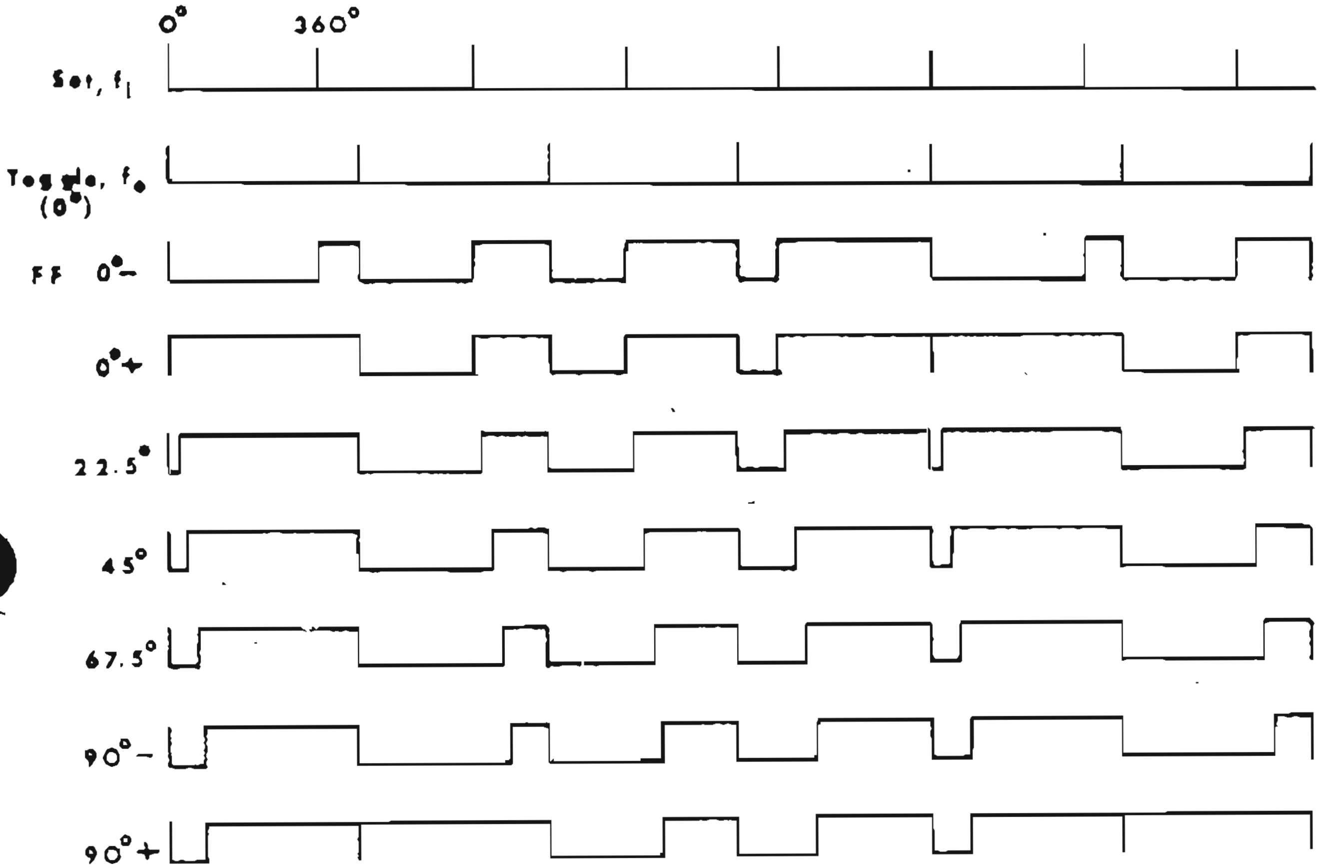
(continued)



Bandpass Limiter

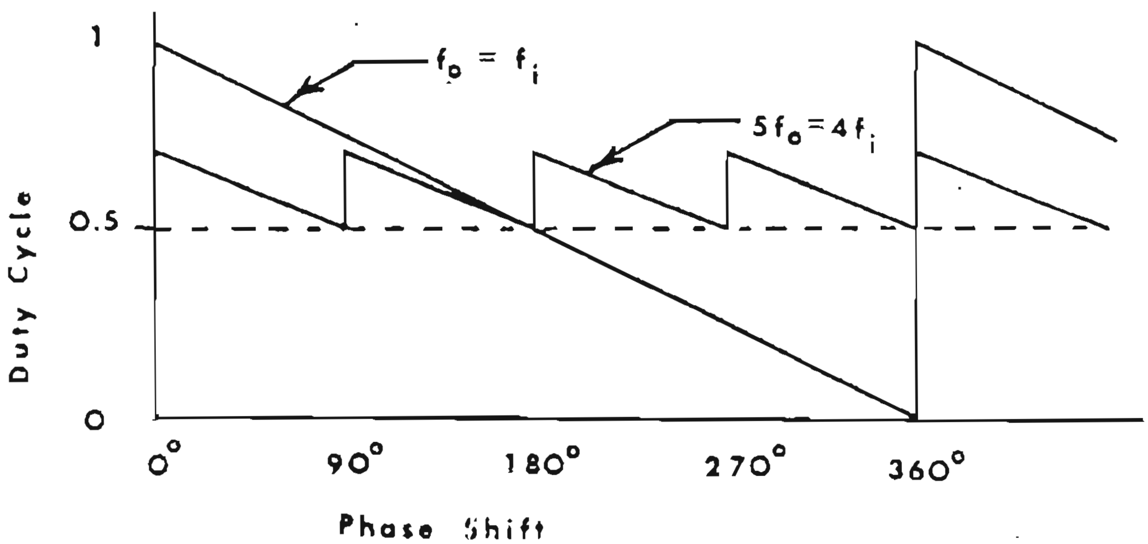


v_d
0
θ_e

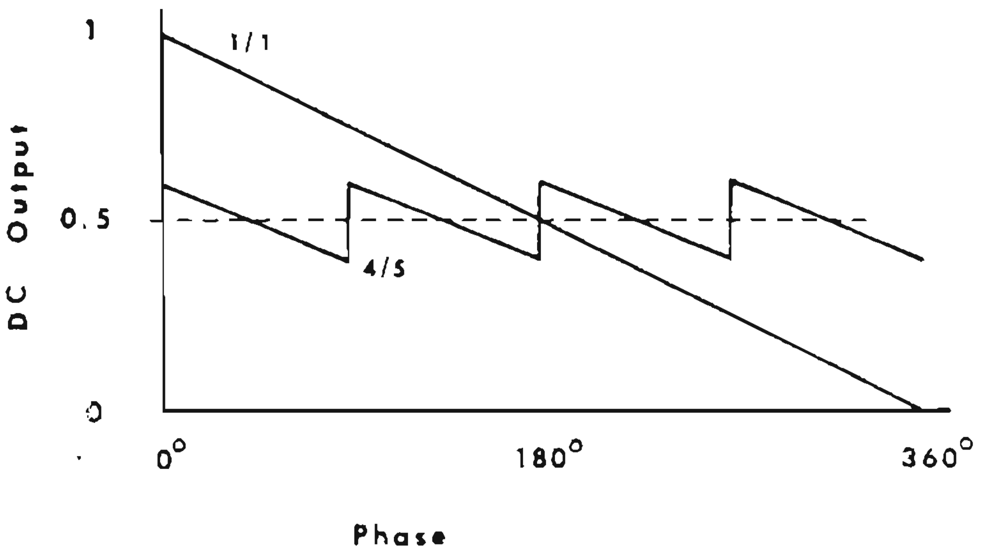


WAVEFORMS

FF PD IN 4/5 MODE

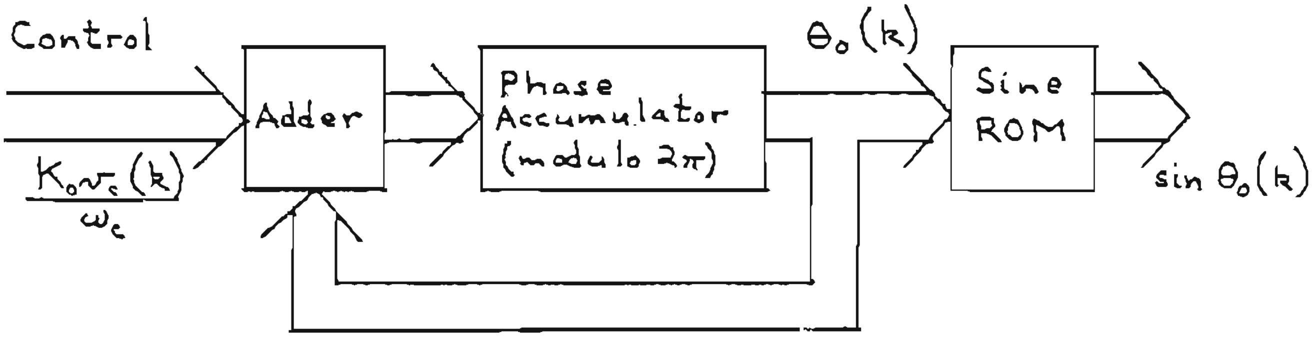


FF PD CHARACTERISTIC



SAMPLE AND HOLD PD CHARACTERISTIC

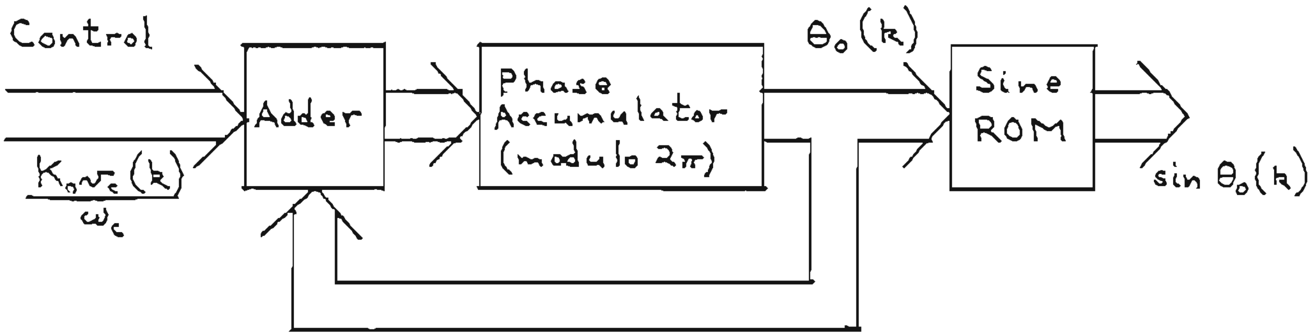
Number-controlled Oscillator



ω_c = clock frequency

$$\theta_o(k) = \theta_o(k-1) + \frac{K_o v_c(k)}{\omega_c}$$

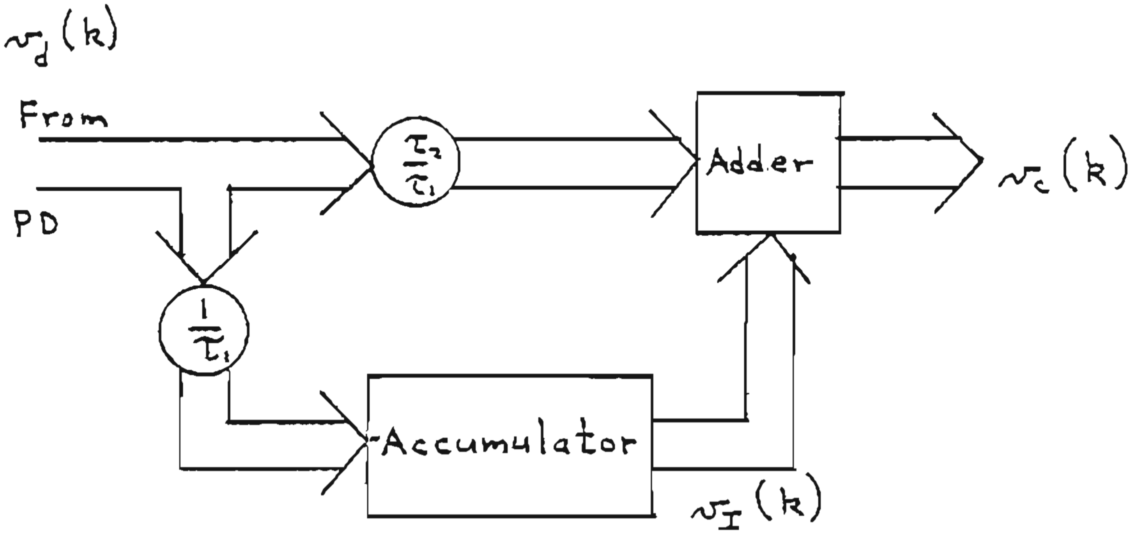
Number-controlled Oscillator



ω_c = clock frequency

$$\theta_o(k) = \theta_o(k-1) + \frac{K_o v_c(k)}{\omega_c}$$

Digital Loop Filter



$$v_I(k) = v_I(k-1) + \frac{v_d(k)}{\tau_1}$$

$$v_c(k) = \frac{\tau_2}{\tau_1} v_d(k) + \frac{1}{\tau_1} \sum_{n=-\infty}^{k} v_d(n)$$

DIGITAL PHASE DETECTORS

1. Multiply digitized input sample by digitized "VCO" sample.

   Need not be synchronized.

