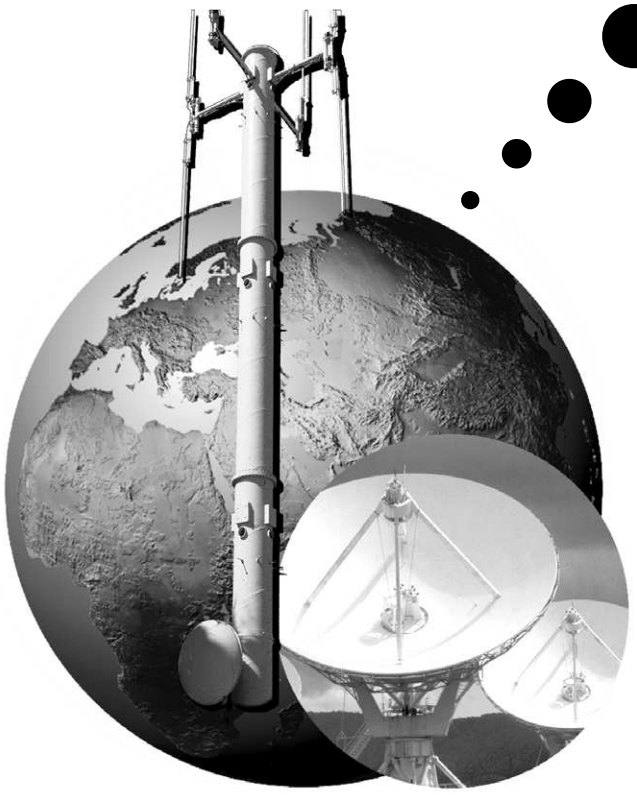
Agilent
Fundamentals of RF and Microwave Noise Figure Measurements

Application Note 57-1



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Chapter 1.
What is Noise Figure?

Introduction

Modern receiving systems must often process very weak signals, but the noise added by the system components tends to obscure those very weak signals. Sensitivity, bit error ratio (BER) and noise figure are system parameters that characterize the ability to process low-level signals. Of these parameters, noise figure is unique in that it is suitable not only for characterizing the entire system but also the system components such as the pre-amplifier, mixer, and IF amplifier that make up the system. By controlling the noise figure and gain of system components, the designer directly controls the noise figure of the overall system. Once the noise figure is known, system sensitivity can be easily estimated from system bandwidth. Noise figure is often the key parameter that differentiates one system from another, one amplifier from another, and one transistor from another. Such widespread application of noise figure specifications implies that highly repeatable and accurate measurements between suppliers and their customers are very important.

The reason for measuring noise properties of networks is to minimize the problem of noise generated in receiving systems. One approach to overcome noise is to make the weak signal stronger. This can be accomplished by raising the signal power transmitted in the direction of the receiver, or by increasing the amount of power the receiving antenna intercepts, for example, by increasing the aperture of the receiving antenna. Raising antenna gain, which usually means a larger antenna, and raising the transmitter power, are eventually limited by government regulations, engineering considerations, or economics. The other approach is to minimize the noise generated within receiver components. Noise measurements are key to assuring that the added noise is minimal. Once noise joins the signals, receiver components can no longer distinguish noise in the signal frequency band from legitimate signal fluctuations. The signal and noise get processed together. Subsequent raising of the signal level with gain, for example, will raise the noise level an equal amount.

This application note is part of a series about noise measurement. Much of what is discussed is either material that is common to most noise figure measurements or background material. It should prove useful as a primer on noise figure measurements. The need for highly repeatable, accurate and meaningful measurements of noise without the complexity of manual measurements and calculations has lead to the development of noise figure measurement instruments with simple user interfaces. Using these instruments does not require an extensive background in noise theory. A little noise background may prove helpful, however, in building confidence and understanding a more complete picture of noise in RF and microwave systems. Other literature to consider for additional information on noise figure measurements is indicated throughout this note. Numbers appearing throughout this document in square brackets [ ] correspond to the same numerical listing in the References section. Related Agilent Technologies literature and web resources appear later in this application note.



NFA simplifies noise figure measurements

The Importance of Noise in Communication Systems

The signal-to-noise (S/N) ratio at the output of receiving systems is a very important criterion in communication systems. Identifying or listening to radio signals in the presence of noise is a commonly experienced difficulty. The ability to interpret the audio information, however, is difficult to quantify because it depends on such human factors as language familiarity, fatigue, training, experience and the nature of the message. Noise figure and sensitivity are measurable and objective figures of merit. Noise figure and sensitivity are closely related (see Sensitivity in the glossary). For digital communication systems, a quantitative reliability measure is often stated in terms of bit error ratio (BER) or the probability P(e) that any received bit is in error. BER is related to noise figure in a non-linear way. As the S/N ratio decreases gradually, for example, the BER increases suddenly near the noise level where l's and 0's become confused. Noise figure shows the health of the system but BER shows whether the system is dead or alive. Figure 1-1, which shows the probability of error vs. carrier-to-noise ratio for several types of digital modulation, indicates that BER changes by several orders of magnitude for only a few dB change in signal-to-noise ratio.

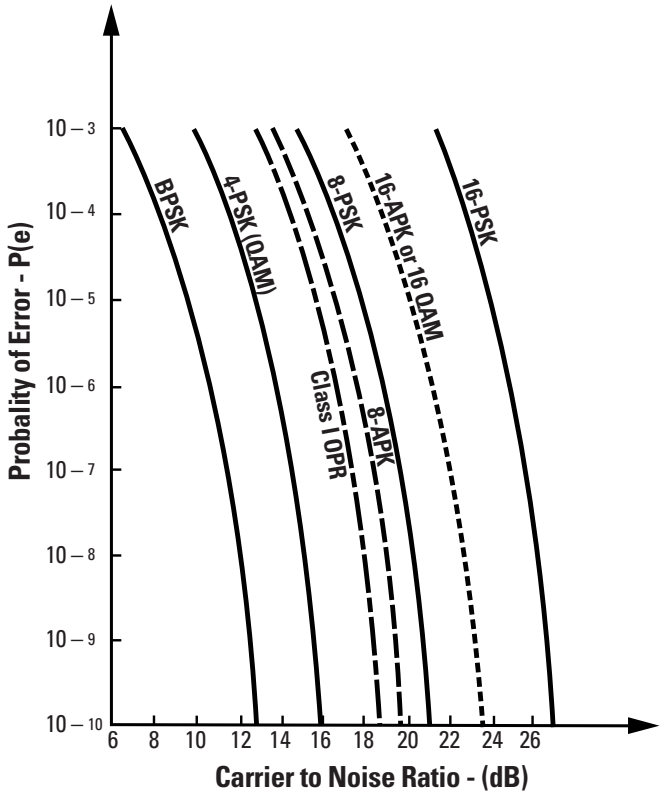

Figure 1-1. Probability of error, P(e), as a function of carrier-to-noise ratio, C/N (which can be interpreted as signal-to-noise ratio), for various kinds of digital modulation. From Kamilo Feher, DIGITAL COMMUNICATIONS: Microwave Applications, ©1981, p.71. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, NJ

The output signal-to-noise ratio depends on two things—the input signal-to-noise ratio and the noise figure. In terrestrial systems the input signal-to-noise ratio is a function of the transmitted power, transmitter antenna gain, atmospheric transmission coefficient, atmospheric temperature, receiver antenna gain, and receiver noise figure. Lowering the receiver noise figure has the same effect on the output signal-to-noise ratio as improving any one of the other quantities.

In satellite systems, noise figure may be particularly important. Consider the example of lowering a direct broadcast satellite (DBS) receiver's noise figure from 2 dB to 1dB by improving the LNA (low noise amplifier) in the receiver. This can have nearly the same effect on the signal-to-noise ratio as doubling the transmitter power. Doubling the satellite transmitter power, if allowed, can be very costly compared to the small cost of improving the LNA.



DBS receiver

In the case of a production line that produces satellite receivers, it may be quite easy to reduce the noise figure 1 dB by adjusting impedance levels or carefully selecting specific transistors. A 1dB reduction in noise figure has approximately the same effect as increasing the antenna diameter by 40%. But increasing the diameter could change the design and significantly raise the cost of the antenna and support structure.

Sometimes noise is an important parameter of transmitter design. For example, if a linear, broadband, power amplifier is used on a base station, excess broadband noise could degrade the signal-to-noise ratio at the adjacent channels and limit the effectiveness of the system. The noise figure of the power amplifier could be measured to provide a figure of merit to insure acceptable noise levels before it is installed in the system.

Sources of Noise

The noise being characterized by noise measurements consists of spontaneous fluctuations caused by ordinary phenomena in the electrical equipment. Thermal noise arises from vibrations of conduction electrons and holes due to their finite temperature. Some of the vibrations have spectral content within the frequency band of interest and contribute noise to the signals. The noise spectrum produced by thermal noise is nearly uniform over RF and microwave frequencies. The power delivered by a thermal source into an impedance matched load is kTB watts, where k is Boltzmann's constant (1.38×10^{-23} joules/K), T is the temperature in K, and B is the systems noise bandwidth. The available power is independent of the source impedance. The available power into a matched load is directly proportional to the bandwidth so that twice the bandwidth would allow twice the power to be delivered to the load. (see Thermal Noise in the glossary)

Shot noise arises from the quantized nature of current flow (see Shot Noise in the glossary). Other random phenomena occur in nature that are quantized and produce noise in the manner of shot noise. Examples are the generation and recombination of hole/electron pairs in semiconductors (G-R noise), and the division of emitter current between the base and collector in transistors (partition noise). These noise generating mechanisms have the characteristic that like thermal noise, the frequency spectra is essentially uniform, producing equal power density across the entire RF and microwave frequency range.

There are many causes of random noise in electrical devices. Noise characterization usually refers to the combined effect from all the causes in a component. The combined effect is often referred to as if it all were caused by thermal noise. Referring to a device as having a certain noise temperature does not mean that the component is that physical temperature, but merely that it's noise power is equivalent to a thermal source of that temperature. Although the noise temperature does not directly correspond with physical temperature there may be a dependence on temperature. Some very low noise figures can be achieved when the device is cooled to a temperature below ambient.

Noise as referred to in this application note does not include human-generated interference, although such interference is very important when receiving weak signals. This note is not concerned with noise from ignition, sparks, or with undesired pick-up of spurious signals. Nor is this note concerned with erratic disturbances like electrical storms in the atmosphere. Such noise problems are usually resolved by techniques like relocation, filtering, and proper shielding. Yet these sources of noise are important here in one sense—they upset the measurements of the spontaneous noise this note is concerned with. A manufacturer of LNAs may have difficulty measuring the noise figure because there is commonly a base station nearby radiating RF power at the very frequencies they are using to make their sensitive measurements. For this reason, accurate noise figure measurements are often performed in shielded rooms.

The Concept of Noise Figure

The most basic definition of noise figure came into popular use in the 1940's when Harold Friis [8] defined the noise figure F of a network to be the ratio of the signal-to-noise power ratio at the input to the signal-to-noise power ratio at the output.

$$F = \frac{S_i/N_i}{S_o/N_o} \qquad (1\text{-}1)$$

Thus the noise figure of a network is the decrease or degradation in the signal-to-noise ratio as the signal goes through the network. A perfect amplifier would amplify the noise at its input along with the signal, maintaining the same signal-to-noise ratio at its input and output (the source of input noise is often thermal noise associated with the earth's surface temperature or with losses in the system). A realistic amplifier, however, also adds some extra noise from its own components and degrades the signal-to-noise ratio. A low noise figure means that very little noise is added by the network. The concept of noise figure only fits networks (with at least one input and one output port) that process signals. This note is mainly about two-port networks; although mixers are in general three-port devices, they are usually treated the same as a two-port device with the local oscillator connected to the third port.

It might be worthwhile to mention what noise figure does not characterize. Noise figure is not a quality factor of networks with one port; it is not a quality factor of terminations or of oscillators. Oscillators have their own quality factors like "carrier-to-noise ratio" and "phase noise". But receiver noise generated in the sidebands of the local oscillator driving the mixer, can get added by the mixer. Such added noise increases the noise figure of the receiver.

