Small Transmitting Loop Antennas—A different perspective on determining Q and efficiency

Over the past several decades there have been numerous published articles that describe the construction and testing of small transmitting loop antennas. 'Small', in this context, refers to an antenna that has a circumference that is less than about 0.1 wavelength. In this article I want to focus more on evaluation of the performance of the antenna and little on the construction aspects. I will rely on both models and measurements on a simple loop antenna. To my knowledge, there have been no articles that describe the method of determining the Q of the antenna as presented herein. Note also, that I do not attempt to cover electromagnetic modeling using such tools as NEC and EZNEC. References at the end of this article describe construction methods, NEC models, etc.

The purpose of this paper:
-Describe the electrical behavior of a small transmitting loop antenna near the tuned frequency using an electrical circuit analogue.
-Provide guidance in tuning the antenna for an impedance match to a coaxial transmission line.
-Describe a measurement for determining the 'Q' of the antenna system.
-Estimate the efficiency of the antenna (fraction of the applied power that is radiated to the far field) based on measured Q.
-Compare model data with data obtained from measurements.

Assumptions-
-The circumference of the loop is small compared to wavelength (Current is essentially constant around the loop).
-Coax is connected to a small coupling loop that is magnetically coupled to the main loop. The analysis should also apply to a "delta" or "gamma" match.

The loop antenna to be examined is one that was built several years ago for PSK31 operation on 20 meters. The main loop is approximately 32 inches in diameter, and is constructed from 5/8 inch OD copper tubing. A butterfly capacitor is connected across the gap formed when the length of copper tubing is shaped into a loop. This capacitor tunes the inductance of the loop to resonance at a frequency of about 14.1MHz. A small (7.75inch diameter) loop, formed from ¼ inch OD copper tubing is used to transfer power into and out of the main loop. Coax (50 ohm) is connected across the gap that is formed when the tubing is shaped into a circular loop. See Figure 1 below for a photo of the completed antenna.

The position of the small coupling loop is used to alter the mutual inductance (or coupling coefficient) between the two loops, and this provides a means to adjust the matching between the coax and the antenna for 1:1 VSWR.

Although the butterfly capacitor may not have losses as low as a vacuum variable capacitor, it is what I had available. Likewise, losses in the loop would be lower if using larger diameter, the 5/8 inch tubing was readily available in the local Lowe's. Consequently, some improvements are possible with more attention to reducing losses in the system.

Figure 1a. Small Transmitting Loop Antenna

Figure 1b. Coupling Loop Detail

While the purpose of his article is not to describe the design and construction of small transmitting loop antennas, the above description and photos should enable anyone to replicate the evaluation that I have done. In addition, it should be noted that there are numerous articles available from ARRL, the internet, or other sources to guide one in building such an antenna. However, be warned that there are many misleading claims about such antennas (e.g. "efficiency greater than 90 per cent on 80 and 40 meters" and errors in some of the formulas)