   Take care with aliasing of multiplier products.

2. Use sequential phase detector with U,O, and D states at output.

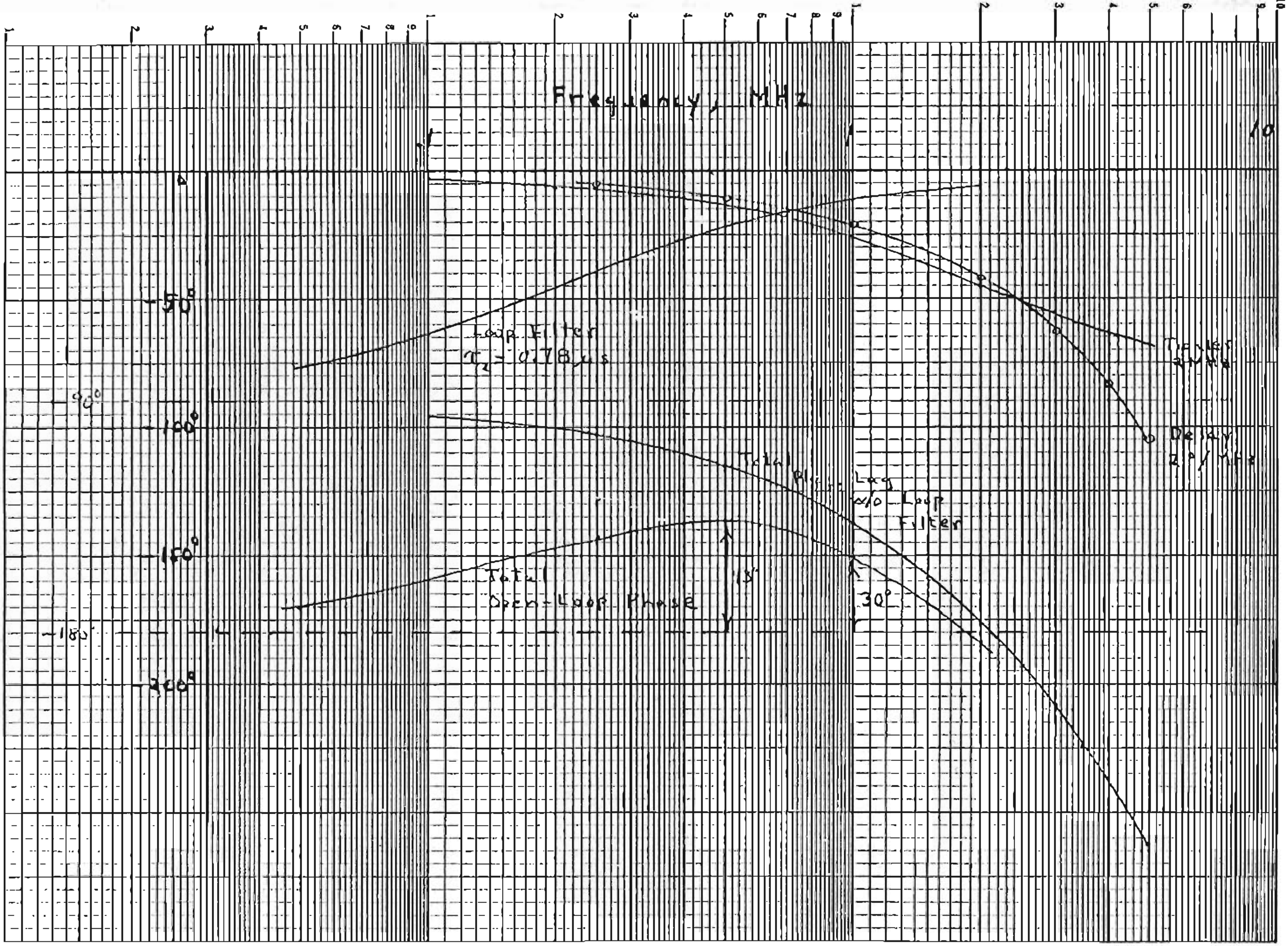
   Count clock cycles in U or D state to determine phase error.

3. Use input edge to sample phase accumulator in "VCO".

   Yields output phase at instant of input crossing, which

   is equivalent to input phase = 0.

CHAPTER 8


Frequency, MHz
.1
1
10
-50°
-90°
-100°
-150°
-180
-200°
Loop Filter
τ_2 = 0.78 μs
Total
w/o Loop
Filter
Total
Open-Loop Phase
18°
30°
Delay

CHAPTER 9

FM with large loop bandwidth

Define:

$$\Omega_{op} = \frac{sH(s)}{K_d} V_p(s)$$

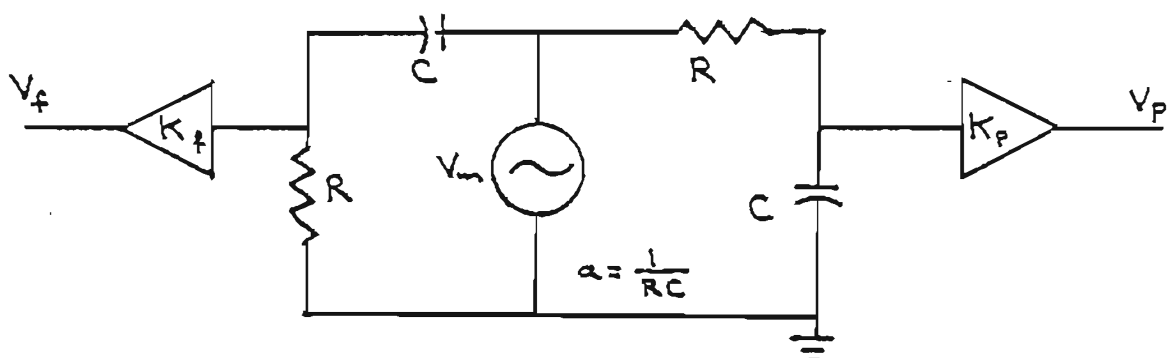
$$\Omega_{of} = K_o[1 - H(s)]\, V_f(s)$$

Let:

$V_m(s)$ be message

$$V_f(s) = K_f \frac{s}{s+a} V_m(s)$$

$$V_p(s) = K_p \frac{a}{s+a} V_m(s)$$

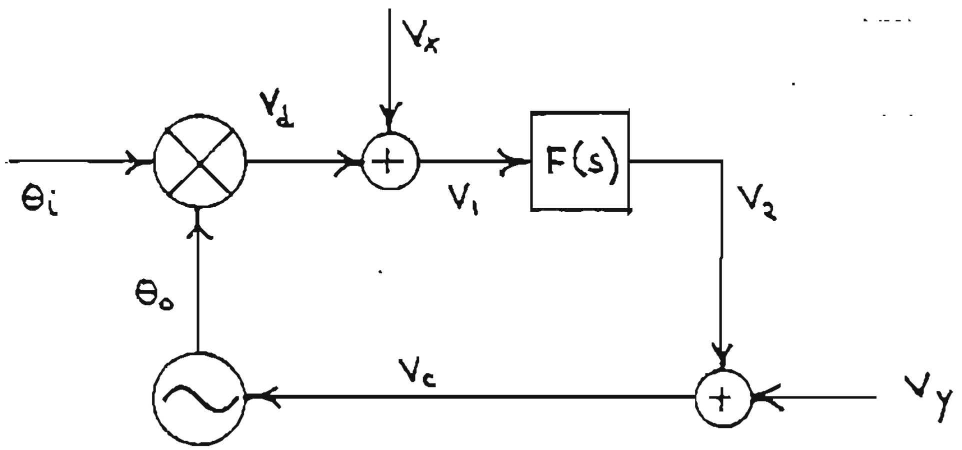


$$\Omega_o(s) = \Omega_{op} + \Omega_{of} =$$

$$\frac{s V_m(s)}{s+a}\left[\frac{aK_p}{K_d} H(s) + K_o K_f [1 - H(s)]\right]$$

If $K_o K_f = aK_p / K_d$, then

$$\boxed{\Omega_o(s) = \frac{K_o K_f\, s}{s+a} V_m(s)}$$



$\theta_i \equiv 0$

<u>If $V_y = 0$</u>

$$V_d = -H(s)\,V_x$$

$$V_1 = [1 - H(s)]\,V_x$$

<u>If $V_x = 0$</u>

$$V_2 = -H(s)\,V_y$$

$$V_c = [1 - H(s)]\,V_y$$

$$H(s) = \frac{G(s)}{1 + G(s)} \qquad (2.15)$$

$H(s)$ = Closed-loop transfer function (eq. 2.6)

$G(s)$ = Open-loop transfer function (eq. 2.14)

$1 - H(s)$ = Error transfer function (eq. 2.7)

$$G(s) = \frac{H(s)}{1 - H(s)}$$

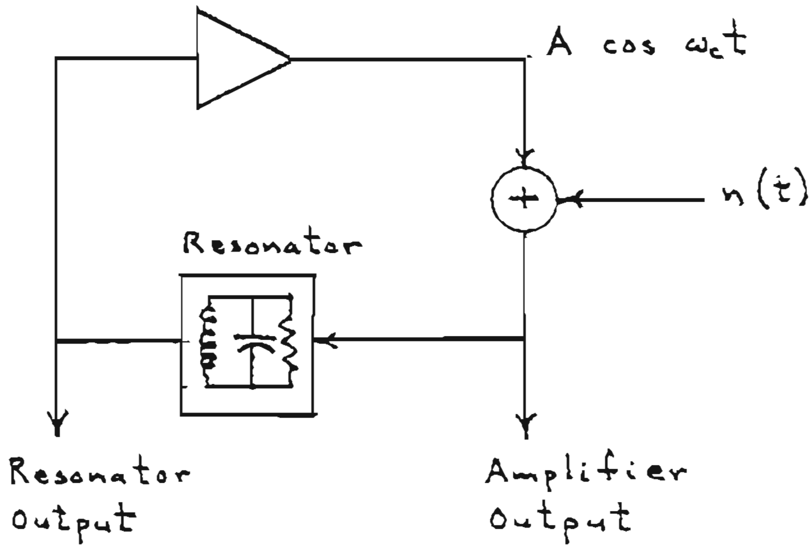
$$G(j\omega) = -\frac{V_d(j\omega)}{V_1(j\omega)} \qquad (V_x\text{-test})$$

$$= -\frac{V_2(j\omega)}{V_c(j\omega)} \qquad (V_y\text{-test})$$

CHAPTER 10

Oscillator Noise

Leeson's Model (Proc. IEEE; Feb. 1966)



Conditions for stable oscillation:
(In absence of noise)

Loop gain = 1

Loop phase shift = 0

Resonator Characteristics:

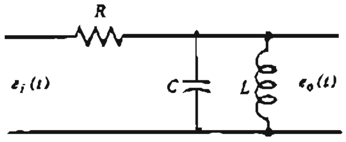


Figure A.8 Tuned circuit.

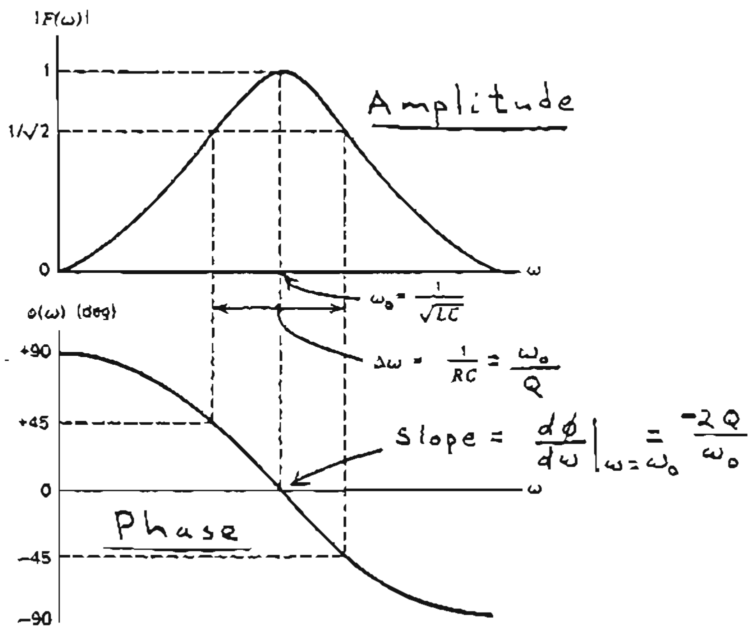


Figure A.9 Frequency response of tuned circuit.