Noise figure has nothing to do with modulation or demodulation. It is independent of the modulation format and of the fidelity of modulators and demodulators. Noise figure is, therefore, a more general concept than noise-quieting used to indicate the sensitivity of FM receivers or BER used in digital communications.

Noise figure should be thought of as separate from gain. Once noise is added to the signal, subsequent gain amplifies signal and noise together and does not change the signal-to-noise ratio.

Figure 1-2(a) shows an example situation at the input of an amplifier. The depicted signal is 40 dB above the noise floor: Figure 1-2(b) shows the situation at the amplifier output. The amplifier's gain has boosted the signal by 20 dB. It also boosted the input noise level by 20 dB and then added its own noise. The output signal is now only 30 dB above the noise floor. Since the degradation in signal-to-noise ratio is 10 dB, the amplifier has a 10 dB noise figure.

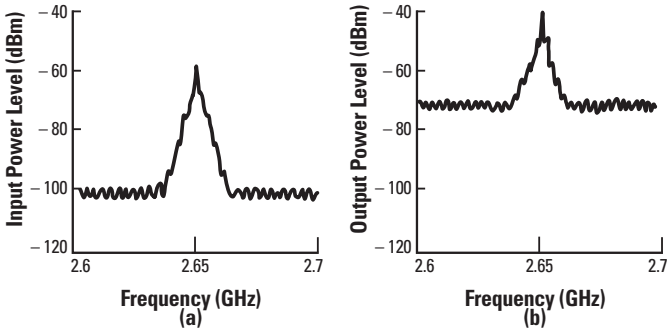

Figure 1-2. Typical signal and noise levels vs. frequency (a) at an amplifier's input and (b) at its output. Note that the noise level rises more than the signal level due to added noise from amplifier circuits. This relative rise in noise level is expressed by the amplifier noise figure.

Note that if the input signal level were 5 dB lower (35 dB above the noise floor) it would also be 5 dB lower at the output (25 dB above the noise floor), and the noise figure would still be 10 dB. Thus noise figure is independent of the input signal level.

A more subtle effect will now be described. The degradation in a network's signal-to-noise ratio is dependent on the temperature of the source that excites the network. This can be proven with a calculation of the noise figure F, where S_i and N_i represent the signal and noise levels

$$F = \frac{S_i/N_i}{S_o/N_o}$$

$$= \frac{S_i/N_i}{GS_i/(N_a + GN_i)}$$

$$= \frac{N_a + GN_i}{GN_i} \qquad (1\text{-}2)$$

available at the input to the device under test (DUT), S_o and N_o represent the signal and noise levels available at the output, N_a is the noise added by the DUT, and G is the gain of the DUT. Equation (1-2) shows the dependence on noise at the input N_i. The input noise level is usually thermal noise from the source and is referred to by kT_oB. Friis [8] suggested a reference source temperature of 290K (denoted by T_o), which is equivalent to 16.8° C and 62.3° F. This temperature is close to the average temperature seen by receiving antennas directed across the atmosphere at the transmitting antenna.

The power spectral density kT_o, furthermore, is the even number 4.00 x 10^{-21} watts per hertz of bandwidth (–174 dBm/Hz). The IRE (forerunner of the IEEE) adopted 290K as the standard temperature for determining noise figure [7]. Then equation (1-2) becomes

$$F = \frac{N_a + kT_oBG}{kT_oBG} \qquad (1\text{-}3)$$

which is the definition of noise figure adopted by the IRE.

Noise figure is generally a function of frequency but it is usually independent of bandwidth (so long as the measurement bandwidth is narrow enough to resolve variations with frequency). Noise powers N_a and N_i of equation (1-2) are each proportional to bandwidth. But the bandwidth in the numerator of (1-2) cancels with that of the denominator—resulting in noise figure being independent of bandwidth.

In summary, the noise figure of a DUT is the degradation in the signal-to-noise ratio as a signal passes through the DUT. The specific input noise level for determining the degradation is that associated with a 290K source temperature. The noise figure of a DUT is independent of the signal level so long as the DUT is linear (output power vs. input power).

The IEEE Standard definition of noise figure, equation (1-3), states that noise figure is the ratio of the total noise power output to that portion of the noise power output due to noise at the input when the input source temperature is 290K.

While the quantity F in equation (1-3) is often called "noise figure", more often it is called "noise factor" or sometimes "noise figure in linear terms". Modern usage of "noise figure" usually is reserved for the quantity NF, expressed in dB units:

$$NF = 10 \log F \qquad (1\text{-}4)$$

This is the convention used in the remainder of this application note.

Noise Figure and Noise Temperature

Sometimes "effective input noise temperature", T_e, is used to describe the noise performance of a device rather than the noise figure, (NF). Quite often temperature units are used for devices used in satellite receivers. T_e is the equivalent temperature of a source impedance into a perfect (noise-free) device that would produce the same added noise, N_a. It is often defined as

$$T_e = \frac{N_a}{kGB} \qquad (1\text{-}5)$$

It can be related to the noise factor F:

$$T_e = T_o(F\text{-}1), \quad \text{where } T_o \text{ is 290K} \qquad (1\text{-}6)$$

The input noise level present in terrestrial VHF and microwave communications is often close to the 290K reference temperature used in noise figure calculations due to the earth's surface temperature. When this is the case, a 3 dB change in noise figure will result in a 3 dB change in the signal-to-noise ratio.

In satellite receivers the noise level coming from the antenna can be far less, limited by sidelobe radiation and the background sky temperature to values often below 100K. In these situations, a 3 dB change in the receiver noise figure may result in much more than 3 dB signal-to-noise change. While system performance may be calculated using noise figure without any errors (the 290K reference temperature need not correspond to actual temperature), system designers may prefer to use T_e as a system parameter.

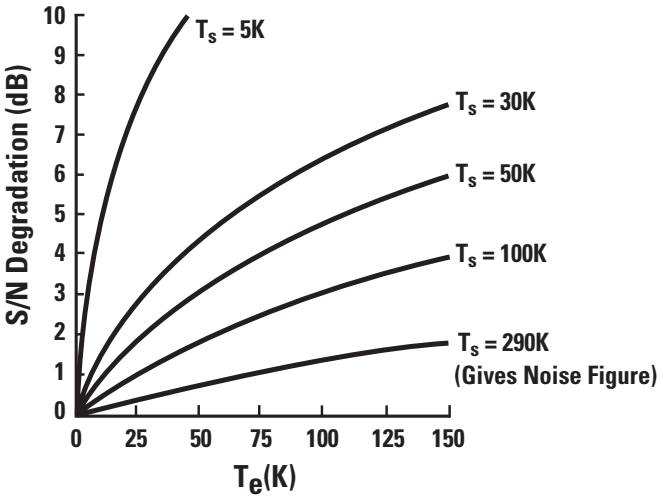

Figure 1-3. Degradation in the S/N ratio vs T_e of a device for various values of temperature for the source impedance. Noise figure is defined for a source temperature of 290K.

Chapter 2.
Noise Characteristics of Two-Port Networks

The Noise Figure of Multi-stage Systems

The noise figure definition covered in Chapter 1 can be applied to both individual components such as a single transistor amplifier, or to a complete system such as a receiver. The overall noise figure of the system can be calculated if the individual noise figures and gains of the system components are known. To find the noise figure of each component in a system, the internal noise added by each stage, N_a, must be found. The gain must also be known. The actual methods used to determine noise and gain are covered in Chapter 3: The Measurement of Noise Figure. The basic relationship between the individual components and the system will be discussed here.

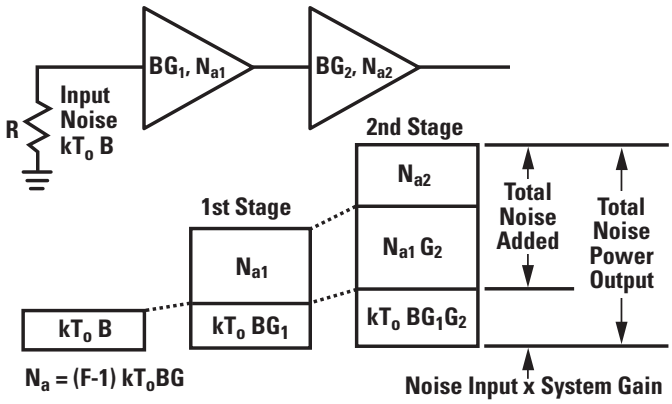

Figure 2-1. The effect of second stage contribution.

For two stages see Figure 2-1, the output noise will consist of the kT_oB source noise amplified by both gains, G_1G_2, plus the first amplifier output noise, N_{a1}, amplified by the second gain, G_2, plus the second amplifiers output noise, N_{a2}. The noise power contributions may be added since they are uncorrelated. Using equation (1-3) to express the individual amplifier noise contributions, the output noise can be expressed in terms of their noise factors, F.

$$N_o = kT_oBG_1G_2\left[F_1 + \frac{F_2 - 1}{G_1}\right] \qquad (2\text{-}1)$$

With the output noise known, the noise factor of the combination of both amplifiers can be calculated using equation (1-1). This is the overall system noise figure of this two-stage example.

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} \qquad (2\text{-}2)$$

The quantity $(F_2\text{-}1)/G_1$ is often called the second stage contribution. One can see that as long as the first stage gain is high, the second stage contribution will be small. This is why the pre-amplifier gain is an important parameter in receiver design. Equation (2-2)can be re-written to find F_1 if the gain and overall system noise factor is known. This is the basis of corrected noise measurements and will be discussed in the next chapter.

This calculation may be extended to a n-stage cascade of devices and expressed as

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \ldots \quad \frac{F_n - 1}{G_1G_2\ldots G_{n-1}} \qquad (2\text{-}3)$$

Equation (2-3) is often called the cascade noise equation.

Gain and Mismatch

The device gain is an important parameter in noise calculations. When an input power of kT_oB is used in these calculations, it is an available power, the maximum that can be delivered to a matched load. If the device has a large input mismatch (not unusual for low-noise amplifiers), the actual power delivered to the device would be less. If the gain of the device is defined as the ratio of the actual power delivered to the load to the maximum power available from the source we can ignore the mismatch loss present at the input of the device since it is taken into account in our gain definition. This definition of gain is called transducer gain, G_t. When cascading devices, however, mismatch errors arise if the input impedance of the device differs from the load impedance. In this case the total gain of a cascaded series of devices does not equal the product of the gains.

Available gain, (G_a), is often given as a transistor parameter, it is the gain that will result when a given source admittance, Y_s, drives the device and the output is matched to the load. It is often used when designing amplifiers. Refer to the glossary for a more complete description of the different definitions of gain.

Most often insertion gain, G_i, or the forward transmission coefficient, $(S_{21})^2$, is the quantity specified or measured for gain in a 50 ohm system. If the measurement system has low reflection coefficients and the device has a good output match there will be little error in applying the cascade noise figure equation (2-3) to actual systems. If the device has a poor output match or the measurement system has significant mismatch errors, an error between the actual system and calculated performance will occur. If, for example, the output impedance of the first stage was different from the 50 ohm source impedance that was used when the second stage was characterized for noise figure, the noise generated in the second stage could be altered. Fortunately, the second stage noise contribution is reduced by the first stage gain so that in many applications errors involving the second stage are minimal. When the first stage has low gain ($G \leq F_2$), second stage errors can become significant. The complete analysis of mismatch effects in noise calculations is lengthy and generally requires understanding the dependence of noise figure on source impedance. This effect, in addition to the gain mismatch effect, will be discussed in the next section (Noise Parameters). It is because of this noise figure dependence that S-parameter correction is not as useful as it would seem in removing the errors associated with mismatch [4]

Noise Parameters

Noise figure is, in principle, a simplified model of the actual noise in a system. A single, theoretical noise element is present in each stage. Most actual amplifying devices such as transistors can have multiple noise contributors; thermal, shot, and partition as examples. The effect of source impedance on these noise generation processes can be a very complex relationship. The noise figure that results from a noise figure measurement is influenced by the match of the noise source and the match of the measuring instrument; the noise source is the source impedance for the DUT, and the DUT is the source impedance for the measuring instrument. The actual noise figure performance of the device when it is in its operating environment will be determined by the match of other system components.