Circuit Model

Another assumption, for the purpose of analysis, is that the loop antenna may be modeled using a simple first order system electronic circuit analogue. That is, the antenna is equivalent to a transformer with a series RC circuit across the secondary of the transformer. The proposed model is shown in Figure 2. Such a model has also been suggested by others in the literature, for example, ref (1).
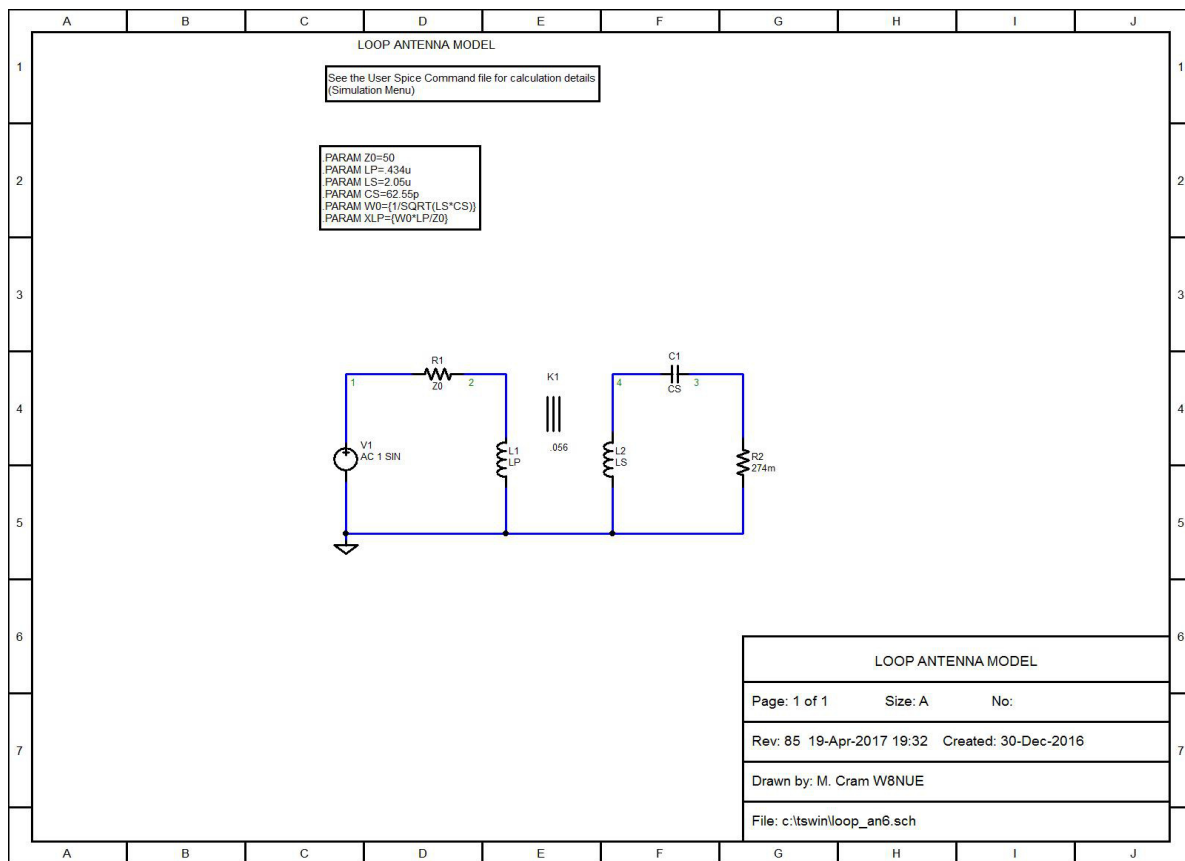
LOOP ANTENNA MODEL

See the User Spice Command file for calculation details
(Simulation Menu)

PARAM Z0=50
PARAM LP=.434u
PARAM LS=2.05u
PARAM CS=62.55p
PARAM W0={1/SQRT(LS*CS)}
PARAM XLP={W0*LP/Z0}

R1
Z0

K1

C1
CS

V1
AC 1 SIN

L1
LP

.056

L2
LS

R2
274m

LOOP ANTENNA MODEL

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Figure 2. TopSpice model of STL antenna

The primary of the transformer is assumed to be the small coupling loop to which the transmission line is connected. The main loop of the antenna is modeled as the secondary of the transformer, loaded with a resistor and capacitor. The inductance of both the primary and secondary of the transformer are calculated with an equation found at many places on the internet (ref 3 is the one I used)---

$$L = \mu r \mathbf{ln}(8r/a-2) \text{ Henries} \qquad \qquad 1)$$

where μ is the permeability of free space, r is the radius of the loop in meters, and a is the radius of the conductor in meters. **ln** is the symbol for the natural logarithm.

In the model described above, the primary inductance is calculated to be about .434 microHenries. The secondary inductance is calculated to be approximately 2.05 microHenries. The value of the butterfly capacitor, to make the secondary inductance resonate at 14.1 MHz, is calculated to be approximately 62.5 picoFarads.

The secondary resistance used in the model is assumed to include the radiation resistance of the antenna AND the effect of all losses in the antenna system (i.e. both ohmic losses in the material of the antenna and losses due to the environment—ground and other nearby lossy materials). It also includes the effect of radiated power. The value for the total resistance was set to .27 ohm as a result of measured data (to be explained later). With this value of resistance and the inductances given above,

electrical simulations and measured results are in close agreement.

To analyze the circuit in Figure 2, I use the free (demo) Version 8 of TopSpice (www.penzar.com). This program was chosen because of previous experience with TopSpice and its ability to generate SmithCharts. Although the demo version is limited in the number of components and nodes, it suffices for the simple model we want to analyze.

To gain further insight into the the model, I also derived the equations that apply to the circuit. Using simple current loop equations, the input impedance to the antenna (at the point the coax is connected to the coupling loop) is found to be given by:

$Zin(\omega) = j\omega Lp + (\omega M)^2/Zs(\omega)$, where $Zs(\omega)$ is the series impedance of the secondary.   2)

$Zs(\omega) = Rs + j\omega Ls + 1/j\omega Cs$ 3)

In this equation, Lp is the inductance of the coupling loop, Ls is the inductance of the main loop, Cs is the capacitance value of the butterfly tuning capacitance (including any "stray" capacitance) and Rs is the resistance that accounts for all power "loss" in the antenna system (including that which is radiated). In order to more easily understand the behavior of the antenna near resonance, I have used a change of variables in the equation for the input impedance. I have also employed the commonly used expression for the 'Q' of the tuned circuit.

