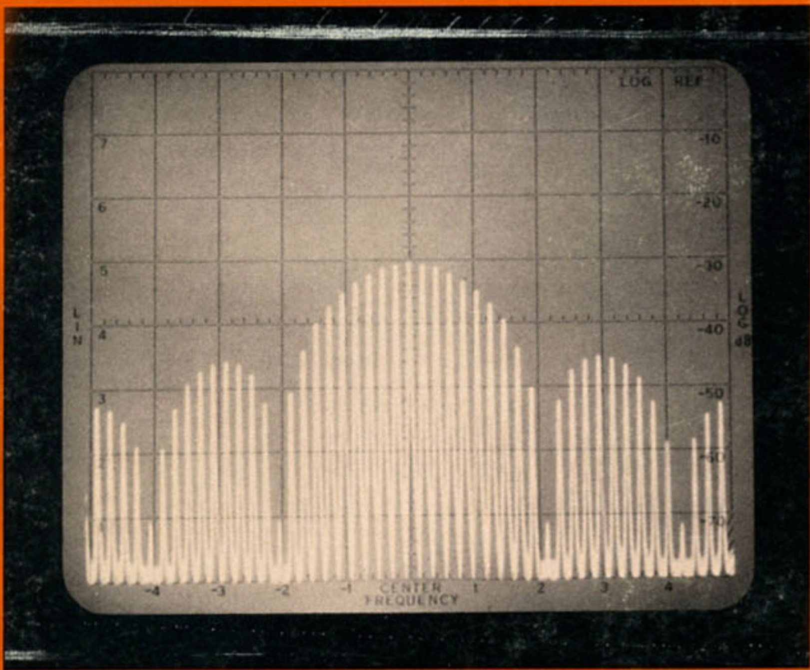


Spectrum Analyzer Series

APPLICATION NOTE 150–2

# SPECTRUM ANALYSIS. . . . Pulsed RF

HEWLETT hp PACKARD

November 1971

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# CONTENTS

# CHAPTER 1
# INTRODUCTION

## THE BASIC PULSE SPECTRUM

The spectrum analyzer was originally designed to look at the output of radar transmitters. A pulse radar signal is a train of RF pulses with a constant repetition rate, constant pulse width and shape, and constant amplitude. By looking at the characteristic spectra, all important properties of the pulsed signal such as pulse width, occupied bandwidth, duty cycle, peak and average power, etc., can be measured easily and with high accuracy. Perhaps an even more important application of the spectrum analyzer is the detection of transmitter misfiring and frequency pulling effects. This application note is intended as an aid for the operation of the spectrum analyzer and the interpretation of the displayed pulse spectra.

The formation of a square wave from a fundamental sine wave and its odd harmonics is a good way to start an explanation of the spectral display for nonsinusoidal waveforms. You will recall perhaps at one time plotting a sine wave and its odd harmonics on a sheet of graph paper, then adding up all the instantaneous values. If there were enough harmonics plotted at their correct amplitudes and phases, the resultant waveform began to approach a square wave. The fundamental frequency determined the square wave rate, and the amplitudes of the harmonics varied inversely to their number.

A rectangular pulse is merely an extension of this principle, and by changing the relative amplitudes and phases of harmonics, both odd and even, we can plot an infinite number of waveshapes. The spectrum analyzer effectively "unplots" waveforms and presents the fundamental and each harmonic contained in the waveform.

Consider a perfect rectangular pulse train as shown in Figure 1a, perfect in the respect that rise time is zero and there is no overshoot or other aberrations. This pulse is shown in the time domain and we wish to examine its spectrum so it must be broken down into its individual frequency components. Figure 1b superimposes the fundamental and its second harmonic plus a constant voltage to show how the pulse begins to take shape as more harmonics are plotted. If an infinite number of harmonics were plotted, the resulting pulse would be perfectly rectangular. A spectral plot of this would be as shown in Figure 2.

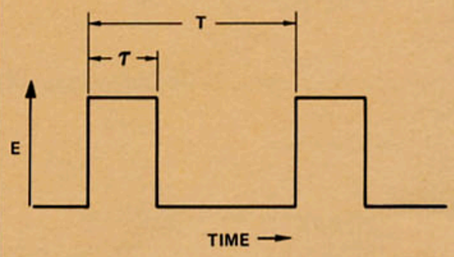


Figure 1a. Periodic rectangular pulse train.

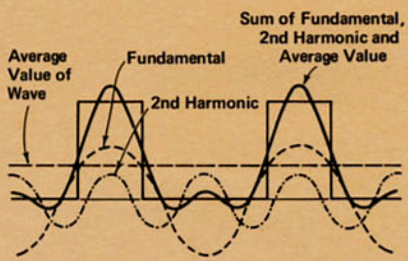


Figure 1b. Addition of a fundamental cosine wave and its harmonics to form rectangular pulses.

The envelope of this plot follows a function of the basic form: $y = \frac{\sin x}{x}$

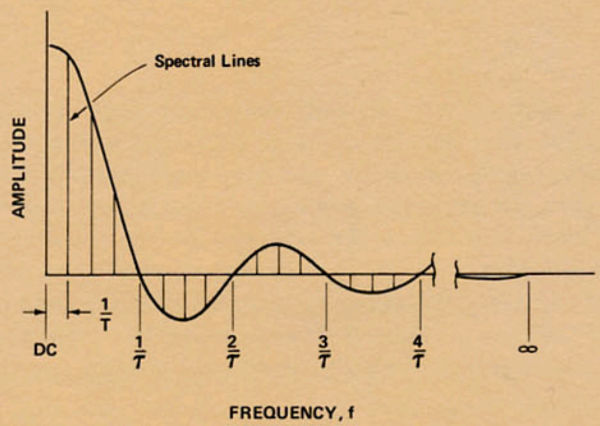


**Figure 2.** Spectrum of a perfectly rectangular pulse. Amplitudes and phases of an infinite number of harmonics are plotted, resulting in smooth envelope as shown.

There is one major point that must be made clear before going into the analyzer display further. We have been talking about a square wave and a pulse without any relation to a carrier or modulation. With this background we now apply the pulse waveform as amplitude modulation to an RF carrier. This produces sums and differences of the carrier and all of the harmonic components contained in the modulating pulse.

We know from single tone AM how the sidebands are produced above and below the carrier frequency. The idea is the same for a pulse, except that the pulse is made up of many tones, thereby producing multiple sidebands which are commonly referred to as spectral lines on the analyzer display. In fact, there will be twice as many sidebands or spectral lines as there are harmonics contained in the modulating pulse.

Figure 3 shows the spectral plot resulting from rectangular amplitude pulse modulation of a carrier. The individual lines represent the modulation product of the carrier and the modulating pulse repetition frequency with its harmonics. Thus, the lines will be spaced in frequency by whatever the pulse repetition frequency might

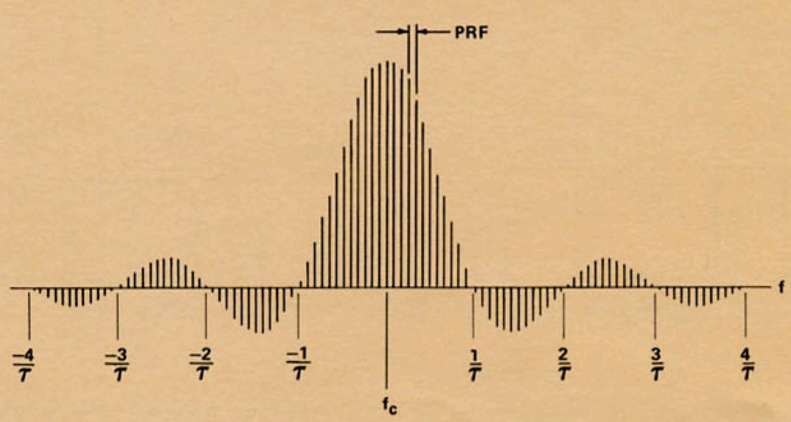


**Figure 3.** Resultant spectrum of a carrier amplitude modulated with a rectangular pulse.

happen to be. The spectral line frequencies may be expressed as:

$$F_L = F_c \pm n \cdot \mathrm{PRF}$$

where $F_c$ = carrier frequency
$PRF$ = pulse repetition frequency
$n = 0, 1, 2, 3, \ldots\ldots$

The "mainlobe" in the center and the "sidelobes" are shown as groups of spectral lines extending above and below the baseline. For perfectly rectangular pulses and other functions whose derivatives are discontinuous at some point, the number of sidelobes is infinite.