Designing low noise amplifiers requires tradeoffs between the gain of a stage and its corresponding noise figure. These decisions require knowledge of how the active device's gain and noise figure change as a function of the source impedance or admittance. The minimum noise figure does not necessarily occur at either the system impedance, Z_o, or at the conjugate match impedance that maximizes gain.

To fully understand the effect of mismatch in a system, two characterizations of the device-under-test (DUT) are needed, one for noise figure and another for gain. While S-parameter correction can be used to calculate the available gain in a perfectly matched system, it can not be used to find the optimum noise figure. A noise parameter characterization uses a special tuner to present different complex impedances to the DUT. [29]

The dependence of noise factor on source impedance presented by the tuner is described by

$$F = F_{min} + \frac{4R_n}{Z_o}\left(\frac{|\Gamma_{opt} - \Gamma_s|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_s|^2)}\right) \qquad (2\text{-}4)$$

where the Γ_s is the source reflection coefficient that results in the noise factor F. In the equation, F_{min} is the minimum noise factor for the device that occurs when $\Gamma_s = \Gamma_{opt}$. R_n is the noise resistance (the sensitivity of noise figure to source admittance changes). F_{min}, R_n, and Γ_{opt} are frequently referred to as the "noise parameters", and it is their determination which is called "noise characterization". When Γ_s is plotted on a Smith chart for a set of constant noise factors, F, the result is "noise circles". Noise circles are a convenient format to display the complex relation between source impedance and noise figure.

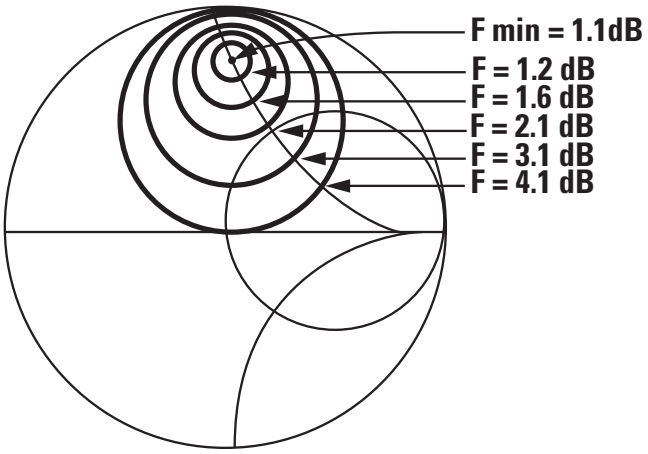

Figure 2-2. Noise circles

The available gain, G_a, provided by a device when it is driven by a specified source impedance, can be calculated from the S-parameters of the device [35, 40] and the source reflection coefficient, Γ_s, using equation (2-5). S-parameters are commonly measured with a network analyzer.

$$G_a = \frac{(1 - |\Gamma_s|^2)|S_{21}|^2}{|1 - S_{11}\Gamma_s|^2 (1 - |S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}|^2)} \qquad (2\text{-}5)$$

When the source reflection coefficient, Γ_s, is plotted on a Smith chart corresponding to a set of fixed gains, "gain circles" result. Gain circles are a convenient format to display the relation between source impedance and gain.

The Effect of Bandwidth

Although the system bandwidth is an important factor in many systems and is involved in the actual signal-to-noise calculations for demodulated signals, noise figure is independent of device bandwidth. A general assumption made when performing noise measurements is that the device to be tested has an amplitude-versus-frequency characteristic that is constant over the measurement bandwidth. This means that noise measurement bandwidth should be less than the device bandwidth. When this is not the case, an error will be introduced [34]. The higher end Agilent NFA series noise figure analyzers have variable bandwidths to facilitate measurement of narrow-band devices, as do spectrum analyzer-based measurement systems.

Most often the bandwidth-defining element in a system, such as a receiver, will be the IF or the detector. It will usually have a bandwidth much narrower than the RF circuits. In this case noise figure is a valid parameter to describe the noise performance of the RF circuitry. In the unusual case where the RF circuits have a bandwidth narrower than the IF or detector, noise figure may still be used as a figure of merit for comparisons, but a complete analysis of the system signal-to-noise ratio will require the input bandwidth as a parameter.

Chapter 3.
The Measurement of Noise Figure

Noise Power Linearity

The basis of most noise figure measurements depends on a fundamental characteristic of linear two-port devices, noise linearity. The noise power out of a device is linearly dependent on the input noise power or temperature as shown in Figure 3-1. If the slope of this characteristic and a reference point is known, the output power corresponding to a noiseless input power, N_a can be found. From N_a the noise figure or effective input noise temperature can be calculated as described in Chapter 1. Because of the need for linearity, any automatic gain control (AGC) circuitry must be deactivated for noise figure measurements.

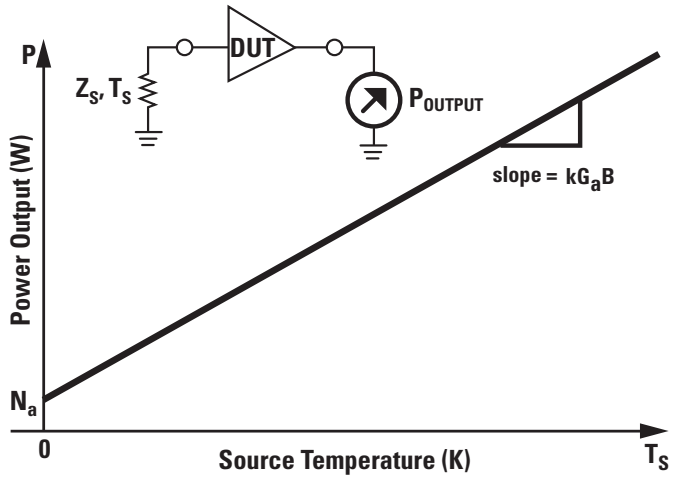

Figure 3-1. The straight-line power output vs. source temperature characteristic of linear, two-port devices. For a source impedance with a temperature of absolute zero, the power output consists solely of added noise N_a from the device under test (DUT). For other source temperatures the power output is increased by thermal noise from the source amplified by the gain characteristic of the DUT.

Noise Sources

One way of determining the noise slope is to apply two different levels of input noise and measure the output power change. A noise source is a device that will provide these two known levels of noise. The most popular noise source consists of a special low-capacitance diode that generates noise when reverse biased into avalanche breakdown with a constant current [5]. Precision noise sources such as the Agilent SNS-series have an output attenuator to provide a low SWR to minimize mismatch errors in measurements. If there is a difference between the on and off state impedance an error can be introduced into the noise figure measurement [23]. The N4000A noise source has a larger value of attenuation to minimize this effect.

When the diode is biased, the output noise will be greater than kT_cB due to avalanche noise generation in the diode [11, 12, 13, 15, 20, 21]; when unbiased, the output will be the thermal noise produced in the attenuator, kT_cB. These levels are sometimes called T_h and T_c corresponding to the terms “hot” and “cold”. The N4001A produces noise levels approximately equivalent to a 10,000K when on and 290K when off. Diode noise sources are available to 50 GHz from Agilent.



SNS-Series Noise Source

To make noise figure measurements a noise source must have a calibrated output noise level, represented by excess noise ratio (ENR). Unique ENR calibration information is supplied with the noise source and, in the case of the SNS-Series, is stored internally on EEPROM. Other noise sources come with data on a floppy disk, or hard-copy. ENR_{dB} is the ratio, expressed in dB of the difference between T_h and T_c, divided by 290K. It should be noted that a 0 dB ENR noise source produces a 290K temperature change between its on and off states. ENR is not the “on” noise relative to kTB as is often erroneously believed.

$$ENR_{dB} = 10 \log\left(\frac{T_h - T_c}{T_o}\right) \qquad (3\text{-}1)$$

T_c in equation (3-1) is assumed to be 290K when it is calibrated. When the noise source is used at a different physical temperature, compensation must be applied to the measurement. The SNS-Series noise sources contain a temperature sensor which can be read by Agilent's NFA analyzers. The temperature compensation will be covered in the next section of this chapter.

In many noise figure calculations the linear form of ENR will be used.

$$ENR = 10^{\frac{ENR_{dB}}{10}} \qquad (3\text{-}2)$$

Noise sources may be calibrated from a transfer standard noise source (calibrated traceable to a top level National Standards laboratory) or by a primary physical standard such as a hot/cold load. Most noise sources will be supplied with an ENR characterized versus frequency.

Hot and cold loads are used in some special applications as a noise source. Ideally the two loads need to be kept at constant temperatures for good measurement precision. One method immerses one load into liquid nitrogen at a temperature of 77K, the other may be kept at room temperature or in a temperature controlled oven. The relatively small temperature difference compared to noise diode sources and potential SWR changes resulting from switching to different temperature loads usually limits this method to calibration labs and millimeter-wave users.

Gas discharge tubes imbedded into waveguide structures produce noise due to the kinetic energy of the plasma. Traditionally they have been used as a source of millimeter-wave noise. They have been essentially replaced by solid-state noise diodes at frequencies below 50 GHz. The noise diode is simpler to use and generally is a more stable source of noise. Although the noise diode is generally a coaxial device, integral, precision waveguide adapters may be used to provide a waveguide output.



R/Q 347B waveguide noise sources

The Y-Factor Method

The Y-Factor method is the basis of most noise figure measurements whether they are manual or automatically performed internally in a noise figure analyzer. Using a noise source, this method allows the determination of the internal noise in the DUT and therefore the noise figure or effective input noise temperature.

With a noise source connected to the DUT, the output power can be measured corresponding to the noise source on and the noise source off (N_2 and N_1). The ratio of these two powers is called the Y-factor. The power detector used to make this measurement may be a power meter, spectrum analyzer, or a special internal power detector in the case of noise figure meters and analyzers. The relative level accuracy is important. One of the advantages of modern noise figure analyzers is that the internal power detector is very linear and can very precisely measure level changes. The absolute power level accuracy of the measuring device is not important since a ratio is to be measured.

$$Y = \frac{N_2}{N_1} \qquad (3\text{-}3)$$

Sometimes this ratio is measured in dB units, in this case:

$$Y = 10^{\frac{Y_{db}}{10}} \qquad (3\text{-}4)$$

The Y-factor and the ENR can be used to find the noise slope of the DUT that is depicted in Figure 3-1.

Since the calibrated ENR of the noise source represents a reference level for input noise, an equation for the DUT internal noise, N_a can be derived. In a modern noise figure analyzer, this will be automatically determined by modulating the noise source between the on and off states and applying internal calculations.

$$N_a = kT_oBG\left(\frac{ENR}{Y-1} - 1\right) \qquad (3\text{-}5)$$

From this we can derive a very simple expression for the noise factor. The noise factor that results is the total "system noise factor", F_{sys}. System noise factor includes the noise contribution of all the individual parts of the system. In this case the noise generated in the measuring instrument has been included as a second stage contribution. If the DUT gain is large ($G_1 >> F_2$), the noise contribution from this second stage will be small. The second stage contribution can be removed from the calculation of noise figure if the noise figure of the second stage and the gain of the DUT is known. This will be covered in the section on corrected noise figure and gain. Note that the device gain is not needed to find F_{sys}.

$$F_{sys} = \frac{ENR}{Y - 1} \tag{3-6}$$

When the noise figure is much higher than the ENR, the device noise tends to mask the noise source output. In this case the Y-factor will be very close to 1. Accurate measurement of small ratios can be difficult. Generally the Y-factor method is not used when the noise figure is more than 10 dB above the ENR of the noise source, depending on the measurement instrument.

