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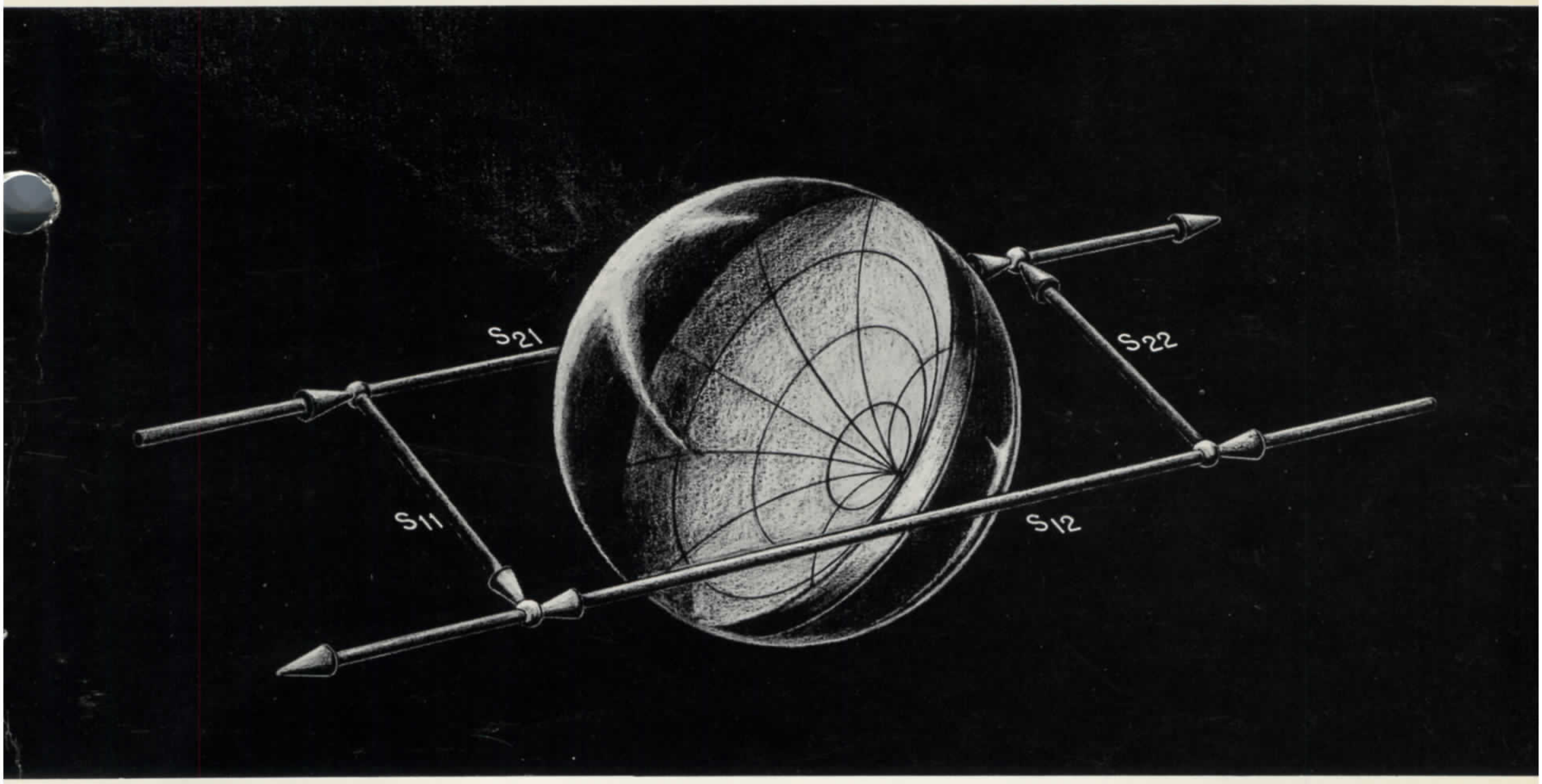
S-Parameters....

circuit analysis and design



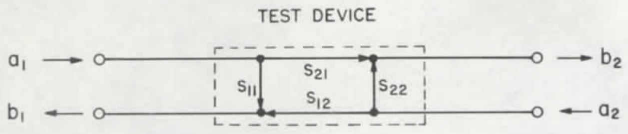
HEWLETT PACKARD

application note 95A

WHAT ARE "S" PARAMETERS?

"S" parameters are reflection and transmission coefficients, familiar concepts to RF and microwave designers. Transmission coefficients are commonly called gains or attenuations; reflection coefficients are directly related to VSWR's and impedances.

Conceptually they are like "h," "y," or "z" parameters because they describe the inputs and outputs of a black box. The inputs and outputs are in terms of power for "s" parameters, while they are voltages and currents for "h," "y," and "z" parameters. Using the convention that "a" is a signal into a port and "b" is a signal out of a port, the figure below will help to explain "s" parameters.



In this figure, "a" and "b" are the square roots of power; $(a_1)^2$ is the power incident at port 1, and $(b_2)^2$ is the power leaving port 2. The diagram shows the relationship between the "s" parameters and the "a's" and "b's." For example, a signal a_1 is partially reflected at port 1 and the rest of the signal is transmitted through the device and out of port 2. The fraction of a_1 that is reflected at port 1 is s_{11}, and the fraction of a_1 that is transmitted is s_{21}. Similarly, the fraction of a_2 that is reflected at port 2 is s_{22}, and the fraction s_{12} is transmitted.

The signal b_1 leaving port 1 is the sum of the fraction of a_1 that was reflected at port 1 and the fraction of a_2 that was transmitted from port 2.

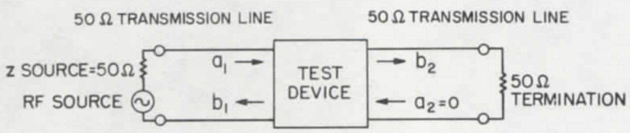
Thus, the outputs can be related to the inputs by the equations:

$$b_1 = s_{11}\,a_1 + s_{12}\,a_2$$
$$b_2 = s_{21}\,a_1 + s_{22}\,a_2$$

When $a_2 = 0$, and when $a_1 = 0$,

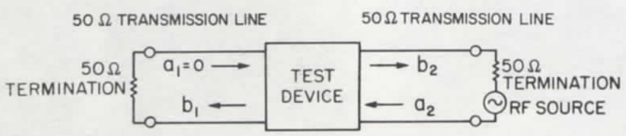
$$s_{11} = \frac{b_1}{a_1},\ s_{21} = \frac{b_2}{a_1} \qquad s_{12} = \frac{b_1}{a_2},\ s_{22} = \frac{b_2}{a_2}$$

The setup below shows how s_{11} and s_{21} are measured.



Port 1 is driven and a_2 is made zero by terminating the 50 Ω transmission line coming out of port 2 in its characteristic 50 Ω impedance. This termination ensures that none of the transmitted signal, b_2, will be reflected toward the test device.

Similarly, the setup for measuring s_{12} and s_{22} is:



If the usual "h," "y," or "z" parameters are desired, they can be calculated readily from the "s" parameters. Electronic computers *and calculators* make these conversions especially easy.

WHY "S" PARAMETERS

Total Information

"S" parameters are vector quantities; they give magnitude and phase information. Most measurements of microwave components, like attenuation, gain, and VSWR, have historically been measured only in terms of magnitude. Why? Mainly because it was too difficult to obtain both phase and magnitude information.

"S" parameters are measured so easily that obtaining accurate phase information is no longer a problem. Measurements like electrical length or dielectric coefficient can be determined readily from the phase of a transmission coefficient. Phase is the difference between only knowing a VSWR and knowing the exact impedance. VSWR's have been useful in calculating mismatch uncertainty, but when components are characterized with "s" parameters there is no mismatch uncertainty. The mismatch error can be precisely calculated.

Easy To Measure

Two-port "s" parameters are easy to measure at high frequencies because the device under test is terminated in the characteristic impedance of the measuring system. The characteristic impedance termination has the following advantages:

1. The termination is accurate at high frequencies . . . it is possible to build an accurate characteristic impedance load. "Open" or "short" terminations are required to determine "h," "y," or "z" parameters, but lead inductance and capacitance make these terminations unrealistic at high frequencies.

2. No tuning is required to terminate a device in the characteristic impedance . . . positioning an "open" or "short" at the terminals of a test device requires precision tuning. A "short" is placed at the end of a transmission line, and the line length is precisely varied until an "open" or "short" is reflected to the device terminals. On the other hand, if a characteristic impedance load is placed at the end of the line, the device will see the characteristic impedance regardless of line length.

3. Broadband swept frequency measurements are possible . . . because the device will remain terminated in the characteristic impedance as frequency changes. However, a carefully reflected "open" or "short" will move away from the device terminals as frequency is changed, and will need to be "tuned-in" at each frequency.

4. The termination enhances stability . . . it provides a resistive termination that stabilizes many negative resistance devices, which might otherwise tend to oscillate.

An advantage due to the inherent nature of "s" parameters is:

5. Different devices can be measured with one setup . . . probes do not have to be located right at the test device. Requiring probes to be located at the test device imposes severe limitations on the setup's ability to adapt to different types of devices.

Easy To Use

Quicker, more accurate microwave design is possible with "s" parameters. When a Smith Chart is laid over a polar display of s_{11} or s_{22}, the input or output impedance is read directly. If a swept-frequency source is used, the display becomes a graph of input or output impedance versus frequency. Likewise, CW or swept-frequency displays of gain or attenuation can be made.

"S" parameter design techniques have been used for some time. The Smith Chart and "s" parameters are used to optimize matching networks and to design transistor amplifiers. Amplifiers can be designed for maximum gain, or for a specific gain over a given frequency range. Amplifier stability can be investigated, and oscillators can be designed.

These techniques are explained in the literature listed at the bottom of this page. Free copies can be obtained from your local Hewlett-Packard Sales Representative.

References:

1. "Transistor Parameter Measurements," Application Note 77-1, February 1, 1967.
2. "Scattering Parameters Speed Design of High Frequency Transistor Circuits," by Fritz Weinert, Electronics, September 5, 1966.
3. "S" Parameter Techniques for Faster, More Accurate Network Design," by Dick Anderson, Hewlett-Packard Journal, February 1967.
4. "Two Port Power Flow Analysis Using Generalized Scattering Parameters," by George Bodway, Microwave Journal, May 1967.
5. "Quick Amplifier Design with *Scattering Parameters*," by William H. Froehner, Electronics, October 16, 1967.

Additional copies of this article can be obtained from your local Hewlett-Packard Sales Representative.

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S-Parameter Techniques for Faster, More Accurate Network Design

ABSTRACT. Richard W. Anderson describes s-parameters and flowgraphs and then relates them to more familiar concepts such as transducer power gain and voltage gain. He takes swept-frequency data obtained with a network analyzer and uses it to design amplifiers. He shows how to calculate the error caused by assuming the transistor is unilateral. Both narrow band and broad band amplifier designs are discussed. Stability criteria are also considered.

This article originally appeared in the February 1967 issue of the Hewlett-Packard Journal. It is also included in Hewlett-Packard Application Note 95.

LINEAR NETWORKS, OR NONLINEAR NETWORKS operating with signals sufficiently small to cause the networks to respond in a linear manner, can be completely characterized by parameters measured at the network terminals (ports) without regard to the contents of the networks. Once the parameters of a network have been determined, its behavior in any external environment can be predicted, again without regard to the specific contents of the network.

S-parameters are being used more and more in microwave design because they are easier to measure and work with at high frequencies than other kinds of parameters. They are conceptually simple, analytically convenient, and capable of providing a surprising degree of insight into a measurement or design problem. For these reasons, manufacturers of high-frequency transistors and other solid-state devices are finding it more meaningful to specify their products in terms of s-parameters than in any other way. How s-parameters can simplify microwave design problems, and how a designer can best take advantage of their abilities, are described in this article.

Two-Port Network Theory

Although a network may have any number of ports, network parameters can be explained most easily by considering a network with only two ports, an input port and an output port, like the network shown in Fig. 1. To characterize the performance of such a network, any of several parameter sets can be used, each of which has certain advantages.

Each parameter set is related to a set of four variables associated with the two-port model. Two of these variables represent the excitation of the network (independent variables), and the remaining two represent the response of the network to the excitation (dependent variables). If the network of Fig. 1 is excited by voltage sources V_1 and V_2, the network currents I_1 and I_2 will be related by the following equations (assuming the network behaves linearly):

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad (1)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad (2)$$

In this case, with port voltages selected as independent variables and port currents taken as dependent variables, the relating parameters are called short-circuit admittance parameters, or y-parameters. In the absence of additional information, four measurements are required to determine the four parameters y_{11}, y_{21}, y_{12}, and y_{22}. Each measurement is made with one port of the network excited by a voltage source while the other port is short circuited. For example, y_{21}, the forward transadmittance, is the ratio of the current at port 2 to the voltage at port 1 with port 2 short circuited as shown in equation 3.

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \text{ (output short circuited)} \quad (3)$$

If other independent and dependent variables had been chosen, the network would have been described, as before, by two linear equations similar to equations 1 and 2, except that the variables and the parameters describing their relationships would be different. However, all parameter sets contain the same information about a network, and it is always possible to calculate any set in terms of any other set.

Fig. 1. *General two-port network.*

S-Parameters

The ease with which scattering parameters can be measured makes them especially well suited for describing transistors and other active devices. Measuring most other parameters calls for the input and output of the device to be successively opened and short circuited. This is difficult to do even at RF frequencies where lead inductance and capacitance make short and open circuits difficult to obtain. At higher frequencies these measurements typically require tuning stubs, separately adjusted at each measurement frequency, to reflect short or open circuit conditions to the device terminals. Not only is this inconvenient and tedious, but a tuning stub shunting the input or output may cause a transistor to oscillate, making the measurement difficult and invalid. S-parameters, on the other hand, are usually measured with the device imbedded between a 50Ω load and source, and there is very little chance for oscillations to occur.

Another important advantage of s-parameters stems from the fact that traveling waves, unlike terminal voltages and currents, do not vary in magnitude at points along a lossless transmission line. This means that scattering parameters can be measured on a device located at some distance from the measurement transducers, provided that the measuring device and the transducers are connected by low-loss transmission lines.

Generalized scattering parameters have been defined by K. Kurokawa.[1] These parameters describe the interrelationships of a new set of variables (a_i, b_i). The variables a_i and b_i are normalized complex voltage waves incident on and reflected from the i^{th} port of the network. They are defined in terms of the terminal voltage V_i, the terminal current I_i, and an arbitrary reference impedance Z_i, as follows

[1] K. Kurokawa, 'Power Waves and the Scattering Matrix,' IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-13, No. 2, March, 1965.

Fig. 2. *Two-port network showing incident* (a_1, a_2) *and reflected* (b_1, b_2) *waves used in s-parameter definitions.*

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}} \tag{4}$$

$$b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{|\mathrm{Re}\, Z_i|}} \tag{5}$$

where the asterisk denotes the complex conjugate.

For most measurements and calculations it is convenient to assume that the reference impedance Z_i is positive and real. For the remainder of this article, then, all variables and parameters will be referenced to a single positive real impedance Z_o.