Additive noise $n(t)$ causes amplitude and phase perturbations on fictitious clean signal $A \cos \omega_c t$. Neglect AM (suppressed by limiting action of amplifier).

Slow phase fluctuations $\Delta\phi_L$ could pass through resonator. If uncorrected, would violate phase criterion of oscillation. Circuit automatically compensates by shifting oscillation frequency by

$$\Delta\omega = \frac{-\Delta\phi_L}{\text{Phase Slope}} = \frac{\Delta\phi_L\,\omega_0}{2Q} \quad \text{rad/s}$$

Resulting spectrum of <u>frequency</u> disturbance is

$$\Phi_{\Delta\omega}(\omega) = \frac{\Phi_{\Delta\phi_L}(\omega)\,\omega_0^2}{4Q^2} \quad (\text{rad/s})^2/\text{Hz}$$

where $\Phi_{\Delta\phi_L}(\omega)$ is the spectral density of $\Delta\phi_L$ (in rad^2/Hz). Valid only for $\omega < \omega_0/2Q$.

Fast phase fluctuations $\Delta\phi_H$ are suppressed by resonator and do not influence frequency of oscillation. Spectrum of <u>phase</u> disturbance at amplifier output is $\Phi_{\phi_H}(\omega)$, the spectral density of ϕ_H (in rad^2/Hz). Valid for $\omega > \omega_0/2Q$.

If output is taken after resonator, high-frequency fluctuations are filtered by $(\omega_0/2Q\omega_m)^2$, where $\omega_m = |\omega - \omega_0|$, while low-frequency fluctuations (close-in to carrier) are unaffected.

PM caused by additive noise:

Gaussian noise

$$s(t) = A \cos \omega_c t + n(t)$$

$$= A \cos \omega_c t + n_c \cos \omega_c t - n_s \sin \omega_c t$$

$$\phi(t) \triangleq \tan^{-1}\left[\frac{n_s(t)}{A + n_c(t)}\right]$$

phase noise

$$\approx \frac{n_s(t)}{A} \qquad \text{if} \quad n_c, n_s \ll A$$

Spectral density

$$\Phi_\phi(\omega) = \frac{\Phi_{n_s}(\omega)}{A^2} \qquad \text{rad}^2/\text{Hz}$$

For white noise, spectral density of $n(t)$ is N_0 V^2/Hz (W_0 watts/Hz)

so $\Phi_{n_s}(\omega) = 2N_0$ and

$$\Phi_\phi(\omega) = \frac{2N_0}{A^2} \qquad \text{(eq. 3.13a)}$$

$$= \frac{W_0}{P_s} \quad \text{rad}^2/\text{Hz}$$

Phase spectrum is related to frequency spectrum by $\Phi_0 = \Phi_\omega / \omega^2$.

<u>Within</u> resonator passband:

$$\Phi_\omega = \Phi_{\Delta\phi}(\omega)\left(\frac{\omega_0}{2Q}\right)^2$$

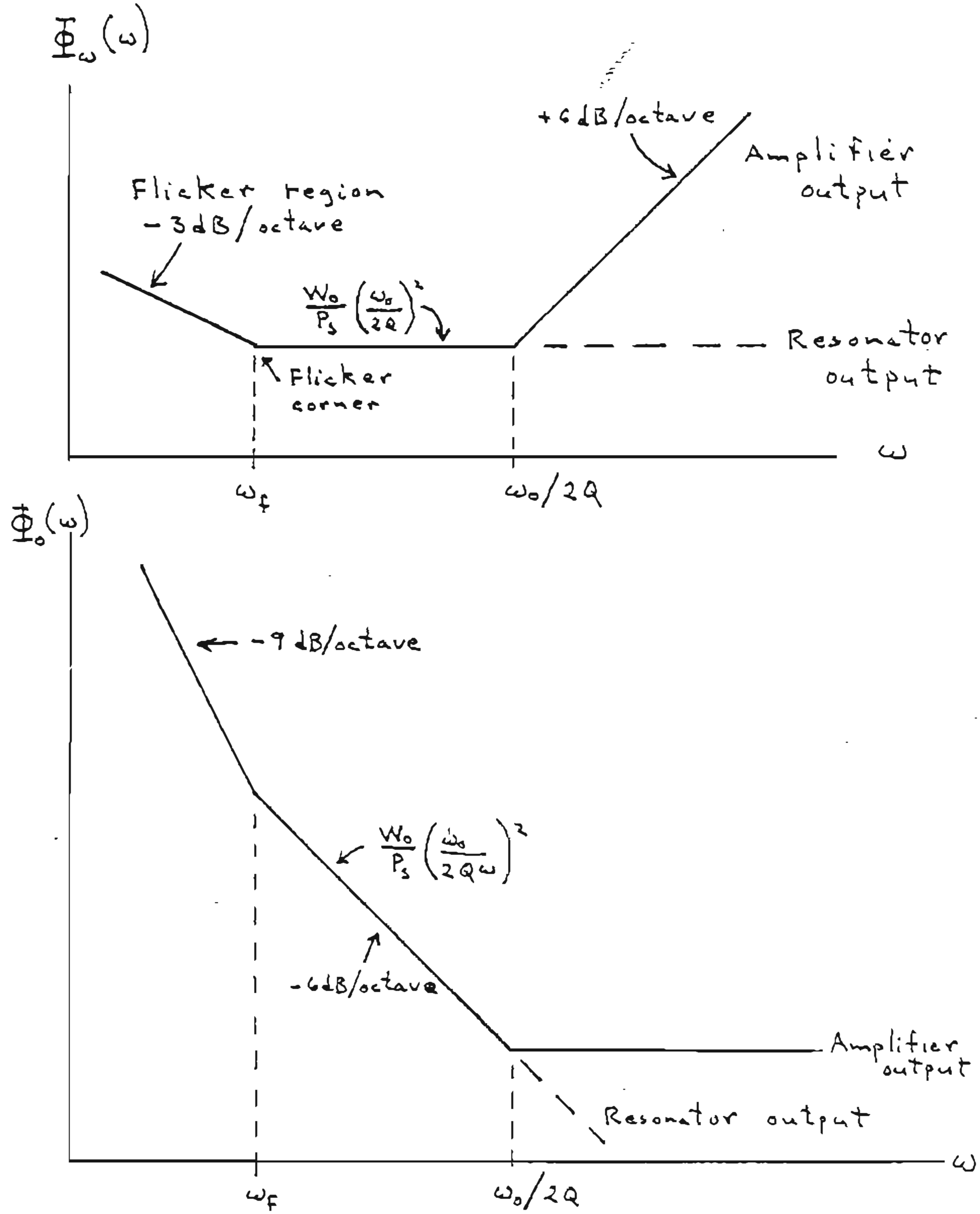
$$= \frac{2N_0}{A^2}\left(\frac{\omega_0}{2Q}\right)^2 = \frac{W_0}{P_s}\left(\frac{\omega_0}{2Q}\right)^2 \quad (\text{rad/s})^2/\text{Hz}$$

$$\Phi_0(\omega) = \Phi_\omega / \omega^2 = \frac{W_0}{P_s}\left(\frac{\omega_0}{2Q\omega}\right)^2 \quad \text{rad}^2/\text{Hz}$$

<u>Outside</u> resonator passband:

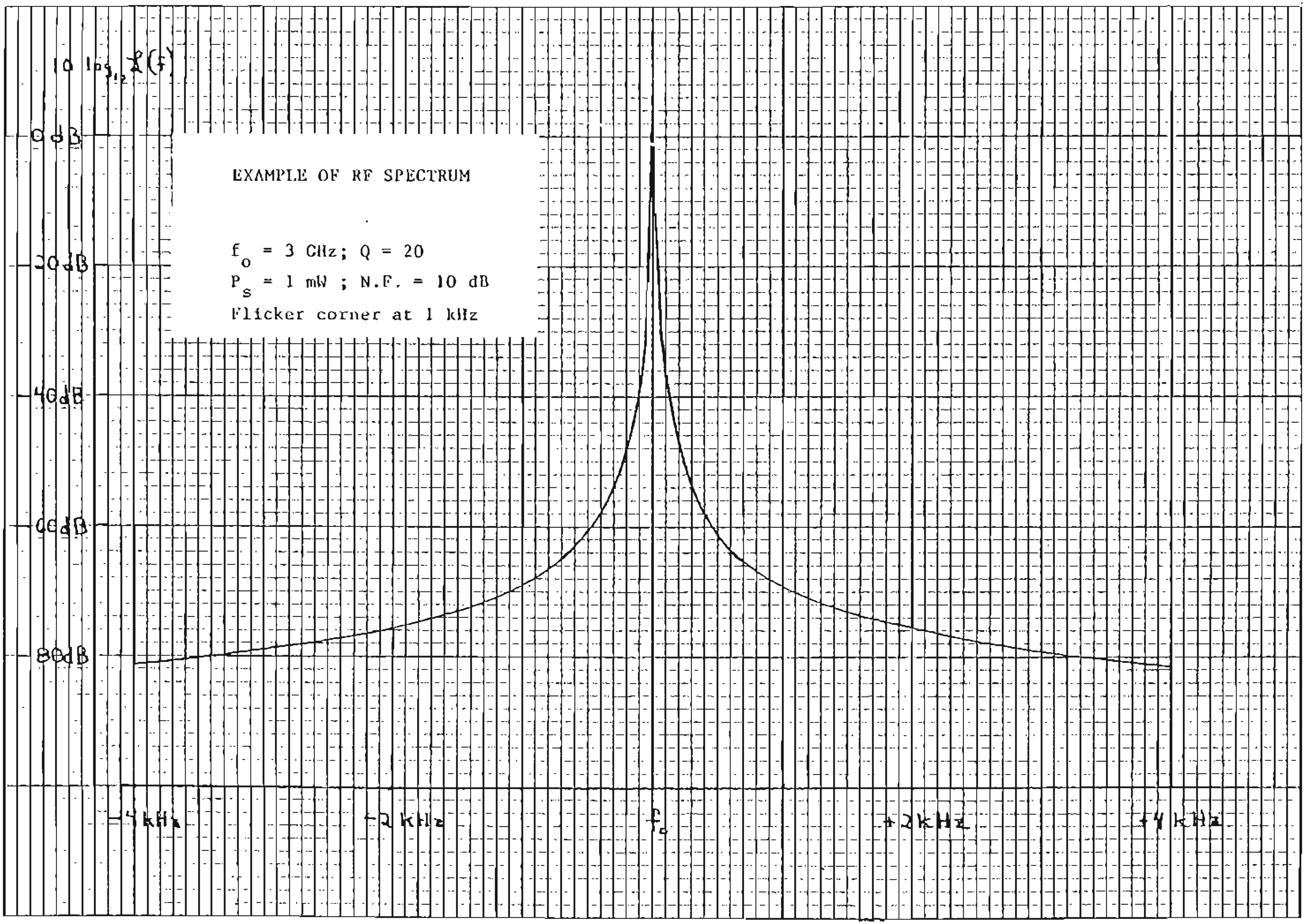
$$\Phi_0 = \Phi_\phi(\omega) = \frac{2N_0}{A^2} = \frac{W_0}{P_s} \quad \text{rad}^2/\text{Hz}$$