The mainlobe contains the carrier frequency represented by the longest spectral line in the center. Amplitude of the spectral lines forming the lobes varies as a function of frequency according to the expression $\dfrac{\sin \omega \frac{\tau}{2}}{\omega \frac{\tau}{2}}$ for a perfectly rectangular pulse. Thus, for a given carrier frequency the points where these lines go through zero amplitude are determined by the *modulating pulse width only*. As pulse width becomes shorter, minima of the envelope become further removed in frequency from the carrier, and the lobes become wider. The *sidelobe widths* in frequency are related to the modulating pulse width by the expression $f = 1/\tau$. Since the mainlobe contains the origin of the spectrum (the carrier frequency), the upper and lower sidebands extending from this point form a *main* lobe $2/\tau$ wide. Remember, however, that the total *number* of sidelobes remains constant so long as the pulse quality, or shape, is unchanged and only its repetition rate is varied. Figure 4 compares the spectral plots

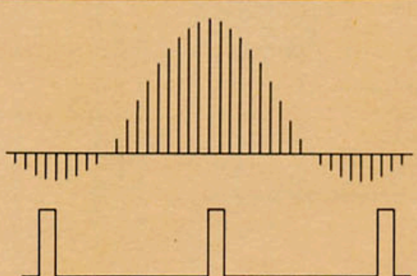

**Figure 4a.** Narrow pulse width causes wide spectrum lobes, high PRF results in low spectral line density.

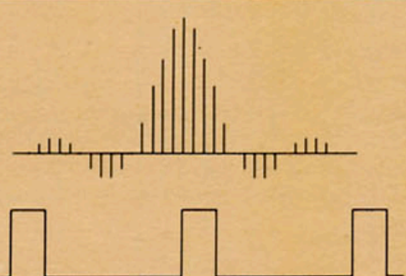

**Figure 4b.** Wider pulse than 4a causes narrower lobes, but line density remains constant since PRF is unchanged.

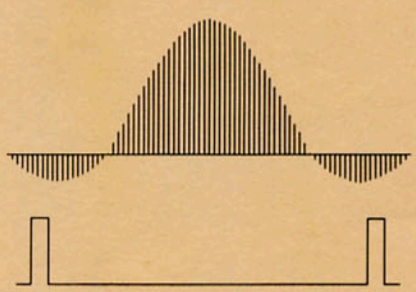

**Figure 4c.** PRF lower than 4a results in higher spectral density. Lobe width is same as 4a since pulse widths are identical.

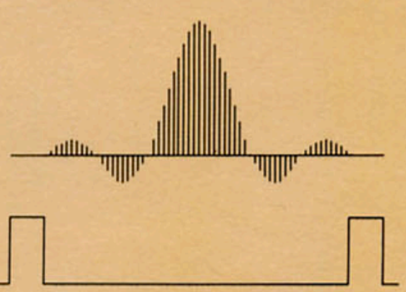

**Figure 4d.** Spectral density and PRF unchanged from 4c, but lobe widths are reduced by wider pulse.

for two pulse lengths, each at two repetition rates with carrier frequency held constant.

Notice in the drawings how the spectral lines extend **below** the baseline as well as above. This corresponds to the harmonics in the modulating pulse, having a phase relationship of 180° with respect to the fundamental of the modulating waveform. Since the spectrum analyzer can only detect amplitudes and not phase, it will invert the negative-going lines and display all amplitudes **above** the baseline.

Because a pulsed RF signal has unique properties we have to be careful to interpret the display on a spectrum analyzer correctly. The response that a spectrum analyzer (or any swept receiver) can have to a periodically pulsed RF signal can be of two kinds, resulting in displays which are similar but of completely different significance. One response is called a "line spectrum" and the other is called a "pulse spectrum." We must keep in mind that these are both responses to the same periodically pulsed RF input signal, and the "line" and "pulse" spectrum refer solely to the response or display on the spectrum analyzer.

We will discuss both types of response to a signal with the basic appearance as shown in Figure 5 with the aid of pictures, and then summarize all formulas and rules for proper operation of the analyzer.

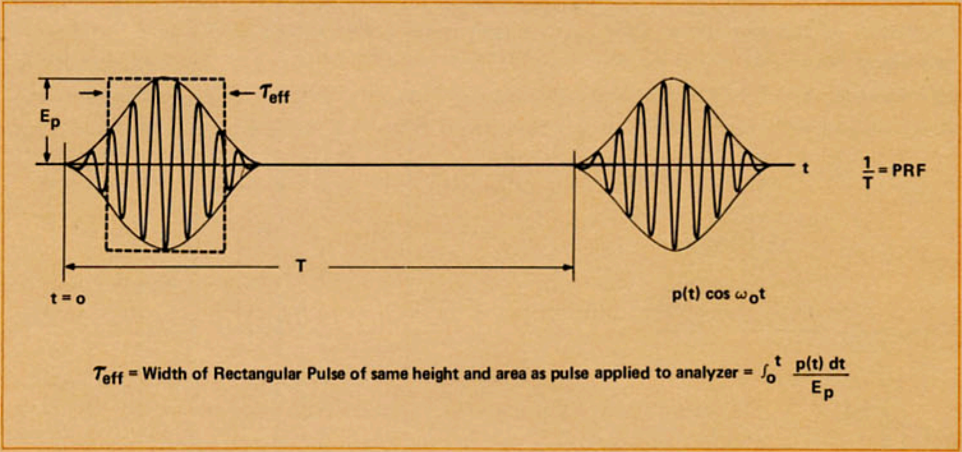


**Figure 5.** Basic RF Pulse.

# CHAPTER 2
# "LINE" SPECTRUM

## GENERAL RULES AND EXPLANATION

A "line" spectrum occurs when the spectrum analyzer's 3 dB bandwidth $B$ is narrow compared to the frequency spacing of the input signal components. Since the individual spectral components are spaced at the pulse repetition frequency (PRF) of the periodically pulsed RF, we can say:

$$B < \text{PRF} \qquad (1)$$

In this case all individual frequency components can be resolved. Only one is within the bandwidth at a time as shown in Figure 6.

The display is a frequency domain display of the actual Fourier components of the input signal. Each component behaves as a CW signal. The display has the normal true frequency domain characteristics:

1. The spacing between lines on the display will NOT change when the analyzer scan time ("sec/Div") is changed.
2. The amplitude of each line will not change when the bandwidth B is changed as long as B remains considerably below the PRF.

We will now look at CRT pictures on page 7A of a pulsed RF signal to see how different scan times, scan widths, and IF bandwidths of the spectrum analyzer influence the appearance of the signal on the display.

A carrier signal with a CW amplitude of −30 *dBm* (Figure 7) is modulated by a pulse train with a PRF of 1 *kHz* and an effective pulse width $\tau_{eff}$ of 0.1 *ms* (Figure 8). In Figure 9 we see the resulting pulse spectrum in a linear display. The analyzer bandwidth is 100 *Hz*, one-tenth of the PRF.

The logarithmic display (Figure 10) allows a much better evaluation of the signal spectrum, because the lower amplitudes of the higher order sidelobes can now be easily measured.

Each Fourier component is resolved and the line spacing is measured as 1 *kHz*, which is the PRF. We can also see that the spacing of the sidelobe minima is 10 *kHz*, according to the relation $\frac{1}{\tau_{eff}} = \frac{1}{0.1\ ms} = 10\ kHz$.

