SPECTRUM ANALYSIS . . .
Noise Figure Measurement

Introduction

Noise figure measurement can easily be accomplished with spectrum analyzers, an approach which has several advantages over conventional noise figure meters:

1. Noise figure can be measured at any frequency within a spectrum analyzer's multi-decade frequency range. This enables measurement at the device's operating frequency without changes in the test set-up.
2. Frequency selective noise figure measurements independent of device bandwidth or spurious responses.
3. Standard spectrum analyzers can make a variety of frequency domain measurements (power, frequency, distortion, etc.) as well as noise figure.

This does not mean to say that a spectrum analyzer can replace a noise figure meter. An analyzer's sensitivity and accuracy become limiting factors in noise figure measurements just as with any other measurement. Nevertheless, the three advantages listed above may make a spectrum analyzer the best choice for a noise figure measurement.

Noise Figure Theory

The ideal limit of a system's sensitivity is set by the noise present at its input. In practice, however, the sensitivity is often limited by noise generated within the system itself. The number used to indicate how closely the ideal is approached is called noise factor, F, defined as the input to output signal-to-noise ratio:

$$F = \frac{S_i/N_i}{S_o/N_o}$$

where $S_{i,o}$ = signal power input and output, $N_{i,o}$ = noise power input and output.

This number, F, indicates the change in signal-to-noise ratio which occurs as a signal passes through a device. Thus, F is a figure of merit (ideally equal to one) which can be used to compare different amplifiers and receivers. So, being a dimensionless quantity independent of bandwidth, noise factor is a better basis for comparison of receivers than sensitivity. Furthermore, with knowledge of a system's noise factor and bandwidth, we can predict its sensitivity and how it might be improved by the addition of pre-amplifiers.*

The terms of the definition can be conveniently rearranged as shown below:

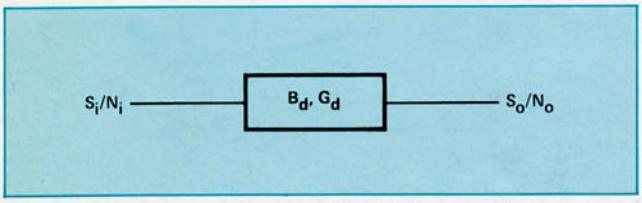


Figure 1

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i/kTB_d}{S_iG_d/N_o} = \frac{N_o}{G_dkTB_d}$$

*The subject of noise measurements is covered in greater detail in Application Notes 150-4 and 150-7.

Noise figure, the logarithmic equivalent of noise factor is given as $N_{fig} = 10 \log F$
where:

N_o = noise power output (delivered to a matched load with the input terminated in its characteristic impedance)
k = Boltzman's constant, 1.374×10^{-23} joule/°K
T = 290°K (room temp.) kT = 3.98×10^{-21} watts/Hz, equivalent to −174 dBm in a 1 Hz bandwidth
B_d = device noise power bandwidth in Hertz
G_d = device gain.

We can rearrange the noise figure equation in terms of noise power output, gain and noise input using values in dB for convenience:

$$N_{fig} = \underbrace{10 \log F_dG_dkTB}_{\text{Noise Power Output}} - \underbrace{10 \log G_d}_{\text{Gain}} - \underbrace{10 \log kTB}_{\text{Noise Input}}$$

Thus, determination of a device's noise figure can be made with knowledge of its noise power output, gain, and bandwidth. The noise power bandwidth for the measurement can be set by selecting the resolution bandwidth, B, of the spectrum analyzer sufficiently narrow so the analyzer determines the system bandwidth. The device gain and noise power output are both readily measured so the equation in terms of the unknowns now becomes:

$$N_{fig} = \underbrace{10 \log N_o}_{\text{Noise Output}} - \underbrace{10 \log G_d}_{\text{Gain}} - \underbrace{(-174 \text{ dB} + 10 \log B)}_{\text{Equivalent Noise Input in Bandwidth B}}$$