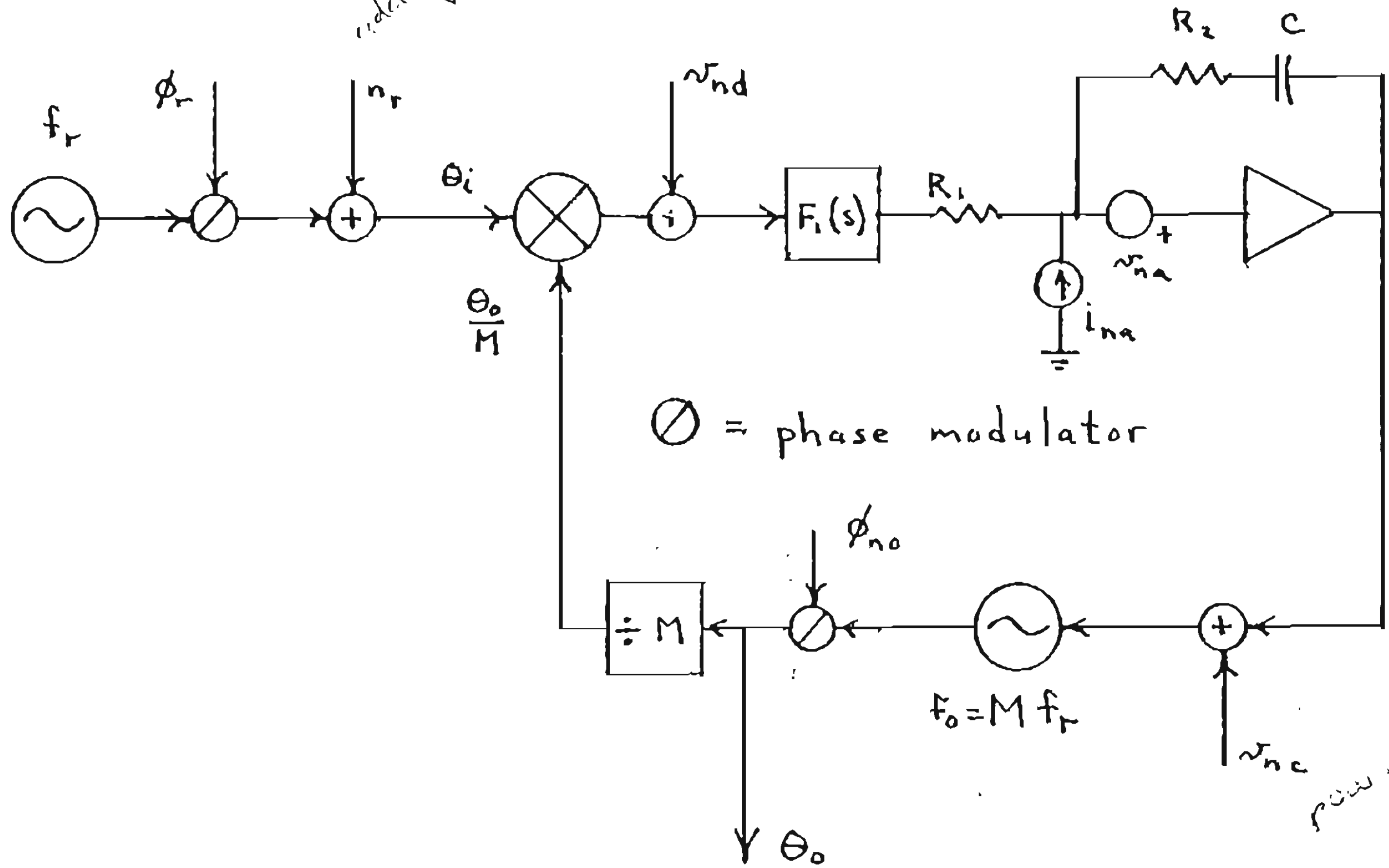
$$\Phi_\omega(\omega) = \omega^2 \Phi_0(\omega) = \frac{\omega^2 W_0}{P_s} \quad (\text{rad/s})^2/\text{Hz}$$



Oscillator Noise Spectra

(Log-Log scales)


EXAMPLE OF RF SPECTRUM
f_o = 3 GHz; Q = 20
P_s = 1 mW ; N.F. = 10 dB
Flicker corner at 1 kHz
$10 \log_{10} \mathcal{L}(f)$
0 dB
20 dB
40 dB
60 dB
80 dB
−4 kHz
−2 kHz
f_o
+2 kHz
+4 kHz



$$H(s) \triangleq \frac{\frac{1}{M}\theta_o(s)}{\theta_i(s)} = \frac{\frac{K_o K_d F_1(s)}{M}\left(\frac{sCR_2+1}{sCR_1}\right)}{s + \frac{K_o K_d F_1(s)}{M}\left(\frac{sCR_2+1}{sCR_1}\right)}$$

<u>Noise Model of Locked Oscillator</u>

Name	Symbol	Spectral Density	Spectral Contribution to Jitter of θ_o. (rad²/Hz)
Output Phase	θ_o	$\Phi_\theta(\omega)$ rad²/Hz	——
Reference Amplitude	V_R	——	——
Reference Jitter	ϕ_r	$\Phi_r(\omega)$ rad²/Hz	$M^2\lvert H(j\omega)\rvert^2\,\Phi_r(\omega)$
Additive Reference Noise	n_r	$\Phi_{nr}(\omega_r\pm\omega)$ rad²/Hz	$M^2\lvert H(j\omega)\rvert^2\cdot\frac{1}{V_R^2}\left[\Phi_{nr}(\omega_r+\omega)+\Phi_{nr}(\omega_r-\omega)\right]$
(If white noise)	"	N_0 V²/Hz or W_0 W/Hz	$M^2\lvert H(j\omega)\rvert^2\cdot\frac{2N_0}{V_R^2}$ or $M^2\lvert H(j\omega)\rvert^2\cdot\frac{W_0}{P_s}$
PD Noise including ripple	v_{nd}	$\Phi_d(\omega)$ V²/Hz	$M^2\lvert H(j\omega)\rvert^2\cdot\frac{\Phi_d(\omega)}{K_d^2}$
Op-amp Noises	v_{na} i_{na}	$\Phi_{va}(\omega)$ V²/Hz $\Phi_{Ia}(\omega)$ A²/Hz	$\frac{M^2\lvert H(j\omega)\rvert^2}{K_d^2\lvert F_1(j\omega)\rvert^2}\left[R_1^2\Phi_{Ia}(\omega)+\Phi_{va}(\omega)\right]$ $+\frac{\lvert 1-H(j\omega)\rvert^2K_o^2}{\omega^2}\Phi_{va}(\omega)$
Control Noise	v_{nc}	$\Phi_{nc}(\omega)$ V²/Hz	$\frac{\lvert 1-H(j\omega)\rvert^2K_o^2}{\omega^2}\Phi_{nc}(\omega)$
VCO Jitter	ϕ_{no}	$\Phi_{no}(\omega)$ rad²/Hz	$\lvert 1-H(j\omega)\rvert^2\,\Phi_{no}(\omega)$

Transfer Function Approximations

Inside passband:

$$|H(j\omega)|^2 \approx 1$$

$$|1 - H(j\omega)|^2 \approx \frac{\omega^2 M^2}{(K_o K_d)^2 |F_1(j\omega)|^2}$$

$$\approx \frac{\omega^4 \tau_1^2}{(K_o K_d)^2} = \frac{\omega^4}{\omega_n^4}$$

$$|F_1(j\omega)|^2 \approx 1$$

Outside passband:

$$|H(j\omega)|^2 \approx \frac{(K_o K_d)^2 |F(j\omega)|^2}{M^2 \omega^2}$$

$$\approx \frac{(K_o K_d)^2 \left(\frac{\tau_2}{\tau_1}\right)^2 |F_1(j\omega)|^2}{M^2 \omega^2}$$