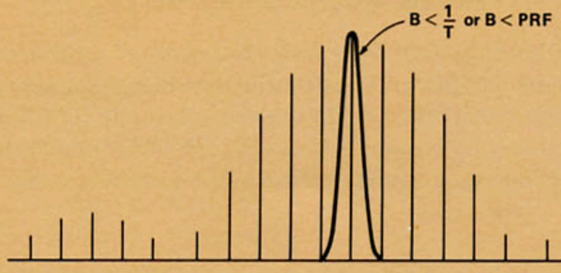


**Figure 6.** IF bandwidths smaller than PRF.

We thus can count ten spectral lines in each sidelobe or twenty lines plus the carrier line in the mainlobe, according to the duty cycle of the pulsed signal.

$$\frac{\tau_{eff}}{T} \text{ or } \tau_{eff} \cdot \text{PRF} = 0.1$$

(The fact that the amplitude of the spectral lines on the lobe minima reach zero for each integer ratio of $\frac{\tau_{eff}}{T}$ can be used to adjust the duty cycle very accurately.)

The display in Figure 10 does not change for different scan times, unless we select a scan time too short for the given scan width and bandwidth.

The new HP Spectrum Analyzer systems have a built-in logic with a warning light that enables us to avoid any wrong combination of these control settings.

For spectrum analyzers without this feature we have to satisfy an additional equation to avoid display errors:

$$\frac{\text{Scan Width } [Hz/\text{Div}]}{\text{Scan Time } [\text{sec/Div}]} < (B[Hz])^2 \qquad (2)$$

(See Appendix B)

In Figure 11 the bandwidth of the analyzer has been changed to 300 Hz. Although the resolution of the spectral lines is reduced (minima) we still have a true Fourier line spectrum display. From this experience we can derive a rule of thumb for the analyzer's bandwidth to obtain a line spectrum:

$$B < 0.3 \text{ PRF (preferably } B < 0.1 \text{ PRF)} \qquad (3)$$

This rule is valid for the shape factors (1:10 to 1:30) of the IF filters used in HP Spectrum Analyzers.

In Figure 12 we have changed the spectrum width from 100 *kHz* (10 *kHz*/Div) to 50 *kHz* (5 *kHz*/Div). We see that the spectrum envelope and the line spacing have changed, but the number of lines in each lobe remains constant.

In Figure 13 the pulse width has been altered from $\tau_{eff} = 0.1\ ms$ to $\tau_{eff} = 0.05\ ms$. Comparing with Figure 10 (same control settings on the analyzer), we find three differences:

1. The sidelobe minima are spaced by 20 *kHz*.
2. The number of lines in each sidelobe is 20. (The line spacing is still 1 kHz since we did not change the PRF.)
3. The amplitude of the spectrum envelope is 6 *dB* lower.

The last point reveals a very important fact which has not been mentioned yet, but can easily be seen in the pictures of the calibrated logarithmic displays on page 7A: The amplitude of the carrier component (highest amplitude in the spectrum envelope) of a pulse modulated signal is considerably lower than the CW amplitude of the unmodulated carrier. This effect is commonly called pulse desensitization.

## PULSE DESENSITIZATION $\alpha_L$

The expression "pulse desensitization" is quite misleading since the sensitivity of the spectrum analyzer is not reduced by a pulse modulated signal. The apparent reduction in peak amplitude can be explained in the following manner: Pulsing a CW carrier results in its power being distributed over a number of spectral components (carrier and sidebands). Each of these spectral components then contains only a fraction of the total power.

In Figure 10, where we have a duty cycle $\frac{\tau_{eff}}{T}$ of 0.1, we measure a display amplitude which has a difference of $-20\ dB$ compared to the CW amplitude of the carrier. In Figure 13, with a duty cycle of 0.05, we measure $-26\ dB$. This leads to the equation for the line spectrum pulse desensitization factor $\alpha_L$:

$$\alpha_L\ [dB] = 20 \log_{10} \frac{\tau_{eff}}{T} = 20 \log_{10} \tau_{eff} \cdot \mathrm{PRF} \tag{4}$$

This relation is only valid for a true Fourier line spectrum ($B < 0.3$ PRF). We can see that $\alpha_L$ is only dependent on the duty cycle $\frac{\tau_{eff}}{T}$ of the pulsed signal.

The average power $P_{avg}$ of the signal is also dependent on the duty cycle:

$$P_{avg} = P_{peak} \cdot \frac{\tau_{eff}}{T} \text{ or } P_{avg} = P_{peak} \cdot \tau_{eff} \cdot \mathrm{PRF}$$

Written as a ratio in dB:

$$\frac{P_{avg}}{P_{peak}}\ [dB] = 10 \log_{10} \tau_{eff} \cdot \mathrm{PRF} \tag{4a}$$

Figure 14 represents equations (4) and (4a) in a diagram.

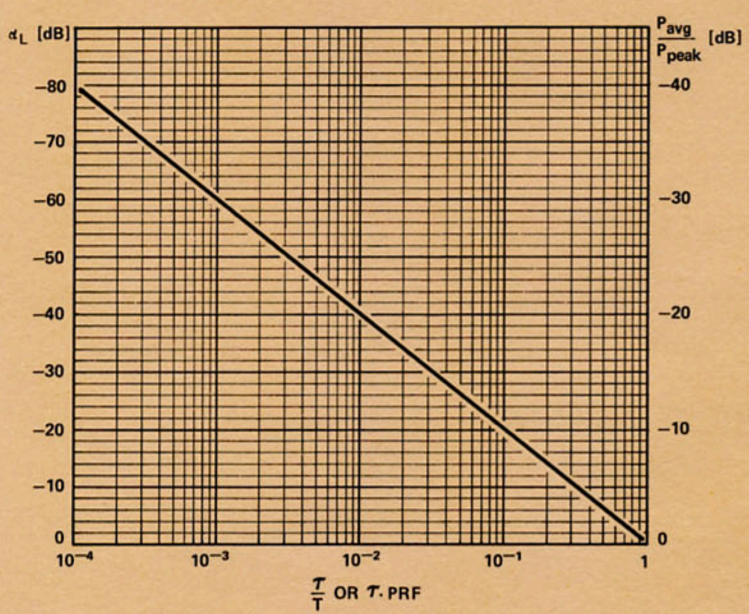


**Figure 14.** Pulse desensitization $\alpha_L$ (line spectrum).

## LINE SPECTRA OF A PULSED MODULATED 50 MHz CARRIER

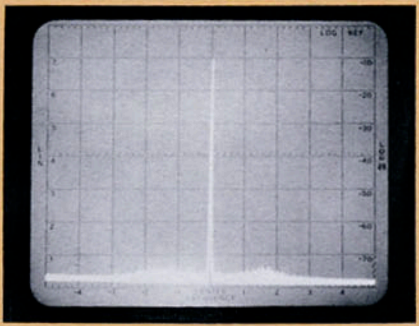

**Figure 7.** CW signal 50 *MHz*, −30 *dBm*, scan width 10 *kHz*/Div, bandwidth 100 *Hz* Log Ref. −20 *dBm*, 10 *dB*/Div.

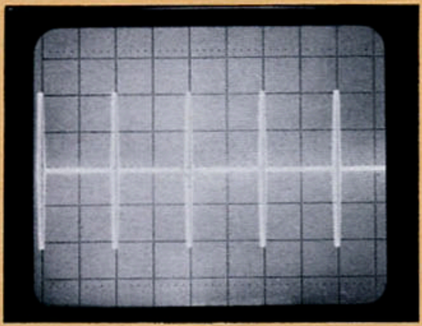

**Figure 8.** Time domain display of the 50 *MHz* signal pulse modulated with $\tau_{eff}$ = 0.1 *ms* and PRF = 1 *kHz* (0.5 *ms*/Div).

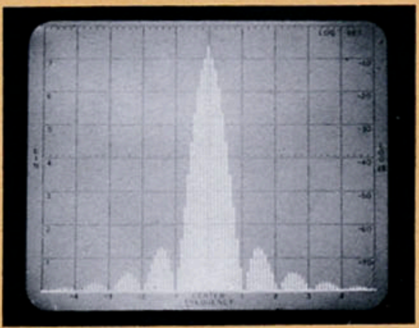

**Figure 9.** Line spectrum of the pulsed 50 *MHz* signal. Linear display 100 *μV*/ Div, scan 10 *kHz*/Div.

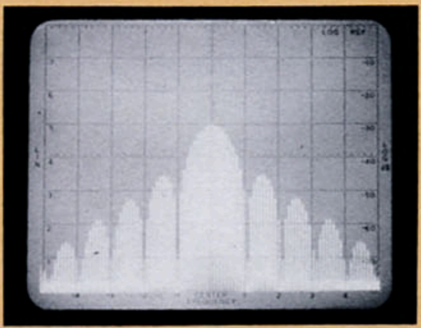

**Figure 10.** Same spectrum in logarithmic display scan width 10 *kHz*/Div, bandwidth 100 *Hz*, Log Ref. −20 *dBm*, 10 *dB*/Div.