Additionally, the mutual inductance between the primary and secondary will be represented by the coupling coefficient k using the formula

$$M^2 = k^2 LpLs$$ 4)

The resonant frequency of a series tuned circuit is given by $f_0 = 1/2\pi\sqrt{LsCs}$

The Q factor is $Q = \omega_0 Ls/Rs$ where $\omega_0 = 2\pi f_0$

Since we are interested in the behavior of the antenna near its resonant frequency, we make the substitution $f = f_0*(1+\delta)$, where $|\delta|$ is much smaller than unity.
Without going into the detailed manipulation of the equation for the input impedance, it may be shown that

$$Zin(\delta) = j\omega_0 Lp(1+\delta) + ((\omega_0^2 M^2(1+\delta)^2)/Zs(\delta)$$ 5)

$$Zs(\delta) = Rs + j\omega_0 Ls(1+\delta) + 1/(j\omega_0 Cs(1+\delta))$$ 6)

Using the common definition of Q, shown above, the equation for the input impedance may be simplified to

$$Zin(\delta) = j\omega_0 Lp(1+\delta) + (1+2\delta)k^2 Q(1+j2\delta Q)$$ 7)

Note: In the above expression, we have made the substitution $(1+\delta)^2 = 1+2\delta$ and $1/(1+\delta) = 1-\delta$. As long as $|\delta|$ is <<1, this should introduce negligible error. As you will see later, the calculated

impedance matches measured data very closely. These approximations imply that we have a high Q system.

This equation will be used to examine the Real and Imaginary parts of the input impedance as a function of the deviation of the frequency from the self-resonant frequency of the loop. The actual frequency deviation is δ times the resonant frequency f_0.

The real part of $Zin(\delta)$ is given by

$$Re(Zin(\delta)) = \omega_0 Lp(1+2\delta)k^2Q/(1+4\delta^2Q^2) \qquad\qquad 8)$$

The imaginary part of $Zin(\delta)$ is given by

$$Im(Zin(\delta)) = \omega_0 Lp((1+\delta) - (1+2\delta)2\delta k^2Q^2/(1+4\delta^2Q^2)) \qquad\qquad 9)$$

At resonance, we also want to satisfy two conditions--

$$Re(Zin(\delta)) = Z_0 \text{ and} \qquad\qquad 10)$$
$$Im(Zin(\delta)) = 0 \qquad\qquad 11)$$

Z_0 is the characteristic impedance of the coax transmission line, nominally 50 ohms.

From the condition on the real part we have

$$\omega_0 Lp(1+2\delta)k^2Q/(1+4\delta^2Q^2) = Z_0 \qquad\qquad 12)$$

or, $\quad (1+2\delta)k^2Q/(1+4\delta^2Q^2) = Z_0/ \omega_0 Lp \qquad\qquad 13)$

If we now insert expression 13) into 9) for the imaginary part of $Zin(\delta)$ we obtain the following-

$$Im(Zin(\delta)) = \omega_0 Lp((1+\delta) - 2\delta QZ_0/ \omega_0 Lp) \qquad\qquad 14)$$

Setting this expression to 0, as required for resonance, we have
$$\delta_{res} = 1/(2QZ_0/ \omega_0 Lp - 1) \qquad\qquad 15)$$

Therefore, when the frequency f is $f_0(1+ \delta_{res})$, the real part of the impedance will be Z_0 and the imaginary part will be 0.

We can now return to the expression 8) for the real part of the impedance to determine what coupling coefficient is needed to achieve Z_0 ohms for the real part. After some manipulation, we obtain-

$$k^2 = Z_0(1+4 \delta_{res}^2Q^2)/\omega_0 Lp(1+2\delta_{res})Q \qquad\qquad 16)$$

In practice, we do not have to solve this equation for the coupling coefficient as we usually adjust the coupling to achieve the desired resonance condition using either an impedance analyzer, or an SWR bridge. However, it may be enlightening to make the calculation once we have an estimate of the Q.

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Determination of the Q of the antenna

The Q of the antenna is of interest for two main reasons. First, the usable bandwidth (without the need for retuning) is determined by the Q and, second, the efficiency of the antenna is dependent on Q.