This equation can be modified to correct for the condition when the noise source cold temperature, T_c, is not at the 290K reference temperature, T_o.

$$F_{sys} = \frac{ENR - Y\left(\frac{T_c}{T_o} - 1\right)}{Y - 1} \tag{3-7}$$

This often used equation assumes that T_h is unaffected by changes in T_c as is the case with hot and cold loads. With solid-state noise sources, T_h will likely be affected by changes in T_c. Since the physical noise source is at a temperature of T_c, the internal attenuator noise due to T_c is added both when the noise source is on and off. In this case it is better to assume that the noise change between the on and off state remains constant (T_h-T_c). This distinction is most important for low ENR noise sources when T_h is less than 10 T_c. An alternate equation can be used to correct for this case.

$$F_{sys} = \frac{ENR\left(\frac{T_c}{T_o}\right)}{Y - 1} \tag{3-8}$$

The Signal Generator Twice-power Method

Before noise sources were available this method was popular. It is still particularly useful for high noise figure devices where the Y-factors can be very small and difficult to accurately measure. First, the output power is measured with the device input terminated with a load at a temperature of approximately 290K. Then a signal generator is connected, providing a signal within the measurement bandwidth. The generator output power is adjusted to produce a 3 dB increase in the output power. If the generator power level and measurement bandwidth are known we can calculate the noise factor. It is not necessary to know the DUT gain.

$$F_{sys} = \frac{P_{gen}}{kT_oB} \tag{3-9}$$

There are some factors that limit the accuracy of this method. The noise bandwidth of the power-measuring device must be known, perhaps requiring a network analyzer. Noise bandwidth, B, is a calculated equivalent bandwidth, having a rectangular, "flat-top" spectral shape with the same gain bandwidth product as the actual filter shape. The output power must be measured on a device that measures true power since we have a mix of noise and a CW signal present. Thermal-based power meters measure true power very accurately but may require much amplification to read a low noise level and will require a bandwidth-defining filter. Spectrum analyzers have good sensitivity and a well-defined bandwidth but the detector may respond differently to CW signals and noise. Absolute level accuracy is not needed in the power detector since a ratio is being measured.

The Direct Noise Measurement Method

This method is also useful for high noise figure devices. The output power of the device is measured with an input termination at a temperature of approximately 290K. If the gain of the device and noise bandwidth of the measurement system is known, the noise factor can be determined.

$$F_{sys} = \frac{N_o}{kT_oBG} \tag{3-10}$$

Again with this method the noise bandwidth, B, must be known and the power-measuring device may need to be very sensitive. Unlike the twice-power method, the DUT gain must be known and the power detector must have absolute level accuracy.

Corrected Noise Figure and Gain

The previous measurements are used to measure the total system noise factor, F_{sys}, including the measurement system. Generally it is the DUT noise figure that is desired. From the cascade noise-figure equation it can be seen that if the DUT gain is large, the measurement system will have little effect on the measurement. The noise figure of high gain DUTs can be directly measured with the previously discussed methods. When a low gain DUT is to be measured or the highest accuracy is needed, a correction can be applied if we know the gain of the DUT and the noise figure of the system. Using equation (2-2) and re-writing to solve for F_1 gives the equation for the actual DUT noise factor.

$$F_1 = F_{sys} - \frac{F_2 - 1}{G_1} \qquad (3\text{-}11)$$

Both the gain of the DUT and the measurement system noise factor, F_2, can be determined with an additional noise source measurement. This step is called a system calibration. With a noise-figure analyzer this calibration is usually performed before connecting the DUT so that all subsequent measurements can use the corrections and the corrected noise figure can be displayed. The necessary calculations to find the gain and the corrected noise figure are automatically performed internally. When manual measurements are made with alternative instruments, a calibrated noise figure measurement can be performed as follows:

1. Connect the noise source directly to the measurement system and measure the noise power levels corresponding to the noise source "on" and "off". These levels; N_2 and N_1 respectively, can then be used to calculate the measurement system noise factor F_2 using the Y-factor method.

2. The DUT is inserted into the system. The noise levels N_2' and N_1' are measured when the noise source is turned on and off. The DUT gain can be calculated with the noise level values.

$$G_1 = \frac{N_2' - N_1'}{N_2 - N_1} \qquad (3\text{-}12)$$

   The gain is usually displayed in dB terms: $G_{db}=10 \log G$

3. The overall system noise factor, F_{sys}, can be calculated by applying the Y-factor method to the values N_2' and N_1'.

4. The DUT noise factor, F_1, can be calculated with equation (3-11). The DUT noise figure is $10 \log F_1$.

Jitter

Noise can be thought of as a series of random events, electrical impulses in this case. The goal of any noise measurement is to find the mean noise level at the output of the device. These levels can be used, with appropriate corrections, to calculate the actual noise figure of the device. In theory, the time required to find the true mean noise level would be infinite. In practice, averaging is performed over some finite time period. The difference between the measured average and the true mean will fluctuate and give rise to a repeatability error.

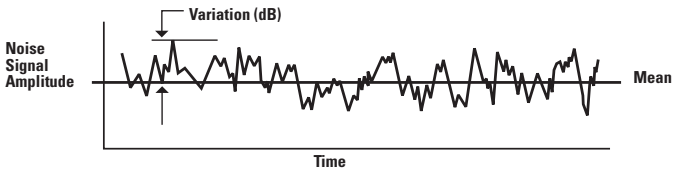

Figure 3-2. Noise jitter

For small variations, the deviation is proportional to $1/\sqrt{(t)}$ so that longer averaging times will produce better averages. Because the average includes more events it is closer to the true mean. The variation is also proportional to $1/\sqrt{(B)}$. Larger measurement bandwidths will produce a better average because there are more noise events per unit of time in a large bandwidth; therefore, more events are included in the average. Usually noise figure should be measured with a bandwidth as wide as possible but narrower than the DUT.

Frequency Converters

Frequency converters such as receivers and mixers usually are designed to convert an RF frequency band to an IF frequency band. While the noise figure relationships discussed in this application note apply to converters as well as non-converters, there are some additional characteristics of these devices that can affect noise figure measurements. In addition to DUTs that are frequency converters, sometimes the noise measurement system uses mixing to extend the measurement frequency range.

Loss

Amplifiers usually have a gain associated with them, while passive mixers have loss. All the equations for noise figure still apply; however, the linear gain values used will be less than one. One implication of this can be seen by applying the cascade noise figure equation; the second stage noise contribution can be major (See equation 2-2). Another is that passive mixers, if measured using the Y-factor technique, can have small Y-factors owing to their high noise figures. This may increase measurement uncertainty. High ENR noise sources can be used to provide a larger Y-factor.

LO Noise

Receivers and mixers have local oscillator (LO) signals that may have noise present. This noise can be converted in the mixer to the IF frequency band and become an additional contribution to the system's noise figure. The magnitude of this effect varies widely depending on the specific mixer type and how much noise is in the LO. It is possible to eliminate this noise in fixed frequency LO systems with a band-pass filter on the LO port of the mixer. A filter that rejects noise at f_{LO}+/-f_{IF}, f_{IF}, and f_{RF} while passing f_{LO} will generally eliminate this noise. There may also be higher order noise conversions that could contribute if the LO noise level is very high. A lowpass filter can be used to prevent noise conversions at harmonics of the LO frequency.

LO Leakage

A residual LO signal may be present at the output (IF) of a mixer or converter. The presence of this signal is generally unrelated to the noise performance of the DUT and may be acceptable when used for the intended application. When a noise figure measurement is made, this LO signal may overload the noise measurement instrument or create other spurious mixing products. This is most likely to be an issue when the measuring system has a broadband amplifier or other unfiltered circuit at it's input. Often a filter can be added to the instrument input to filter out the LO signal while passing the IF.

Unwanted Responses

Sometimes the desired RF frequency band is not the only band that converts to the IF frequency band. Unwanted frequency band conversions may occur if unwanted frequencies are present at the RF port in addition to the desired RF signal. Some of these are: the image response ($f_{LO} + f_{IF}$ or $f_{LO} - f_{IF}$ depending on the converter), harmonic responses ($2f_{LO} \pm f_{IF}$, $3f_{LO} \pm f_{IF}$, etc.), spurious responses, and IF feed-through response. Often, particularly in receivers, these responses are negligible due to internal filtering. With many other devices, especially mixers, one or more of these responses may be present and may convert additional noise to the IF frequency band.

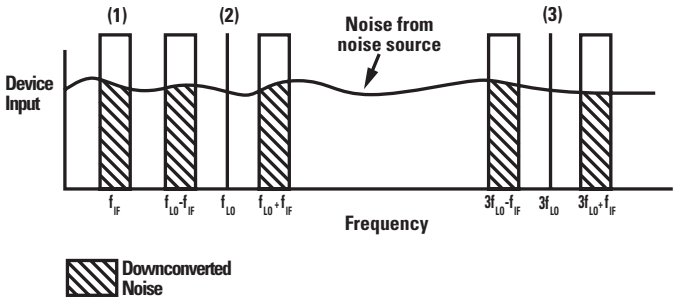

Figure 3-3. Possible noise conversion mechanisms with mixers and converters. (1) IF feedthrough response, (2) double sideband response, (3) harmonic response.

Mixers having two main responses ($f_{LO} + f_{IF}$ and $f_{LO} - f_{IF}$) are often termed double side-band (DSB) mixers. $f_{LO} + f_{RF}$ is called the upper side-band (USB). $f_{LO} - f_{IF}$ is called the lower side-band (LSB). They convert noise in both frequency bands to the IF frequency band. When such a mixer is part of the noise measurement system, the second response will create an error in noise figure measurements unless a correction, usually +3dB, is applied. Ideally filtering is used at the RF port to eliminate the second response so that single side-band (SSB) measurements can be made.

When a DSB mixer is the DUT we have a choice when measuring the noise figure. Usually the user wants to measure the equivalent SSB noise figure. In passive mixers that do not have LO noise, the equivalent SSB noise figure is often close in value to the conversion loss measured with a CW signal. There are two ways to make this measurement; an input filter can be used, or the +3dB correction can be applied. There are accuracy implications with these methods that must be considered if precision measurements are to be made; an input filter will add loss that should be corrected for, the +3dB correction factor assumes equal USB and LSB responses.

Converters used in noise receivers, such as radiometers and radiometric sensors are often designed to make use of both main responses, in which case it is desirable to know the DSB noise figure. In this case, no correction or input filter is used; the resulting noise figure measured will be in DSB terms.

Noise Figure Measuring Instruments

Noise Figure Analyzers

The noise figure analyzer represents the most recent evolution of noise figure measurement solutions. A noise figure analyzer in its most basic form consists of a receiver with an accurate power detector and a circuit to power the noise source. It provides for ENR entry and displays the resulting noise figure value corresponding to the frequency it is tuned to. Internally a noise figure analyzer computes the noise figure using the Y-factor method.

A noise figure analyzer allows the display of swept frequency noise figure and gain and associated features such as markers and limit lines. The Agilent NFA series noise figure analyzers, combined with the SNS-Series noise sources offer improvements in accuracy and measurement speed, important factors in manufacturing environments. The NFA is specifically designed and optimized for one purpose: to make noise figure measurements. Combination products that must make other measurements usually compromises accuracy to some degree.



NFA Series noise figure analyzer

Features:
- Flexible, intuitive user interface makes it easy to characterize amplifiers and frequency-converting devices
- Measurement to 26.5 GHz in a single instrument eliminates the need for a separate system downconverter
- Accurate and repeatable results allow tighter specification of device performance.

Spectrum Analyzers

Spectrum analyzers are often used to measure noise figure, because they are already present in the test racks of many RF and microwave production facilities performing a variety of tasks. With software and a controller they can be used to measure noise figure using any of the methods outlined in this product note. They are particularly useful for measuring high noise figure devices using the signal generator or direct power measurement method. The variable resolution bandwidths allow measurement of narrow-band devices.

The 8590-series, option 119 spectrum analyzers were the first to provide built-in noise figure measurements with downloadable personality software, performing the measurement and calculations internally, allowing the user to measure noise figure directly.



8591E Spectrum analyzer

One of the advantages of a spectrum analyzer-based noise figure analyzer is the multi-functionality. It can, for example, make distortion measurements on an amplifier. Also it can locate spurious or stray signals and then the noise figure of the device can be measured at frequencies where the signals will not interfere with noise measurements.

Spectrum analyzers generally require the addition of a low noise pre-amplifier to improve sensitivity. The user must take care not to overload the system with broadband noise power or stray signals. The firmware-based measurement algorithms of a noise figure analyzer can be faster than external software control of a spectrum analyzer, and the detector accuracy is not as good as the best dedicated noise figure analyzers/meters [24].