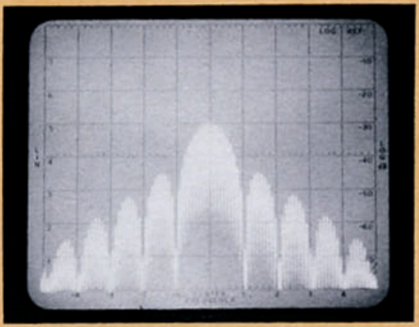

**Figure 11.** Same spectrum with 300 *Hz* analyzer bandwidth. Scan width 10 *kHz*/ Div, Log Ref. −20 *dBm*, 10 *dB*/Div.

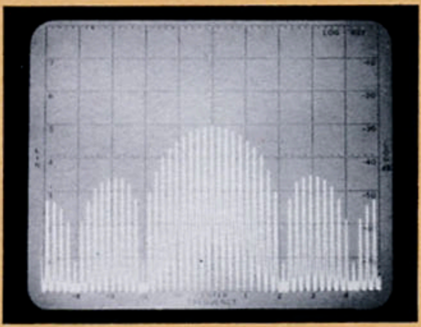

**Figure 12.** Same signal but scan width changed to 5 *kHz*/Div. Bandwidth 100 *Hz*, Log Ref. −20 *dBm*, 10 *dB*/Div.

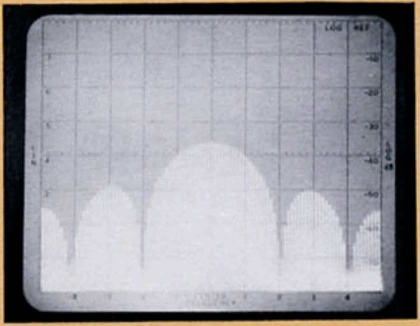

**Figure 13.** Carrier now modulated with a pulse width of $\tau_{eff}$ = 0.05 *ms* (PRF = 1 *kHz*) scan width 10 *kHz*/Div, bandwidth 100 *Hz*, Log Ref. −20 *dBm*, 10 *dB*/Div.

We read from the diagram that for a duty cycle of 0.1 we will get a display desensitization of −20 *dB*, and for a ratio of 0.05 we get −26 *dB* as shown in Figure 10 and Figure 13. The diagram also shows that the desensitization factor $\alpha_L$ becomes very large for low duty cycles. In this case, the sensitivity of the analyzer and the maximum signal level at the broadband front end mixer become important factors. We shall describe the necessary considerations for these analyzer properties in the next chapter about the more important "Pulse" spectrum display.

## TRANSITION TO THE "PULSE" RESPONSE

If we increase the IF bandwidth in our example further to 1 *kHz*, we get the display shown in Figure 15. We notice that the analyzer has lost the ability to resolve the spectral lines since B = PRF. The lines now displayed are generated in the time domain by the single pulses of the signal. We also see that the displayed amplitude of the spectrum envelope has increased. This is due to the fact that the IF filter is now sampling a broader part of the spectrum at a time, thus collecting the power of several spectral lines.

A pulse repetition rate equal to the resolution bandwidth is the demarcation line between a true Fourier-series spectrum, where each line is a response representing the energy contained in that harmonic, and a "pulse" or Fourier-transform response.

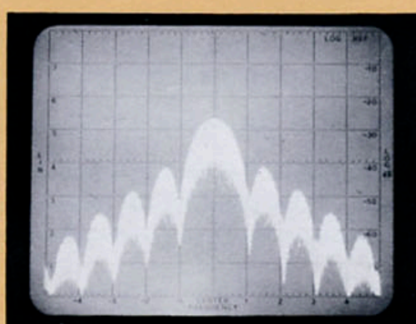

**Figure 15.** Bandwidth 1 kHz = PRF.

# CHAPTER 3
# "PULSE" SPECTRUM

## GENERAL RULES AND EXPLANATION

A "pulse" spectrum occurs when the bandwidth B of the spectrum analyzer is equal to/or greater than the PRF. The spectrum analyzer in this case cannot resolve the actual individual Fourier frequency domain components, since several lines are within its bandwidth. However, if the bandwidth is narrow compared to the spectrum envelope, then the envelope can be resolved (Figure 16). The resultant display is not a true frequency domain display, but a combination of time and frequency display. It is a time domain display of the pulse lines, since each line is displayed when a pulse occurs, regardless of the frequency within the pulse spectrum to which the analyzer is tuned at that moment. It is a frequency domain display of the spectrum envelope. The display has three distinguishing characteristics:

1. The spacing between the pulse lines and their number will change when the scan time of the analyzer is changed. The lines are spaced in real time by 1/PRF. The shape of the spectrum envelope will not change with the scan time.
2. The spacing between the lines will not change when the scan width ("*MHz*/Div" or "*kHz*/Div") is changed. The spectrum envelope will change horizontally as we would expect.
3. The amplitude of the display envelope will increase linearly as the bandwidth $B$ is increased. This means an amplitude increase of 6 $dB$ for doubling $B$. This is true as long as $B$ does not exceed $\frac{0.2}{\tau_{eff}}$. When the bandwidth equals $\frac{1}{\tau_{eff}}$ (or ½ of the mainlobe width), the display amplitude is practically the peak amplitude of the signal. At this point the IF filter covers nearly all significant spectral components. But then we have lost the ability to resolve the spectrum envelope.

We show these characteristics in the following pictures:

In Figure 17 we modulate the $-30\ dBm$ CW carrier by a pulse train with a PRF of 100 $Hz$ and $\tau_{eff} = \frac{1}{10\ kHz} = 100\ \mu s$. The analyzer's IF bandwidth is 1 $kHz$; i.e., $B = \frac{0.1}{\tau_{eff}}$.

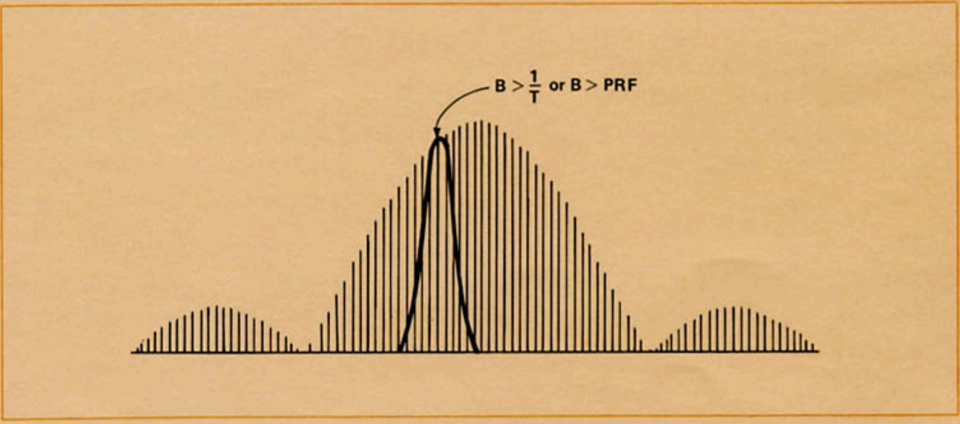


**Figure 16.** If bandwidth greater than PRF.

We can see the spectrum envelope with the mainlobe and sidelobes and the minima in between. The lines which form the envelope are not spectral lines but pulse lines in the time domain.

We can verify this by changing the scan time (Figure 18). If we reduce the scan time further, we lose the information about the shape of the spectrum envelope; i.e., the frequency domain information. But we now can easily measure the PRF in the time domain (Figure 19 and Figure 20).

In Figure 21 we changed the scan/width to 5 *kHz*/Div. The scan time is the same as in Figure 18. We can see that the spectrum envelope changed (frequency domain), but the line spacing remains constant (time domain).