The wave functions used to define s-parameters for a two-port network are shown in Fig. 2. The independent variables a_1 and a_2 are normalized incident voltages, as follows:

$$a_1 = \frac{V_1 + I_1 Z_o}{2\sqrt{Z_o}} = \frac{\text{voltage wave incident on port 1}}{\sqrt{Z_o}} = \frac{V_{i1}}{\sqrt{Z_o}} \tag{6}$$

$$a_2 = \frac{V_2 + I_2 Z_o}{2\sqrt{Z_o}} = \frac{\text{voltage wave incident on port 2}}{\sqrt{Z_o}} = \frac{V_{i2}}{\sqrt{Z_o}} \tag{7}$$

Dependent variables b_1 and b_2 are normalized reflected voltages:

$$b_1 = \frac{V_1 - I_1 Z_o}{2\sqrt{Z_o}} = \frac{\text{voltage wave reflected (or emanating) from port 1}}{\sqrt{Z_o}} = \frac{V_{r1}}{\sqrt{Z_o}} \tag{8}$$

$$b_2 = \frac{V_2 - I_2 Z_o}{2\sqrt{Z_o}} = \frac{\text{voltage wave reflected (or emanating) from port 2}}{\sqrt{Z_o}} = \frac{V_{r2}}{\sqrt{Z_o}} \tag{9}$$

The linear equations describing the two-port network are then:

$$b_1 = s_{11} a_1 + s_{12} a_2 \tag{10}$$

$$b_2 = s_{21} a_1 + s_{22} a_2 \tag{11}$$

The s-parameters s_{11}, s_{22}, s_{21}, and s_{12} are:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0} \tag{12}$$
= Input reflection coefficient with the output port terminated by a matched load ($Z_L = Z_o$ sets $a_2 = 0$).

$$s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0} \tag{13}$$
= Output reflection coefficient with the input terminated by a matched load ($Z_S = Z_o$ and $V_S = 0$).

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0} = \text{Forward transmission (insertion) gain with the output port terminated in a matched load.} \quad (14)$$

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0} = \text{Reverse transmission (insertion) gain with the input port terminated in a matched load.} \quad (15)$$

Notice that

$$s_{11} = \frac{b_1}{a_1} = \frac{\frac{V_1}{I_1} - Z_o}{\frac{V_1}{I_1} + Z_o} = \frac{Z_1 - Z_o}{Z_1 + Z_o} \quad (16)$$

and
$$Z_1 = Z_o \frac{(1 + s_{11})}{(1 - s_{11})} \quad (17)$$

where $Z_1 = \frac{V_1}{I_1}$ is the input impedance at port 1.

This relationship between reflection coefficient and impedance is the basis of the Smith Chart transmission-line calculator. Consequently, the reflection coefficients s_{11} and s_{22} can be plotted on Smith charts, converted directly to impedance, and easily manipulated to determine matching networks for optimizing a circuit design.

The above equations show one of the important advantages of s-parameters, namely that they are simply gains and reflection coefficients, both familiar quantities to engineers. By comparison, some of the y-parameters described earlier in this article are not so familiar. For example, the y-parameter corresponding to insertion gain s_{21} is the 'forward transadmittance' y_{21} given by equation 3. Clearly, insertion gain gives by far the greater insight into the operation of the network.

Another advantage of s-parameters springs from the simple relationships between the variables a_1, a_2, b_1, and b_2, and various power waves:

$|a_1|^2$ = Power incident on the input of the network.
= Power available from a source of impedance Z_o.

$|a_2|^2$ = Power incident on the output of the network.
= Power reflected from the load.

$|b_1|^2$ = Power reflected from the input port of the network.
= Power available from a Z_o source minus the power delivered to the input of the network.

$|b_2|^2$ = Power reflected or emanating from the output of the network.
= Power incident on the load.
= Power that would be delivered to a Z_o load.

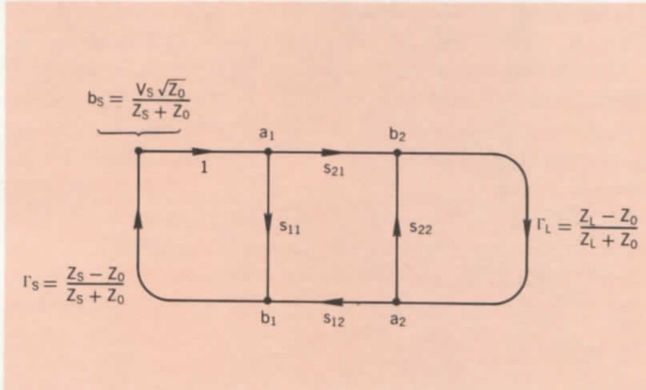


Fig. 3. *Flow graph of network of Fig. 2.*

Hence s-parameters are simply related to power gain and mismatch loss, quantities which are often of more interest than the corresponding voltage functions:

$$|s_{11}|^2 = \frac{\text{Power reflected from the network input}}{\text{Power incident on the network input}}$$

$$|s_{22}|^2 = \frac{\text{Power reflected from the network output}}{\text{Power incident on the network output}}$$

$$|s_{21}|^2 = \frac{\text{Power delivered to a } Z_o \text{ load}}{\text{Power available from } Z_o \text{ source}}$$
= Transducer power gain with Z_o load and source

$|s_{12}|^2$ = Reverse transducer power gain with Z_o load and source.

Network Calculations with Scattering Parameters

Scattering parameters turn out to be particularly convenient in many network calculations. This is especially true for power and power gain calculations. The transfer parameters s_{12} and s_{21} are a measure of the complex insertion gain, and the driving point parameters s_{11} and s_{22} are a measure of the input and output mismatch loss. As dimensionless expressions of gain and reflection, the parameters not only give a clear and meaningful physical interpretation of the network performance but also form a natural set of parameters for use with signal flow graphs[2,3]. Of course, it is not necessary to use signal flow graphs in order to use s-parameters, but flow graphs make s-parameter calculations extremely simple, and I recommend them very strongly. Flow graphs will be used in the examples that follow.

In a signal flow graph each port is represented by two nodes. Node a_n represents the wave coming into the device from another device at port n and node b_n represents the wave leaving the device at port n. The complex scattering coefficients are then represented as multipliers on branches connecting the nodes within the network and in adjacent networks. Fig. 3 is the flow graph representation of the system of Fig. 2.

Fig. 3 shows that if the load reflection coefficient Γ_L is zero ($Z_L = Z_o$) there is only one path connecting b_1 to a_1 (flow graph rules prohibit signal flow against the forward direction of a branch arrow). This confirms the definition of s_{11}:

$$s_{11} = \frac{b_1}{a_1}\bigg|_{a_2 = \Gamma_L b_2 = 0}$$

The simplification of network analysis by flow graphs results from the application of the "non-touching loop rule." This rule applies a generalized formula to determine the transfer function between any two nodes within a complex system. The non-touching loop rule is explained in footnote 4.

[2] J. K. Hunton, 'Analysis of Microwave Measurement Techniques by Means of Signal Flow Graphs,' IRE Transactions on Microwave Theory and Techniques, Vol. MTT-8, No. 2, March, 1960.
[3] N. Kuhn, 'Simplified Signal Flow Graph Analysis,' Microwave Journal, Vol. 6, No. 11, Nov., 1963.

[4] The nontouching loop rule provides a simple method for writing the solution of any flow graph by inspection. The solution T (the ratio of the output variable to the input variable) is

$$T = \frac{\sum_k T_k \Delta_k}{\Delta}$$

where T_k = path gain of the k^{th} forward path

$\Delta = 1 -$ (sum of all individual loop gains) + (sum of the loop gain products of all possible combinations of two nontouching loops) − (sum of the loop gain products of all possible combinations of three nontouching loops) + ...

Δ_k = The value of Δ not touching the k_{th} forward path.

A path is a continuous succession of branches, and a forward path is a path connecting the input node to the output node, where no node is encountered more than once. Path gain is the product of all the branch multipliers along the path. A loop is a path which originates and terminates on the same node, no node being encountered more than once. Loop gain is the product of the branch multipliers around the loop.

For example, in Fig. 3 there is only one forward path from b_s to b_2 and its gain is s_{21}. There are two paths from b_s to b_1; their path gains are $s_{21}s_{12}\Gamma_L$ and s_{11} respectively. There are three individual loops, only one combination of two nontouching loops, and no combinations of three or more nontouching loops; therefore, the value of Δ for this network is

$$\Delta = 1 - (s_{11}\Gamma_s + s_{21}s_{12}\Gamma_L\Gamma_s + s_{22}\Gamma_L) + (s_{11}s_{22}\Gamma_L\Gamma_s).$$

The transfer function from b_s to b_2 is therefore

$$\frac{b_2}{b_s} = \frac{s_{21}}{\Delta}$$

Using scattering parameter flow-graphs and the non-touching loop rule, it is easy to calculate the transducer power gain with arbitrary load and source. In the following equations the load and source are described by their reflection coefficients Γ_L and Γ_S, respectively, referenced to the real characteristic impedance Z_o.

Transducer power gain

$$G_T = \frac{\text{Power delivered to the load}}{\text{Power available from the source}} = \frac{P_L}{P_{avS}}$$

$$P_L = P(\text{incident on load}) - P(\text{reflected from load})$$
$$= |b_2|^2 (1 - |\Gamma_L|^2)$$

$$P_{avS} = \frac{|b_S|^2}{(1 - |\Gamma_S|^2)}$$

$$G_T = \left|\frac{b_2}{b_s}\right|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)$$

Using the non-touching loop rule,

$$\frac{b_2}{b_S} = \frac{s_{21}}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L - s_{21}s_{12}\Gamma_L\Gamma_S + s_{11}\Gamma_S s_{22}\Gamma_L}$$
$$= \frac{s_{21}}{(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{21}s_{12}\Gamma_L\Gamma_S}$$

$$G_T = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{21}s_{12}\Gamma_L\Gamma_S|^2} \quad (18)$$

Two other parameters of interest are:

1) Input reflection coefficient with the output termination arbitrary and $Z_S = Z_0$.

$$s'_{11} = \frac{b_1}{a_1} = \frac{s_{11}(1 - s_{22}\Gamma_L) + s_{21}s_{12}\Gamma_L}{1 - s_{22}\Gamma_L}$$
$$= s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1 - s_{22}\Gamma_L} \quad (19)$$

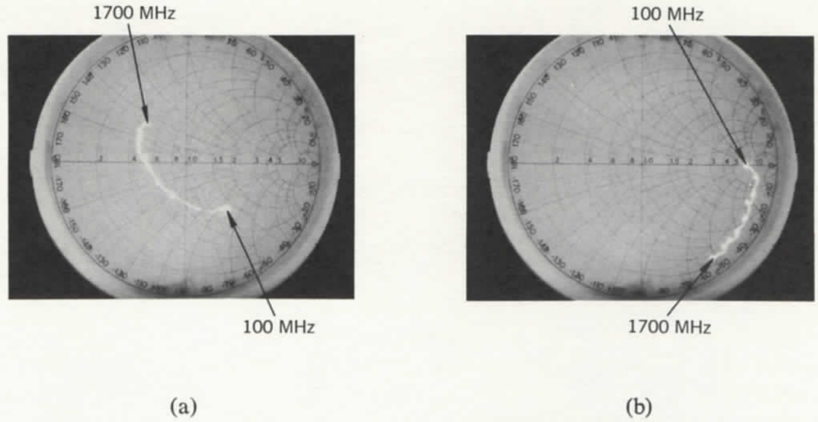
2) Voltage gain with arbitrary source and load impedances

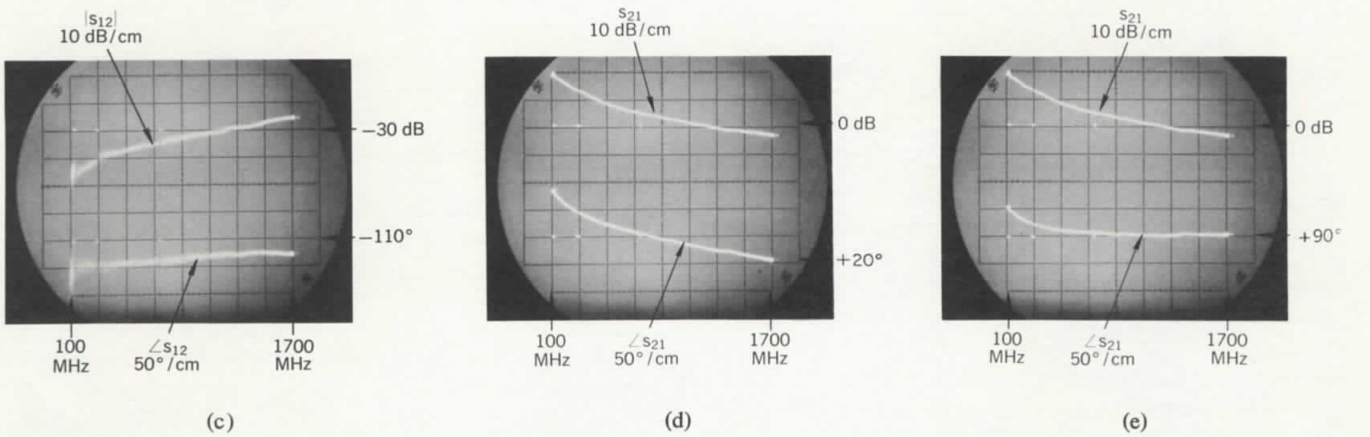
$$A_V = \frac{V_2}{V_1} \qquad V_1 = (a_1 + b_1)\sqrt{Z_o} = V_{i1} + V_{r1}$$
$$V_2 = (a_2 + b_2)\sqrt{Z_o} = V_{i2} + V_{r2}$$
$$a_2 = \Gamma_L b_2$$
$$b_1 = s'_{11} a_1$$

$$A_V = \frac{b_2(1 + \Gamma_L)}{a_1(1 + s'_{11})} = \frac{s_{21}(1 + \Gamma_L)}{(1 - s_{22}\Gamma_L)(1 + s'_{11})} \quad (20)$$

On p. 11 is a table of formulas for calculating many often-used network functions (power gains, driving point characteristics, etc.) in terms of scattering parameters. Also included in the table are conversion formulas between s-parameters and h-, y-, and z-parameters, which are other parameter sets used very often for specifying transistors at

Fig. 4. *S parameters of 2N3478 transistor in common-emitter configuration, measured by –hp– Model 8410A Network Analyzer.* (a) s_{11}. *Outermost circle on Smith Chart overlay corresponds to* $|s_{11}| = 1$. (b) s_{22}. *Scale factor same as* (a). (c) s_{12}. (d) s_{21}. (e) s_{21} *with line stretcher adjusted to remove linear phase shift above 500 MHz.*





lower frequencies. Two important figures of merit used for comparing transistors, f_t and f_{max}, are also given, and their relationship to s-parameters is indicated.

Amplifier Design Using Scattering Parameters

The remainder of this article will show by several examples how s-parameters are used in the design of transistor amplifiers and oscillators. To keep the discussion from becoming bogged down in extraneous details, the emphasis in these examples will be on s-parameter design *methods,* and mathematical manipulations will be omitted wherever possible.

Measurement of S-Parameters

Most design problems will begin with a tentative selection of a device and the measurement of its s-parameters. Fig. 4 is a set of oscillograms containing complete s-parameter data for a 2N3478 transistor in the common-emitter configuration. These oscillograms are the results of swept-frequency measurements made with the new microwave network analyzer described elsewhere in this issue. They represent the actual s-parameters of this transistor between 100 MHz and 1700 MHz.