It has been proposed (3-5) that the Q of an RLC circuit can be determined from VSWR measurement. I have confirmed mathematically, using the circuit model, that VSWR measurements can, indeed, be used to estimate the Q. However, for the method to work, the VSWR must be very close to 1:1 at the operating frequency. When this condition is met, if one determines frequencies on either side of resonance, where the VSWR reaches a specific level, the difference in frequencies at the specified VSWR level, divided into the frequency at 1:1 VSWR provides a number that is proportional to the Q of the antenna. For example, when the target VSWR is chosen to be 3:1, $f_0/(f_{1\_3}-f_{2\_3})$ is $Q/1.154$. When the target VSWR is 2.618, $f_0/(f_{1\_2.618}-f_{2\_2.618})$ is numerically equal to Q. At a target VSWR of 2:1, $f_0/(f_{1\_2}-f_{2\_2}) = .707Q$. These expressions agree with ones published in reference 5. In the expressions above, f_{1-xx} and f_{2-xx} are the two frequencies on either side of resonance where the VSWR reaches the value xx.

Since using the VSWR method to determine Q requires a good match (VSWR very close to 1:1), I decided to look at an alternative method. Using plots of the real and imaginary part of Zin obtained from the Spice model, Figure 3, it is obvious that changes in the coupling coefficient between the small and large loops have very small effect on the frequencies at which the imaginary part of Z has its maximum and minimum values (close to the resonant frequency f0). This led me to further analyze the expression for the imaginary part of Zin, particularly, in the vicinity of f_0.

Recall equation 9) for the imaginary part of $\mathrm{Zin}(\delta)$

$$\mathrm{Im}(\mathrm{Zin}(\delta)) = \omega_0 Lp((1+\delta) - (1+2\delta)2\delta k^2 Q^2/(1+4\delta^2 Q^2)) \qquad \qquad 17)$$

Since δ is very small, we can approximate $\mathrm{Im}(\mathrm{Zin}(\delta))$ with $\omega_0 Lp(1-2\delta k^2 Q^2/(1+4\delta^2 Q^2))$. We then differentiate this expression with respect to δ and set the derivative to zero to locate the maximum and minimum values as a function of δ. This yields the result that when $\delta = +/- 1/2Q$, the imaginary part of the input impedance will have either a maximum or minimum value.

Pursuing this further, we will now focus on the Spice model results.

The following figure 3 is an output from the TopSpice program showing the behavior of the real and imaginary parts of the input impedance close to the resonant frequency f_0.

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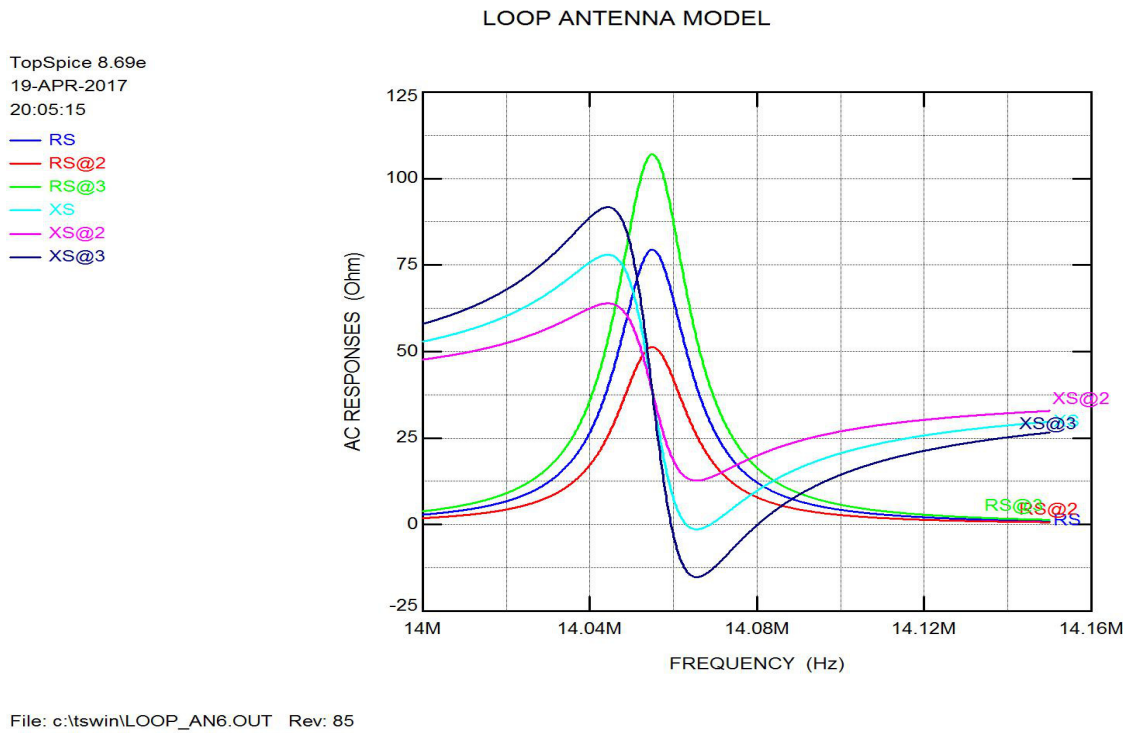LOOP ANTENNA MODEL