In Figure 22 we use an IF bandwidth of 300 *Hz*. We can measure an amplitude decrease of approximately 10 *dB* compared to Figure 17, which shows the linear relationship between IF bandwidth and display amplitude. We also can see that the minima are better resolved than in Figure 17. In Figure 23 the bandwidth is increased to 3 *kHz*. The display amplitude increase compared to Figure 22 is not 20 *dB* but only 18 *dB*. We lost the linear relationship between bandwidth and display amplitude because $B$ is greater than $\frac{0.2}{\tau_{eff}}$ in this case. Also the resolution of the sidelobes is lost to a great extent.

If we increase $B$ to 10 *kHz* $\left(\text{which is equal to } \frac{1}{\tau_{eff}}\right)$, we get a display with an amplitude practically equal to the peak amplitude of the pulsed signal (Figure 24).

Some additional rules of thumb are of importance:

1. For a sufficient resolution of the spectrum envelope the bandwidth should be less than 5% of the mainlobe width or:

$$B < \frac{0.1}{\tau_{eff}} \tag{5}$$

   For higher resolution into the lobe minima (20 to 30 dB) we should use:

$$B < \frac{0.03}{\tau_{eff}} \tag{6}$$

2. The system must respond to each pulse independently. The effects of one pulse must decay out before the next pulse occurs. The IF amplifier decay time constant is approximately $0.3/B$. A decay of the pulse effect down to 1% (−40 *dB*) requires five time constants. This leads to the rule:

$$B > 1.7 \text{ PRF} \tag{7}$$

   However, we get less than 1 *dB* error if $B = \text{PRF}$, where the baseline is only 20 to 25 *dB* below the spectrum envelope (see Figure 15).
   The range between $B < 0.3$ PRF (line spectrum) and $B >$ PRF (pulse spectrum) shows properties of both response types and should be avoided.

3. The number of pulse lines which form the spectrum envelope display is determined by the PRF and the scan time. For a display with useful resolution, i.e., a sufficient number of lines, the scan time should be selected to:

$$\text{scan time } [s/\text{Div}] \geqslant \frac{10}{\text{PRF } [Hz]} \tag{8}$$

We then have more than 100 lines forming the spectrum envelope, thus assuring that the mainlobe peak is displayed on each scan (see Figure 18 and Figure 22).

## PULSED RF SIGNAL IN "PULSED" SPECTRUM DISPLAY
(All pictures show the same Log Ref of −20 dBm).

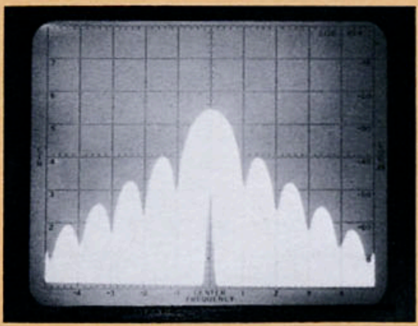

**Figure 17.** Signal (peak amplitude −30 *dBm*) pulsed with PRF = 100 *Hz*, $\tau_{eff}$ = 1/10 *kHz* = 100 $\mu$s. Scan width 10 *kHz*/Div, *B* = 1 *kHz* scan time 0.5 s/Div.

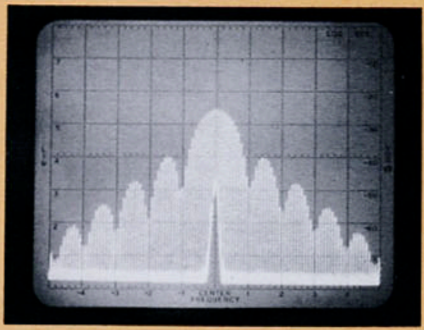

**Figure 18.** Same signal, but scan time changed to 0.1 s/Div.

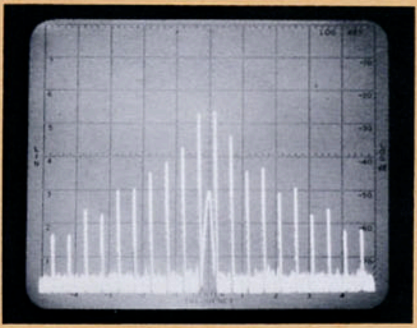

**Figure 19.** Same signal with a scan time of 20 *ms*/Div.

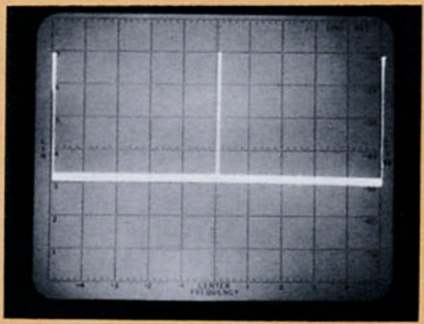

**Figure 20.** Same signal, but *B* = 300 *kHz* and scan time 2 *ms*/Div. The PRF can be measured to 1/10 *ms* = 100 *Hz*.

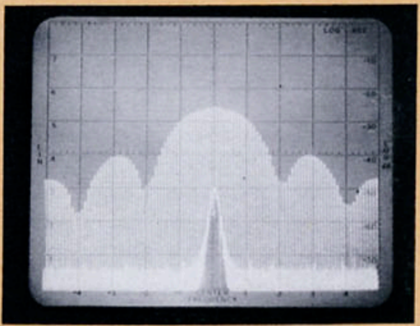

**Figure 21.** Same signal with scan width 5 *kHz*/Div, *B* = 1 *kHz*, scan time 0.1 s/Div.

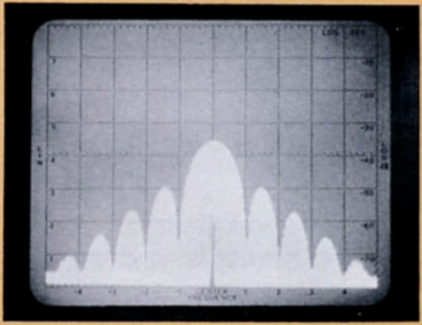

**Figure 22.** Same signal with *B* = 300 *Hz* scan width 10 *kHz*/Div, scan time 0.2 s/Div.

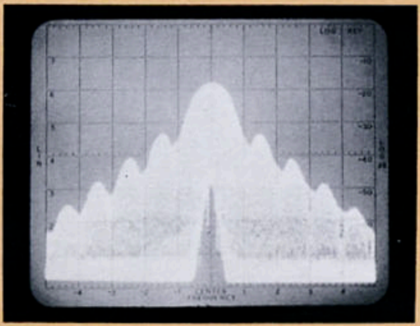

**Figure 23.** Same signal with *B* = 3 *kHz*.

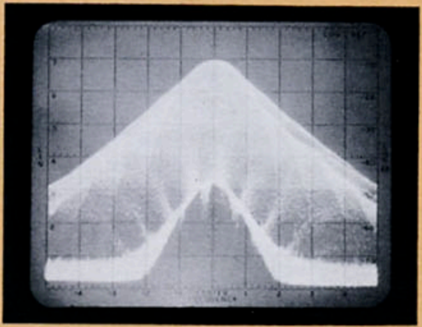

**Figure 24.** Same signal with *B* = 10 *kHz*.

The signal in the center of the displays (baseline lift) is the residual carrier which is still present during the "off" periods of the pulsed signal. Because it is essentially a CW signal, the on-off ratio of the pulse modulator can only be measured directly if $B \geqq \frac{1}{\tau_{eff}}$ as shown in Figure 24. We measure an on-off ratio of 38 *dB*.

In the other displays where $B < \frac{1}{\tau_{eff}}$ we again have to consider a "pulse desensitization" factor since we compare a CW signal with a pulsed signal. This factor will be extensively discussed later.

## WHY USE A "PULSE" SPECTRUM DISPLAY?

In many instances, it is neither possible nor desirable to make a fine grain line-by-line analysis of a spectrum. A good example of such a case is a train of short RF pulses at a low repetition frequency as normally used in radar transmitters. Not only must the IF bandwidth become inconveniently narrow, but often the frequency modulation on the pulsed carrier is so excessive that the resulting display is confusing.