In Fig. 5, the magnitude of s_{21} from Fig. 4(d) is replotted on a logarithmic frequency scale, along with additional data on s_{21} below 100 MHz, measured with a vector voltmeter. The magnitude of s_{21} is essentially constant to 125 MHz, and then rolls off at a slope of 6 dB/octave. The phase angle of s_{21}, as seen in Fig. 4(d), varies linearly with frequency above about 500 MHz. By adjusting a calibrated line stretcher in the network analyzer, a compensating linear phase shift was introduced, and the phase curve of Fig. 4(e) resulted. To go from the phase curve of Fig. 4(d) to that of Fig. 4(e) required 3.35 cm of line, equivalent to a pure time delay of 112 picoseconds.

After removal of the constant-delay, or linear-phase, component, the phase angle of s_{21} for this transistor [Fig. 4(e)] varies from 180° at dc to +90° at high frequencies, passing through +135° at 125 MHz, the −3 dB point of the magnitude curve. In other words, s_{21} behaves like a single pole in the frequency domain, and it is possible to write a closed expression for it. This expression is

$$s_{21} = \frac{-s_{21o}e^{-j\omega T_o}}{1 + j\frac{\omega}{\omega_o}} \qquad (21)$$

where
$$T_o = 112 \text{ ps}$$
$$\omega = 2\pi f$$
$$\omega_o = 2\pi \times 125 \text{ MHz}$$
$$s_{21o} = 11.2 = 21 \text{ dB}$$

The time delay $T_o = 112$ ps is due primarily to the transit time of minority carriers (electrons) across the base of this npn transistor.

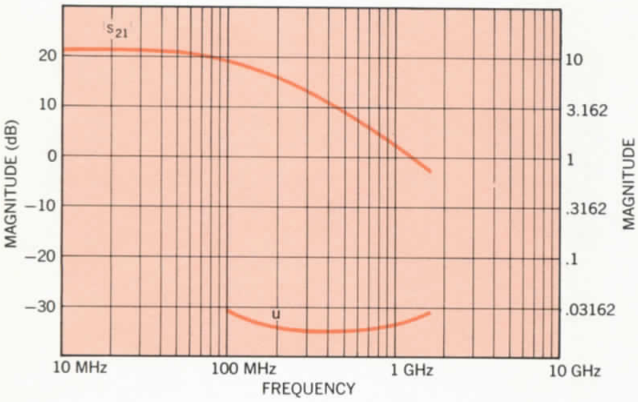


Fig. 5. *Top curve:* $|s_{21}|$ *from Fig. 4 replotted on logarithmic frequency scale. Data below 100 MHz measured with -hp- 8405A Vector Voltmeter. Bottom curve: unilateral figure of merit, calculated from s parameters (see text).*

Narrow-Band Amplifier Design

Suppose now that this 2N3478 transistor is to be used in a simple amplifier, operating between a 50Ω source and a 50Ω load, and optimized for power gain at 300 MHz by means of lossless input and output matching networks. Since reverse gain s_{12} for this transistor is quite small — 50 dB smaller than forward gain s_{21}, according to Fig. 4 — there is a possibility that it can be neglected. If this is so, the design problem will be much simpler, because setting s_{12} equal to zero will make the design equations much less complicated.

In determining how much error will be introduced by assuming $s_{12} = 0$, the first step is to calculate the unilateral figure of merit u, using the formula given in the table on p. 11, i.e.

$$u = \frac{|s_{11}s_{12}s_{21}s_{22}|}{|(1 - |s_{11}|^2)(1 - |s_{22}|^2)|} . \tag{22}$$

A plot of u as a function of frequency, calculated from the measured parameters, appears in Fig. 5. Now if G_{Tu} is the transducer power gain with $s_{12} = 0$ and G_T is the actual transducer power gain, the maximum error introduced by using G_{Tu} instead of G_T is given by the following relationship:

$$\frac{1}{(1+u)^2} < \frac{G_T}{G_{Tu}} < \frac{1}{(1-u)^2} . \tag{23}$$

From Fig. 5, the maximum value of u is about 0.03, so the maximum error in this case turns out to be about ±0.25 dB at 100 MHz. This is small enough to justify the assumption that $s_{12} = 0$.

Incidentally, a small reverse gain, or feedback factor, s_{12}, is an important and desirable property for a transistor to have, for reasons other than that it simplifies amplifier design. A small feedback factor means that the input characteristics of the completed amplifier will be independent of the load, and the output will be independent of the source impedance. In most amplifiers, isolation of source and load is an important consideration.

Returning now to the amplifier design, the unilateral expression for transducer power gain, obtained either by setting $s_{12} = 0$ in equation 18 or by looking in the table on p. 11, is

$$G_{Tu} = \frac{|s_{21}|^2(1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - s_{11}\Gamma_s|^2|1 - s_{22}\Gamma_L|^2} . \tag{24}$$

When $|s_{11}|$ and $|s_{22}|$ are both less than one, as they are in this case, maximum G_{Tu} occurs for $\Gamma_S = s^*_{11}$ and $\Gamma_L = s^*_{22}$ (table, p. 11).

The next step in the design is to synthesize matching networks which will transform the 50Ω load and source impedances to the impedances corresponding to reflection coefficients of s^*_{11} and s^*_{22}, respectively. Since this is to be a single-frequency amplifier, the matching networks need not be complicated. Simple series-capacitor, shunt-inductor networks will not only do the job, but will also provide a handy means of biasing the transistor — via the inductor — and of isolating the dc bias from the load and source.

Values of L and C to be used in the matching networks are determined using the Smith Chart of Fig. 6. First, points corresponding to s_{11}, s^*_{11}, s_{22}, and s^*_{22} at 300 MHz are plotted. Each point represents the tip of a vector leading away from the center of the chart, its length equal to the magnitude of the reflection coefficient being plotted, and its angle equal to the phase of the coefficient. Next, a combination of constant-resistance and constant-conductance circles is found, leading from the center of the chart, representing 50Ω, to s^*_{11} and s^*_{22}. The circles on the Smith Chart are constant-resistance circles; increasing series capacitive reactance moves an impedance point counter-clockwise along these circles. In this case, the circle to be used for finding series C is the one passing through the center of the chart, as shown by the solid line in Fig. 6.

Increasing shunt inductive susceptance moves impedance points clockwise along constant-conductance circles. These circles are like the constant-resistance circles, but they are on another Smith Chart, this one being just the reverse of the one in Fig. 6. The constant-conductance circles for shunt L all pass through the leftmost point of the chart rather than the rightmost point. The circles to be used are those passing through s^*_{11} and s^*_{22}, as shown by the dashed lines in Fig. 6.

Once these circles have been located, the normalized values of L and C needed for the matching networks are calculated from readings taken from the reactance and susceptance scales of the Smith Charts. Each element's reactance or susceptance is the difference between the scale readings at the two end points of a circular arc. Which arc corresponds to which element is indicated in Fig. 6. The final network and the element values, normalized and unnormalized, are shown in Fig. 7.

Broadband Amplifier Design

Designing a broadband amplifier, that is, one which has nearly constant gain over a prescribed frequency range, is a matter of surrounding a transistor with external elements in order to compensate for the variation of forward gain $|s_{21}|$ with frequency. This can be done in either of two ways — first, negative feedback, or second, selective mismatching of the input and output circuitry. We will use the second method. When feedback is used, it is usually convenient to convert to y- or z-parameters (for shunt or series feedback respectively) using the conversion equations given in the table, p. 12, and a digital computer.

Equation 24 for the unilateral transducer power gain can be factored into three parts:

$$G_{Tu} = G_o G_1 G_2$$

where

$$G_o = |s_{21}|^2$$

$$G_1 = \frac{1 - |\Gamma_s|^2}{|1 - s_{11}\Gamma_s|^2}$$

$$G_2 = \frac{1 - |\Gamma_L|^2}{|1 - s_{22}\Gamma_L|^2} .$$

When a broadband amplifier is designed by selective mismatching, the gain contributions of G_1 and G_2 are varied to compensate for the variations of $G_o = |s_{21}|^2$ with frequency.

Suppose that the 2N3478 transistor whose s-parameters are given in Fig. 4 is to be used in a broadband amplifier which has a constant gain of 10 dB over a frequency range of 300 MHz to 700 MHz. The amplifier is to be driven from a 50Ω source and is to drive a 50Ω load. According to Fig. 5,

$$|s_{21}|^2 = 13 \text{ dB at } 300 \text{ MHz}$$
$$= 10 \text{ dB at } 450 \text{ MHz}$$
$$= 6 \text{ dB at } 700 \text{ MHz}.$$

To realize an amplifier with a constant gain of 10 dB, source and load matching networks must be found which will decrease the gain by 3 dB at 300 MHz, leave the gain the same at 450 MHz, and increase the gain by 4 dB at 700 MHz.

Although in the general case both a source matching network and a load matching network would be designed, $G_{1\,max}$ (i.e., G_1 for $\Gamma_S = s^*_{11}$) for this transistor is less than 1 dB over the frequencies of interest, which means there is little to be gained by matching the source. Consequently, for this example, only a load-matching network will be designed. Procedures for designing source-matching networks are identical to those used for designing load-matching networks.

The first step in the design is to plot s^*_{22} over the required frequency range on the Smith Chart, Fig. 8. Next, a set of constant-gain circles is drawn. Each circle is drawn for a single frequency; its center is on a line between the center of the Smith Chart and the point representing s^*_{22} at that frequency. The distance from the center of the Smith Chart to the center of the constant gain circle is given by (these equations also appear in the table, p. 11):

$$r_2 = \frac{g_2|s_{22}|}{1 - |s_{22}|^2(1 - g_2)}$$

where

$$g_2 = \frac{G_2}{G_{2\,max}} = G_2(1 - |s_{22}|^2).$$

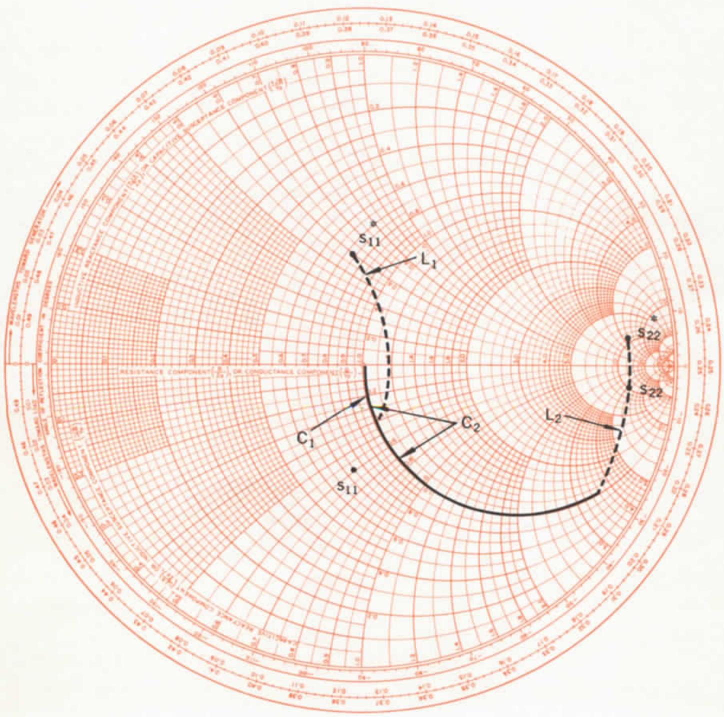
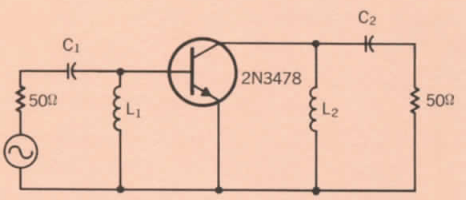


Fig. 6. *Smith Chart for 300-MHz amplifier design example.*



Calculations:

$$X_{L2} = \frac{50}{0.32} = 156\ \Omega$$

$$L_2 = \frac{156}{2\pi\,(0.3 \times 10^9)} = 83\ \text{nH}$$

$$C_2 = \frac{1}{2\pi\,(0.3 \times 10^9)\,(3.5)\,(50)} = 3\ \text{pF}$$

$$C_1 = \frac{3\,(3.5)}{0.42}\ \text{pF} = 25\ \text{pF}$$

$$L_1 = \frac{50}{1.01}\ \text{nH} = 26\ \text{nH}$$

Fig. 7. *300-MHz amplifier with matching networks for maximum power gain.*

The radius of the constant-gain circle is

$$\rho_2 = \frac{\sqrt{1 - g_2}\,(1 - |s_{22}|^2)}{1 - |s_{22}|^2\,(1 - g_2)}.$$

For this example, three circles will be drawn, one for $G_2 = -3$ dB at 300 MHz, one for $G_2 = 0$ dB at 450 MHz, and one for $G_2 = +4$ dB at 700 MHz. Since $|s_{22}|$ for this transistor is constant at 0.85 over the frequency range [see Fig. 4(b)], $G_{2\,max}$ for all three circles is $(0.278)^{-1}$, or 5.6 dB. The three constant-gain circles are indicated in Fig. 8.

The required matching network must transform the center of the Smith Chart, representing 50Ω, to some point on the −3 dB circle at 300 MHz, to some point on the 0 dB circle at 450 MHz, and to some point on the +4 dB circle at 700 MHz. There are undoubtedly many networks that will do this. One which is satisfactory is a combination of two inductors, one in shunt and one in series, as shown in Fig. 9.

Shunt and series elements move impedance points on the Smith Chart along constant-conductance and constant-resistance circles, as I explained in the narrow-band design example which preceded this broadband example. The shunt inductance transforms the 50Ω load along a circle of constant conductance and varying (with frequency) inductive susceptance. The series inductor transforms the combination of the 50Ω load and the shunt inductance along circles of constant resistance and varying inductive reactance.

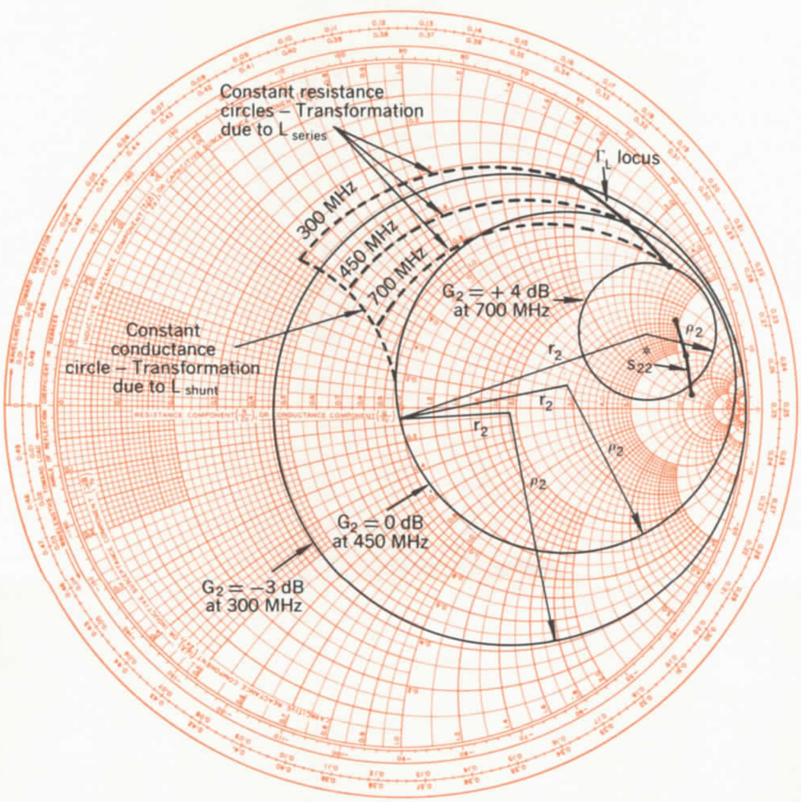


Fig. 8. *Smith Chart for broadband amplifier design example.*

Optimizing the values of shunt and series L is a cut-and-try process to adjust these elements so that

—the transformed load reflection terminates on the right gain circle at each frequency, and

—the susceptance component decreases with frequency and the reactance component increases with frequency. (This rule applies to inductors; capacitors would behave in the opposite way.)