Note that the number used for B in the equation relates to noise power bandwidth. This is because there is a difference in the total noise power which passes through a real vs. ideal filter of bandwidth B as shown in Figure 2:

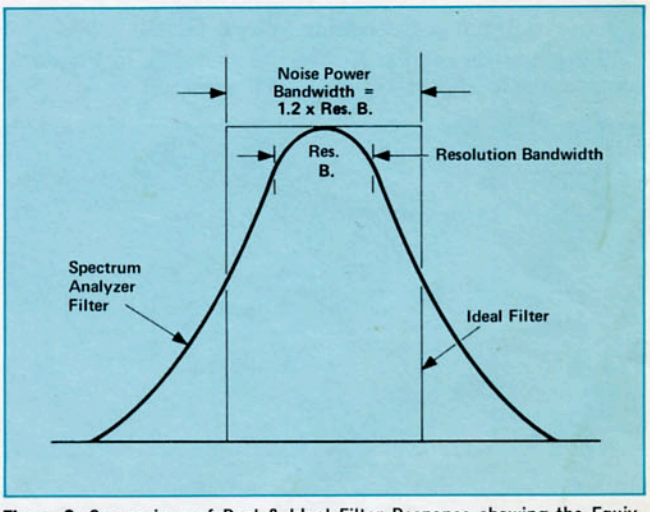


Figure 2. Comparison of Real & Ideal Filter Response showing the Equivalent Noise Power Bandwidth.

In fact, the noise power bandwidth of an HP spectrum analyzer's filter is typically 1.2 times the resolution bandwidth indicated on its bandwidth control (for other analyzers, this ratio may be determined empirically).

Knowing this, we may use the resolution bandwidth setting for B in the equation provided we introduce a 0.8 dB (10 log 1.2) correction term.

Next, the device's gain is measured by noting the effect it has on the power of a signal as displayed by the spectrum analyzer set to the desired frequency.

Finally, noise power measurements on a spectrum analyzer must account for the random fluctuations of its power with time. For that reason, the detected noise should be averaged by suitable video filtering (video bandwidth $\leq$0.01 IF bandwidth) so the average noise power can be read from the display as a smooth line. The spectrum analyzer's log amplifier and detector perform non-linear processes that cause the noise level to be displayed 2.5 dB below its actual value. So 2.5 dB is added to the displayed level to give the proper reading.

The noise output of the device may then be regarded as a signal with a power, $N_o = F_d G_d$ kTB, a rearrangement of the noise figure equation. It follows that N_o must be greater than the analyzer's internal noise level (sensitivity, N_{sa}) so that it can be measured. In fact, N_o should be 6 dB greater than N_{sa} so that N_{sa} adds less than 1 dB to the displayed noise level; minimizing error. If N_o is below the analyzer's sensitivity level, its power must be raised by a low-noise, high-gain (G_p) preamplifier. Then, the noise level measured by the spectrum analyzer is greater than the device's N_o by the preamp's gain G_p in dB. The noise figure of the device, N_{fig}, is then calculated as:

$$N_{fig} = 10 \log F_d$$
$$= N - \underbrace{(G_d + G_p)}_{\text{[Total gain]}} - 10 \log B + 174 \text{ dB} + 1.7 \text{ dB}^*$$

Accuracy, of course, is dependent on the absolute amplitude accuracy of the spectrum analyzer and the accuracy of its IF bandwidth setting. See AN 150-8 for further information on accuracy considerations.

So far, we've shown that noise figure can be measured once the device gain, bandwidth, and noise power output are known. Next, we'll see how a spectrum analyzer is applied to measure these values.

Measurement Procedure:

Noise figure measurement with a spectrum analyzer is a two-step process:

Step 1. Determine System Gain.