$$= \frac{K^2}{\omega^2} |F_1(j\omega)|^2$$

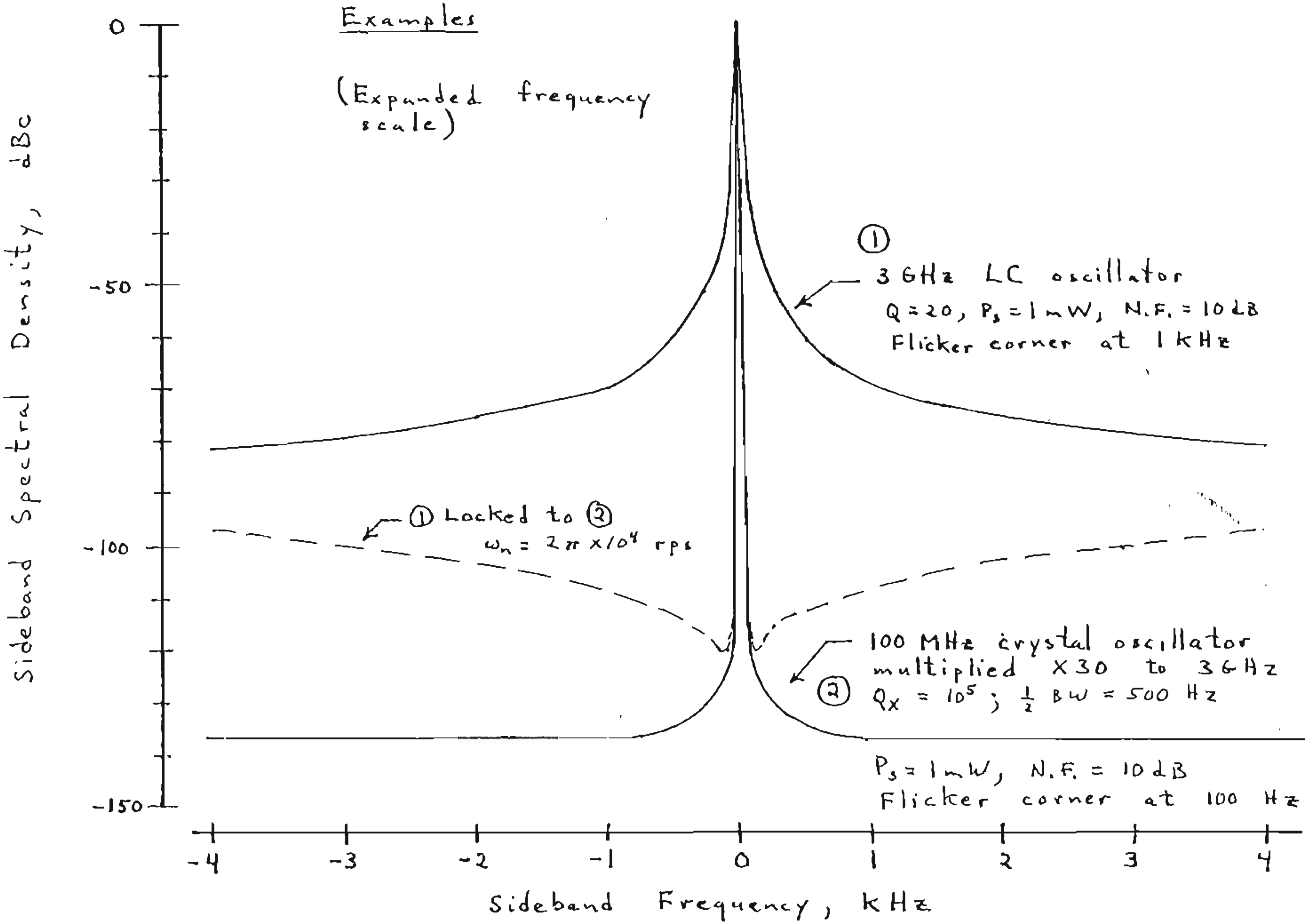
$$|1 - H(j\omega)|^2 \approx 1$$

$$\frac{|H(j\omega)|^2}{|F_1(j\omega)|^2} \approx \frac{K^2}{\omega^2}$$

Oscillator Spectrum

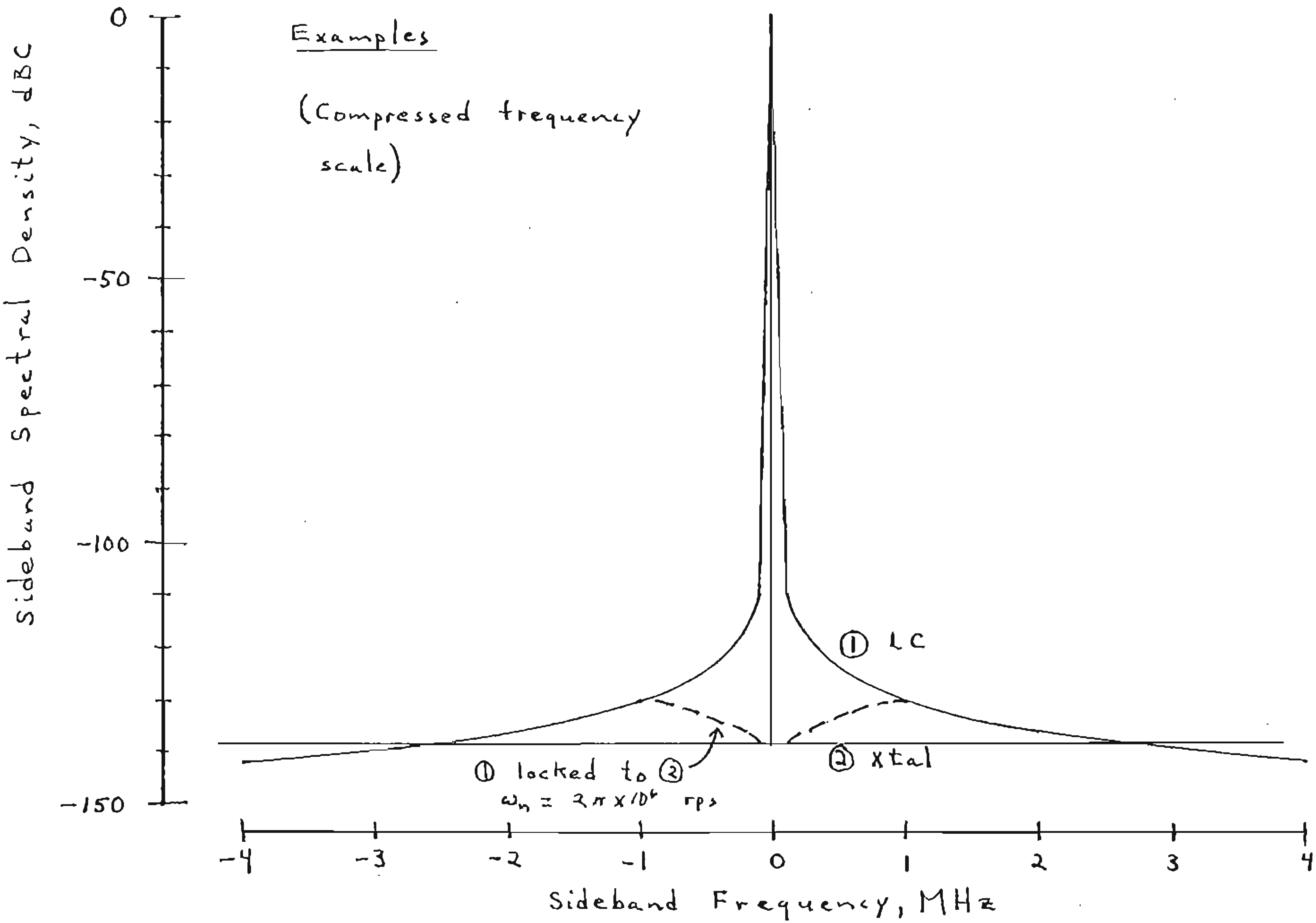
Examples

(Expanded frequency scale)

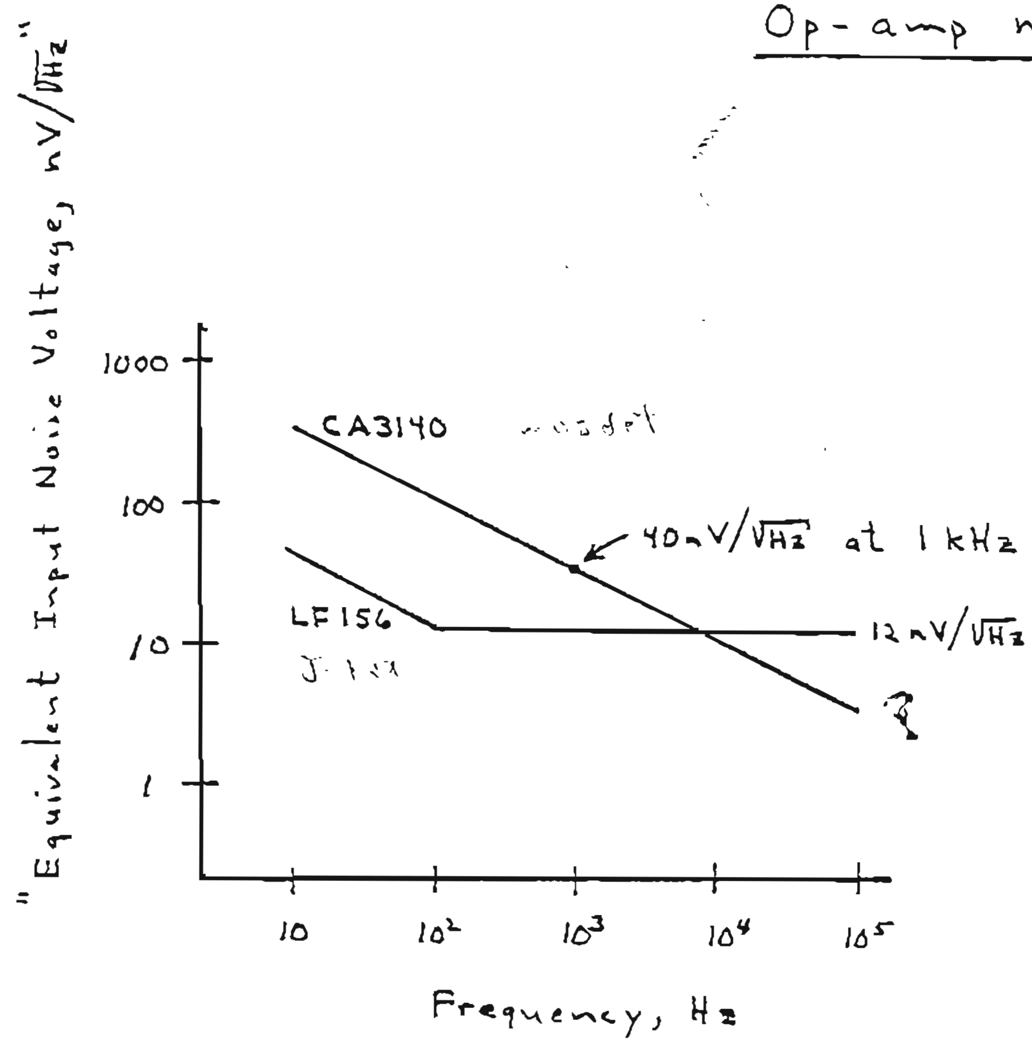


Oscillator Spectrum Examples

(Compressed frequency scale)



Op-amp noise



Interpretation:

"40 nV/√Hz at 1 kHz" means that a spectrum analyzer with 1-Hz bandwidth, tuned to a frequency of 1 kHz, will indicate 40 nV rms noise.

Spectral Density

$$40\ nV/\sqrt{Hz} \longrightarrow (40 \times 10^{-9})^2\ V^2/Hz \quad \text{at } 1000\ Hz$$

$$\Phi_{va}(f) = (40 \times 10^{-9})^2 \left(\frac{1000}{f}\right)$$

$$= \frac{1.6 \times 10^{-12}}{f}\ V^2/Hz \qquad (CA3140)$$

$$\Phi_{va}(f) = 1.44 \times 10^{-16}\left(1 + \frac{10}{f}\right)\ V^2/Hz$$

(LF 156)

Problem:

Given: The two previous example oscillators with ① locked to ② in loop BW $\omega_n = 2\pi \times 10^6$ rps

The two op-amps shown above,

Diode-ring PD with $K_d = 0.3\ V/rad$.

Find: RF spectrum of locked oscillator close-in to carrier.

Solution

Contribution of op-amp noise close-in is

$$\Phi_\theta = \frac{M^2 |H(j\omega)|^2}{K_d^2 |F_1(j\omega)|^2} \Phi_{va}$$

$$\approx \frac{M^2}{K_d^2} \Phi_{va}$$

$$= \frac{(30)^2}{(0.3)^2} \Phi_{va} = 10^4 \Phi_{va} \quad \text{rad}^2/\text{Hz}$$

Phase noise

$$\mathcal{L}(f) = \frac{1}{2} \Phi_\theta(f) = \frac{1}{2} \times 10^4 \Phi_{va}(f)$$

RF sideband noise

For CA 3140:

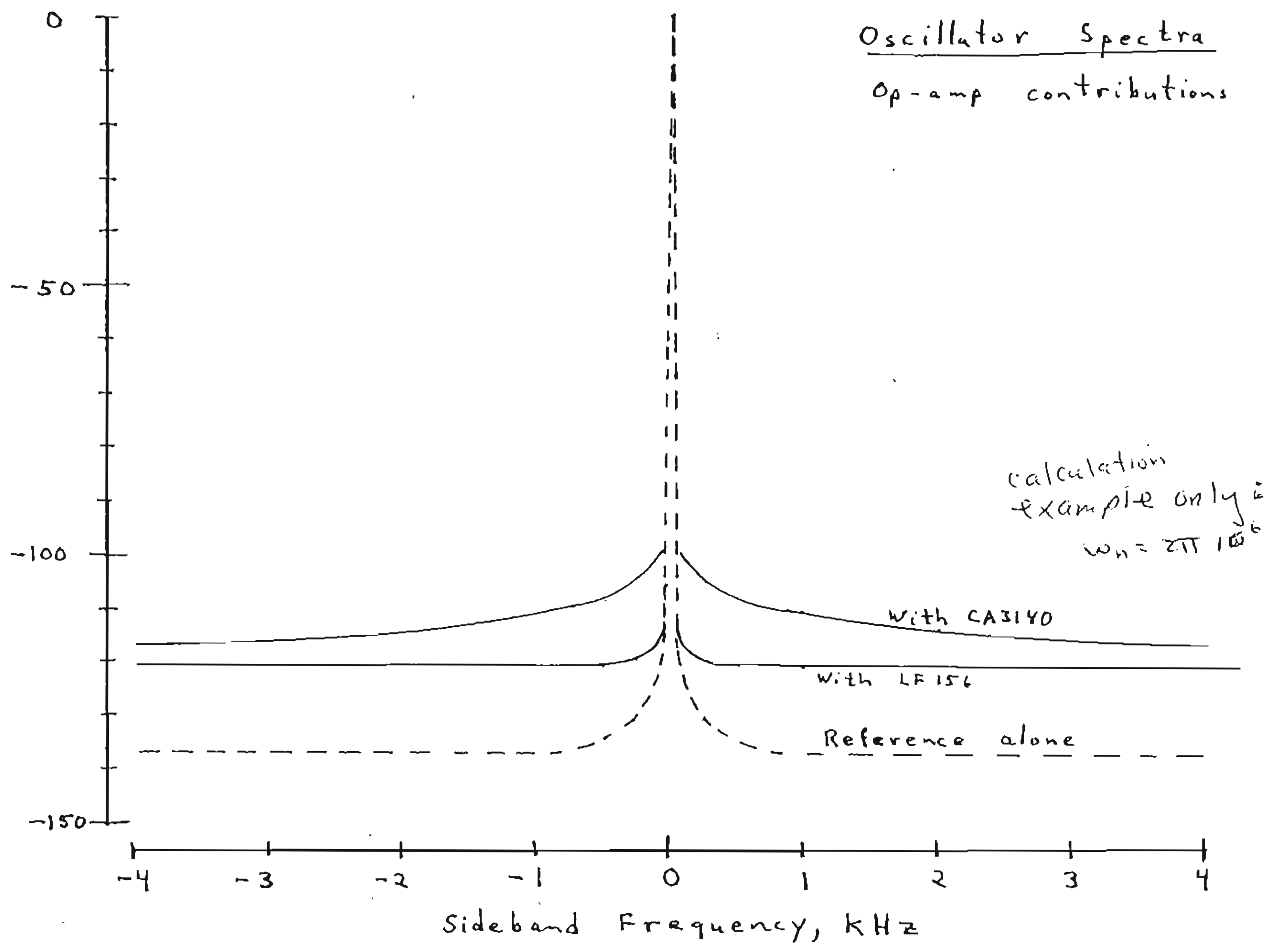
$$\mathcal{L}(f) = \frac{0.8 \times 10^{-8}}{f} \quad \text{rad}^2/\text{Hz}$$