Once appropriate constant-conductance and constant-resistance circles have been found, the reactances and susceptances of the elements can be read directly from the Smith Chart. Then the element values are calculated, the same as they were for the narrow-band design.

Fig. 10 is a schematic diagram of the completed broadband amplifier, with unnormalized element values.

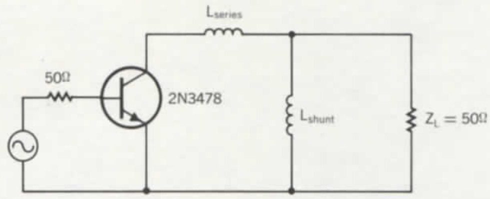


Fig. 9. *Combination of shunt and series inductances is suitable matching network for broadband amplifier.*

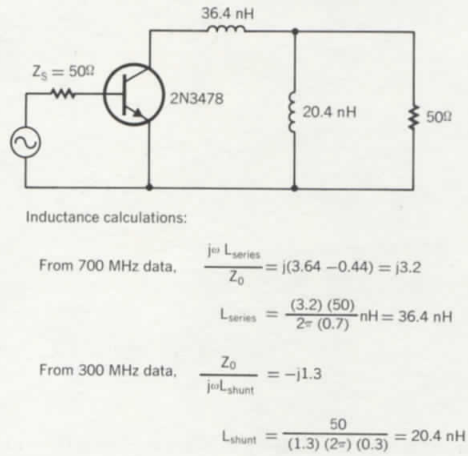
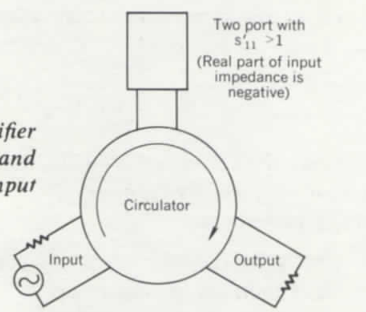


Fig. 10. *Broadband amplifier with constant gain of 10 dB from 300 MHz to 700 MHz.*

Stability Considerations and the Design of Reflection Amplifiers and Oscillators

When the real part of the input impedance of a network is negative, the corresponding input reflection coefficient (equation 17) is greater than one, and the network can be used as the basis for two important types of circuits, reflection amplifiers and oscillators. A reflection amplifier (Fig. 11) can be realized with a circulator—a nonreciprocal three-port device — and a negative-resistance device. The circulator is used to separate the incident (input) wave from the larger wave reflected by the negative-resistance device. Theoretically, if the circulator is perfect and has a positive real characteristic impedance Z_o, an amplifier with infinite gain can be built by selecting a negative-resistance device whose input impedance has a real part equal to $-Z_o$ and an imaginary part equal to zero (the imaginary part can be set equal to zero by tuning, if necessary).

Fig. 11. *Reflection amplifier consists of circulator and transistor with negative input resistance.*



Amplifiers, of course, are not supposed to oscillate, whether they are reflection amplifiers or some other kind. There is a convenient criterion based upon scattering parameters for determining whether a device is stable or potentially unstable with given source and load impedances. Referring again to the flow graph of Fig. 3, the ratio of the reflected voltage wave b_1 to the input voltage wave b_S is

$$\frac{b_1}{b_S} = \frac{s'_{11}}{1 - \Gamma_S s'_{11}}$$

where s'_{11} is the input reflection coefficient with $\Gamma_S = 0$ (that is, $Z_S = Z_o$) and an arbitrary load impedance Z_L, as defined in equation 19.

If at some frequency

$$\Gamma_S s'_{11} = 1 \quad (25)$$

the circuit is unstable and will oscillate at that frequency. On the other hand, if

$$|s'_{11}| < \left|\frac{1}{\Gamma_S}\right|$$

the device is unconditionally stable and will not oscillate, whatever the phase angle of Γ_S might be.

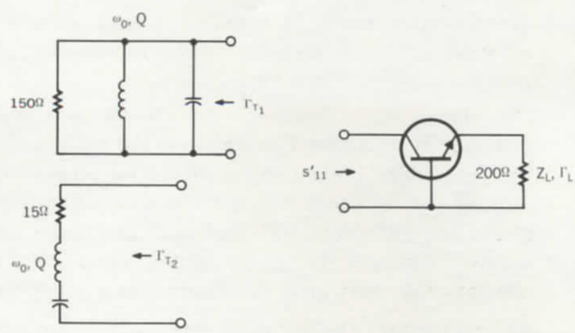


Fig. 12. *Transistor oscillator is designed by choosing tank circuit such that* $\Gamma_T s'_{11} = 1$.

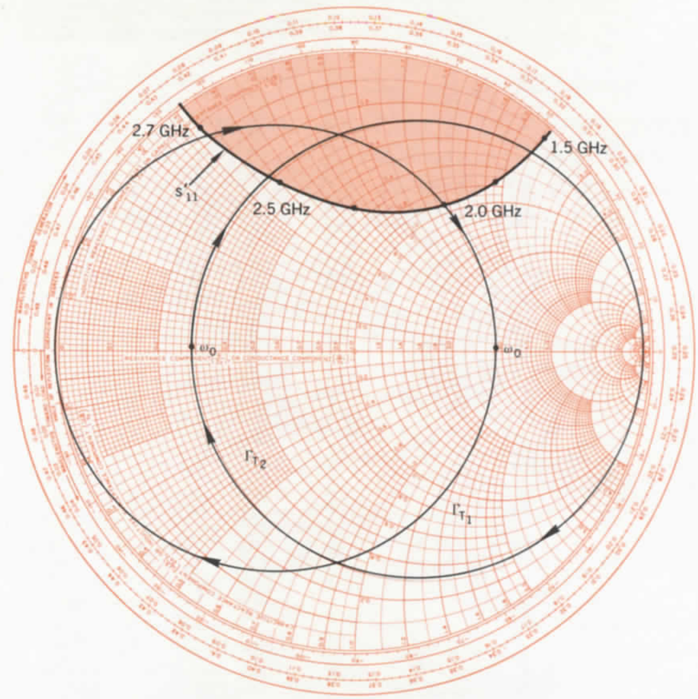


Fig. 13. *Smith Chart for transistor oscillator design example.*

As an example of how these principles of stability are applied in design problems, consider the transistor oscillator design illustrated in Fig. 12. In this case the input reflection coefficient s'_{11} is the reflection coefficient looking into the collector circuit, and the 'source' reflection coefficient Γ_S is one of the two tank-circuit reflection coefficients, Γ_{T1} or Γ_{T2}. From equation 19,

$$s'_{11} = s_{11} + \frac{s_{12}\, s_{21}\, \Gamma_L}{1 - s_{22}\, \Gamma_L}$$

To make the transistor oscillate, s'_{11} and Γ_S must be adjusted so that they satisfy equation 25. There are four steps in the design procedure:

—Measure the four scattering parameters of the transistor as functions of frequency.

—Choose a load reflection coefficient Γ_L which makes s'_{11} greater than unity. In general, it may also take an external feedback element which increases $s_{12}\, s_{21}$ to make s'_{11} greater than one.

—Plot $1/s'_{11}$ on a Smith Chart. (If the new network analyzer is being used to measure the s-parameters of the transistor, $1/s'_{11}$ can be measured directly by reversing the reference and test channel connections between the reflection test unit and the harmonic frequency converter. The polar display with a Smith Chart overlay will then give the desired plot immediately.)

—Connect either the series or the parallel tank circuit to the collector circuit and tune it so that Γ_{T1} or Γ_{T2} is large enough to satisfy equation 25 (the tank circuit reflection coefficient plays the role of Γ_S in this equation).

Fig. 13 shows a Smith Chart plot of $1/s'_{11}$ for a high-frequency transistor in the common-base configuration. Load impedance Z_L is 200Ω, which means that Γ_L referred to 50Ω is 0.6. Reflection coefficients Γ_{T1} and Γ_{T2} are also plotted as functions of the resonant frequencies of the two tank circuits. Oscillations occur when the locus of Γ_{T1} or Γ_{T2} passes through the shaded region. Thus this transistor would oscillate from 1.5 to 2.5 GHz with a series tuned circuit and from 2.0 to 2.7 GHz with a parallel tuned circuit.

—*Richard W. Anderson*

Additional Reading on S-Parameters

Besides the papers referenced in the footnotes of the article, the following articles and books contain information on s-parameter design procedures and flow graphs.

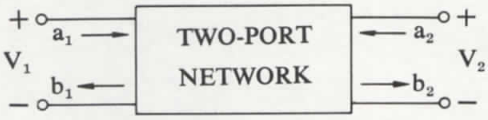
—F. Weinert, 'Scattering Parameters Speed Design of High-Frequency Transistor Circuits,' Electronics, Vol. 39, No. 18, Sept. 5, 1966.

—G. Fredricks, 'How to Use S-Parameters for Transistor Circuit Design,' EEE, Vol. 14, No. 12, Dec., 1966.

—D. C. Youla, 'On Scattering Matrices Normalized to Complex Port Numbers,' Proc. IRE, Vol. 49, No. 7, July, 1961.

—J. G. Linvill and J. F. Gibbons, 'Transistors and Active Circuits,' McGraw-Hill, 1961. (No s-parameters, but good treatment of Smith Chart design methods.)

Useful Scattering Parameter Relationships



$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

Input reflection coefficient with arbitrary Z_L

$$s'_{11} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}$$

Output reflection coefficient with arbitrary Z_S

$$s'_{22} = s_{22} + \frac{s_{12}s_{21}\Gamma_S}{1 - s_{11}\Gamma_S}$$

Voltage gain with arbitrary Z_L and Z_S

$$A_V = \frac{V_2}{V_1} = \frac{s_{21}(1 + \Gamma_L)}{(1 - s_{22}\Gamma_L)(1 + s'_{11})}$$

$$\text{Power Gain} = \frac{\text{Power delivered to load}}{\text{Power input to network}}$$

$$G = \frac{|s_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |s_{11}|^2) + |\Gamma_L|^2 (|s_{22}|^2 - |D|^2) - 2\,\mathrm{Re}\,(\Gamma_L N)}$$

$$\text{Available Power Gain} = \frac{\text{Power available from network}}{\text{Power available from source}}$$

$$G_A = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)}{(1 - |s_{22}|^2) + |\Gamma_S|^2 (|s_{11}|^2 - |D|^2) - 2\,\mathrm{Re}\,(\Gamma_S M)}$$

$$\text{Transducer Power Gain} = \frac{\text{Power delivered to load}}{\text{Power available from source}}$$

$$G_T = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_L\Gamma_S|^2}$$

Unilateral Transducer Power Gain ($s_{12} = 0$)

$$G_{Tu} = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - s_{11}\Gamma_S|^2\,|1 - s_{22}\Gamma_L|^2}$$

$$= G_o G_1 G_2$$

$$G_o = |s_{21}|^2$$

$$G_1 = \frac{1 - |\Gamma_S|^2}{|1 - s_{11}\Gamma_S|^2}$$

$$G_2 = \frac{1 - |\Gamma_L|^2}{|1 - s_{22}\Gamma_L|^2}$$

Maximum Unilateral Transducer Power Gain when $|s_{11}| < 1$ and $|s_{22}| < 1$

$$G_u = \frac{|s_{21}|^2}{|(1 - |s_{11}|^2)(1 - |s_{22}|)^2|}$$

$$= G_o\, G_{1\,max}\, G_{2\,max}$$

$$G_{i\,max} = \frac{1}{1 - |s_{ii}|^2} \qquad i = 1, 2$$

This maximum attained for $\Gamma_S = s^*_{11}$ and $\Gamma_L = s^*_{22}$

Constant Gain Circles (Unilateral case: $s_{12} = 0$)

—center of constant gain circle is on line between center of Smith Chart and point representing s^*_{ii}

—distance of center of circle from center of Smith Chart:

$$r_i = \frac{g_i |s_{ii}|}{1 - |s_{ii}|^2 (1 - g_i)}$$

—radius of circle:

$$\rho_i = \frac{\sqrt{1 - g_i}\,(1 - |s_{ii}|^2)}{1 - |s_{ii}|^2 (1 - g_i)}$$

where: $i = 1, 2$

$$\text{and } g_i = \frac{G_i}{G_{i\,max}} = G_i (1 - |s_{ii}|^2)$$

Unilateral Figure of Merit

$$u = \frac{|s_{11}s_{22}s_{12}s_{21}|}{|(1 - |s_{11}|^2)(1 - |s_{22}|^2)|}$$

Error Limits on Unilateral Gain Calculation

$$\frac{1}{(1 + u^2)} < \frac{G_T}{G_{Tu}} < \frac{1}{(1 - u^2)}$$

Conditions for Absolute Stability

No passive source or load will cause network to oscillate if a, b, and c are all satisfied.

a. $|s_{11}| < 1, |s_{22}| < 1$

b. $\left| \dfrac{|s_{12}s_{21}| - |M^*|}{|s_{11}|^2 - |D|^2} \right| > 1$

c. $\left| \dfrac{|s_{12}s_{21}| - |N^*|}{|s_{22}|^2 - |D|^2} \right| > 1$

Condition that a two-port network can be simultaneously matched with a positive real source and load:

$K > 1$ or $C < 1$

$C =$ Linvill C factor

Linvill C Factor

$C = K^{-1}$

$$K = \frac{1 + |D|^2 - |s_{11}|^2 - |s_{22}|^2}{2\,|s_{12}s_{21}|}$$

Source and Load for Simultaneous Match

$$\Gamma_{mS} = M^* \left[\frac{B_1 \pm \sqrt{B_1{}^2 - 4\,|M|^2}}{2\,|M|^2} \right]$$

$$\Gamma_{mL} = N^* \left[\frac{B_2 \pm \sqrt{B_2{}^2 - 4\,|N|^2}}{2\,|N|^2} \right]$$

Where $B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |D|^2$

$B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |D|^2$

Maximum Available Power Gain

If $K > 1$,

$$G_{A\,max} = \left| \frac{s_{21}}{s_{12}} (K \pm \sqrt{K^2 - 1}) \right|$$

$K = C^{-1}$

$C =$ Linvill C Factor

(Use minus sign when B_1 is positive, plus sign when B_1 is negative. For definition of B_1 see 'Source and Load for Simultaneous Match,' elsewhere in this table.)