Network Analyzers

Like spectrum analyzers, network analyzers are common multi-use instruments in industry. Products are available that offer noise figure measurements in addition to the usual network measurements. An advantage is that they can offer other measurements commonly associated with devices: such as gain and match. Because network measurements are usually made with the same internal receiver architecture, there can be some performance limitations when used in noise figure applications. Often the receiver is of the double side-band (DSB) type, where noise figure is actually measured at two frequencies and an internal correction is applied. When a wide measurement bandwidth is used this may result in error if the device noise figure or gain is not constant over this frequency range. When narrow measurement bandwidth is used to measure narrow-band devices, the unused frequency spectrum between the upper and lower side-band does not contribute to the measurement and a longer measurement time is needed to reduce jitter (see Jitter in this chapter).

Network analyzers have the ability to measure the S-parameters of the device. It has been considered that S-parameter data can reduce noise figure measurement uncertainty by offering mismatch correction. Ideally this mismatch correction would provide a more accurate gain measurement of the device so that the second stage noise contribution can be subtracted with more precision. Unfortunately, the mismatch also effects the noise generation in the second stage which cannot be corrected for without knowing the noise parameters of the device. The same situation occurs at the input of the device when a mismatch is present between the noise source and DUT input. (see noise parameters in Chapter 2 of this note) [4]. Network analyzers do not, by themselves, provide measurement of the noise parameters. The measurement of noise parameters generally requires a tuner and software in addition to the network analyzer. The resulting measurement system can be complex and expensive. Error correction in a network analyzer is primarily of benefit for gain measurements and calculation of available gain.

Noise Parameter Test Sets

A noise parameter test set is usually used in conjunction with software, a vector network analyzer and a noise analyzer to make a series of measurements, allowing the determination of the noise parameters of the device [29] (see Noise Parameters in Chapter 2). Noise parameters can then be used to calculate the minimum device noise figure, the optimum source impedance, and the effect of source impedance on noise figure. The test set has an adjustable tuner to present various source impedances to the DUT. Internal networks provide bias to semi-conductor devices that may be tested. A noise source is coupled to the test set to allow noise figure measurements at different source impedances. The corresponding source impedances are measured with the network analyzer. From this data, the complete noise parameters of the device can be calculated. Generally the complete device S-parameters are also measured so that gain parameters can also be determined. Because of the number of measurements involved, measurement of the full noise parameters of a device is much slower than making a conventional noise figure measurement but yields useful design parameters. Noise parameters are often supplied on low-noise transistor data sheets. Noise parameters are generally not measured on components and assemblies that are intended to be used in well matched 50 (or 75) ohm systems because the source impedance is defined in the application.

Power Meters and True-RMS Voltmeters

As basic level measuring devices, power meters and true-RMS voltmeters can be used to measure noise figure with any of the methods described in this note with the necessary manual or computer calculations. Being broadband devices, they need a filter to limit their bandwidth to be narrower than the DUT. Such a filter will usually be fixed in frequency and allow measurements only at this frequency. Power meters are most often used to measure receiver noise figures where the receiver has a fixed IF frequency and much gain. The sensitivity of power meters and voltmeters is usually poor but the receiver may provide enough gain to make measurements. If additional gain is added ahead of a power meter to increase sensitivity, care should be taken to avoid temperature drift and oscillations.



EPM Series Power Meter

4. Glossary

Symbols and abbreviations

B	Noise Bandwidth
BER	Bit Error Ratio
$\|b_s\|^2$	Power delivered by a generator to a non reflecting load
C/N	Carrier to Noise Ratio
DBS	Direct Broadcast by Satellite
DSB	Double Sideband
DUT	Device Under Test
ENR_{dB}	Excess Noise Ratio
F	Noise Factor
F_1	First Stage Noise Factor
FM	Frequency Modulation
F_{min}	Minimum Noise Factor
F_{sys}	System Noise Factor
1/f	Flicker Noise
G_p	Power Gain
G_{ass}	Associated Gain
G_a	Available Gain
G_i	Insertion Gain
G_t	Transducer Gain
G/T	Gain-to-Temperature Ratio
IEEE	Institute of Electrical and Electronics Engineers
IF	Intermediate Frequency
IRE	Institute of Radio Engineers
K	Kelvins (Unit of Temperature)
k	Boltzmann's Constant
LNA	Low Noise Amplifier
LSB	Lower Sideband
M	Noise Measure
M_u	Mismatch Uncertainty
N_a	Noise Added
NF	Noise Figure
N_{off}	$=N_1$ (see Y Factor)
N_{on}	$=N_2$ (see Y Factor)
N_1	N_{out} for T_c (see Y Factor)
N_2	N_{out} for T_h (see Y Factor)
N_i	Input Noise Power
N_o	Output Noise Power
RF	Radio Frequency
RMS	Root Mean Square
R_n	Equivalent Noise Resistance
r_n	Equivalent Noise Resistance, normalized
RSS	Root Sum-of-the-Squares
S/N	Signal to Noise Ratio
SSB	Single Sideband
$\|S_{21}\|^2$	Forward Transmission Coefficient
S_i	Input Signal Power
S_o	Output Signal Power
T_a	Noise Temperature
T_C, T_c	Cold Temperature (see T_c)
T_e	Effective Input Noise Temperature
T_H, T_h	Hot Temperature (see T_h)
T_{ne}	Effective Noise Temperature
T_{off}	Off Temperature (see T_{off})
T_{on}	On Temperature (see T_{on})
T_{op}	Operating Noise Temperature
T_o	Standard Noise Temperature (290K)
T_s	Effective Source Noise Temperature
USB	Upper Sideband
Γ_{opt}	Optimum Source Reflection Coefficient
Γ_s	Source Reflection Coefficient
Γ_L	Load Reflection Coefficient

Glossary Terms

Associated Gain (G_{ass}). The available gain of a device when the source reflection coefficient is the optimum reflection coefficient Γ_{opt} corresponding with F_{min}.

Available Gain (G_a). [2, 35, 40] The ratio, at a specific frequency, of power available from the output of the network P_{ao} to the power available from the source P_{as}.

$$G_a = \frac{P_{ao}}{P_{as}} \tag{1}$$

For a source with output $|b_s|^2$ and reflection coefficient Γ_s

$$P_{as} = \frac{|b_s|^2}{1 - |\Gamma_s|^2} \tag{2}$$

$$P_{ao} = \frac{|b_s|^2 \, |S_{21}|^2 \, (1 - |\Gamma_2|^2)}{|(1 - \Gamma_s S_{11}) \, (1 - \Gamma_2^* S_{22}) - \Gamma_s \Gamma_2^* S_{12} S_{21}|^2} \tag{3}$$

where

$$\Gamma_2 = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s} \tag{4}$$

An alternative expression for the available output power is

$$P_{ao} = \frac{|b_s|^2 \, |S_{21}|^2}{|1 - \Gamma_s S_{11}|^2 (|1 - |\Gamma_2|^2)} \tag{5}$$

These lead to two expressions for G_a

$$G_a = |S_{21}|^2 \frac{(1 - |\Gamma_s|^2) \, (1 - |\Gamma_2|^2)}{|(1 - \Gamma_s S_{11}) \, (1 - \Gamma_2^* S_{22}) - \Gamma_s \Gamma_2^* S_{12} S_{21}|^2} \tag{6}$$

$$G_a = |S_{21}|^2 \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_s S_{11}|^2 \, (1 - |\Gamma_2|^2)} \tag{7}$$

NOTE: G_a is a function of the network parameters and of the source reflection coefficient Γ_s. G_a is independent of the load reflection coefficient Γ_L.
G_a is often expressed in dB

$$G_a(\text{dB}) = 10 \log \frac{P_{ao}}{P_{as}} \tag{8}$$

Bandwidth (B). See Noise Bandwidth.

Boltzmann's Constant (k). 1.38 x10^{-23} joules/kelvin.

Cascade Effect. [8]. The relationship, when several networks are connected in cascade, of the noise characteristics (F or T_e and G_a) of each individual network to the noise characteristics of the overall or combined network. If F_1, F_2, . . ., F_n (numerical ratios, not dB) are the individual noise figures and G_{a1}, G_{a2}, …,G_{an} (numerical ratios) are the individual available gains, the combined noise figure is

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1}G_{a2}} + \cdots + \frac{F_n - 1}{G_{a1}G_{a2} \cdots G_{a(n-1)}} \quad (1)$$

the combined available gain is

$$G_a = G_{a1}G_{a2} \cdots G_{an} \quad (2)$$

In terms of individual effective input noise temperatures T_{e1}, T_{e2}, …, T_{en} the overall effective input noise temperature is

$$T_e = T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e2}}{G_{a1}G_{a2}} + \cdots + \frac{T_{en}}{G_{a1}G_{a2} \cdots G_{a(n-1)}} \quad (3)$$

NOTE: Each F_i, T_{ei}, and G_{ai} above refers to the value for the source impedance that corresponds to the output impedance of the previous stage.

Diode Noise Source. [11, 12, 13, 15, 20, 21] A noise source that depends on the noise generated in a solid state diode that is reverse biased into the avalanche region. Excess noise ratios of well-matched devices are usually about 15 dB (T_{ne} ≈10000K). Higher excess noise ratios are possible by sacrificing impedance match and flat frequency response.

Double Sideband (DSB). See Single-sideband (SSB).

Effective Input Noise Temperature (T_e). [17] The noise temperature assigned to the impedance at the input port of a DUT which would, when connected to a noise-free equivalent of the DUT, yield the same output power as the actual DUT when it is connected to a noise-free input port impedance. The same temperature applies simultaneously for the entire set of frequencies that contribute to the out put frequency. If there are several input ports, each having a specified impedance, the same temperature applies simultaneously to all the ports. All ports except the output are to be considered input ports for purposes of defining T_e. For a two-port transducer with a single input and a single output frequency, T_e is related to the noise figure F by

$$T_e = 290(F–1) \quad (1)$$

Effective Noise Temperature (T_{ne}). [1] (This is a property of a one-port, for example, a noise source.) The temperature that yields the power emerging from the output port of the noise source when it is connected to a nonreflecting, nonemitting load. The relationship between the noise temperature T_a and effective noise temperature T_{ne} is

$$T_{ne} = T_a (1 - |\Gamma|^2) \quad (1)$$

where G is the reflection coefficient of the noise source. The proportionality factor for the emerging power is kB so that

$$T_{ne} = \frac{P_e}{(kB)} \quad (2)$$

where P_e is the emerging power, k is Boltzmann's constant, and B is the bandwidth of the power measurement. The power spectral density across the measurement bandwidth is assumed to be constant.

Equivalent Noise Resistance (r_n or R_n). See Noise Figure Circles.

Excess Noise Ratio (ENR). [1] A noise generator property calculated from the hot and cold noise temperatures (T_h and T_c) using the equation

$$ENR_{dB} = 10 \log \frac{T_h - T_c}{T_o} \quad (1)$$

where T_o is the standard temperature of 290K. Noise temperatures T_h and T_c should be the "effective" noise temperatures. (See Effective Noise Temperature) [25]. The ENR calibration of diode noise sources assumes T_c=T_o.

A few examples of the relationship between ENR and T_h may be worthwhile. An ENR of 0 dB corresponds to T_h = 580K. T_h of 100°C (373K) corresponds to an ENR of –5.43 dB. T_h of 290K corresponds to an ENR of –• dB.

Flicker Noise and 1/f Noise. [33, 39] Any noise whose power spectral density varies inversely with frequency. Especially important at audio frequencies or with GASFET's below about 100 MHz.

Forward Transmission Coefficient $(S_{21})^2$. The ratio, at a specific frequency, of the power delivered by the output of a network, to the power delivered to the input of the network when the network is terminated by a nonreflecting load and excited by a nonreflecting generator.

The magnitude of this parameter is often given in dB.

$$|S_{21}|^2 \text{ (dB)} = 10 \log |S_{21}|^2 \quad (1)$$

Gain to Temperature Ratio (G/T). [32, 41] A figure of merit for a satellite or radio astronomy receiver system, including the antenna, that portrays the operation of the total system. The numerator is the antenna gain, the denominator is the operating noise temperature of the receiver. The ratio is usually expressed in dB, for example, 10 log (G/T). G/T is often measured by comparing the receiver response when the antenna input is a "hot" celestial noise source to the response when the input is the background radiation of space (≈3K).