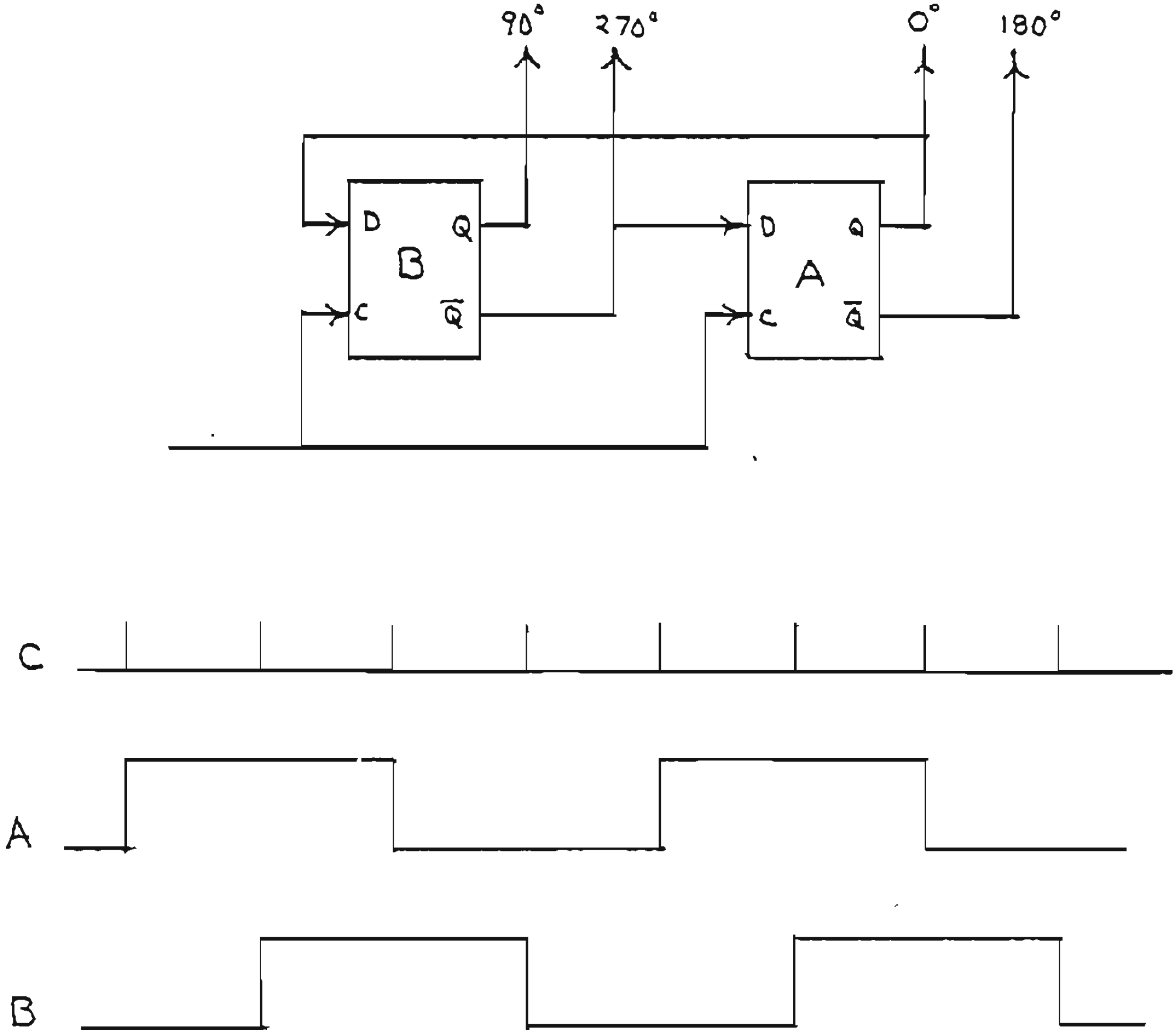
$$\mathcal{L}(1\text{kHz}) = 0.8 \times 10^{-11} \ \text{rad}^2/\text{Hz} \quad (-111 \text{ dBc})$$

For LF 156:

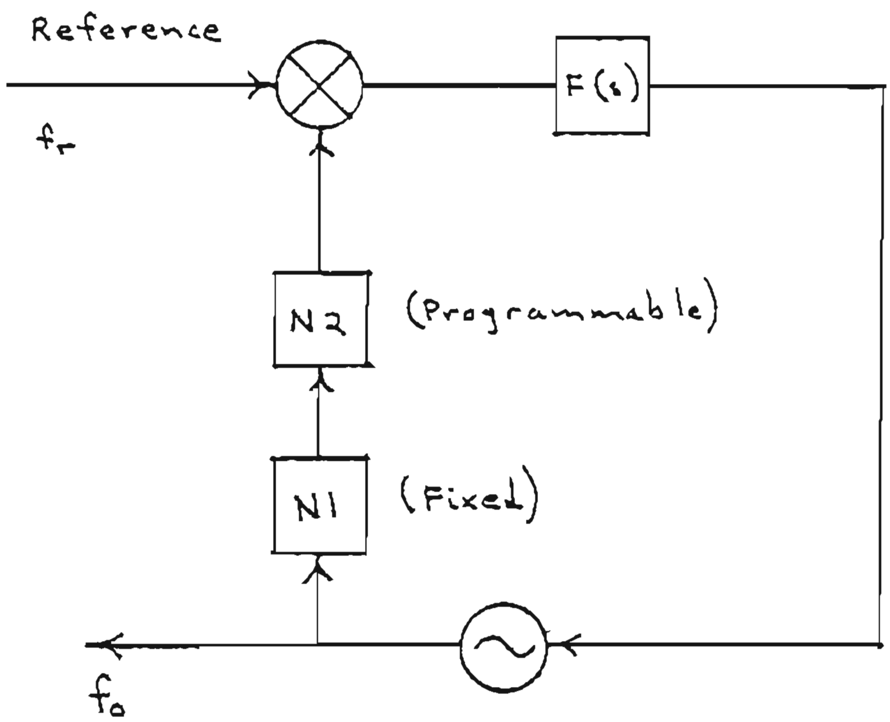
$$\mathcal{L}(f) = 0.72 \times 10^{-12} \left(1 + \frac{100}{f}\right) \ \text{rad}^2/\text{Hz}$$

$$\mathcal{L}(1\text{kHz}) = 0.72 \times 10^{-12} \text{rad}^2/\text{Hz} \quad (-121 \text{ dBc})$$


Oscillator Spectra
Op-amp contributions
calculation example only
$\omega_n = 2\pi \, 10^6$
With CA3140
With LF156
Reference alone
0
-50
-100
-150
-4
-3
-2
-1
0
1
2
3
4
Sideband Frequency, KHz



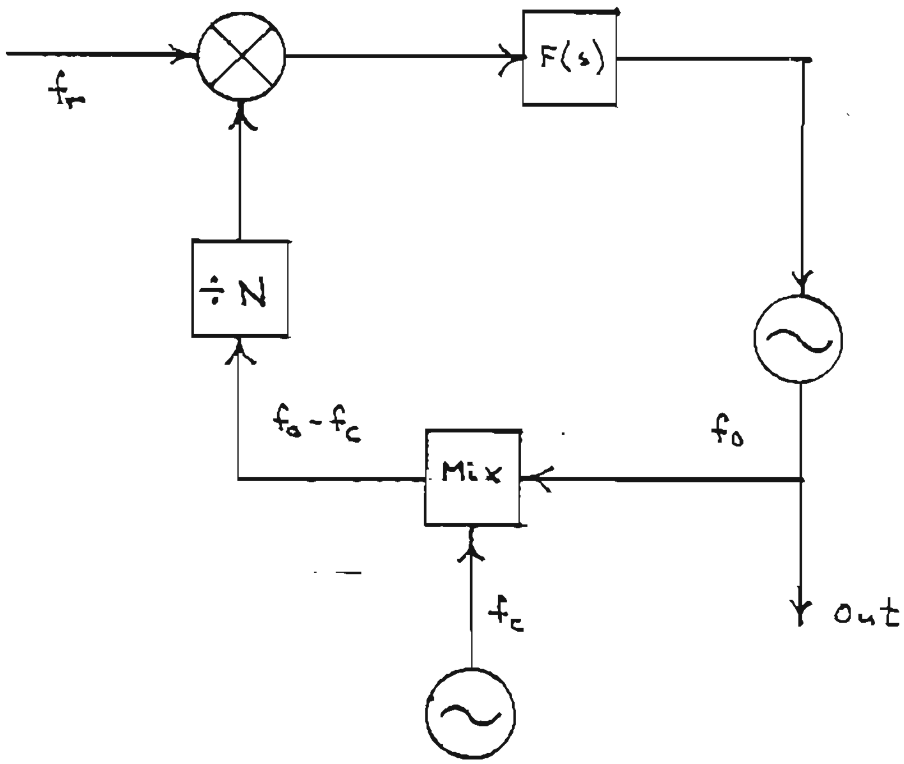
pg 206
Fig 10.7



Prescalar

$$f_o = N_1 N_2 f_r$$

Frequency increments = $N_1 f_r$

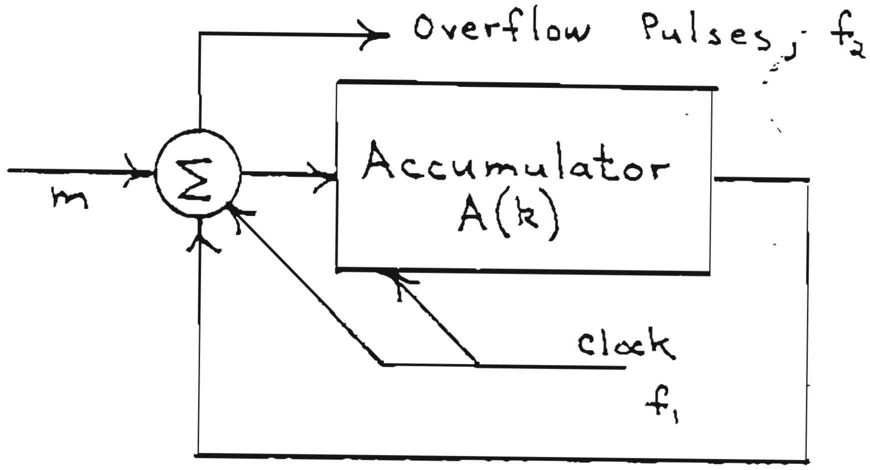


Mixer Loop

$$f_o = N f_r + f_c$$

PLL filters out unwanted mixer products

Counter Representation



$$0 \leq A(k) \leq Q-1$$

$$A(k+1) = [m + A(k)] \text{ modulo } Q$$

$$0 \leq m < Q$$

Examples:

1) $Q = 5$; $m = 1$.

$\{A(k)\} = 0, 1, 2, 3, 4, 0, 1 \cdots\cdots$

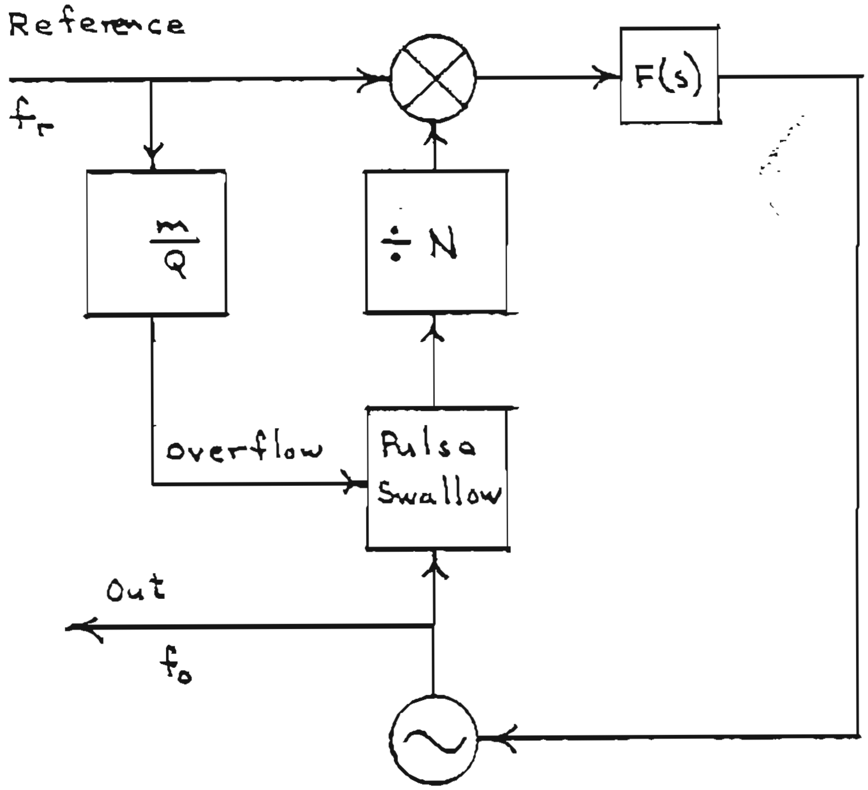
2) $Q = 8$; $m = 3$

$\{A(k)\} = 0, 3, 6, 1, 4, 7, 2, 5, 0, 3 \cdots\cdots$

(Arrows show overflows)

On long-time average:

$$\boxed{f_2 = \frac{m}{Q} f_1}$$



"Digiphase"
or
"Fractional-N"

Counting

Assume $\frac{Q}{m}$ is an integer.