$$D = s_{11}s_{22} - s_{12}s_{21}$$

$$M = s_{11} - D\,s^*_{22}$$

$$N = s_{22} - D\,s^*_{11}$$

s-parameters in terms of h-, y-, and z-parameters	h-, y-, and z-parameters in terms of s-parameters
$s_{11} = \dfrac{(z_{11} - 1)(z_{22} + 1) - z_{12}z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}}$	$z_{11} = \dfrac{(1 + s_{11})(1 - s_{22}) + s_{12}s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12}s_{21}}$
$s_{12} = \dfrac{2z_{12}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}}$	$z_{12} = \dfrac{2s_{12}}{(1 - s_{11})(1 - s_{22}) - s_{12}s_{21}}$
$s_{21} = \dfrac{2z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}}$	$z_{21} = \dfrac{2s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12}s_{21}}$
$s_{22} = \dfrac{(z_{11} + 1)(z_{22} - 1) - z_{12}z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}}$	$z_{22} = \dfrac{(1 + s_{22})(1 - s_{11}) + s_{12}s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12}s_{21}}$
$s_{11} = \dfrac{(1 - y_{11})(1 + y_{22}) + y_{12}y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$	$y_{11} = \dfrac{(1 + s_{22})(1 - s_{11}) + s_{12}s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$
$s_{12} = \dfrac{-2y_{12}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$	$y_{12} = \dfrac{-2s_{12}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$
$s_{21} = \dfrac{-2y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$	$y_{21} = \dfrac{-2s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$
$s_{22} = \dfrac{(1 + y_{11})(1 - y_{22}) + y_{12}y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$	$y_{22} = \dfrac{(1 + s_{11})(1 - s_{22}) + s_{12}s_{21}}{(1 + s_{22})(1 + s_{11}) - s_{12}s_{21}}$
$s_{11} = \dfrac{(h_{11} - 1)(h_{22} + 1) - h_{12}h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12}h_{21}}$	$h_{11} = \dfrac{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}$
$s_{12} = \dfrac{2h_{12}}{(h_{11} + 1)(h_{22} + 1) - h_{12}h_{21}}$	$h_{12} = \dfrac{2s_{12}}{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}$
$s_{21} = \dfrac{-2h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12}h_{21}}$	$h_{21} = \dfrac{-2s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}$
$s_{22} = \dfrac{(1 + h_{11})(1 - h_{22}) + h_{12}h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12}h_{21}}$	$h_{22} = \dfrac{(1 - s_{22})(1 - s_{11}) - s_{12}s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}$

The h-, y-, and z-parameters listed above are all normalized to Z_o. If h′, y′, and z′ are the actual parameters, then

$z_{11}' = z_{11}Z_o$	$y_{11}' = \dfrac{y_{11}}{Z_o}$	$h_{11}' = h_{11}Z_o$
$z_{12}' = z_{12}Z_o$	$y_{12}' = \dfrac{y_{12}}{Z_o}$	$h_{12}' = h_{12}$
$z_{21}' = z_{21}Z_o$	$y_{21}' = \dfrac{y_{21}}{Z_o}$	$h_{21}' = h_{21}$
$z_{22}' = z_{22}Z_o$	$y_{22}' = \dfrac{y_{22}}{Z_o}$	$h_{22}' = \dfrac{h_{22}}{Z_o}$

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Scattering parameters speed design of high-frequency transistor circuits

By Fritz Weinert
Hewlett-Packard Co., Palo Alto, Calif.

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VECTOR VOLTMETER

Specifications

Input Characteristics

Instrument Type: Two-channel sampling rf millivoltmeter-phasemeter which measures voltage of two signals and simultaneously displays the phase angle between the two signals.

Frequency Range: 1 MHz to 1 GHz in 21 overlapping octave bands (lowest band covers two octaves).

Tuning: Automatic within each band. Automatic phase control (APC) circuit responds to the channel-A input signal. Search and lock time, approximately 10 millisec; maximum sweep speed, 15 MHz/sec.

Voltage Range:

Channel A:
1 to 10 MHz: 1.5 mV to 1 V rms;
10 to 500 MHz: 300 μV to 1 V rms;
500 to 1,000 MHz: 500 μV to 1 V rms;
can be extended by a factor of 10 with 10214A 10:1 Divider.

Channel B: 100 μV to 1 V rms full scale (input to channel A required); can be extended by a factor of 10 with 10214A 10:1 Divider.

Input Impedance (nominal): 0.1 megohm shunted by approximately 2.5 pF; 1 megohm shunted by approximately 2 pF when 10214A 10:1 Divider is used; 0.1 megohm shunted by approximately 5 pF when 10216A Isolator is used. AC coupled.

Isolation Between Channels:
1 to 400 MHz: greater than 100 dB;
400 to 1,000 MHz: greater than 75 dB.

Maximum AC Input (for proper operation): 3 V p-p (30 V p-p when 10214A 10:1 Divider is used).

Maximum DC Input: ± 150 V.

Voltmeter Characteristics

Meter Ranges: 100 μV to 1 V rms full scale in 10-dB steps. Meter indicates amplitude of the fundamental component of the input signal.

Voltage Accuracy (at the probes):
1 to 100 MHz: within ± 2% of full scale;
100 to 400 MHz: within ± 6% of full scale;
400 to 1,000 MHz: within 12% of full scale;
not including response to test-point impedance.*

Voltage Response to Test-Point Impedance:* + 0, — 2% from 25 to 1,000 ohms. Effects of test-point impedance are eliminated when 10214A 10:1 Divider or 10216A Isolator is used.

Residual Noise: Less than 10 μV as indicated on the meter.

Bandwidth: 1 kHz.

Phasemeter Characteristics

Phase Range: 360°, indicated on zero-center meter with end-scale ranges of ± 180, ± 60, ± 18, and ± 6°. Meter indicates phase difference between the fundamental components of the input signals.

Resolution: 0.1° at any phase angle.

Meter Offset: ± 180° in 10° steps.

Phase Accuracy: Within ± 1°, not including phase response vs. frequency, amplitude, and test-point impedance.*

Phase Response vs. Frequency:
1 to 100 MHz: less than ± 0.2°;
100 to 1,000 MHz: less than ± 3°.

Phase Response vs. Signal Amplitude:
1 V to 3 mV rms: less than ±2°;
1 V to 100 μV rms: less than ± 3°
(add an additional ± 10° from 0.1 to 1 V rms between 500 and 1,000 MHz, + for changes affecting channel A only, — for channel B only; effects tend to cancel when signals to both channels change equally).

Phase Response vs. Test-Point Impedance:*
0 to 50 ohms: less than ± 2°;
25 to 1,000 ohms: less than— 0°, + 9° for channel A only, less than + 0°, — 9° for channel B only.

Phase Jitter vs. Channel B Input Level:
Greater than 700 μV: typically less than 0.1° p-p;
125 to 700 μV: typically less than 0.5° p-p;
20 to 125 μV: typically less than 2° p-p.

General

20 kHz IF Output (each channel): Reconstructed signals, with 20 kHz fundamental components, having the same amplitude, waveform, and phase relationship as the input signals. Output impedance, 1,000 ohms in series with 2,000 pF; BNC female connectors.

Recorder Output:
Amplitude: 0 to + 1 V dc ± 6% open circuit, proportional to voltmeter reading. Output tracks meter reading within ± 0.5% of full scale. Output impedance, 1,000 ohms; BNC female connector.

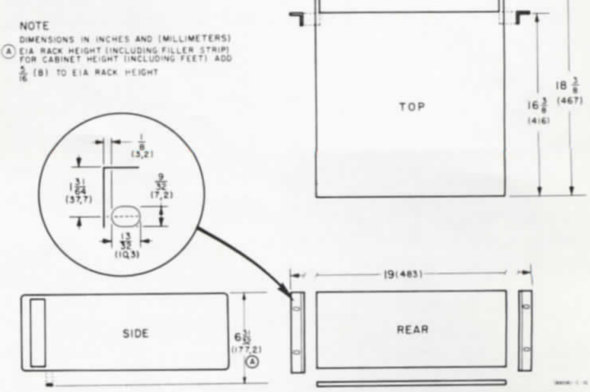
Phase: 0 to ± 0.5 V dc ± 6%, proportional to phasemeter reading. External load greater than 10,000 ohms affects recorder output and meter reading less than 1%. Output tracks meter reading within ± 1.5% of end scale; BNC female connector.

RFI: Conducted and radiated leakage limits are below those specified in MIL-I-6181D and MIL-I-16910C except for pulses emitted from probes. Spectral intensity of these pulses is approximately 60 μV/MHz; spectrum extends to approximately 2 GHz. Pulse rate varies from 1 to 2 MHz.

Power: 115 or 230 V ± 10%, 50 to 400 Hz, 35 watts.

Weight: Net, 30 lbs. *(13,5 kg)*. Shipping 35 lbs. *(15,8 kg)*.

Dimensions:



Price: Model 8405A, $2,500.00. *Prices f.o.b. factory*

Data subject to change without notice

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* Variation in the high-frequency impedance of test points as a probe is shifted from point to point influences the samplers and can cause the indicated amplitude and phase errors. These errors are different from the effects of any test-point loading due to the input impedance of the probes.

Scattering parameters speed design of high-frequency transistor circuits

At frequencies above 100 Mhz scattering parameters are easily measured and provide information difficult to obtain with conventional techniques that use h, y or z parameters

By Fritz Weinert

Hewlett-Packard Co., Palo Alto, Calif.

Performance of transistors at high frequencies has so improved that they are now found in all solid-state microwave equipment. But operating transistors at high frequencies has meant design problems:

- Manufacturers' high-frequency performance data is frequently incomplete or not in proper form.
- Values of h, y or z parameters, ordinarily used in circuit design at lower frequencies, can't be measured accurately above 100 megahertz because establishing the required short and open circuit conditions is difficult. Also, a short circuit frequently causes the transistor to oscillate under test.

These problems are yielding to a technique that uses scattering or s parameters to characterize the high-frequency performance of transistors. Scattering parameters can make the designer's job easier.

- They are derived from power ratios, and consequently provide a convenient method for measuring circuit losses.
- They provide a physical basis for understanding what is happening in the transistor, without need for an understanding of device physics.
- They are easy to measure because they are based on reflection characteristics rather than short- or open-circuit parameters.

The author

Fritz K. Weinert, who joined the technical staff of Hewlett-Packard in 1964, is project leader in the network analysis section of the microwave laboratory. He holds patents and has published papers on pulse circuits, tapered-line transformers, digital-tuned circuits and shielding systems.

Like other methods that use h, y or z parameters, the scattering-parameter technique does not require a suitable equivalent circuit to represent the transistor device. It is based on the assumption that the transistor is a two-port network—and its terminal behavior is defined in terms of four parameters, s_{11}, s_{12}, s_{21} and s_{22}, called s or scattering parameters.

Since four independent parameters completely define any two-port at any one frequency, it is possible to convert from one known set of parameters to another. At frequencies above 100 Mhz, however, it becomes increasingly difficult to measure the h, y or z parameters. At these frequencies it is difficult to obtain well defined short and open circuits and short circuits frequently cause the device to oscillate. However, s parameters may be measured directly up to a frequency of 1 gigahertz. Once obtained, it is easy to convert the s parameters into any of the h, y or z terms by means of tables.

Suggested measuring systems

To measure scattering parameters, the unknown transistor is terminated at both ports by pure resistances. Several measuring systems of this kind have been proposed. They have these advantages:

- Parasitic oscillations are minimized because of the broadband nature of the transistor terminations.
- Transistor measurements can be taken remotely whenever transmission lines connect the semiconductor to the source and load—especially when the line has the same characteristic impedance as the source and load respectively.
- Swept-frequency measurements are possible instead of point-by-point methods. Theoretical work shows scattering parameters can simplify design.

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Scattering-parameter definitions

To measure and define scattering parameters the two-port device, or transistor, is terminated at both ports by a pure resistance of value Z_0, called the reference impedance. Then the scattering parameters are defined by s_{11}, s_{12}, s_{21} and s_{22}. Their physical meaning is derived from the two-port network shown in first figure below.

Two sets of parameters, (a_1, b_1) and (a_2, b_2), represent the incident and reflected waves for the two-port network at terminals 1-1′ and 2-2′ respectively. Equations 1a through 1d define them.

$$a_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_0}} + \sqrt{Z_0}\, I_1\right) \qquad (1a)$$

$$b_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_0}} - \sqrt{Z_0}\, I_1\right) \qquad (1b)$$

$$a_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} + \sqrt{Z_0}\, I_2\right) \qquad (1c)$$

$$b_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} - \sqrt{Z_0}\, I_2\right) \qquad (1d)$$

The scattering parameters for the two-port network are given by equation 2.

$$\begin{aligned} b_1 &= s_{11}\, a_1 + s_{12}\, a_2 \\ b_2 &= s_{21}\, a_1 + s_{22}\, a_2 \end{aligned} \qquad (2)$$

In matrix form the set of equations of 2 becomes

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad (3)$$

where the matrix

$$[s] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \qquad (4)$$

is called the scattering matrix of the two-port network. Therefore the scattering parameters of the two-port network can be expressed in terms of the incident and reflected parameters as:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0} \qquad s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0}$$

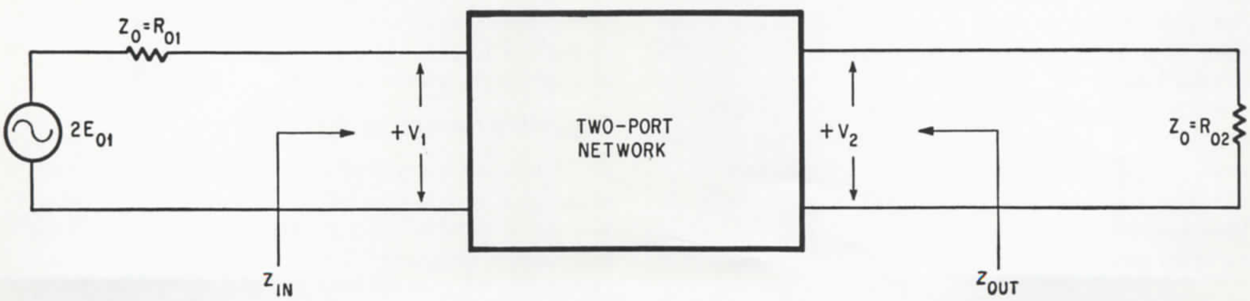
$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0} \qquad s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0} \qquad (5)$$

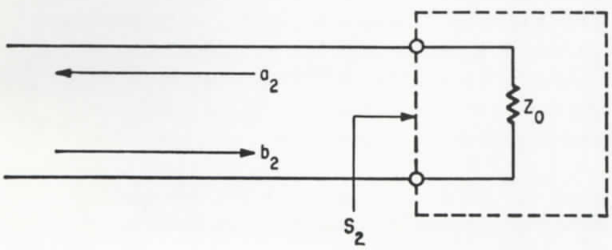
In equation 5, the parameter s_{11} is called the input reflection coefficient; s_{21} is the forward transmission coefficient; s_{12} is the reverse transmission coefficient; and s_{22} is the output reflection coefficient. All four scattering parameters are expressed as ratios of reflected to incident parameters.

Physical meaning of parameters

The implications of setting the incident parameters a_1 and a_2 at zero help explain the physical

Scattering parameters are defined by this representation of a two-port network. Two sets of incident and reflected parameters (a_1, b_1) and (a_2, b_2) appear at terminals 1-1′ and 2-2′ respectively.





By setting a_2 equal to zero the s_{11} parameter can be found. The Z_0 resistor is thought of as a one-port network. The condition $a_2 = 0$ implies that the reference impedance R_{02} is set equal to the load impedance Z_0. By connecting a voltage source, $2\,E_{01}$, with the source impedance, Z_0, parameter s_{21} can be found using equation 5 meaning of these scattering parameters.

By setting $a_2 = 0$, expressions for s_{11} and s_{22} can be found. The terminating section of the two-port network is at bottom of page 79 with the parameters a_2 and b_2 of the 2-2′ port. If the load resistor Z_0 is thought of as a one-port network with a scattering parameter

$$s_2 = \frac{Z_0 - R_{02}}{Z_0 + R_{02}} \quad (6)$$

where R_{02} is the reference impedance of port 2, then a_2 and b_2 are related by

$$a_2 = s_2 b_2 \quad (7)$$

When the reference impedance R_{02} is set equal to the local impedance Z_0, then s_2 becomes

$$s_2 = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0 \quad (8)$$

so that $a_2 = 0$ under this condition. Similarly, when $a_1 = 0$, the reference impedance of port 1 is equal to the terminating impedance; that is, $R_{01} = Z_0$. The conditions $a_1 = 0$ and $a_2 = 0$ merely imply that the reference impedances R_{01} and R_{02} are chosen to be equal to the terminating resistors Z_0.