Gas Discharge Noise Source. [25, 26] A noise source that depends on the temperature of an ionized noble gas. This type of noise source usually requires several thousand volts to begin the discharge but only about a hundred volts to sustain the discharge. Components of the high turn-on voltage sometimes feed through the output to damage certain small, frail, low-noise, solid-state devices. The gas discharge noise source has been replaced by the avalanche diode noise source in most applications. Gas discharge tubes are still used at millimeter wavelengths. Excess noise ratios (ENR) for argon tubes is about 15.5 dB (10000K).

Gaussian Noise. [6] Noise whose probability distribution or probability density function is gaussian, that is, it has the standard form

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \quad (1)$$

where σ is the standard deviation. Noise that is steady or stationary in character and originates from the sum of a large number of small events, tends to be gaussian by the central limit theorem of probability theory. Thermal noise and shot noise are gaussian.

Hot/Cold Noise Source. In one sense most noise figure measurements depend on noise power measurements at two source temperatures—one hot and one cold. The expression "Hot/Cold," however, frequently refers to measurements made with a cold termination at liquid nitrogen temperatures (77K) or even liquid helium (4K), and a hot termination at 373K (100°C). Such terminations are sometimes used as primary standards and for highly accurate calibration laboratory measurements.

Insertion Gain (G_i). The gain that is measured by inserting the DUT between a generator and load. The numerator of the ratio is the power delivered to the load while the DUT is inserted, P_d. The denominator, or reference power P_r, is the power delivered to the load while the source is directly connected. Measuring the denominator might be called the calibration step.

$$G_i = \frac{P_d}{P_r} \quad (1)$$

The load power while the source and load are directly connected is

$$P_r = |b_r|^2 \frac{1 - |\Gamma_l|^2}{|1 - \Gamma_l\Gamma_s|^2} \quad (2)$$

where the subscript "r" denotes the source characteristics while establishing the reference power, i.e., during the calibration step. The load power while the DUT is inserted is

$$P_d = |b_d|^2|S_{21}|^2 \frac{1 - |\Gamma_l|^2}{|(1-\Gamma_{sd}S_{11})(1-\Gamma_l S_{22}) - \Gamma_l\Gamma_{sd}S_{12}S_{21}|^2} \quad (3)$$

or

$$P_d = |b_d|^2 |S_{21}|^2 \frac{1 - |\Gamma_l|^2}{|1-\Gamma_{sd}S_{11}|^2 |1-\Gamma_l\Gamma_2|^2} \quad (4)$$

$$\Gamma_2 = S_{22} + \frac{S_{12}S_{21}\Gamma_{sd}}{1 - \Gamma_{sd}S_{11}} \quad (5)$$

In equations (3,4, and 5) the subscript "d" denotes the source characteristics while the DUT is inserted. The S-parameters refer to the DUT. The source characteristics while calibrating and while the DUT is inserted are some times different. Consider that the DUT, for example, is a microwave receiver with a waveguide input and an IF output at 70 MHz. During the calibration step, the source has a coaxial output at 70 MHz, but while the DUT is inserted the source has a waveguide output at the microwave frequency. Using the above equations, insertion gain is

$$G_i = |S_{21}|^2 \frac{|b_d|^2}{|b_r|^2} \frac{|1 - \Gamma_l\Gamma_{sr}|^2}{|(1-\Gamma_{sd}S_{11})(1-\Gamma_l S_{22}) - \Gamma_l\Gamma_{sd}S_{12}S_{21}|^2} \quad (6)$$

or

$$G_i = |S_{21}|^2 \frac{|b_d|^2}{|b_r|^2} \frac{|1 - \Gamma_l\Gamma_{sr}|^2}{|1-\Gamma_{sd}S_{11}|^2 |1-\Gamma_l\Gamma_2|^2} \quad (7)$$

In those situations where the same source at the same frequency is used during the calibration step and DUT insertion, $|b_d|^2 = |b_r|^2$ and $G_{sr} = G_{sd}$. This is usually the case when measuring amplifiers.

Instrument Uncertainty. The uncertainty caused by errors within the circuits of electronic instruments. For noise figure analyzers/meters this includes errors due to the detector, A/D converter, math round-off effects, any mixer non-linearities, saturation effects, and gain instability during measurement. This uncertainty is often mistakingly taken as the overall measurement accuracy because it can be easily found on specification sheets. With modern techniques, however, it is seldom the most significant cause of uncertainty.

Johnson Noise. [19] The same as thermal noise.

Minimum Noise Factor (F_{min}). See Noise Figure Circles.

Mismatch Uncertainty (M_u). Mismatch uncertainty is caused by re-reflections between one device (the source) and the device that follows it (the load). The re-reflections cause the power emerging from the source (incident to the load) to change from its value with a reflectionless load.

An expression for the power incident upon the load, which includes the effects of re-reflections, is

$$P_i = \frac{|b_s|^2}{|1 - \Gamma_s\Gamma_l|^2} \quad (1)$$

where $|b_s|^2$ is the power the source delivers to a non-reflecting load, G_s is the source reflection coefficient, and G_l is the load reflection coefficient. If accurate evaluation of the power incident is needed when $|b_s|^2$ is given or vice versa, then the phase and magnitude of G_s and G_l is needed—probably requiring a vector network analyzer.

When the phase of the reflection coefficients is not known, the extremes of $|1 - G_sG_l|^2$ can be calculated from the magnitudes of G_s and G_l, for example, P_s and P_l. The extremes of $|1 - G_sG_l|^2$ in dB can be found from the nomograph (Figure 4-l).

$$M_u = 20 \log(1 \pm P_sP_l)$$

The effect of mismatch on noise figure measurements is extremely complicated to analyze. Consider, for example, a noise source whose impedance is not quite 50 ohms.

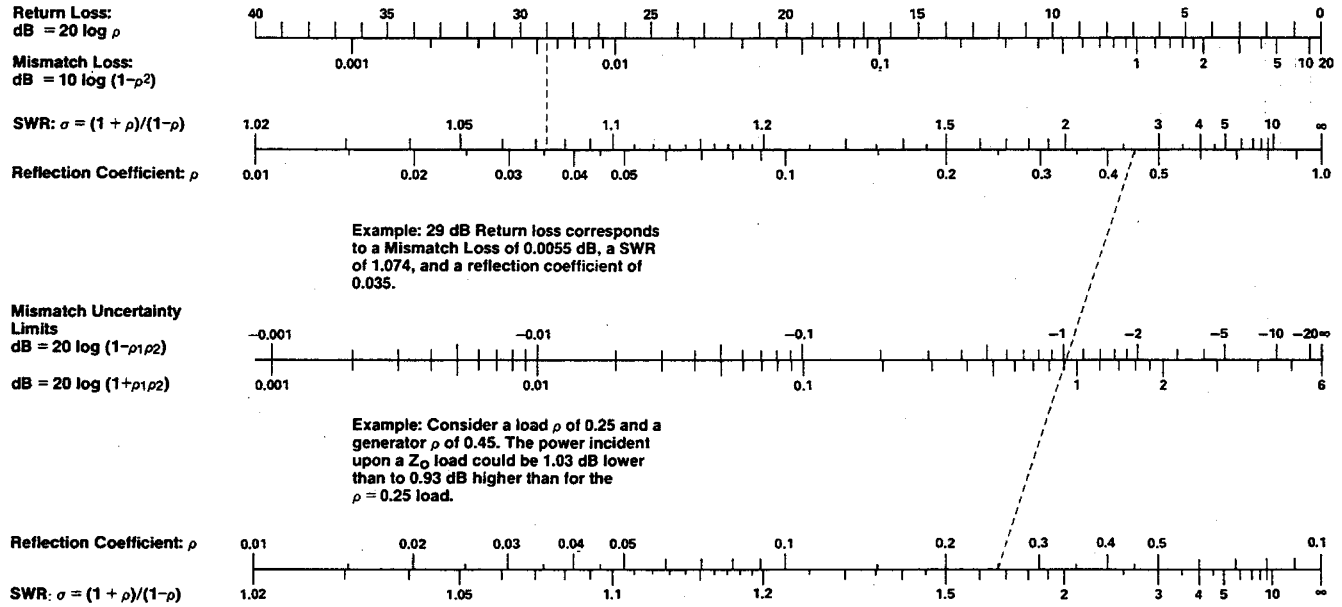

Figure 4-1. This nomograph gives the extreme effects of re-reflections when only the reflection coefficient magnitudes are known. Mismatch uncertainty limits of this nomograph apply to noise figure measurement accuracy for devices that include an isolator at the input.

The source takes part in re-reflections of its own generated noise, but it also reflects noise originating in the DUT and emerging from the DUT input (noise added by a DUT, after all, is a function of the source impedance). The changed source impedance also causes the DUT's available gain to change (remember that available gain is also a function of source impedance). The situation can be complicated further because the source impedance can change between the hot state and the cold state. [23] Many attempts have been made to establish a simple rule-of-thumb for evaluating the effect of mismatch—all with limited success. One very important case was analyzed by Strid [36] to have a particularly simple result. Strid considered the DUT to include an isolator at the input with sufficient isolation to prevent interaction of succeeding devices with the noise source. The effect of noise emerging from the isolator input and re-reflections between the isolator and noise source are included in the final result. The result is that the error in noise figure is

$$\Delta F\,(dB) = F_{act}(dB) - F_{ind}(dB) \quad (2)$$

$$= 10 \log \frac{1}{|1 - S_{11}\Gamma_{sh}|^2} \quad (3)$$

where F_{act} is the noise figure for a reflectionless noise source, F_{ind} is the measured noise figure, S_{11} is the reflection coefficient looking into the DUT, for example, into the isolator input, and G_{sh} is the reflection coefficient looking back into the noise source when in the hot or on condition. Strid also assumed that the isolator and T_{cold} are both 290K. Note that the result is independent of the DUT noise figure, Y factor, and the noise source reflection coefficient for T_{cold}.

Mismatch uncertainty may also occur while characterizing the noise contribution of the measurement system and also at the output of DUT during gain measurement. Gain measurement mismatch effects can be calculated by evaluating the difference between available gain and insertion gain.

Mismatch uncertainty is often the most significant uncertainty in noise figure measurements. Correction usually requires full noise characterization (see Noise Figure Circles) and measurement of phase and amplitude of the reflection coefficients.

N_1	See "Y Factor".
N_2	See "Y Factor".
N_{off}	Same as N1. See "Y factor".
N_{on}	Same as N2. See "Y factor."

Noise Added (N_a). The component of the output noise power that arises from sources within the network under test. This component of output noise is usually differentiated from the component that comes from amplifying the noise that originates in the input source for the network. Occasionally the noise added is referred to the input port, the added noise power at the output is divided by G.

Noise Bandwidth (B). [18, 26] An equivalent rectangular pass band that passes the same amount of noise power as the actual system being considered. The height of the pass band is the transducer power gain at some reference frequency. The reference frequency is usually chosen to be either the band center or the frequency of maximum gain. The area under the equivalent (rectangular) gain vs. frequency curve is equal to the area under the actual gain vs. frequency curve. In equation form

$$B = \int_o^\infty \frac{G(f)df}{G_o} \quad (1)$$

where G_o is the gain at the reference frequency. For a multistage system, the noise bandwidth is nearly equal to the 3 dB bandwidth.

Noise Figure and Noise Factor (NF and F). [7] At a specified input frequency, noise factor is the ratio of (1) the total noise power/hertz at a corresponding output frequency available at the output port when the noise temperature of the input termination is standard (290K) at all frequencies, to (2) that portion of the output power due to the input termination.

The output noise power is often considered to have two components—added noise from the device, N_a, and amplified input noise, for example, the output power from the input termination amplified by the DUT, kT_oBG. Then noise figure can be written

$$F = \frac{N_a + kT_oBG}{kT_oBG} \quad (1)$$

Note: Characterizing a system by noise figure is meaningful only when the impedance (or its equivalent) of the input termination is specified.