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Figure 3. R and X plots for the Small Transmitting Loop Tuned to approx 14.07MHz

The three sets of plots in the above figure correspond to three different values of coupling coefficient. It is particularly interesting that the location (in frequency) of the maximum and minimum values of the imaginary (X) part of the impedance do not appear to move as the coupling coefficient is changed, as suggested earlier. This tends to confirm that the previous analysis is a valid means to calculate the Q. That is, we can use the expression

$$Q = 1/|f_{max} - f_{min}|$$ 18)

where $f_{max} = f_0-1/2Q$ and $f_{min} = f_0+1/2Q$

If we further examine the above plots, we find that on the middle R,X set, X is zero very close to 14.07MHz and R is very close to 50 ohms. When the coupling coefficient is reduced, X never passes through zero, so there can be no resonance. When the coupling coefficient is increased, we again have two frequencies for which X is zero, but now the R value is either too high, or too low. Consequently, there is only one value of coupling coefficient that achieves perfect match.

To further demonstrate the effect of coupling coefficient changes on the input impedance, the Spice model is used to generate SmithChart plots, shown in Figure 4 below.
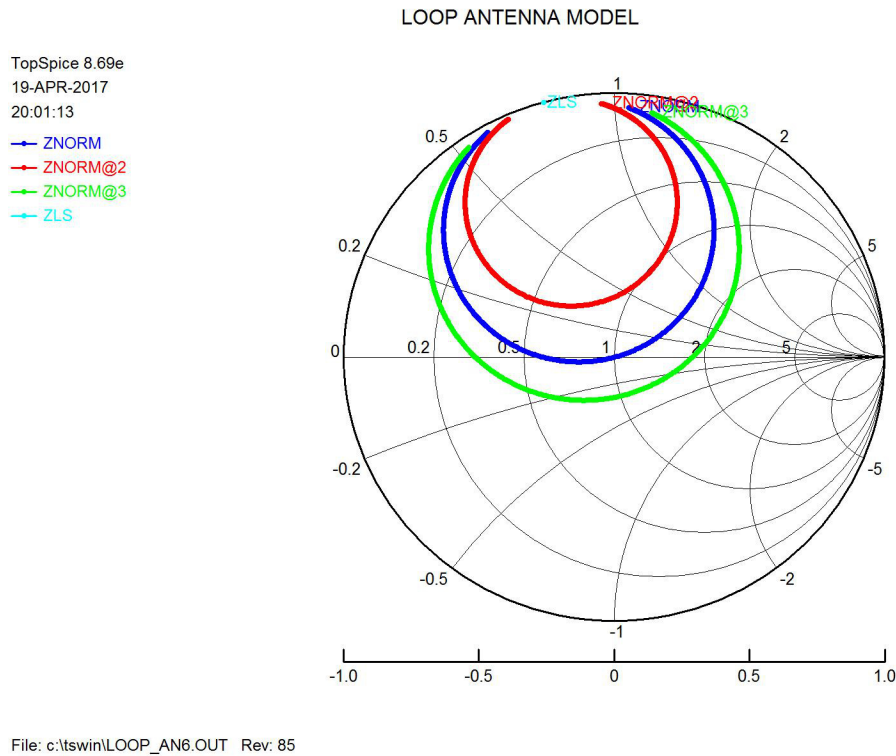
LOOP ANTENNA MODEL

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→ ZNORM@3
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Figure 4. Smith Chart for three coupling coefficient values.

In this figure, only the blue curve passes through the center of the Smith Chart where R=50 and X = 0. This corresponds to a VSWR of 1:1, or a reflection coefficient of zero.

This figure also demonstrates that if one uses an impedance analyzer that is capable of plotting a Smith Chart, it is easy to determine which way the coupling must be changed to achieve a perfect match. The red curve above requires an increase in coupling, while the green curve requires a decrease in coupling. Such a conclusion is not obvious when using the impedance or VSWR plot.