In the "pulse" spectrum mode we can get all information we need: the spectrum envelope and amplitude in the frequency domain and the PRF in the time domain. We also have two advantages over the "line" spectrum display:

1. We can use shorter scan times because of the greater bandwidth.
2. We can increase the display amplitude of the pulsed signal by choosing a broader bandwidth. We know that the display amplitude increases linearly with the bandwidth $B$. The noise level of the analyzer increases only proportional to $\sqrt{B}$. So we can increase the signal-to-noise ratio proportional to $\sqrt{B}$.

   This is opposite to the CW and "line" case where we have to use narrower bandwidths to decrease the noise level, thus increasing the signal-to-noise ratio. Figure 25 and Figure 26 show these effects clearly.

From the preceding discussion about the "pulse" spectrum response we can find another important fact: The spectrum analyzer must provide independent controls for bandwidth, scan width, and scan time to optimize the display according to the rules of thumb given for this type of response. Also the variable persistence CRT offers a great advantage if we want to have a flicker-free display of pulsed signals with low PRF.

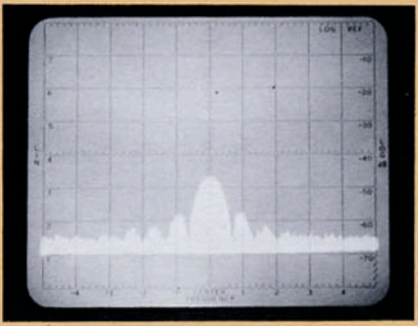

**Figure 25.** A carrier with −50 *dBm* amplitude is modulated by a pulse train with PRF = 400 *Hz*, $\tau_{eff}$ = 3 *μs*. The bandwidth $B$ = 3 *kHz*, scan width = 0.5 *MHz*/Div, scan time = 0.1 *s*/Div. Only the low order sidelobes can be seen but not measured accurately. Log Ref. −40 *dBm*.

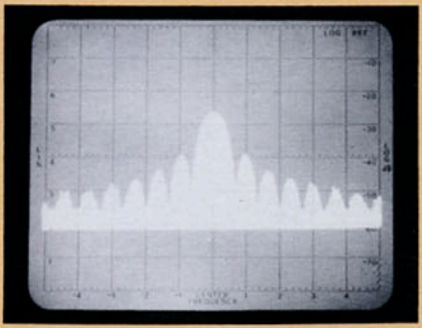

**Figure 26.** The same signal displayed with 30 *kHz* bandwidth. The noise level increased by 10 *dB*, but the signal level by 20 *dB*. The lobes and minima can be measured easily. Log Ref. −40 *dBm*.

## PEAK PULSE RESPONSE – PULSE DESENSITIZATION $\alpha_p$

In the "pulse" spectrum just described, the response of the spectrum analyzer to each RF input pulse is in essence the pulse response of the analyzer's IF amplifier.

The peak pulse response of the HP Spectrum Analyzers has been established and is relatively independent of pulse shape and pulse repetition frequency (for $B > PRF$). The expression relating the peak pulse response to a CW signal response is the pulse desensitization factor $\alpha_p$.

This factor $\alpha_p$ for the "pulse" response depends on different physical conditions compared to $\alpha_L$ in the "line" spectrum:

$$\alpha_p = 20 \log_{10} \cdot \tau_{eff} \cdot B_{imp} \, [dB]$$

In this equation we find a new expression: the Effective Impulse Bandwidth, $B_{imp}$. This can be visualized as the bandwidth of an ideal, rectangularly shaped filter with a pulse response equivalent to the actual filter with the 3 $dB$ bandwidth $B$ (Figure 27). Since the impulse bandwidth $B_{imp}$ of the IF amplifier is not the same as its 3 dB bandwidth $B$, a correction factor K has been introduced. This factor K represents an empirical approach defining $B_{imp}$ relative to $B$:

$$K = \frac{B_{imp}}{B} \tag{9}$$

K can be determined with a pulsed signal with known properties (AN 122, "EMI Measurement Procedure," p. 125).

For synchronuously tuned filters as used in the HP IF sections, the value of $K$ has been measured as approximately 1.5.* The error introduced by different shape factors of the IF filter and different pulse shapes is normally less than 1 $dB$.

We can now write:

$$\alpha_p \, [dB] = 20 \log_{10} \cdot \tau_{eff} \cdot K \cdot B; \qquad K = 1.5 \tag{10}$$

*If the 300 $kHz$ filter is used in the 8552A/$B$ IF sections, a measurement of its impulse bandwidth is recommended, since a number of these plug-ins have a $B_{imp}$ smaller than 450 $kHz$ ($1.5 \times 300$ $kHz$). This results in an additional desensitization of 1 to 2 $dB$. A quick and simple check can be made by measuring the 6 $dB$ bandwidth, which is approximately equal to $B_{imp}$.

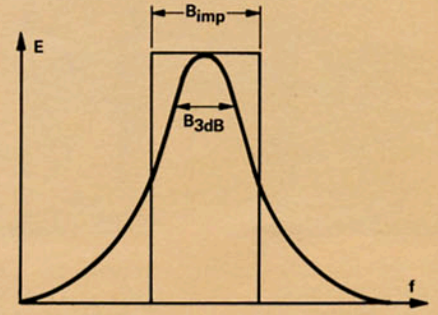


**Figure 27.** Equivalent $B_{imp}$ of Gaussian filter.

There are several conditions which must be satisfied if Eq. (10) is to be valid:

1. The IF bandwidth-pulse width product must be less than two-tenths:

$$B \cdot \tau_{eff} < 0.2 \text{ or } B < \frac{0.2}{\tau_{eff}}$$

2. The normalized scan rate (NSR) of the analyzer must be less than one:

$$\text{NSR} = \frac{\text{Scan Width } [Hz/\text{Div}]}{\text{Scan Time } [s/\text{Div}] \cdot (B[Hz])^2} < 1$$

3. The IF bandwidth must be greater than the PRF: $B > \text{PRF}$

The conditions in 1 to 3 are automatically accomplished if the equations (5), (8), and (7) are satisfied.

4. The peak pulse amplitude at the broadband input mixer of the analyzer must stay below the saturation point (1 dB compression). The typical saturation point for HP spectrum analyzers is between −10 dBm and −5 dBm:

$$P_{\text{peak}} \leqq -10\ dBm \tag{11}$$

Figure 28 is a diagram showing the pulse desensitization $\alpha_p$ in relation to IF bandwidth $B$ and pulse width $\tau_{eff}$. We see that the PRF does not appear, since it is of no significance for the display amplitude as long as $B > \text{PRF}$. The shaded area between the $B = \frac{0.03}{\tau_{eff}}$ and $B = \frac{0.1}{\tau_{eff}}$ represents the optimum bandwidth range for an analysis of a pulsed signal. There are also three dotted lines which show different noise levels of an analyzer for a fast determination of the dynamic range.

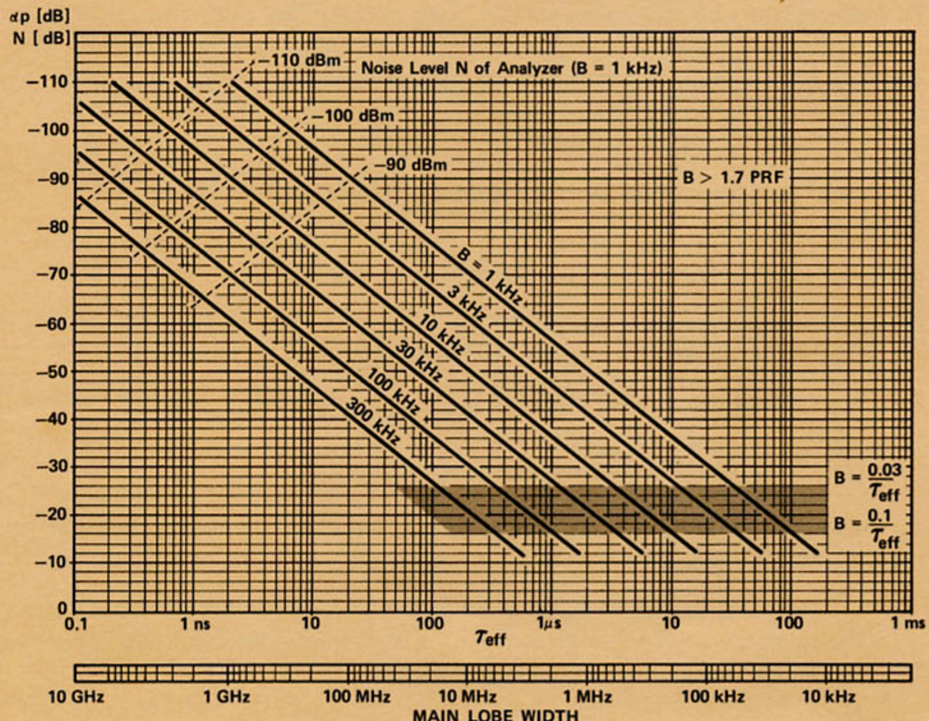


**Figure 28.** Pulse desensitization $\alpha p$ (pulse spectrum).

We will now take a few examples to show how the diagram is used:

1. We assume a pulsed signal with the following characteristics: $P_{peak} = -30$ $dBm$, $\tau_{eff} = 1\ \mu s$, PRF = 1 $kHz$. The noise level of the analyzer is $N = -100$ $dBm$ for 1 $kHz$ bandwidth. We find on the diagram for $\tau_{eff} = 1\ \mu s$ an optimum bandwidth of 100 $kHz$ ($\rightarrow B >$ PRF). We then can read a pulse desensitization of $\alpha_p \approx -16\ dB$. The displayed amplitude of the spectrum envelope will be $\approx -46\ dBm$. We also read from the crossing point of the line for $N = -100\ dBm$ and the line for $B = 100\ kHz$ a resultant noise level of $-80\ dBm$. We thus get a usable display range ($S/N$ ratio) of only 34 $dB$. Although this range is sufficient in most cases for evaluation of the pulse spectrum, this example shows how important a spectrum analyzer with a low noise level is.

2. PULSE POWER MEASUREMENTS: We see on the spectrum analyzer display the spectrum envelope of a pulsed signal with the following characteristics: the display amplitude is $-50\ dBm$, the mainlobe width is 10 $MHz$. The analyzer's bandwidth is 300 $kHz$. What are the peak and the average powers of the signal?

   The effective pulse duration $\tau_{eff}$ is calculated from the lobe width or read from the diagram:

   $$\tau_{eff} = \frac{2}{10\ MHz} = 0.2\ \mu s \text{ or } 200\ ns$$

   In the diagram, we find a pulse desensitization of $-21\ dB$ for $\tau_{eff} = 200$ ns and $B = 300\ kHz$. The peak power is 21 $dB$ greater than the displayed amplitude, and we can calculate the peak power to $P_{peak} = -29\ dBm$.

   To find the average power, we first have to measure the PRF. This is done by reducing the scan time until we can easily measure the pulse line spacing in time domain. Assume we measured the line spacing to 1 ms which equals a PRF of 1 $kHz$, we then can calculate the average power $P_{avg} = P_{peak} \cdot \tau_{eff} \cdot \text{PRF}$.

   $$\tau_{eff} \cdot \text{PRF} = 2.10^{-7} s \cdot 10^3\ Hz = 2.10^{-4}$$

   Using the diagram for $\alpha_L$, Figure 14 on page 7, we find a factor $\frac{P_{avg}}{P_{peak}}$ of $-37\ dB$. Thus, with the peak power $P_{peak}$ of $-29\ dBm$ and the factor of $-37$ $dB$, we can calculate the average power $P_{avg} = -66\ dBm$.

3. We want to calculate the peak power of a signal displayed with an amplitude of $-30\ dBm$ and a mainlobe width of 100 $MHz$. The analyzer bandwidth is 300 $kHz$. The signal has a pulse duration $\tau_{eff} = \frac{2}{100\ MHz} = 20\ ns$. We find a desensitization factor of $-41\ dB$.

   This would yield a signal peak power of $+11\ dBm$, far beyond the saturation level of $-10\ dBm$. Thus, the calculation is not valid. We have to insert at least 20 $dB$ attenuation before the input mixer.

   To check that the input signal level at the front end mixer is below the saturation point, we have to observe that for a 10 $dB$ step of the input attenuator the display amplitude must also change by exactly 10 $dB$.

## VERY SHORT RF PULSES

We know from the diagram for $\alpha_p$ (Figure 28) that the desensitization of the analyzer display becomes very high for very short RF pulses, even with the widest bandwidth. If we assume that we can provide the maximum usable input signal level of $-10$ *dBm* (which is normally possible when we measure in the proximity of the radar transmitter to be investigated), we are then limited only by the sensitivity of the analyzer. For a sufficient evaluation of a pulsed RF signal we should have a display range of at least 30 *dB* above the noise level. Figure 29 is a diagram which shows the maximum usable display range as a function of pulse width and analyzer sensitivity for a maximum input level of $-10$ *dBm* and a bandwidth of 300 *kHz*.*

We can easily see that for a pulse width of, for example, 1 *ns*, an analyzer must have a sensitivity of $-110$ *dBm* (specified for $B = 1$ *kHz*) or better to yield a usable display. It is not possible to improve the signal-to-noise ratio with a low noise preamplifier, since we are already limited by the saturation level of the input mixer. The new generation of HP spectrum analyzers offers exceptionally high sensitivities which allow measurements of such extremely short RF pulses.

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*See note on page 14.

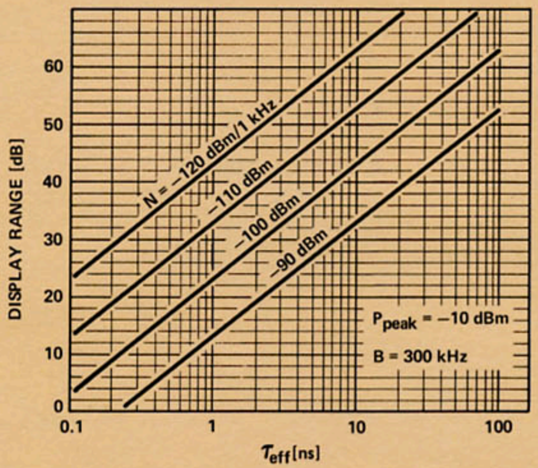
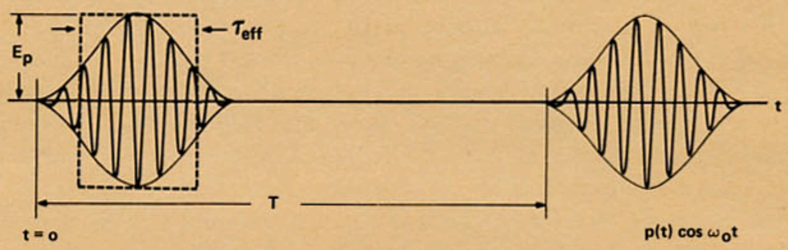



**Figure 29.** Display range vs. sensitivity.

# CHAPTER 4
# SUMMARY OF PULSE SPECTRA CHARACTERISTICS




| Type of Response | "Line" Spectrum (Fourier Series) | "Pulse" Spectrum (Fourier Transform) |
|---|---|---|
| Requirements for each Type of Spectrum: | | $B>1.7$ PRF |
| Bandwidth | $B<0.3$ PRF | $B<\frac{0.1}{\tau_{eff}}$ |
| Scan Time | $T_s > \frac{F_s}{B^2}$ | $T_s>10/\text{PRF}$ |
| Peak Input Power | $P_{peak} \leqq -10$ dBm | $P_{peak} \leqq -10\ dBm$ |
| Desensitization Factor | $\alpha_L = 20 \log_{10} \cdot \frac{\tau_{eff}}{T}$ | $\alpha_p = 20 \log_{10} \cdot \tau_{eff} \cdot K \cdot B$ |
| Amplitude of Spectrum Display at $\omega = \omega_0$ | $A = E_p' \cdot \frac{\tau_{eff}}{T}$ $= E_p' \cdot \tau_{eff} \cdot \text{PRF}$ | $A = E_p' \cdot \tau_{eff} \cdot K \cdot B$ |
| Type of Display Used for Duty Cycle $\frac{\tau_{eff}}{T}$ of | Fourier or spectral lines $> 0.05$ | Pulse repetition rate lines $< 0.05$ |
| Number of Lines/ Division | Changes with scan width not scan time | Changes with scan time not scan width |