Noise figure and noise factor are sometimes differentiated by [31]

$$\text{Noise Figure} = 10 \log (\text{Noise Factor}) \quad (2)$$

so that noise figure is in dB and noise factor is the numerical ratio. Other times the terms are used interchangeably. There should be no confusion, however, because the symbol "dB" seems to be invariably used when 10 log (NF) has been taken. No "dB" symbol implies that the numerical ratio is meant.

Noise Figure Circles. [9, 18] This refers to the contours of constant noise figure for a network when plotted on the complex plane of the source impedance, admittance, or reflection coefficient seen by the network. The general equation expressing the noise factor of a network as a function of source reflection coefficient Γ_s is

$$F = F_{min} + \frac{4R_n}{Z_o} \cdot \frac{|\Gamma_{opt} - \Gamma_s|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_s|^2)} \quad (1)$$

where Γ_{opt} is the source reflection coefficient that results in the minimum noise figure of the network, F_{min} is the minimum noise figure, Z_o is the reference impedance for defining Γ_s (usually 50 ohms) and R_n is called the equivalent noise resistance. Sometimes R_n/Z_o, is given as the single parameter r_n, called the normalized equivalent noise resistance. Loci of constant F, plotted as a function of Γ_s, form circles on the complex plane. Noise figure circles with available gain circles are highly useful for circuit designer insights into optimizing the overall network for low noise figure and flat gain.

Noise Measure (M). [14] A quality factor that includes both the noise figure and gain of a network as follows

$$M = \frac{(F - 1)}{1 - \frac{1}{G}} \quad (1)$$

If two amplifiers with different noise figures and gains are to be cascaded, the amplifier with the lowest M should be used at the input to achieve the smallest overall noise figure. Like noise figure and available power gain, a network's noise measure generally varies with source impedance [9]. To make the decision as to which amplifier to place first, the source impedances must be such that F and G for each amplifier are independent of the order of cascading.

Noise measure is also used to express the overall noise figure of an infinite cascade of identical networks. The overall noise figure is

$$F_{tot} = F + \frac{F - 1}{G_a} + \frac{F - 1}{G_a^2} + \frac{F - 1}{G_a^3} + \cdots \quad (2)$$

$$F_{tot} = 1 + \frac{F - 1}{1 - \frac{1}{G_a}} \quad (3)$$

$$F_{tot} = 1 + M \quad (4)$$

Sometimes F_{tot} of equation (2) is called the noise measure instead of M in equation (1). Care should be exercised as to which definition is being used because they differ by 1.

Noise Temperature (T_a). [1] The temperature that yields the available power spectral density from a source. It is obtained when the corresponding reflection coefficients for the generator and load are complex conjugates. The relationship to the available power P_a is

$$T_a = \frac{P_a}{kB} \quad (1)$$

where k is Boltzmann's constant and B is the bandwidth of the power measurement. The power spectral density across the measurement band is to be constant. Also see Effective Noise Temperature (T_{ne})

Noise temperature can be equivalently defined [26] as the temperature of a passive source resistance having the same available noise power spectral density as that of the actual source.

Nyquist's Theorem. See Thermal Noise.

Operating Noise Temperature (T_{op}). [7] The temperature in kelvins given by:

$$T_{op} = \frac{N_o}{kG_s} \quad (1)$$

where N_o is the output noise power/hertz from the DUT at a specified output frequency delivered into the output circuit under operating conditions, k is Boltzmann's constant, and G_s is the transducer power gain for the signal. NOTE: In a linear two-port transducer with a single input and a single output frequency, T_{op} is related to the noise temperature of the input termination T_a, and the effective input noise temperature T_e, by:

$$T_{op} = T_a + T_e \quad (2)$$

Optimum Reflection Coefficient (Γ_{opt}). See Noise Figure Circles.

Partition Noise. [26, 39] An apparent additional noise source due to the random division of current among various electrodes or elements of a device.

Power Gain (G_P). [2, 35, 40] The ratio, at a specific frequency, of power delivered by a network to an arbitrary load P_l to the power delivered to the network by the source P_s.

$$G_P = \frac{P_l}{P_s} \quad (1)$$

The words "power gain" and the symbol G are often used when referring to noise, but what is probably intended is "available power gain (G_a)", or "transducer power gain (G_t)", or "insertion power gain (G_i)". For an arbitrary source and load, the power gain of a network is given by

$$G_P = |S_{21}|^2 \frac{1 - |\Gamma_l|^2}{|1 - \Gamma_l S_{22}|^2 (1 - |\Gamma_l|^2)} \quad (2)$$

where

$$\Gamma_l = S_{11} + \frac{S_{12} S_{21} \Gamma_l}{1 - \Gamma_l S_{22}} \quad (3)$$

NOTE 1: G_P is function of the load reflection coefficient and the scattering parameters of the network but is independent of the source reflection coefficient.

NOTE 2: The expression for G_P is the same as that for G_a if Γ_l is substituted for Γ_s, and S_{11} is substituted for S_{22}.

G_P is often expressed in dB

$$G_P(\text{dB}) = 10 \log \frac{P_l}{P_s} \quad (4)$$

Root Sum-of-the Squares Uncertainty (RSS). A method of combining several individual uncertainties of known limits to form an overall uncertainty. If a particular measurement has individual uncertainties ±A, ±B, ±C, etc, then the RSS uncertainty is

$$U_{RSS} = (A^2 + B^2 + C^2 + \ldots)^{1/2} \quad (1)$$

The RSS uncertainty is based on the fact that most of the errors of measurement, although systematic and not random, are independent of each other. Since they are independent they are random with respect to each other and combine like random variables.

Second-Stage Effect. A reference to the cascade effect during measurement situations where the DUT is the first stage and the measurement equipment is the second stage. The noise figure measured is the combined noise figure of the DUT cascaded to the measurement equipment. If F_2 is the noise factor of the measurement system alone, and F_{sys} is the combined noise factor of the DUT and system, then F_1, the noise factor of the DUT, is

$$F_1 = F_{sys} - \frac{F_2 - 1}{G} \quad (1)$$

where G is the gain of the DUT.

NOTE: F_2 in equation (1) is the noise factor of the measurement system for a source impedance corresponding to the output impedance of the DUT.

Sensitivity. The smallest signal that a network can reliably detect. Sensitivity specifies the strength of the smallest signal at the input of a network that causes the output signal power to be M times the output noise power where M must be specified. M=1 is very popular. For a source temperature of 290K, the relationship of sensitivity to noise figure is

$$S_i = MkT_oBF \quad (1)$$

In dBm

$$S_i\,(\text{dBm}) = -174\ \text{dBm} + F(\text{dB}) + 10 \log B + 10 \log M \quad (2)$$

Thus sensitivity is related to noise figure in terrestrial systems once the bandwidth is known.

Shot Noise. [6, 39] Noise is caused by the quantized and random nature of current flow. Current is not continuous but quantized, being limited by the smallest unit of charge ($e=1.6 \times 10^{-19}$ coulombs). Particles of charge also flow with random spacing. The arrival of one unit of charge at a boundary is independent of when the previous unit arrived or when the succeeding unit will arrive. When dc current Io flows, the average current is Io but that does not indicate what the variation in the current is or what frequencies are involved in the random variations of current. Statistical analysis of the random occurrence of particle flow yields that the mean square current variations are uniformly distributed in frequency up to the inverse of the transit time of carriers across the device. Like thermal noise, the noise power resulting from this noise current, produces power in a load resistance that is directly proportional to bandwidth.

$$i_n^2\,(f) = 2eI_o\ A^2/\text{Hz} \quad (2)$$

This formula holds for those frequencies which have periods much less than the transit time of carriers across the device. The noisy current flowing through a load resistance forms the power variations known as shot noise.

Single-sideband (SSB). Refers to using only one of the two main frequency bands that get converted to an IF. In noise figure discussions, single-sideband is derived from the meaning attached to modulation schemes in communication systems where energy on one side of the carrier is suppressed to more optimally utilize the radio spectrum. Many noise figure measurements are in systems that include down conversion using a mixer and local oscillator at frequency f_{LO} to generate an intermediate frequency f_{IF}. The IF power from the mixer is usually increased by an amplifier having bandwidth B. Some of these down converting systems respond only to signals over bandwidth B centered at $f_{LO} + f_{IF}$. These are single-sideband measurements at the upper sideband (USB). Some other systems respond only to signals over bandwidth B centered at $f_{LO} - f_{IF}$. These are single-sideband measurements at the lower sideband (LSB). Other systems respond to signals in both bands. Such measurements are called double-side-band (DSB). SSB systems usually use pre-selection filtering or image rejection to eliminate the unwanted sideband.

Confusion often arises when DSB noise figure measurement results for receivers or mixers are to be interpreted for single-sideband applications. The cause of the confusion is that the definition of noise figure (see the notes under Noise Figure in this glossary) states that the numerator should include noise from all frequency transformations of the system, including the image frequency and other spurious responses, but the denominator should only include the principal frequency transformation of the system. For systems that respond equally to the upper sideband and lower sideband, but where the intended frequency translation is to be for only one sideband, the denominator noise power in the definition should be half the total measured output power due to the input noise (assuming gain and bandwidth are the same in both bands). Double-sideband noise figure measurements normally do not make the distinction. Since the noise source contains noise at all frequencies, all frequency transformations are included in both the numerator and denominator. Thus, if the final application of the network being measured has desired signals in only one sideband but responds to noise in both sidebands, the denominator of DSB measurements is too large and the measured noise figure is too small—usually a factor of about two (3 dB).

There are occasions when the information in both side-bands is desired and processed. The measured DSB noise figure is proper and no correction should be performed. In many of those applications, the signal being measured is radiation so the receiver is called a radiometer. Radiometers are used in radio astronomy. Noise figure measurements of amplifiers made with measurement systems that respond to both sidebands should not include a 3 dB correction factor. In this case, the noise figure measurement system is operating as a radiometer because it is using the information in both sidebands.

Spot Noise Figure and Spot Noise Factor. A term used when it is desired to emphasize that the noise figure or noise factor pertains to a single frequency as opposed to being averaged over a broad band.

Standard Noise Temperature (T_0). [7] The standard reference temperature for noise figure measurements. It is defined to be 290K.

T_C, T_c, or T_{cold}. The colder of two noise source temperatures, usually in kelvins, used to measure a network's noise characteristics .

T_H, T_h,or T_{hot}. The hotter of two noise source temperatures, usually in kelvins, used to measure a network's noise characteristics .

T_{off}. The temperature, usually in kelvins, of a noise source when it is biased off. This corresponds to T_{cold}.

T_{on}. The temperature, usually in kelvins, of a noise source when it is biased on. This corresponds to T_{hot}.

Thermal Noise. [19, 26, 30] Thermal noise refers to the kinetic energy of a body of particles as a result of its finite temperature. If some particles are charged (ionized), vibrational kinetic energy may be coupled electrically to another device if a suitable transmission path is provided. The probability distribution of the voltage is gaussian with mean square voltage

$$\overline{e_n^2} = 4kT \int_{f_1}^{f_2} R(f)p(f)df \qquad (1)$$

$$p(f) = \frac{hf}{kT}\left(e^{\frac{hf}{kT}} - 1\right)^{-1} \qquad (2)$$
$$\approx 1$$

where k is Boltzmann's constant (1.38×10^{-23} joules/kelvin), T is the absolute temperature in kelvins, R is the resistance in ohms, f is the frequency in hertz, f_1 and f_2 specify the band over which the voltage is observed, and h is the Planck's constant (6.62×10^{-34}) joule seconds.

For frequencies below 100 GHz and for T = 290K, l>p(f)>0.992, so p(f)=1 and equation (l) becomes

$$\overline{e_n^2} = 4\,kTR(f_2 - f_1)$$

$$= 4\,kTRB \qquad (3)$$

The power available, that is, the power delivered to a complex conjugate load at absolute zero, is

$$P_a = \frac{\overline{e_n^2}}{4R}$$

$$= kT\,(f_2 - f_1)$$

$$= kTB \qquad (4)$$

The units of kTB are usually joules/second, which are the same as watts.