Using a set-up similar to that shown below in Figure 3, measure the total gain, $G_d + G_p$ of the system:

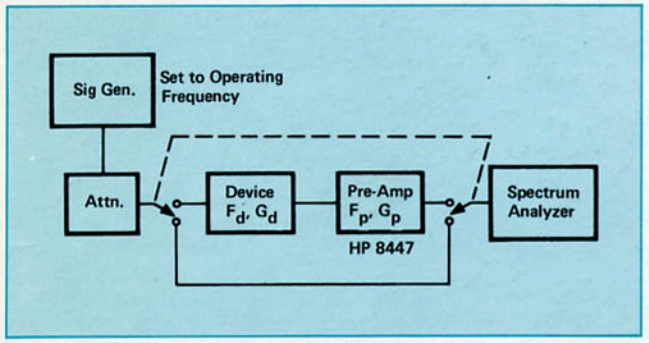


Figure 3.

The total gain is the change in signal power displayed with and without the device and pre-amp in use. Either the signal generator output attenuator or the spectrum analyzer IF attenuator may be used for substitution measurements to improve accuracy.

Step 2. Measure Noise Power.

Disconnect the signal generator and terminate the input of the device in its characteristic impedance and set the analyzer input attenuator to 0 dB attenuation. Read the average noise power, N_o, directly from the display by using sufficient video averaging or smoothing. Record the analyzer's IF bandwidth setting, B.

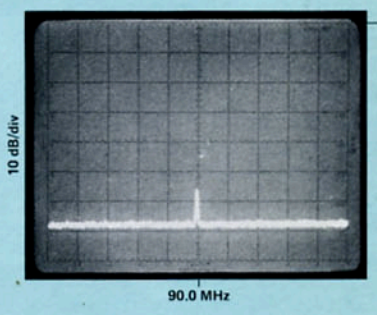
Then, substitute the measured values into the noise figure equation.

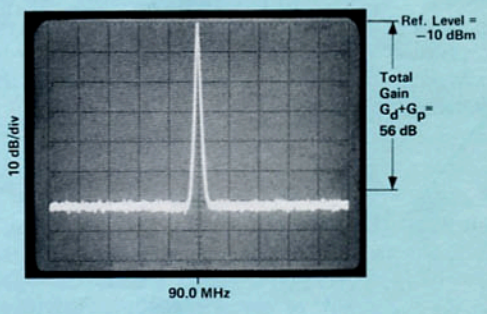
Example:

We wish to measure the noise figure of an amplifier at 90.0 MHz. Following the two-step procedure we:

Step 1

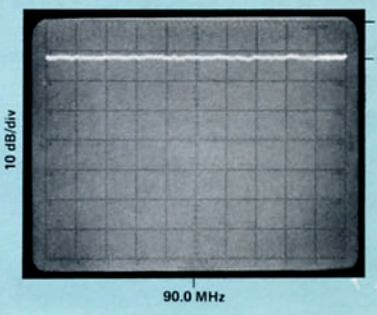
Use a set-up similar to that shown in Figure 3. The total gain at 90 MHz is 56 dB as shown in the photos below:





Step 2

Next, with the test amplifier's input terminated and 0 dB input attenuation, the noise power is at −72 dBm in a 10 kHz bandwidth.



The device's noise figure is then:

N_{fig} = −72 dBm −56 dB −40 dB +175.7 dB
= 8 dB
F = 6.3

*This correction factor is the sum of bandwidth, log amplifier, and detector corrections to the noise power.

For more information, call your local HP Sales Office or East (301) 948-6370 • Midwest (312) 677-0400 • South (404) 434-4000 • West (213) 877-1282. Or, write: Hewlett-Packard, 1501 Page Mill Road, Palo Alto, California 94304. In Europe, Post Office Box 349, CH-1217 Meyrin 1, Geneva, Switzerland. In Japan, Yokogawa-Hewlett-Packard, 1-59-1, Yoyogi, Shibuya-Ku, Tokyo, 151.

5952 - 9229 APRIL 1976 PRINTED IN U.S.A.