In the relationship between the driving-point impedances at ports 1 and 2 and the reflection coefficients s_{11} and s_{22}, the driving-point impedances can be denoted by:

$$Z_{in} = \frac{V_1}{I_1}; \qquad Z_{out} = \frac{V_2}{I_2} \quad (9)$$

From the relationship

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0}$$

$$s_{11} = \frac{\frac{1}{2}\,[(V_1/\sqrt{Z_0}) - \sqrt{Z_0}\,I_1]}{\frac{1}{2}\,[(V_1/\sqrt{Z_0}) + \sqrt{Z_0}\,I_1]} \quad (10)$$

which reduces to

$$s_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad (11)$$

Similarly,

$$s_{22} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} \quad (12)$$

These expressions show that if the reference impedance at a given port is chosen to equal the ports driving-point impedance, the reflection coefficient will be zero, provided the other port is terminated in its reference impedance.

In the equation

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0}$$

the condition $a_2 = 0$ implies that the reference impedance R_{02} is set equal to the load impedance R_2, center figure page 79. If a voltage source 2 E_{01} is connected with a source impedance $R_{01} = Z_0$, a_1 can be expressed as:

$$a_1 = \frac{E_{01}}{\sqrt{Z_0}} \quad (13)$$

Since $a_2 = 0$, then

$$a_2 = 0 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} + \sqrt{Z_0}\,I_2\right)$$

from which

$$\frac{V_2}{\sqrt{Z_0}} = -\sqrt{Z_0}\,I_2$$

Consequently,

$$b_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} - \sqrt{Z_0}\,I_2\right) = \frac{V_2}{\sqrt{Z_0}}$$

Finally, the forward transmission coefficient is expressed as:

$$s_{21} = \frac{V_2}{E_{01}} \quad (14)$$

Similarly, when port 1 is terminated in $R_{01} = Z_0$ and when a voltage source 2 E_{02} with source impedance Z_0 is connected to port 2,

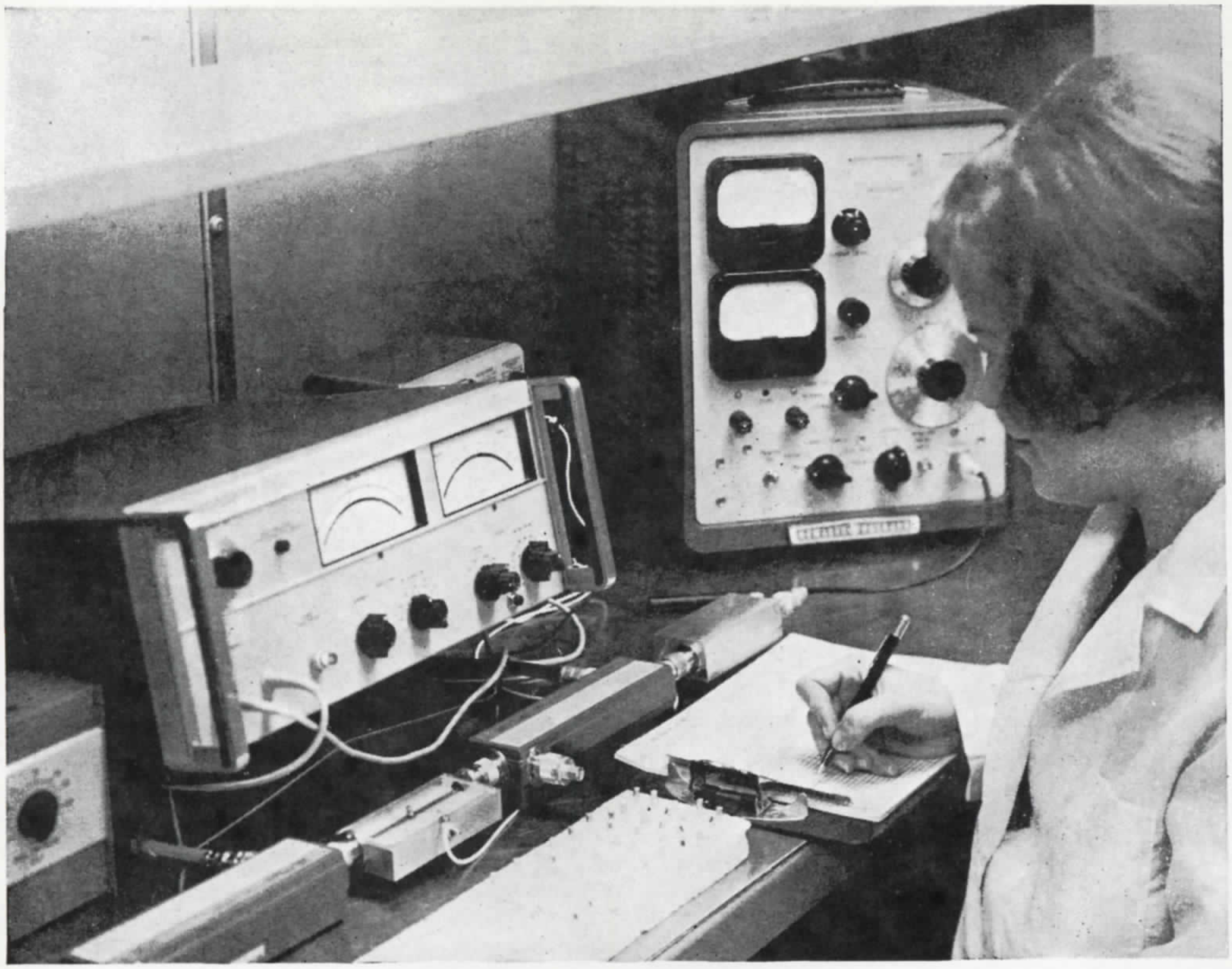
$$s_{12} = \frac{V_1}{E_{02}} \quad (15)$$

Both s_{12} and s_{21} have the dimensions of a voltage-ratio transfer function. And if $R_{01} = R_{02}$, then s_{12} and s_{21} are simple voltage ratios. For a passive reciprocal network, $s_{21} = s_{12}$.

Scattering parameters s_{11} and s_{22} are reflection coefficients. They can be measured directly by means of slotted lines, directional couplers, voltage-standing-wave ratios and impedance bridges. Scattering parameters s_{12} and s_{21} are voltage transducer gains. All the parameters are frequency-dependent, dimensionless complex numbers. At any one frequency all four parameters must be known to describe the two-port device completely.

There are several advantages for letting $R_{01} = R_{02} = Z_0$.

- The s_{11} and s_{22} parameters are power reflection coefficients that are difficult to measure under normal loading. However, if $R_{01} = R_{02} = Z_0$, the parameters become equal to voltage reflection coefficients and can be measured directly with available test equipment.
- The s_{12} and s_{21} are square roots of the transducer power gain, the ratio of power absorbed in the load over the source power available. But for $R_{01} = R_{02} = Z_0$, they become a voltage ratio and can be measured with a vector voltmeter.
- The actual measurement can be taken at a distance from the input or output ports. The measured scattering parameter is the same as the parameter existing at the actual location of the particular port. Measurement is achieved by connecting input and output ports to source and load by means of transmission lines having the same impedance, Z_0,



25°C	100 Mhz	300 Mhz	100°C	100 Mhz	300 Mhz
s_{11}	0.62 < −44.0°	0.305 < −81.0°	s_{11}	0.690 < −40°	0.372 < −71°
s_{12}	0.0115 < +75.0°	0.024 < +93.0°	s_{12}	0.0125 < +76.0°	0.0254 < +89.5°
s_{21}	9.0 < +130°	3.85 < +91.0°	s_{21}	8.30 < +133.0°	3.82 < +94.0°
s_{22}	0.955 < −6.0°	0.860 < −14.0°	s_{22}	0.955 < −6.0°	0.880 < −15.0°

25°C	590 Mhz	1,000 Mhz	100°C	500 Mhz	1,000 Mhz
s_{11}	0.238 < −119.0°	0.207 < +175.0°	s_{11}	0.260 < −96.0°	0.196 < +175.0°
s_{12}	0.0385 < +110.0°	0.178 < +110.0°	s_{12}	0.0435 < +100.0°	0.165 < +103.0°
s_{21}	2.19 < +66.0°	1.30 < +33.0°	s_{21}	2.36 < +69.5°	1.36 < +35.0°
s_{22}	0.830 < −26.0°	0.838 < −49.5°	s_{22}	0.820 < −28.0°	0.850 < −53.0°

Scattering parameters can be measured directly using the Hewlett-Packard 8405A vector voltmeter. It covers the frequency range of 1 to 1,000 megahertz and determines s_{21} and s_{12} by measuring ratios of voltages and phase difference between the input and output ports. Operator at Texas Instruments Incorporated measures s-parameter data for TI's 2N3571 transistor series. Values for $V_{CE} = 10$ volts; $I_C = 5$ milliamperes.

as the source and load. In this way compensation can be made for added cable length.

▪ Transistors can be placed in reversible fixtures to measure the reverse parameters s_{22} and s_{12} with the equipment used to measure s_{11} and s_{21}.

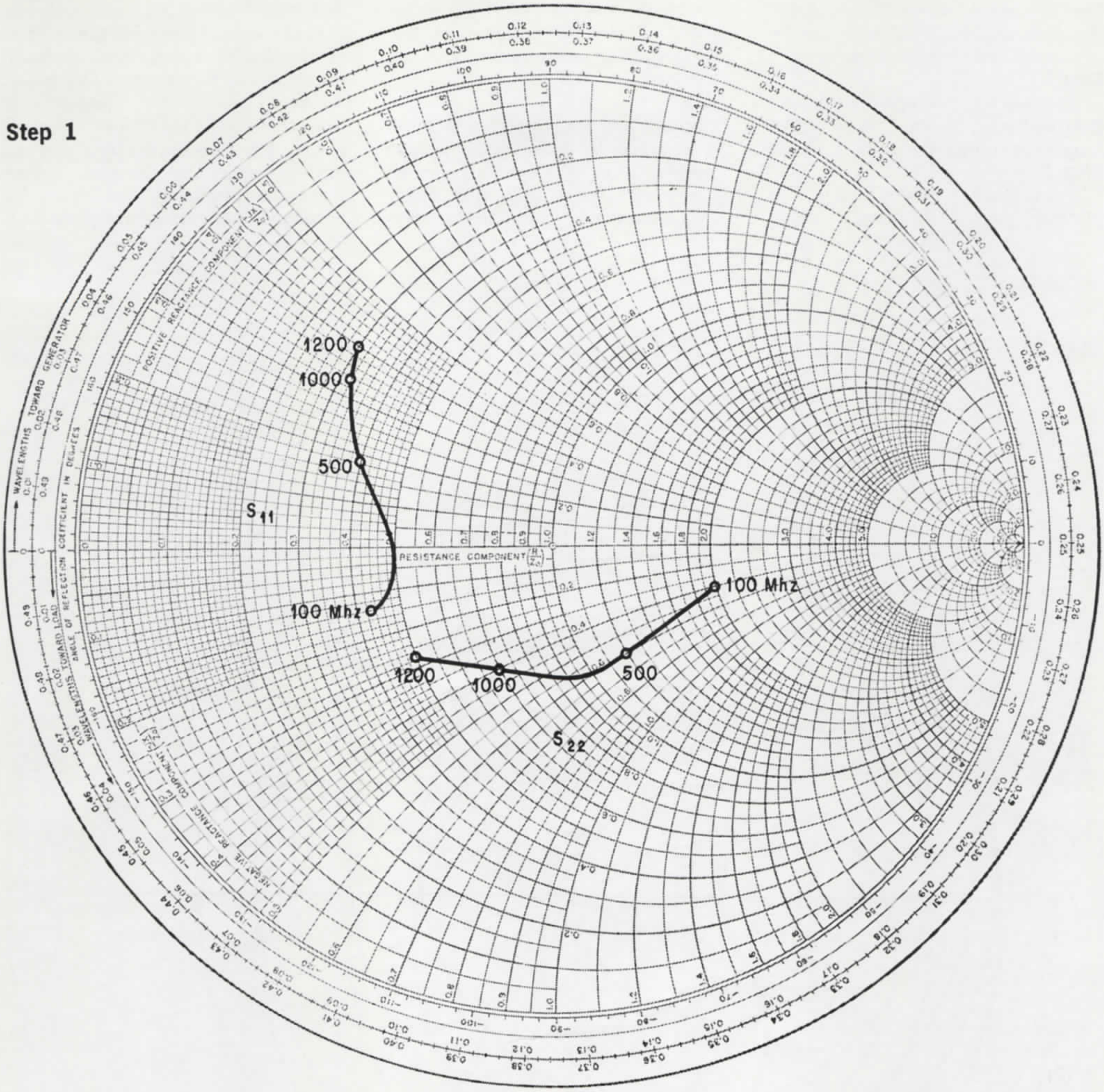
The Hewlett-Packard Co.'s 8405A vector voltmeter measures s parameters. It covers the frequency range of 1 to 1,000 megahertz and determines s_{21} and s_{12} by measuring voltage ratios and phase differences between the input and output ports directly on two meters, as shown above. A dual-directional coupler samples incident and reflected voltages to measure the magnitude and phase of the reflection coefficient.

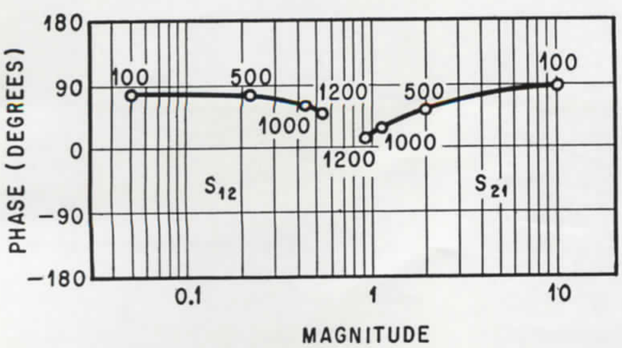
To perform measurements at a distance, the setup on page 86 is convenient. The generator and the load are the only points accessible for measurement. Any suitable test equipment, such as a vector voltmeter, directional coupler or slotted line can be connected. In measuring the s_{21} parameter as shown in the schematic, the measured vector quantity V_2/E_o is the voltage transducer gain or forward gain scattering parameter of the two-port and cables of length L_1 and L_2. The scattering parameter s_{21} of the two-port itself is the same vector V_2/E_o but turned by an angle of $360° (L_1 + L_2)/\lambda$ in a counterclockwise direction.

Plotting s_{11} in the complex plane shows the conditions for measuring s_{11}. Measured vector r_1 is the reflection coefficient of the two-port plus

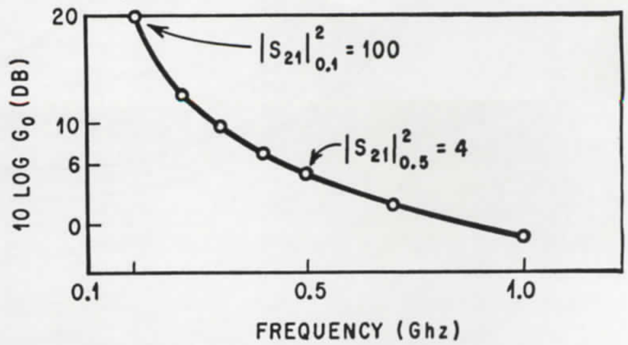
Amplifier design with unilateral s parameters

Step 1





Step 2



From the measured data transducer power gain is plotted as decibels versus frequency. From the plot an amplifier of constant gain is designed. Smith chart is used to plot the scattering parameters.