Out of $\frac{Q}{m}$ reference cycles:

..... $\frac{Q}{m}-1$ have N cycles of f_o

..... One has N+1 cycles of f_o

Total: $N\left(\frac{Q}{m}-1\right)+(N+1)=\frac{QN}{m}+1$ cycles of f_o

Time lapse for $\frac{Q}{m}$ reference cycles $=\frac{Q}{m}\cdot\frac{1}{f_r}$

$$\boxed{f_o=\frac{\frac{QN}{m}+1}{\frac{Q}{m}\cdot\frac{1}{f_r}}=f_r\left[N+\frac{m}{Q}\right]}$$

Phase Modulation from Fractional Counter

Reference phase advances at a rate of 1 cycle/reference interval or Q/m cycles in Q/m reference intervals.

Phase of the N-counter output advances at a rate of

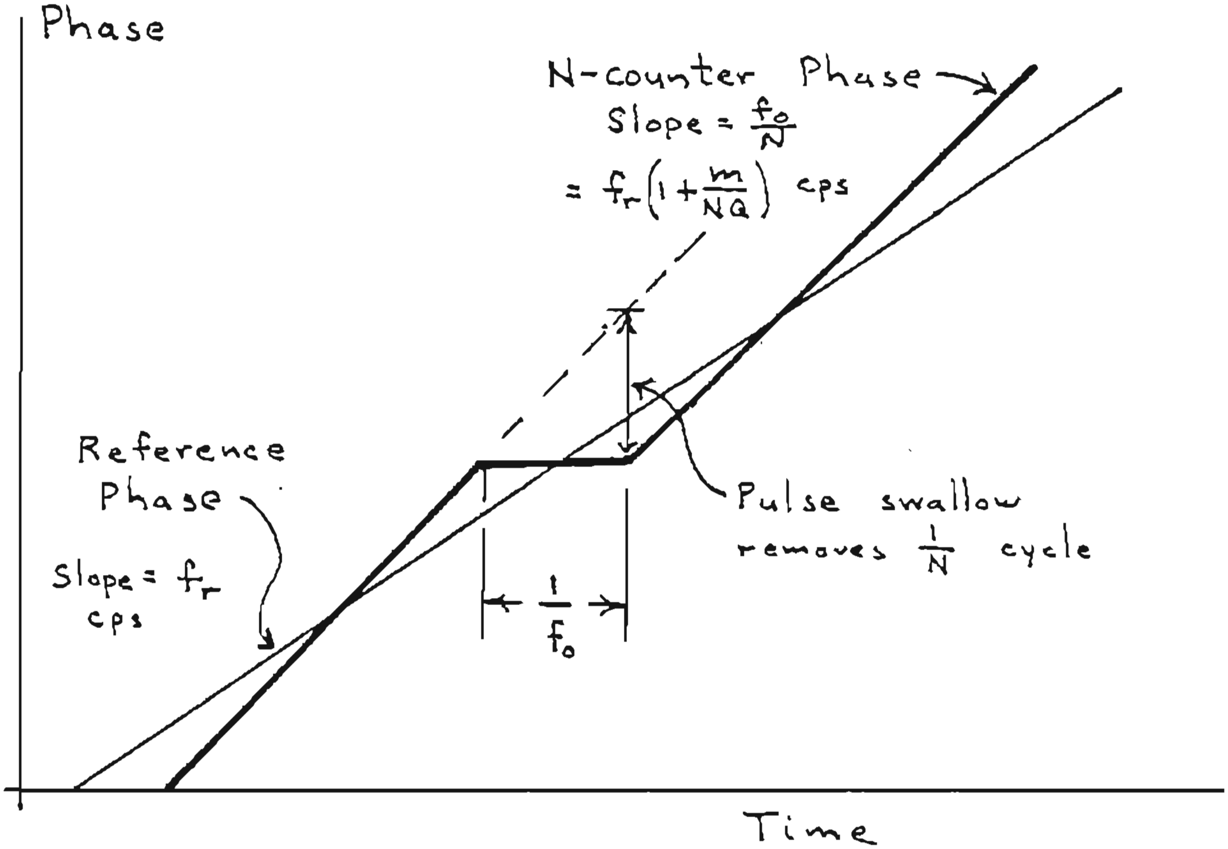
$$\frac{f_o}{Nf_r} = \frac{N + m/Q}{N} \qquad \text{cycles/ reference interval}$$

or Q/m + 1/N cycles in Q/m reference intervals.

The N-counter output advances 1/N cycle more than the reference in Q/m cycles, so f_o has advanced 1 cycle more than Nf_r.

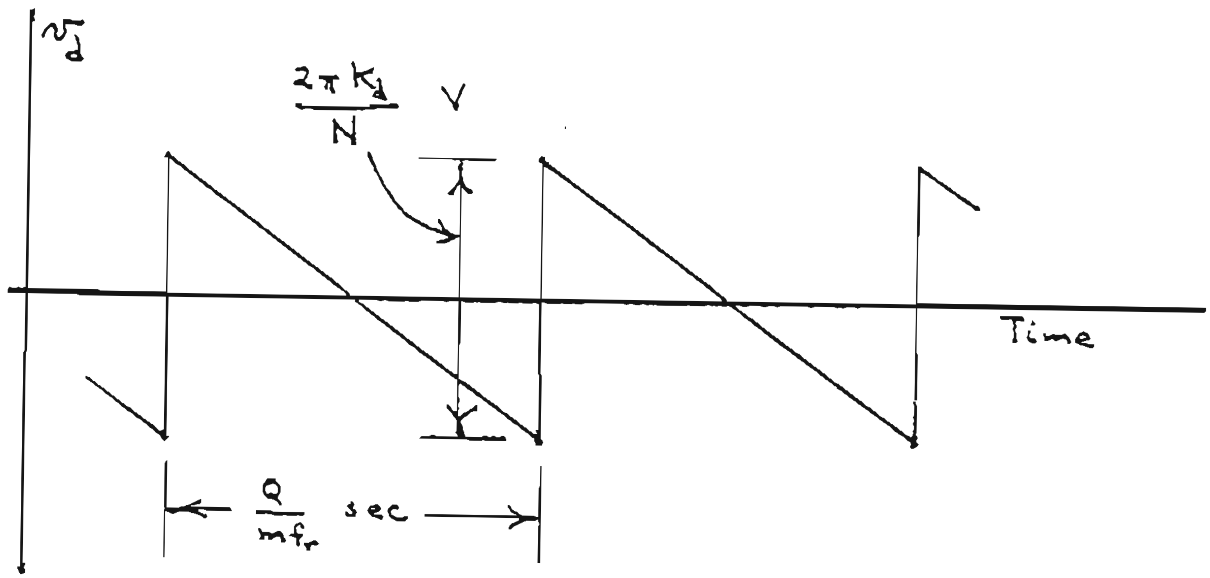
. . . .

Pulse swallowing removes one cycle from N-counter input; in essence resets N-counter output phase to starting value, exactly compensating for excess phase accumulation in Q/m reference cycles.



Phase Accumulation

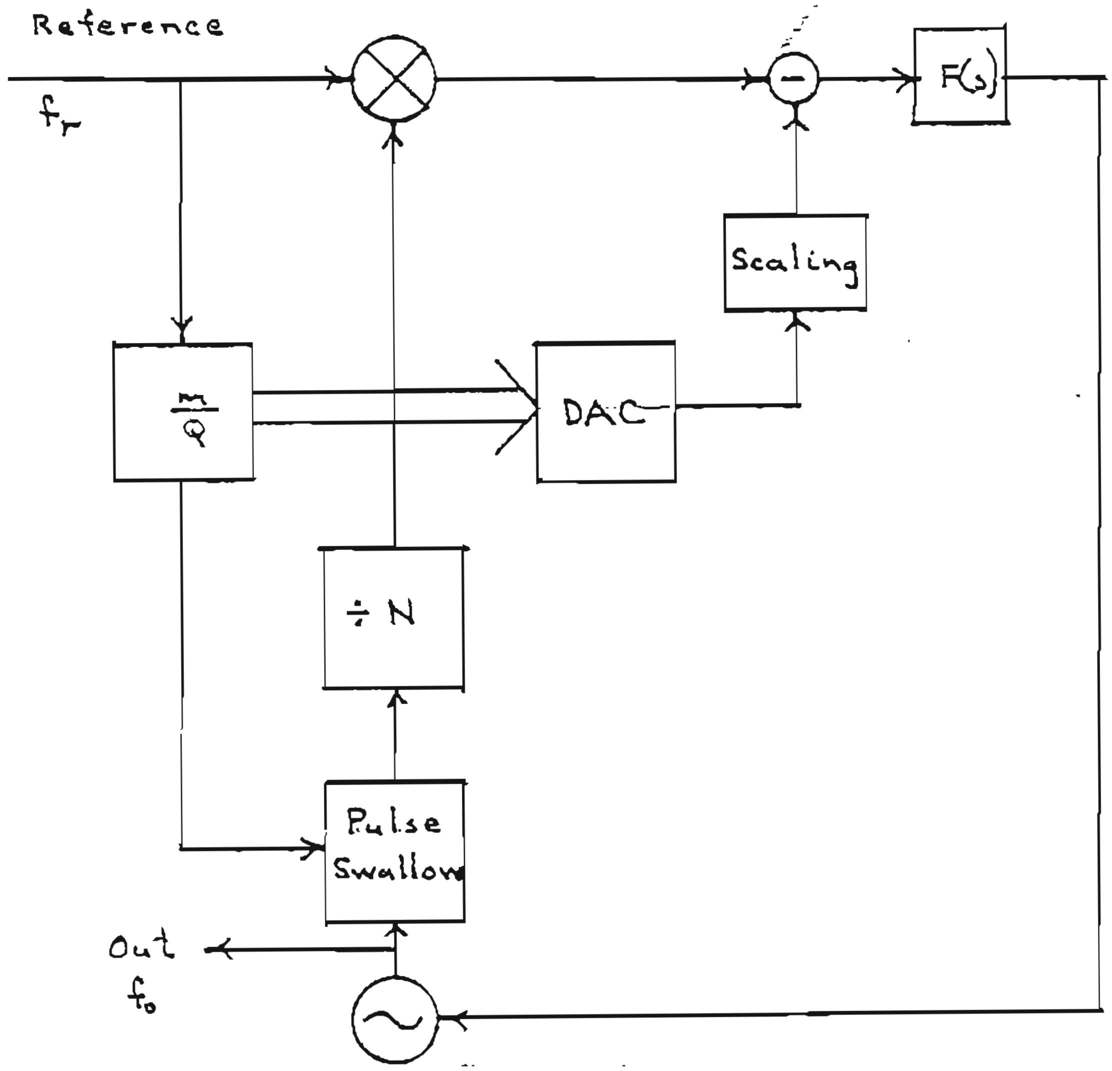
(Phases are really staircases because of inherent sampling in counters; ramps are shown for simplicity. Scale is distorted.)



PD Output

(Loop BW → 0)

This PD error would directly modulate VCO and cause spectra sidebands.