The available power spectral density is kT watts/hertz. Although this development appears to make equation (3) more fundamental than (4), Nyquist [30] first arrived at the value of power spectral density (equation (4)) and then calculated the voltage and current involved (equation (3)). The expression for the voltage generator is

$$e_n^2\,df = 4\,RkT\,df \qquad (5)$$

Equation (5) is frequently referred to as Nyquist's Theorem. This should not be confused with Nyquist's work in other areas such as sampling theory and stability criteria where other relations may also be referred to as Nyquist's Theorem. When T is equal to the standard temperature T_o (290K), $kT_o = 4 \times 10^{-21}$ W/Hz = –174 dBm/Hz.

A brief examination of kTB shows that each of the factors makes sense. Boltzmann's constant k gives the average mechanical energy per particle that can be coupled out by electrical means, per degree of temperature. Boltzmann's constant is thus a conversion constant between two forms of expressing energy—in terms of absolute temperature and in terms of joules.

The power available depends directly on temperature. The more energy that is present in the form of higher temperature or larger vibrations, the more energy that it is possible to remove per second.

It might not be apparent that bandwidth should be part of the expression. Consider the example of a transmission band limited to the 10 to 11 Hz range. Then only that small portion of the vibrational energy in the 10 to 11 Hz band can be coupled out. The same amount of energy applies to the 11 to 12 Hz band (because the energy is evenly distributed across the frequency spectrum). If, however, the band were 10 to 12Hz, then the total energy of the two Hz range, twice as much, is available to be coupled out. Thus it is reasonable to have bandwidth, B, in the expression for available power.

It should be emphasized that kTB is the power available from the device. This power can only be coupled out into the optimum load, a complex-conjugate impedance that is at absolute zero so that it does not send any energy back.

It might seem like the power available should depend on the physical size or on the number of charge carriers and therefore the resistance. A larger body, contains more total energy per degree and more charged particles would seem to provide more paths for coupling energy. It is easy to show with an example that the power available is independent of size or resistance. Consider a system consisting of a large object at a certain temperature, electrically connected to a small object at the same temperature. If there were a net power flow from the large object to the small object, then the large object would become cooler and the small object would become warmer. This violates our common experience—and the second law of thermodynamics. So the power from the large object must be the same as that from the small object. The same reasoning applies to a large resistance and small resistance instead of a large and small object.

This brings up the point that if a source of noise is emitting energy it should be cooling off. Such is generally the case, but for the problems in electrical equipment, any energy removed by noise power transfer is so small that it is quickly replenished by the environment at the same rate. This is because sources of noise are in thermal equilibrium with their environment.

Transducer Power Gain (G_t). [2, 35, 40] The ratio, at a specific frequency, of power delivered by a network to an arbitrary load P_l to the power available from the source P_{as}

$$G_t = \frac{P_l}{P_{as}} \tag{1}$$

For a source of strength $|b_s|^2$ and reflection coefficient Γ_s, and for a load reflection coefficient Γ_l.

$$P_{as} = \frac{|b_s|^2}{1 - |\Gamma_s|^2} \tag{2}$$

$$P_l = \frac{|b_s|^2 |S_{21}|^2 (1 - |\Gamma_l|^2)}{|(1 - \Gamma_s S_{11})(1 - \Gamma_l S_{22}) - \Gamma_l \Gamma_s S_{12} S_{21}|^2} \tag{3}$$

where the S parameters refer to the DUT. An equivalent expression for P_l is

$$P_l = \frac{|b_s|^2 |S_{21}|^2 (1 - |\Gamma_l|^2)}{|1 - \Gamma_s S_{11}|^2 |1 - \Gamma_l \Gamma_2|^2} \tag{4}$$

where

$$\Gamma_2 = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - \Gamma_s S_{11}} \tag{5}$$

Transducer gain is then

$$G_t = |S_{21}|^2 \frac{(1 - |\Gamma_s|^2)(1 - |\Gamma_l|^2)}{|(1 - \Gamma_s S_{11})(1 - \Gamma_l S_{22}) - \Gamma_l \Gamma_s S_{12} S_{21}|^2} \tag{6}$$

$$G_t = |S_{21}|^2 \frac{(1 - |\Gamma_s|^2)(1 - |\Gamma_l|^2)}{|1 - \Gamma_s S_{11}|^2 |1 - \Gamma_l \Gamma_2|^2} \tag{7}$$

Transducer gain is a function of the source and load reflection coefficients as well as the network parameters.

The term “transducer” arises because the result compares the power delivered to an arbitrary load from an arbitrary generator through the DUT with the power delivered to the load through a lossless transducer which transfers all of the available generator power to the load.

Transducer gain is often measured in dB

$$G_t\,(\text{dB}) = 10 \log \frac{P_l}{P_{as}} \tag{8}$$

White Noise. Noise whose power spectral density (watts/hertz) is constant for the frequency range of interest. The term “white” is borrowed from the layman’s concept of white light being a composite of all colors, hence containing all frequencies.

Worst Case Uncertainty. A conservative method of combining several individual measurement uncertainties of known limits to form an overall measurement uncertainty. Each individual uncertainty is assumed to be at its limit in the direction that causes it to combine with the other individual uncertainties to have the largest effect on the measurement result.

Y Factor. The ratio of N_2 to N_1 in noise figure measurements where N_2 is the measured noise power output from the network under test when the source impedance is turned on or at its hot temperature and N_1 is the measured power output when the source impedance is turned off or at its cold temperature.

5. References

[1] *Accuracy Information Sheet*, United States National Bureau of Standards (NBS), enclosure returned with noise sources sent to NBS for calibration.

[2] Anderson, R.W. *S-Parameter Techniques for Faster, More Accurate Network Design*, Hewlett-Packard Application Note 95-1.

[3] Beatty, Robert W. *Insertion Loss Concepts*, Proc. of the IEEE, June, 1964, pp. 663-671.

[4] Boyd, Duncan *Calculate the Uncertainty of NF Measurements.* "Microwaves and RF", October, 1999, p.93.

[5] Chambers, D. R. *A Noise Source for Noise Figure Measurements*, Hewlett-Packard Journal, April, 1983, pp. 26-27.

[6] Davenport, Wilbur B. Jr. and William L. Root. *An Introduction to the Theory of Random Signals and Noise*, McGraw-Hill Book Co., Inc, New York, Toronto, London,1958.

[7] *Description of the Noise Performance of Amplifiers and Receiving System*s, Sponsored by IRE subcommittee 7.9 on Noise, Proc. of the IEEE, March,1963, pp. 436-442.

[8] Friis, *H.T. Noise Figures of Radio Receivers*, Proc. of the IRE, July, 1944, pp. 419-422.

[9] Fukui, H. *Available Power Gain, Noise Figure and Noise Measure of Two-Ports and Their Graphical Representations*, IEEE Trans. on Circuit Theory, June, 1966, pp. 137-143.

[10] Fukui, H. (editor) *Low Noise Microwave Transistors and Amplifiers*, IEEE Press and John Wiley & Sons, New York,1981. (This book of reprints contains many of the articles referenced here.)

[11] Gupta, M-S. *Noise in Avalanche Transit-Time Devices*, Proc. of the IEEE, December, 1971, pp. 1674-1687.

[12] Haitz, R.H. and F.W. Voltmer. Noise Studies in Uniform Avalanche Diodes, Appl. Phys. Lett, 15 November, 1966, pp. 381-383.

[13] Haitz, R.H. and F.W. Voltmer. *Noise of a Self Sustaining Avalanche Discharge in Silicon: Studies at Microwave Frequencies*, J. Appl. Phys., June 1968, pp. 3379-3384.

[14] Haus, H.A. and R.B. Adler. *Optimum Noise Performance of Linear Amplifiers*, Proc. of the IRE, August, 1958, pp. 1517-1533.

[15] Hines, M.E. *Noise Theory for the Read Type Avalanche Diode*, IEEE Trans. on Electron devices, January, 1966, pp. 158-163.

[16] *IRE Standards on Electron Tubes. Part 9, Noise in Linear Two-Ports*, IRE subcommittee 7.9, Noise, 1957.

[17] *IRE Standards on Electron Tubes: Definitions of Terms, 1962 (62 IRE 7.52)*, Proc. of the IEEE, March, 1963, pp. 434-435

[18] *IRE Standards on Methods of Measuring Noise in Linear Twoports, 1959*, IRE Subcommittee on Noise, Proc. of the IRE, January, 1960, pp. 60-68. See also *Representation of Noise in Linear Twoports*, Proc. of the IRE, January,1960, pp. 69-74.

[19] Johnson, J.B. *Thermal Agitation of Electricity in Conductors*, Physical Review, July, 1928, pp. 97-109.

[20] Kanda, M. *A Statistical Measure for the Stability of Solid State Noise Sources*, IEEE Trans. on Micro. Th. and Tech, August, 1977, pp. 676-682.

[21] Kanda, M. *An Improved Solid-State Noise Source*, IEEE Trans. on Micro. Th. and Tech, December, 1976, pp. 990-995.

[22] Kuhn, N.J. *Simplified Signal Flow Graph Analysis*, "Microwave Journal", November 1963, pp. 59-66.

[23] Kuhn, N.J. *Curing a Subtle but Significant Cause of Noise Figure Error*, "Microwave Journal", June, 1984, p. 85.

[24] *Maximizing Accuracy in Noise Figure Measurements*, Hewlett Packard Product Note 85719A-1, July 1992, (5091-4801E).

[25] Mumford, W.W. *A Broadband Microwave Noise Source*, Bell Syst. Tech. J., October,1949, pp.608-618.

[26] Mumford, W.W. and Elmer H. Scheibe. *Noise Performance Factors in Communication Systems*, Horizon House-Microwave, Inc., Dedham, Massachusetts, 1968.

[27] NBS Monograph 142, *The Measurement of Noise Performance Factors: A Metrology Guide*, U.S. Government Printing Office, Washington, D.C.,1974.

[28] NBS Technical Note 640, *Considerations for the Precise Measurement of Amplifier Noise*, U.S. Government Printing Office, Washington, D.C.,1973.

[29] *Noise Parameter Measurement Using the HP 8970B Noise Figure Meter and the ATN NP4 Noise Parameter Test Set*, Hewlett Packard Product Note HP 8970B/S-3, December, 1998, (5952-6639).

[30] Nyquist, H. *Thermal Agitation of Electric Charge in Conductors*, Physical Review, July,1928, pp.110-113.

[31] Oliver, B.M. *Noise Figure and Its Measurement*, Hewlett-Packard Journal, Vol.9, No. 5 (January, 1958), pp.3-5.

[32] Saam, Thomas J. *Small Computers Revolutionize G/T Tests*, "Microwaves", August, 1980, p. 37.

[33] Schwartz, Mischa. *Information Transmission, Modulation and Noise*, McGraw-Hill Book Co., Inc., New York, Toronto, London, 1959.

[34] Slater, Carla *Spectrum-Analyzer-Based System Simplifies Noise Figure Measurement*, "RF Design", December, 1993, p.24.

[35] *S-Parameter Design*, Hewlett Packard Application Note 154, March, 1990, (5952-1087).

[36] Strid, E. *Noise Measurements For Low-Noise GaA FET Amplifier*s, Microwave Systems News, November 1981, pp. 62-70.

[37] Strid, E. *Noise Measurement Checklist Eliminates Costly Errors*, "Microwave Systems News", December, 1981, pp. 88-107.

[38] Swain, H. L. and R. M. Cox *Noise Figure Meter Sets Record for Accuracy, Repeatability, and Convenience*, Hewlett-Packard J., April, 1983, pp. 23-32.

[39] van der Ziel, Aldert. *Noise: Sources, Characterization, Measurement*, Pentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.

[40] Vendelin, George D., *Design of Amplifiers and Oscillators by the S-Parameter Method*, Wiley-Interscience, 1982.

[41] Wait, D.F., *Satellite Earth Terminal G/T Measurements*, "Microwave Journal", April, 1977, p. 49.

6. Additional Agilent Resources, Literature and Tools

10 Hints for Making Successful Noise Figure Measurements, Application Note 1341, literature number 5980-0288E

Noise Figure Measurement Accuracy, Application Note 57-2, literature number 5952-3706

Calculate the Uncertainty of NF Measurements (software and web-based tool available via web)
www.agilent.com/find/nfu

User guides for Agilent noise figure products.

Noise figure analysis web site:
www.agilent.com/find/nf

Component Test web site:
www.agilent.com/find/component_test

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