To design an amplifier stage, a source and load impedance combination must be found that gives the gain desired. Synthesis can be accomplished in three stages.

Step 1

The vector voltmeter measures the scattering parameters over the frequency range desired.

Step 2

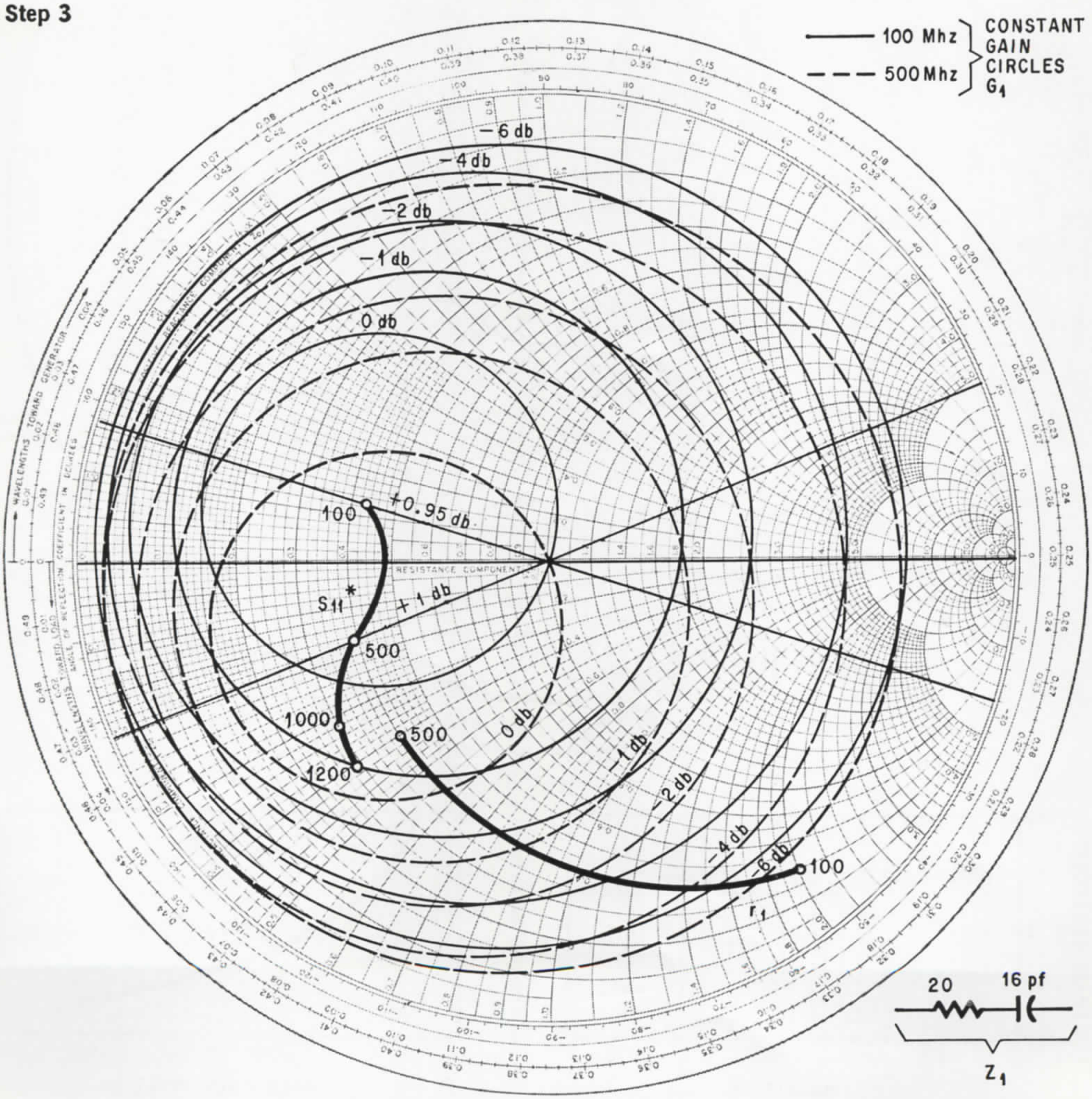
Transducer power gain is plotted versus frequency using equation 19 and the measured data from step 1. This determines the frequency response of the uncompensated transistor network so that a constant-gain amplifier can be designed.

Step 3

Source and load impedances must be selected to provide the proper compensation of a constant power gain from 100 to 500 Mhz. Such a constant-gain amplifier is designed according to the following:

- Plot s_{11}^* on the Smith chart. The magnitude of s_{11}^* is the linear distance measured from the center of the Smith chart. Radius from the center of the chart to any point on the locus of s_{11} represents a reflection coefficient r. The value of r can therefore be determined at any frequency by drawing a line from the origin of the chart to a value of s_{11}^* at the frequency of

Step 3



Source impedance is found by inspecting the input plane for realizable source loci that give proper gain. Phase angle is read on the peripheral scale "angle of reflection coefficient in degrees."

interest. The value of r is scaled proportionately with a maximum value of 1.0 at the periphery of the chart. The phase angle is read on the peripheral scale "angle of reflection coefficient in degrees." Constant-gain circles are plotted using equations 24 and 25 for G_1. These correspond to values of 0, -1, -2, -4 and -6 decibels for $s_{11}°$ at 100 and 500 Mhz. Construction procedure is shown on page 83.

- Constant-gain circles for $s_{22}°$ at 100 and 500 Mhz are constructed similarly to that below.

- The gain G_0 drops from 20 db at 100 Mhz to 6 db at 500 Mhz, a net reduction of 14 db. It is desirable to find source and load impedances that will flatten this slope over this frequency range. For this case it is accomplished by choosing a value of r_1 and r_2 on the constant-gain circle at 100 Mhz, each corresponding to a loss of -7 db. If this value of r_1 and r_2 falls on circles of 0-db gain at 500 Mhz, the over-all gain will be:

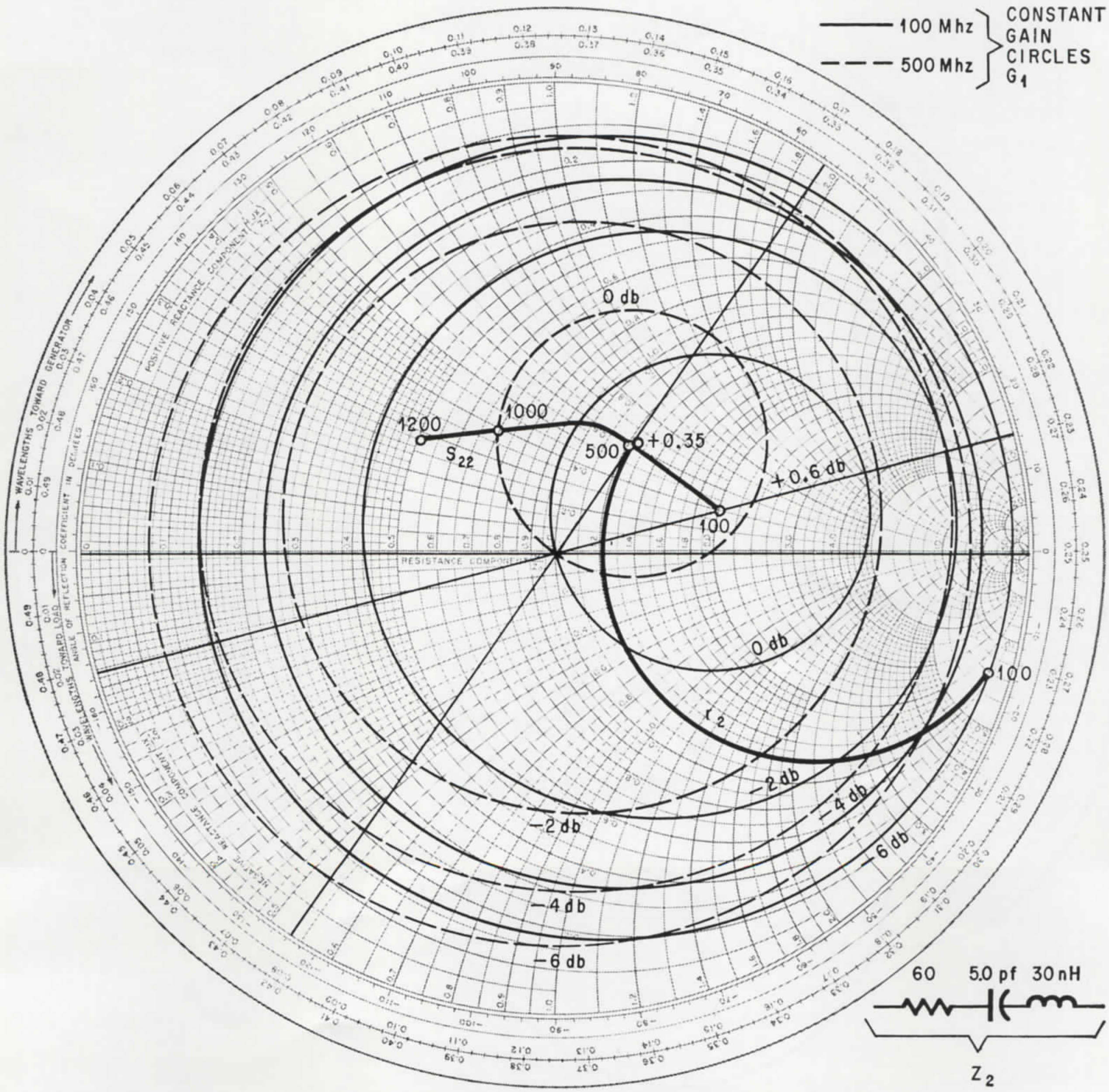
At 100 Mhz,

$$G_T(db) = G_0 + G_1 + G_2 = 20 - 7 - 7 = +6 \text{ db}$$

At 500 Mhz,

$$G_T(db) = 6 + 0 + 0 = +6 \text{ db}$$

- A source impedance of 20 ohms resistance in series with 16 picofarads of capacitance is chosen. Its value is equal to 50 (0.4 − j2) ohms at 100 Mhz. This point crosses the r_1 locus at about the -7 db constant-gain circle of G_1 as illustrated on page 83. At 500 Mhz this impedance combination equals 50 (0.4 — j0.4) ohms and is located at approximately the +0.5 db constant-gain circle. The selection of source impedance is an iterative process of inspection of



Load impedance is found by inspecting the output plane for loci that give proper gain.

the input r_1 plane on the Smith chart. The impedance values at various frequencies between 100 and 500 Mhz are tried until an impedance that corresponds to an approximate constant—gain circle necessary for constant power gain across the band is found.

- At the output port a G_2 of −6 db at 100 Mhz and +0.35 db at 500 Mhz is obtained by selecting a load impedance of 60 ohms in series with 5 pf and 30 nanohenries.
- The gain is:

At 100 Mhz,

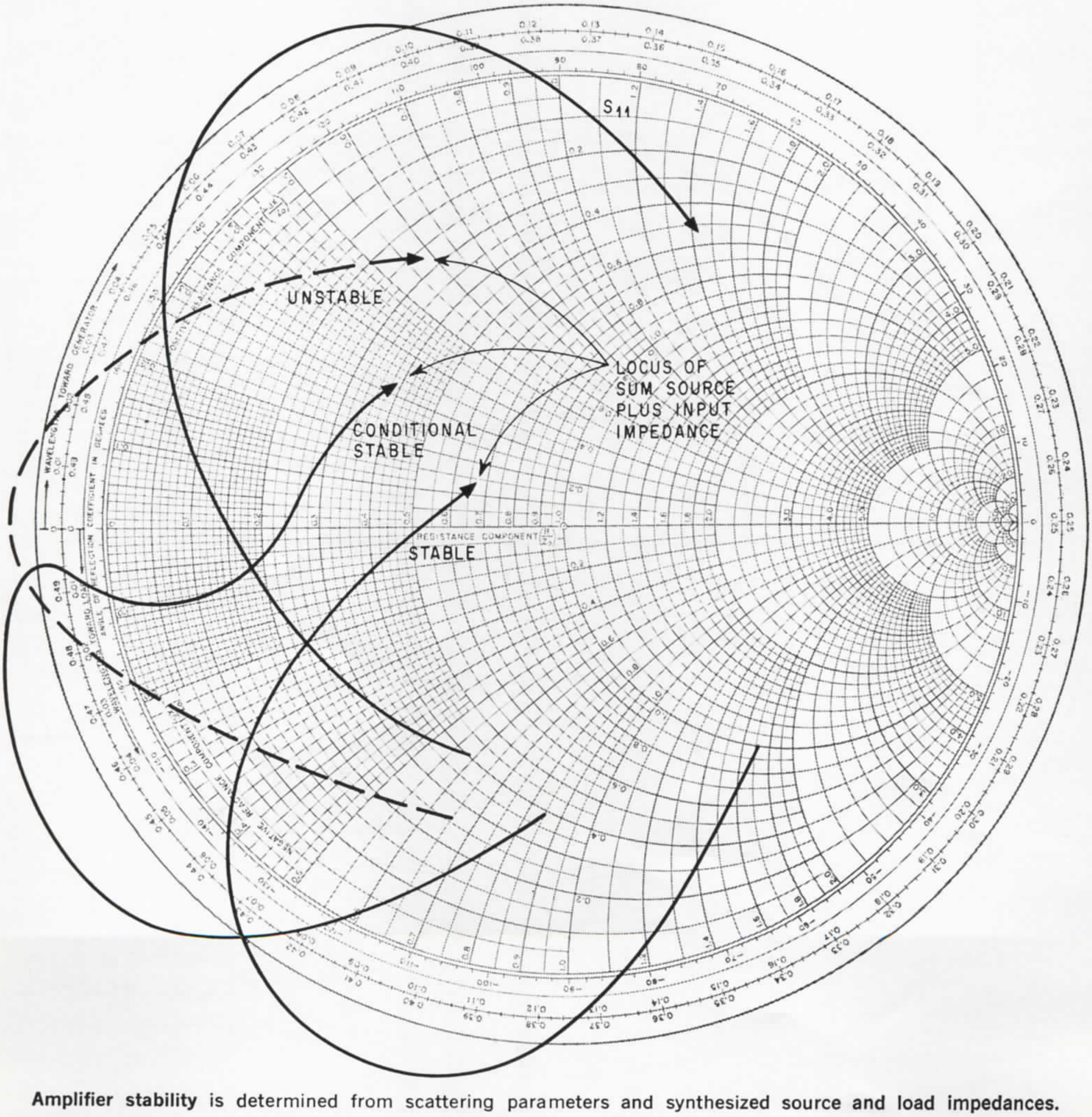
$$G_T(db) = G_0 + G_1 + G_2 = 20 - 7 - 6 = +7 \text{ db}$$

At 500 Mhz,

$$6 + 0.5 + 0.35 = +6.85 \text{ db}$$

Thus the 14-db variation from 100 to 500 Mhz is reduced to 0.15 db by selecting the proper source and load impedances.