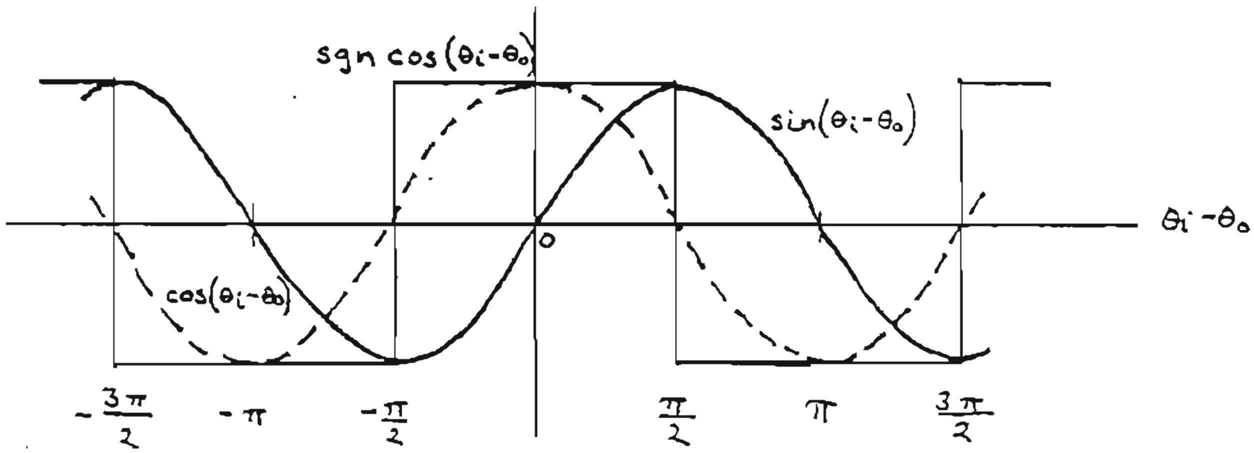
Phase Cancellation

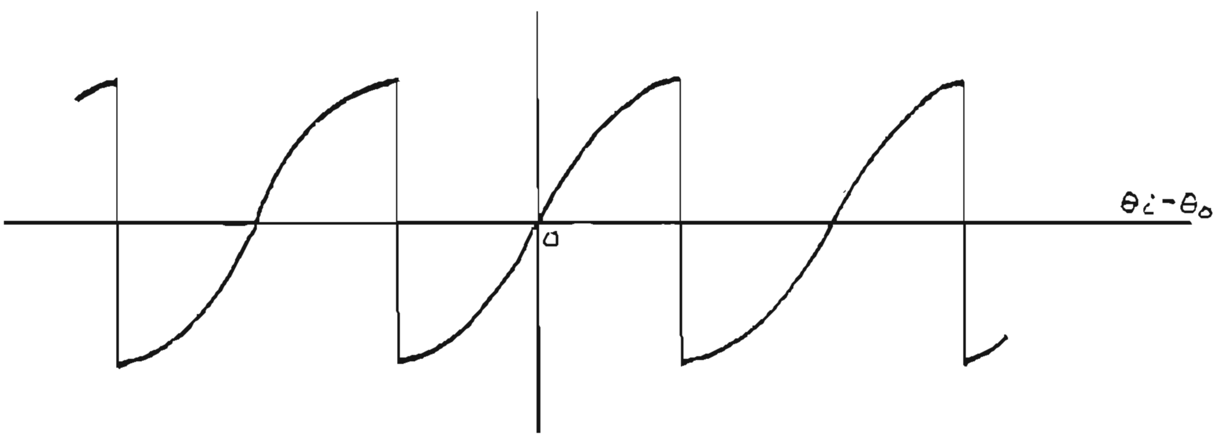
Refs.:

Gorski-Popiel, *Frequency Synthesis*, Chap. 4.

G.C. Gillette, Freq. Tech., Aug. 1969.

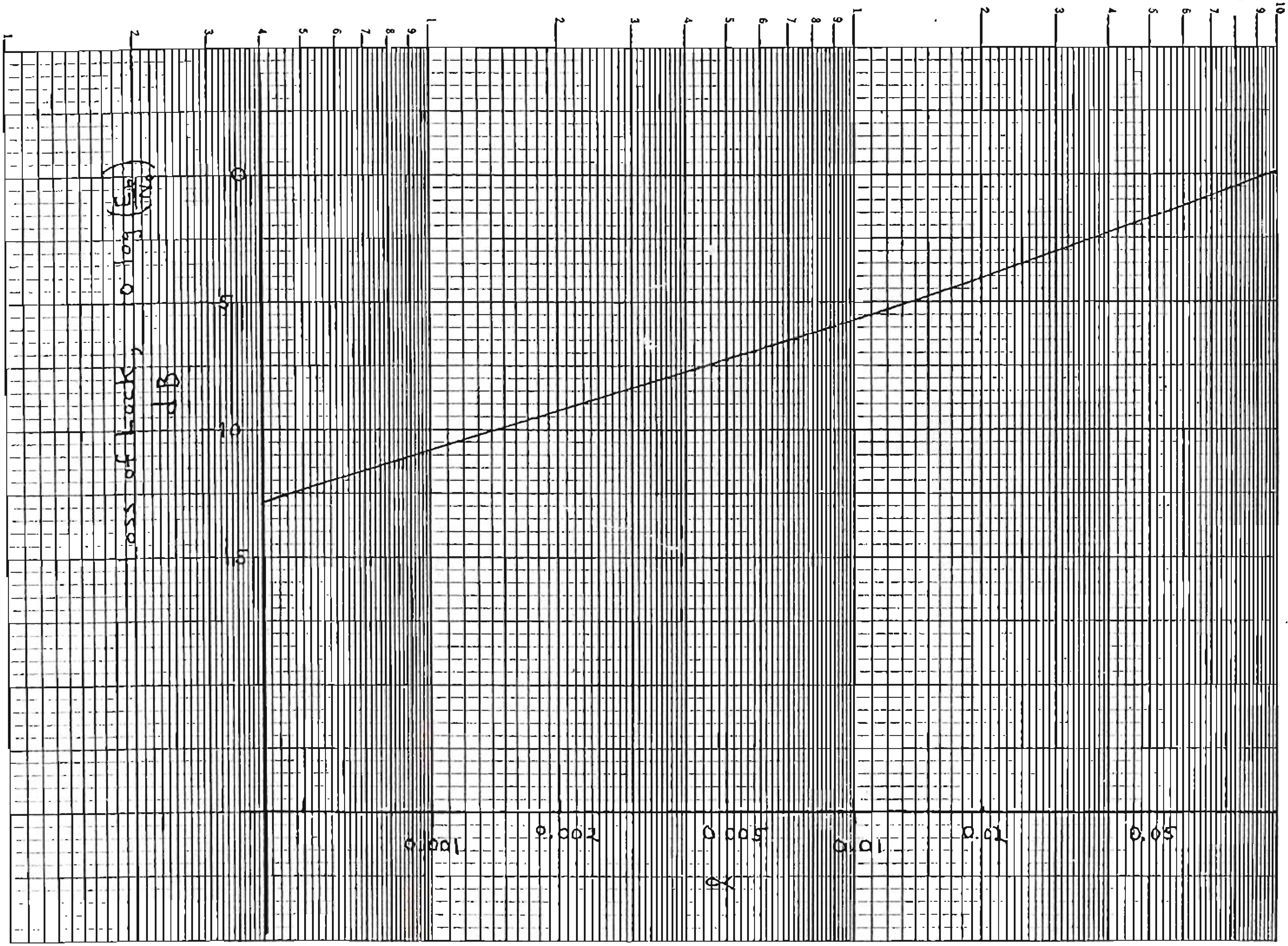
CHAPTER 11

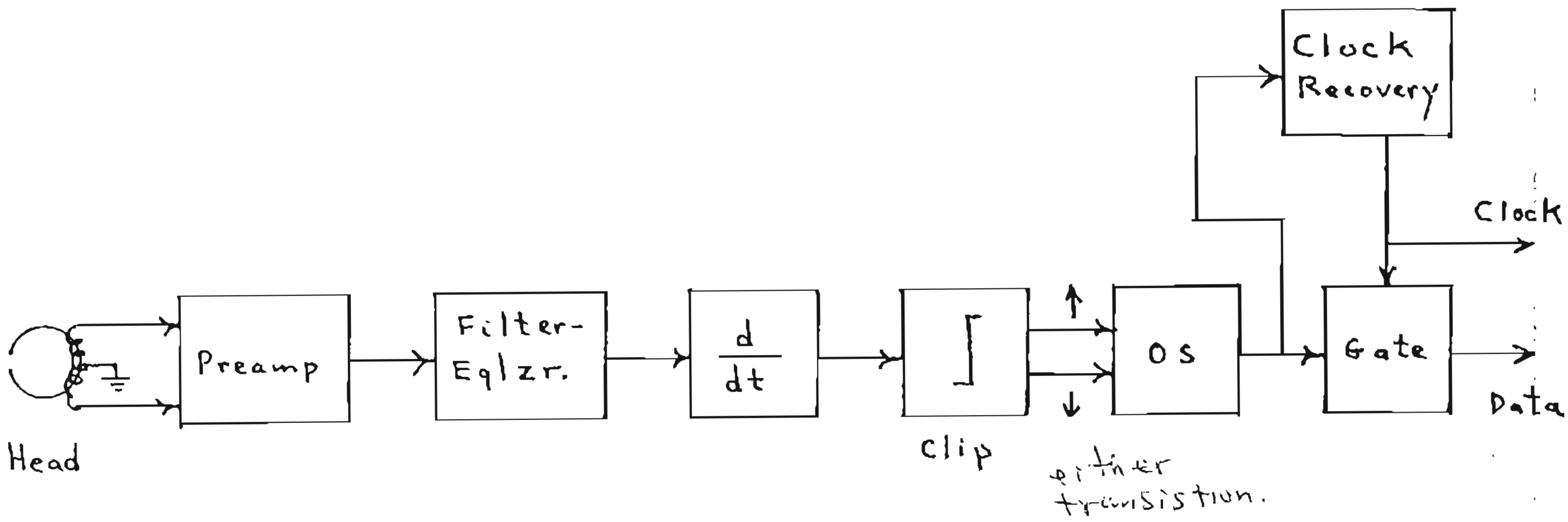




$\sin(\theta_i - \theta_o) \cdot \mathrm{sgn}\left[\cos(\theta_i - \theta_o)\right]$

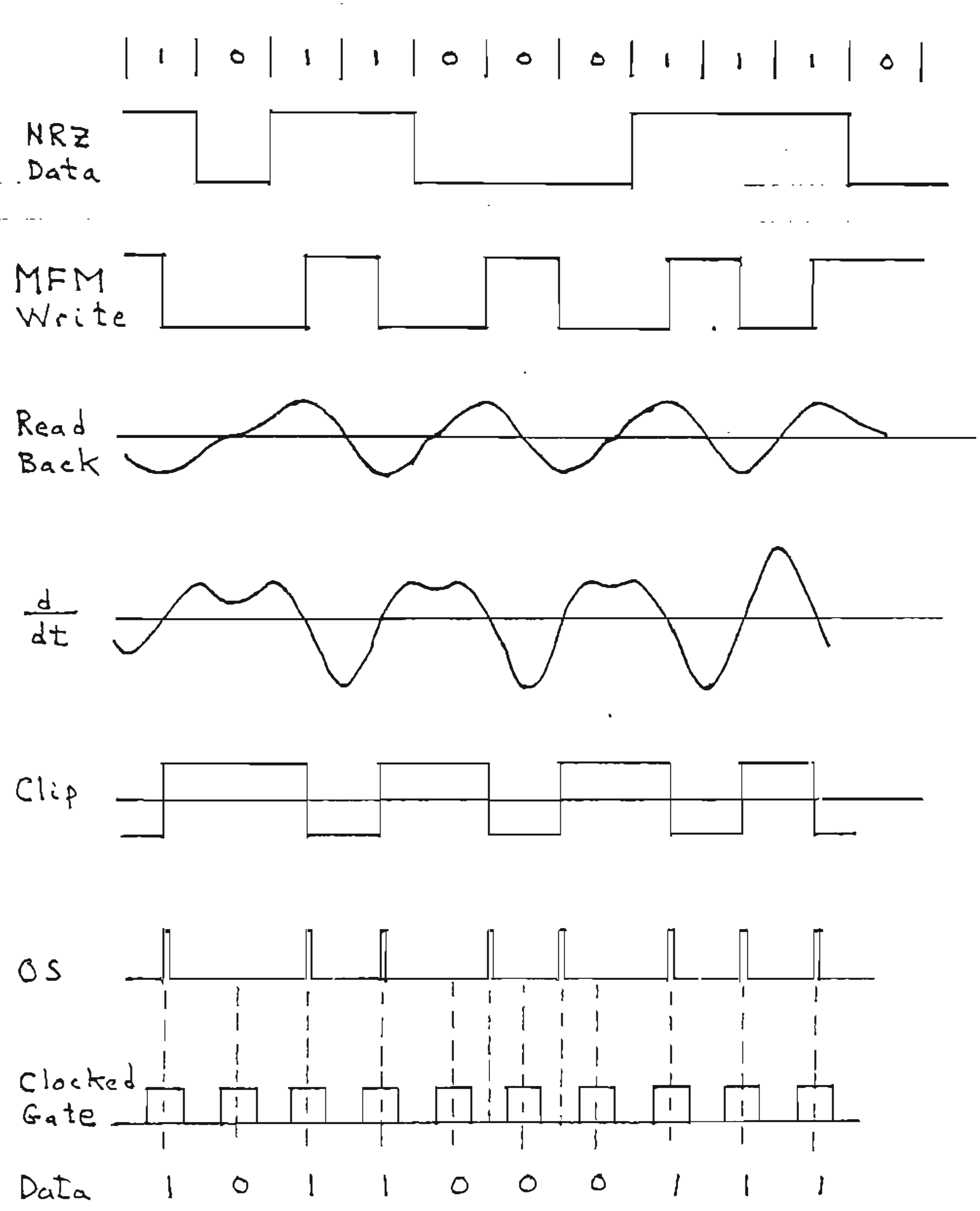
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.





Signal Processor

for Magnetic-Recording Playback



Waveform Processing

for MFM Magnetic Recording

MFM RULES

Write transition in center of cell iff data = 1.

Write transition on cell boundary iff present data = 0 and previous data bit = 0.

Provides high density of transitions.