Note also the point labeled ZLs in the above figure. This is a computed point that represents the normalized impedance $j\omega_0 Lp$ of the primary loop at f0. Each of the circular Smith Chart plots appear to be tangent to the outer circle of the Smith Chart at the point ZLs.

Efficiency

QRPers, in particular, are often interested in the efficiency of their antenna system since they are already operating at very low power levels. In this section, I will provide some estimates of antenna efficiency, based on measurements using a Vector Impedance Analyzer (VIA).

The rationale for my determination of efficiency is that the "effective" series resistance Reff of the loop antenna adequately accounts for all power losses, including that which is radiated. Ideal inductors and capacitors can store energy, but over a complete cycle of the RF, dissipate no power. In this analysis, the losses in the inductor and capacitor are lumped into the single resistance Reff. Further, this effective resistance is related to the Q, as noted earlier. It is also generally accepted that the radiation

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resistance, which is part of the effective resistance, represents power which is radiated by the structure and is therefore lost.

The following formula, found in the literature (2), is used to calculate the radiation resistance Rrad, based on the loop structure.

$$Rrad = 197C_\lambda^4, \hspace{4cm} 19)$$

where C_λ is the circumference of the loop in wavelengths.

Allow Irf to represent the RF current that flows in the loop. Therefore, the power that is dissipated in the loop is given by

$$Ptotal = Irf^2 * Reff \hspace{4cm} 20)$$

and the power radiated is represented by

$$Prad = Irf^2 * Rrad \hspace{4cm} 21)$$

The fraction of the total power that is radiated (or efficiency) is therefore given by the ratio of Rrad to Reff

$$efficiency = 100 \times Prad/Ptotal = 100 \times Rrad/Reff \hspace{2cm} 22)$$

However, since the Q of the antenna is given by

$$Q = \omega Ls/Reff \hspace{0.3cm} \text{(remember Reff is the same as the Spice Model Rs)} \hspace{1cm} 23)$$

we can also express efficiency as

$$efficiency = 100 \times RradQ/\omega Ls \hspace{3.5cm} 24)$$

This tells us that the only options we have for increasing efficiency is to increase Rrad or Q. Rrad may be increased by making the loop larger. Q may be increased by using lower loss materials.

Measurements on a "Real" Loop Antenna

Next we discuss measurements made using a Vector Impedance Analyzer (VIA) offered by the Austin Texas QRP group. For information on the VIA, see http://www.qsl.net/k5bcq/Kits/Kits.html

The VIA is first calibrated using a short piece of 50 ohm coax between the VIA and the open, short, and calibration resistor. This makes the measurement point at the end of the coax, away from the VIA.

The antenna is initially set with the plane of the small loop parallel to the plane of the main loop. The small coupling loop is positioned so that no point on its circumference is closer to the main loop than about 1 inch. Also, as shown in Figure 1, the coupling loop is mounted diametrically opposite from the

tuning capacitor.

A VSWR scan is then run over a wide frequency range to determine the approximate tuned frequency. See Figure 5. The tuning capacitor is adjusted, as needed, to set the operating frequency of the antenna as close to 14.1MHz as possible. Having done this, the scan range of the VIA is reduced to a few hundred kHz and recalibrated (not needed with the latest firmware in the VIA) –Figure 6. We then scan the new range, using the Smith Chart plot option on the VIA-- Figure 7. Note the similarity to Figure 4, above. This plot lets us know whether the coupling between the small and large loops is too little, or too much. If the plot does not encircle the center of the chart, then the coupling is too little. If the plot does encircle the chart center, then the coupling needs to be reduced.
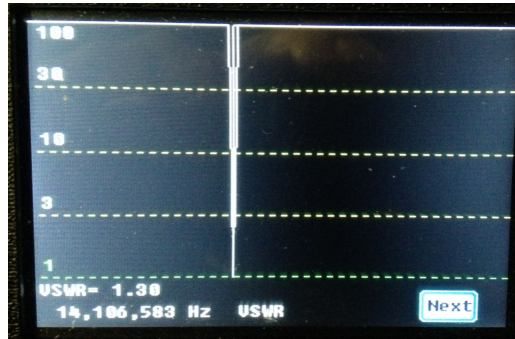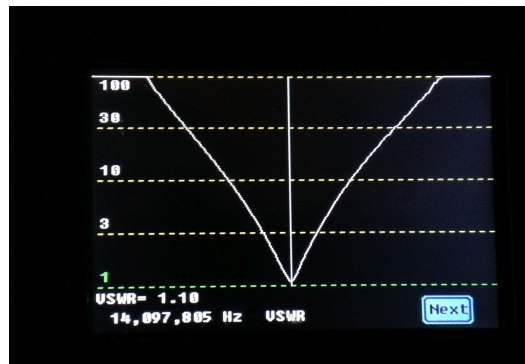

Figure 5. Broad VSWR Scan (10 to 20MHz)

Figure 6. Narrow VSWR Scan (14.0 to 14.2MHz)

The movable cursor in the VIA has been placed at the VSWR minimum, to allow readout of the corresponding frequency.