Stability criterion. Important in the design of amplifiers is stability, or resistance to oscillation. Stability is determined for the unilateral case from the measured s parameters and the synthesized source and load impedances. Oscillations are only possible if either the input or the output port, or both, have negative resistances. This occurs if s_{11} or s_{22} are greater than unity. However, even with negative resistances the amplifier might be stable. The condition for stability is that the locus of the sum of input plus source impedance, or output plus load impedance, does not include zero impedance from frequencies zero to infinity [shown in figure below]. The technique is similar to Nyquist's feedback stability criterion and has been derived directly from it.



Amplifier stability is determined from scattering parameters and synthesized source and load impedances.

input cable L_1 (the length of the output cable has no influence). The scattering parameter s_{11} of the two-port is the same vector r_1 but turned at an angle $720° \, L_1/\lambda$ in a counterclockwise direction.

Using the Smith chart

Many circuit designs require that the impedance of the port characterized by s_{11} or the reflection coefficient r be known. Since the s parameters are in units of reflection coefficient, they can be plotted directly on a Smith chart and easily manipulated to establish optimum gain with matching networks. The relationship between reflection coefficient r and the impedance R is

$$r = \frac{R - Z_0}{R + Z_0} \qquad (16)$$

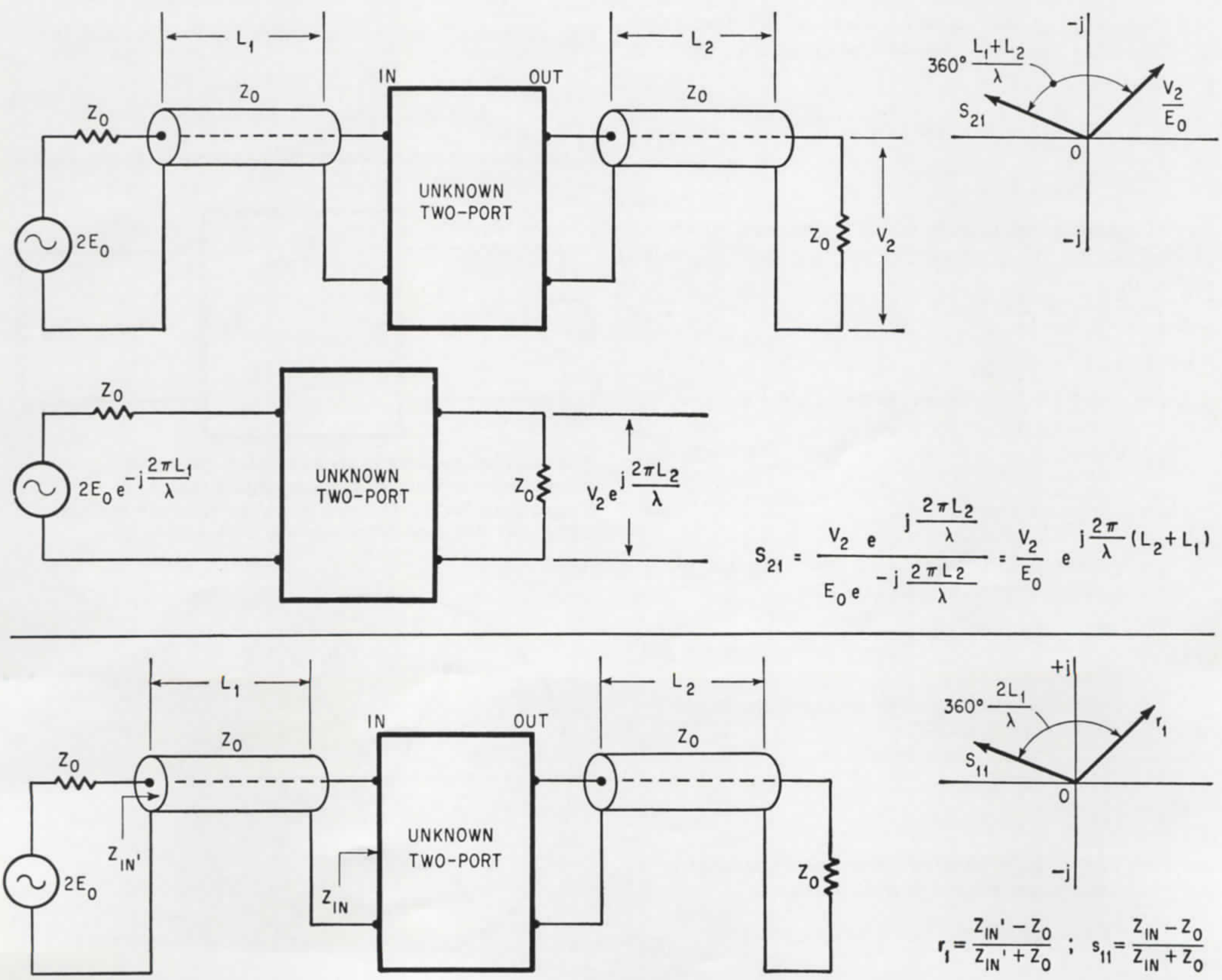
The Smith chart plots rectangular impedance coordinates in the reflection coefficient plane. When the s_{11} or s_{22} parameter is plotted on a Smith chart, the real and imaginary part of the impedance may be read directly.

It is also possible to chart equation 1 on polar coordinates showing the magnitude and phase of the impedance R in the complex reflection coefficient plane. Such a plot is termed the Charter chart. Both charts are limited to impedances having positive resistances, $|r_1| < 1$. When measuring transistor parameters, impedances with negative resistances are sometimes found. Then, extended charts can be used.

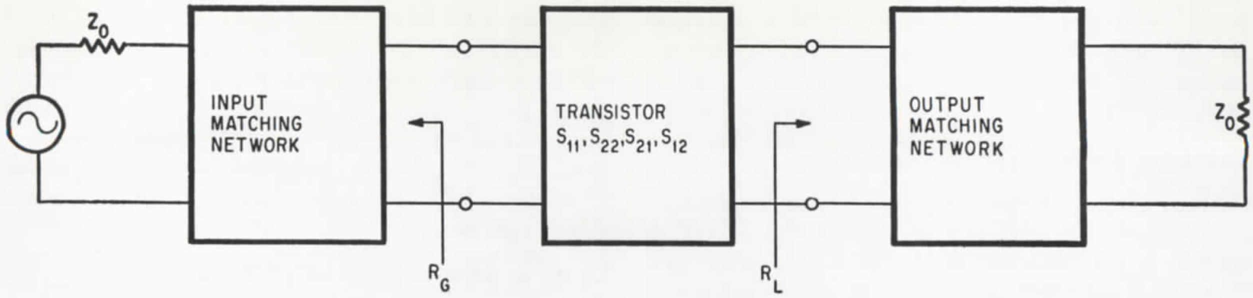
Using a parameter in amplifier design

Measurement of a device's s parameters provides data on input and output impedance and forward and reverse gain. In measuring, a device is inserted between known impedances, usually 50 ohms. In practice it may be desirable to achieve higher gain by changing source or load impedances or both.

An amplifier stage may now be designed in two steps. First, source and load impedances must be found that give the desired gain. Then the impedances must be synthesized, usually as matching networks between a fixed impedance source or between the load and the device [see block diagram top of p. 87].



S parameters can be measured remotely. Top test setup is for measuring s_{21}; bottom, for s_{11}. Measured vector V_2/E_0 is the voltage transducer gain of the two-port and cables L_1 and L_2. The measured vector r_1 is the reflection coefficient of the two-port plus input cable $L_1 + L_2$. Appropriate vectors for r_1 and s parameters are plotted.



To design an amplifier stage, source and load impedances are found to give the gain desired. Then impedances are synthesized, usually as matching networks between a fixed impedance source or the load and the device. When using s parameters to design a transistor amplifier, it is advantageous to distinguish between a simplified or unilateral design for times when s_{12} can be neglected and when it must be used.

When designing a transistor amplifier with the aid of s parameters, it is advantageous to distinguish between a simplified or unilateral design for instances where the reverse-transmission parameter s_{12} can be neglected and the more general case in which s_{21} must be shown. The unilateral design is much simpler and is, for many applications, sufficient.

Unilateral-circuit definitions

Transducer power gain is defined as the ratio of amplifier output power to available source power.

$$G_T = \frac{I_2^2 . R_{e2}}{\frac{E_0^2}{4R_{e1}}} \qquad (17)$$

For the unilateral circuit G_T is expressed in terms of the scattering parameters s_{11}, s_{21} and s_{12} with $s_{12} = 0$.

$$G_T = G_0 . G_1 . G_2 \qquad (18)$$

where:

$G_0 = |s_{21}|^2$ = transducer power gain for $R_1 = Z_0 = R_2$ (19)

$$G_1 = \frac{\left|1 - |r_1|^2\right|}{|1 - r_1 s_{11}|^2} \qquad (20)$$

= power gain contribution from change of source impedance from Z_0 to R_1

$$r_1 = \frac{R_1 - Z_0}{R_1 + Z_0} \qquad (21)$$

= reflection coefficient of source impedance with respect to Z_0

$$G_2 = \frac{\left|1 - |r_2|^2\right|}{|1 - r_2 s_{22}|^2} \qquad (22)$$

= power gain contribution from change of load impedance from Z_0 to R_2

$$r_2 = \frac{R_2 - Z_0}{R_2 + Z_0} \qquad (23)$$

= reflection coefficient of load impedance with respect to Z_0

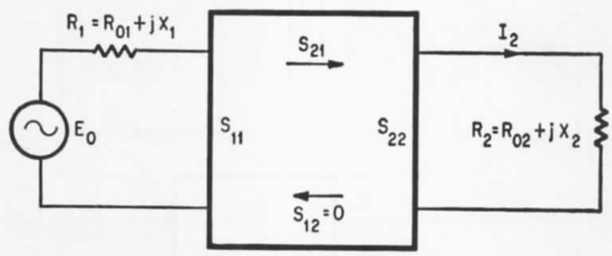
In designing an amplifier stage the graphical procedure shown at the bottom is helpful. The measured values of parameter s_{11} and its complex conjugate s_{11}^* are plotted on the Smith chart together with radius distances. Center of the constant-gain circles located on the line through s_{11}^* and the origin at a distance

$$r_{01} = \frac{G_1}{G_{1\,max}} \left[\frac{|s_{11}|}{1 - |s_{11}|^2 (1 - G_1/G_{1\,max})} \right] \qquad (24)$$

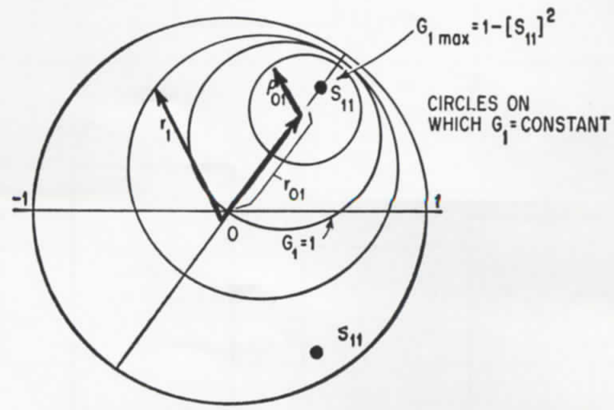
The radius of circles on which G_1 is constant is

$$\rho_{01} = \frac{\sqrt{1 - G_1/G_{1\,max}}\,(1 - |s_{11}|^2)}{1 - |s_{11}|^2 (1 - G_1/G_{1\,max})} \qquad (25)$$

If the source reflection coefficient r_1 is made equal



The two-port network is terminated at the ports by impedances containing resistance and reactance. Expressions for the transducer power gain can then be derived in terms of the scattering parameters.



A graphical plot helps in design of an amplifier stage. Here the measured parameters s_{11} and s_{11}^* are plotted on a Smith chart. The upper point is s_{11}^*.

to s_{11}^*, then the generator is matched to the load and the gain becomes maximum (G_{1max}). Constant-gain circles can be constructed, as shown, in 1- or 2-decibel increments or whatever is practical using equations 9 and 10.

If the source impedance R_1 or its reflection coefficient is plotted, the gain contribution G_1 is read directly from the gain circles. The same method is used to determine G_2 by plotting s_{22}, s_{22}^*, constant-gain circles and r_2.

Examples for the design procedure are given in greater detail in Transistor Parameter Measurements, Hewlett-Packard Application Note 77-1. The procedure is outlined in "Amplifier design with unilateral s parameters," beginning on page 82.

Measuring s parameters

S-parameter measurements of small-signal transistors require fairly sensitive measuring equipment. The input signal often cannot exceed 10 millivolts root mean square. On the other hand, wide frequency ranges are required as well as fast and easy operation. Recent advantages in measuring equipment have provided a fast and accurate measuring system. It is based on the use of a newly developed instrument, the H-P sampling vector voltmeter 8405A [see photo p. 81], and couplers.

The vector voltmeter covers a frequency range of 1 to 1,000 Mhz, a voltage measurement range of 100 microvolts full scale and a phase range of $\pm 180°$ with $0.1°$ resolution. It is tuned automatically by means of a phase-locked loop.

Directional couplers are used to measure reflection coefficients and impedances. A directional coupler consists of a pair of parallel transmission lines that exhibit a magnetic and electric coupling between them. One, called the main line, is connected to the generator and load to be measured. Measurement is taken at the output of the other, called the auxiliary line. Both lines are built to have a well defined characteristic impedance; 50 ohms is usual. The voltage coupled into the auxiliary line consists of components proportional to the voltage and current in the main line. The coupling is arranged so that both components are equal in magnitude when the load impedance equals the characteristic impedance of the line.

Directional couplers using two auxiliary lines in reverse orientation are called dual-directional couplers. A feature of the unit is a movable reference plane; the point where the physical measurement is taken can be moved along the line connecting the coupler with the unknown load. A line stretcher is connected to the output of the first auxiliary line.

The reference plane is set closer to the transistor package than the minimum lead length used with the transistor. Additional lead length is then considered part of the matching networks. The influence of lead length is also measured by changing the location of the reference plane.

Measurement of s_{11} parameter is made when the instrument is switched to one of two positions. The quotient V_B/V_A equals the magnitude of s_{11}. Its phase is read directly on the 8405A meter. When switched to the alternate position, the s_{21} parameter is read directly from the same ratio.

Accuracy and limitations

When measuring small-signal scattering parameters, a-c levels beyond which the device is considered linear must not be exceeded. In a grounded-emitter or grounded-base configuration, input voltage is limited to about 10 millivolts rms maximum (when measuring s_{11} and s_{21}). Much higher voltages can be applied when measuring s_{22} and s_{12} parameters. In uncertain cases linearity is checked by taking the same measurements at a sampling of several different levels.

The system shown is inherently broadband. Frequency is not necessarily limited by the published range of the dual directional couplers. The coupling factor K falls off inversely with frequency below the low-frequency limit of a coupler. The factor K does not appear in the result as long as it is the same for each auxiliary port. Since construction of couplers guarantees this to a high degree, measurements can be made at lower frequencies than are specified for the coupler.

The system's measurement accuracy depends on the accuracy of the vector voltmeter and the couplers. Although it is possible to short circuit the reference planes of the transistors at each frequency, it is not desirable for fast measurements. Hence, broadband tracking of all auxiliary arms of the couplers and tracking of both channels of the vector voltmeter are important. Tracking errors are within about 0.5 db of magnitude and $\pm 3°$ of phase over wide frequency bands. Accuracy of measuring impedances expressed by s_{11} and s_{22} degrade for resistances and impedances having a high reactive component. This is because s_{11} or s_{22} are very close to unity. These cases are usually confined to lower frequencies.

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