Note: Since the magnetic field due to current in the main loop is strongest immediately adjacent to the conductor and weakens towards the center of the loop, moving the small coupling loop closer to the main loop will increase the coupling coefficient. Likewise, moving the small coupling loop closer to the center of the main loop will reduce the coupling coefficient. The coupling coefficient can also be reduced by rotating the plane of the small loop so that it is no longer parallel to the plane of the main loop.
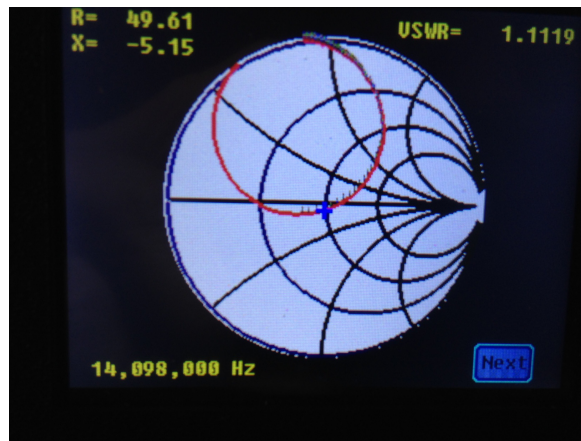
Figure 7. Smith Chart Plot (14.0 to 14.2MHz)

For Figure 7, I did not "tweak" the coupling to make a perfect match at the operating frequency. If I had done so, the curve would have passed exactly over the center point of the Smith Chart.
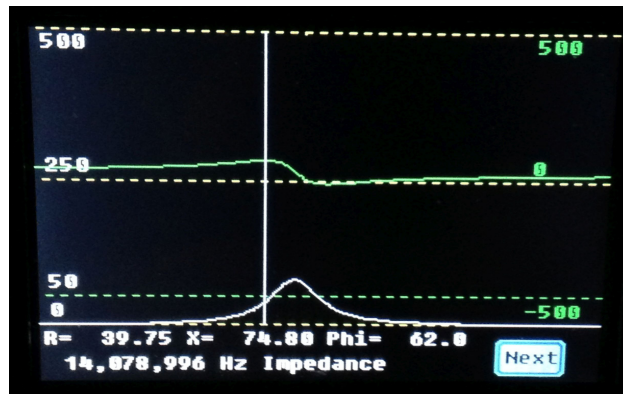

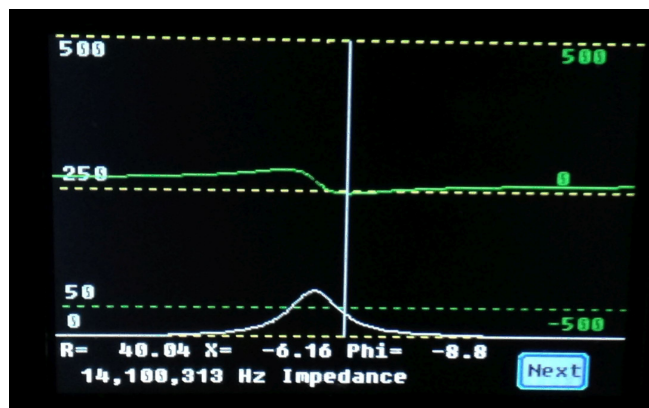Figure 8. Impedance Plot showing maximum X

Figure 9. Impedance Plot showing minimum X

In Figures 8 and 9, the display cursor has been placed over the maximum and minimum points on the X curve (green trace). The corresponding frequency values are 14078996 and 14100313 Hz. The

difference between these two values is 21.3kHz. Since the tuned frequency is about 14100kHz, the calculated Q is 14100/21.3, or 662.

If we now calculate Rrad and Ls using equations 19 and 1, respectively, and measure Q as described above, we can provide an estimate of the efficiency of the antenna.

For the 32 inch loop, Rrad = .04ohm. Q was found to be about 660 (22.3 kHz between the max and min on X) and Ls = 2.05 microHenries. At 14.1MHz, this value of Q gives Reff of .274 ohms. These values yield an efficiency of about 14.5 per cent at 14.1MHz. This value of Reff is what I used in the circuit model.

Note also, in Figure 7, that the point where the circular loop is tangent to the outer circle of the Smith Chart is very close to the corresponding point in Figure 4. This indicates that the calculated and measured coupling loop inductances are in very close agreement.

I've also included a plot of the Reflection Coefficient in Figure 10, below.

Closing comments. The loop Q allows determination of the effective loss resistance (Reff) of the loop, which is assumed to be representative of the total of all power losses, including that which is radiated. The calculated radiation resistance (Rrad) is based on ideal conditions. Hence, it is also assumed that Rrad represents an upper limit on the ability of the loop to radiate power. Consequently, calculation of efficiency using Reff and Rrad will produce an optimistic value for efficiency.

In addition, this analysis may not produce accurate results when the Q is low (<100). Q must be relatively high for the assumption $\delta \gg 1$ to hold true.
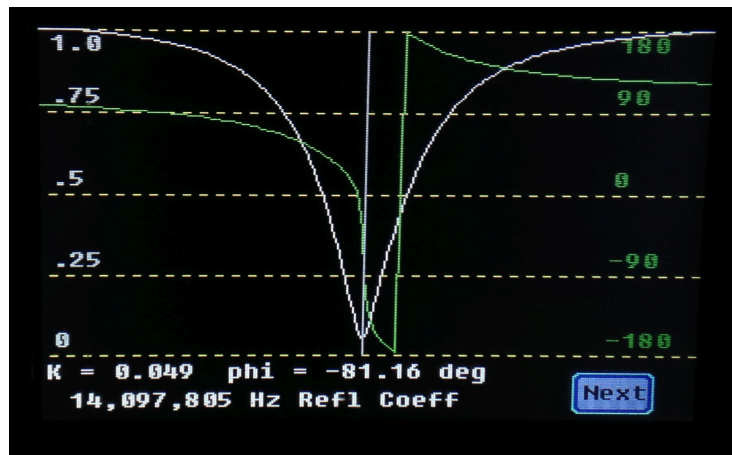

Figure 10. Complex Reflection Coefficient

Conclusions

We have shown that the simple electrical circuit model, with appropriate choice of parameters, produces results that closely match measured data from a physical loop antenna. Using the AQRP Vector Impedance Analyzer (VIA), we have obtained measured data from a 32 in. diameter loop of Copper tubing that are in very good agreement with computed results using the TopSpice Circuit Analysis program.

We have presented a method for using a vector impedance analyzer to facilitate adjustments to the antenna for correct impedance matching. Unlike VSWR and impedance plots, the Smith Chart is shown to be a straightforward method for adjusting the coupling to the antenna. There is no guessing as to which way to adjust coupling for impedance matching.

Plots of the complex input impedance, using a vector impedance analyzer, allow a simple means to determine the Q of the antenna. From this value of Q, the calculated inductance of the loop, and the operating frequency, one can easily estimate the upper limit on the efficiency of the antenna. As shown in this example, do not be surprised if the efficiency is only 10 to 15 per cent.

A brief note about radiation resistance.
I used the equation in J. Kraus "Antennas" book for radiation resistance. I chose this equation as it is based on integrating the Poynting vector over a very large spherical surface. Only the E and H fields that vary inversely with radial distance from the antenna can contribute to total power that is constant (ignoring attenuation) as the radial distance is varied. This is a consequence of the principle of conservation of energy. Fields that vary at a slower rate would imply that power is actually increasing as the radius increases at large distance from the transmitting loop. This is clearly not realistic. Fields that vary more rapidly with increasing radius would result in a decrease in the integrated (total) power as the radius of the sphere is increased. Although power may be radiated in this case, it can not contribute to the true "far field" radiation. Rrad represents only far field power.

There may also be power that is absorbed, and therefore lost, due to currents induced in conductive objects close to the antenna. Even though this represents "radiated power", it does not contribute to power available for communication. Evidence of this power loss is shown by changes in the Q of the loop when moving from inside a building to an outside location that is relatively free of conductive objects near the antenna. The effect of such power loss is included in Reff.

Some authors are apparently including such power when calculating efficiency of their loop antennas and obtaining unrealistic efficiency values by lumping part of Reff into Rrad. The claim is that the difference between the effective resistance Reff, determined from Q measurements, and the calculated AC resistance of the loop is the radiation resistance Rrad. This attributes the effects of enviromental losses to radiation. In addition, the ability to accurately calculate the AC resistance is not easy to achieve.

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June 5, 2017

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7) http://owenduffy.net/blog/?p=4888

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