$E_p'$ = response on CRT due to CW signal $E_p \cos \omega_0 t$

$T_s$ = scan time in sec/Div

$F_s$ = scan width in $Hz$/Div

$\text{PRF} = \frac{1}{T}$ = Pulse repetition frequency in $Hz$

$B$ = IF bandwidth (3 $dB$)

$K$ = constant of IF amplifier ($K \approx 1.5$)

$\tau_{eff}$ = width of rectangular pulse of same height and area as pulse applied to analyzer $= \int_0^t \frac{p(t)\,dt}{E_p'}$

## COMMON PULSE SPECTRA

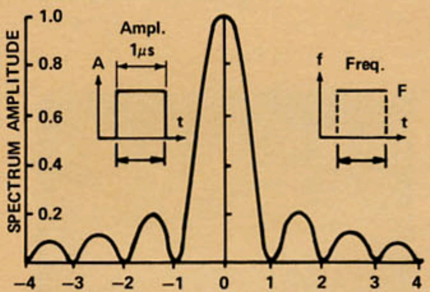
Figure 30 shows some examples of typical spectrum displays for pulse signals with different pulse shapes and with the presence of AM and FM. An extensive mathematical treatment of different pulse forms and their spectra can be found in Appendix A.

The ideal rectangular RF pulse free of FM will produce a symmetrical pulse spectrum as shown in (a). When the pulse is changed to a triangular shape, the spectrum remains symmetrical with decreased amplitude of the sidelobes (b). The pulse spectrum will remain symmetrical even if the pulse shape is distorted or unsymmetrical.

PULSE SPECTRUM IN THE PRESENCE OF FM: A symmetrical pulse with linear coherent FM will produce a symmetrical spectrum with increased sidelobe amplitude and minima not reaching zero, (c), (d).

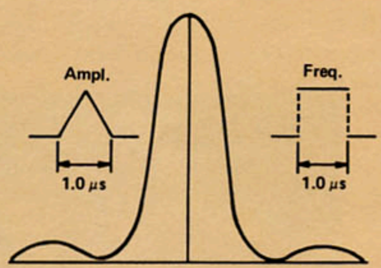
If incidental FM (FM due to amplitude modulation) or coherent FM is introduced together with an unsymmetrical pulse, an unsymmetrical pulse spectrum with the minima not reaching zero will be produced, (e), (f). This is also true for a symmetrical pulse with nonlinear coherent FM.

Pictures of the pulse spectra produced by actual radar transmitters can be found on pages 20 to 23.



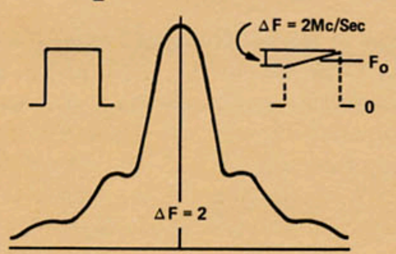

(a)

Spectrum of rectangular pulse without AM or FM occurring during pulse. Shape is that of $\frac{\sin \omega\frac{\tau}{2}}{\omega\frac{\tau}{2}}$ function.



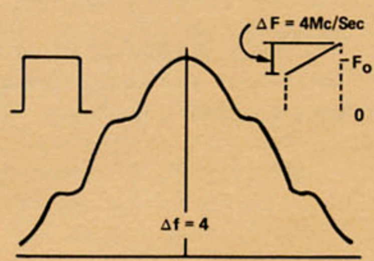

(b)

Triangular pulse spectrum without FM during pulse. Effective pulse width is shorter than (a) causing minimas to occur at wider intervals of frequency.



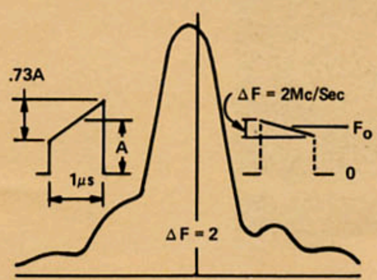

(c)

Spectrum of rectangular pulse with linear FM resulting in increased sidelobe amplitude and minimas not reaching zero.



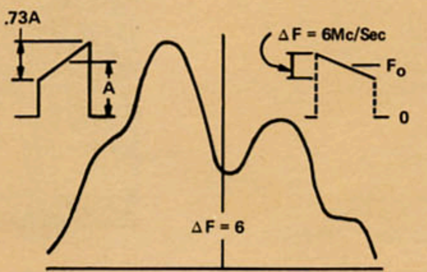

(d)

Same pulse spectrum as (c) with more severe FM.




(e)

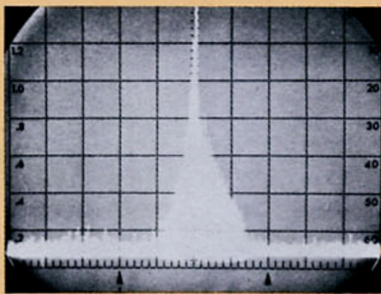
Effect of linear AM and FM during pulse. Note loss in symmetry due to pulse amplitude slope.



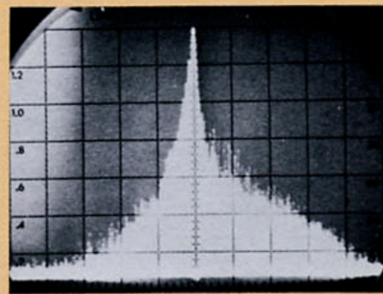

(f)

More severe case of FM and AM occurring during pulse.

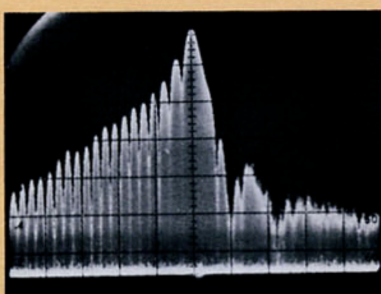
**Figure 30.** Common pulse spectra.



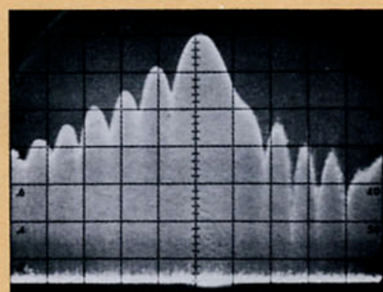
30 *MHz*/Div



10 *MHz*/Div

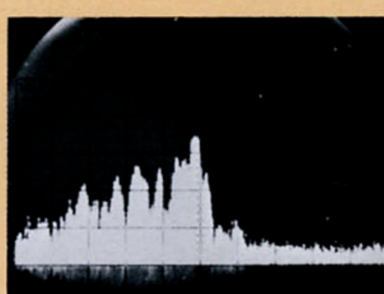


3 *MHz*/Div

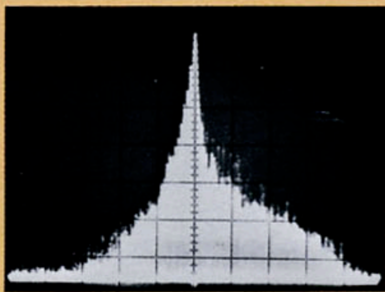


1 *MHz*/Div

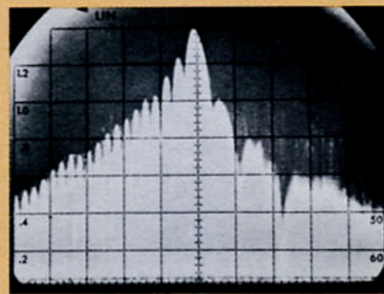
The above is a typical spectrum signature of the fundamental frequencies of an "*L*" band radar with an approximate 1.0 $\mu$sec pulse width. The above pictures were made with a 10 *kHz* bandwidth. Note transients from magnetron along with extreme FM.



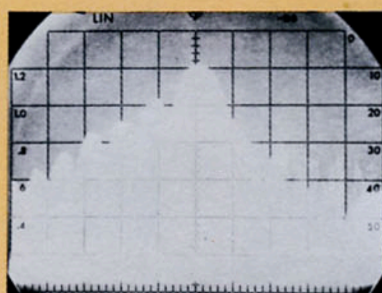
This is the third harmonic output of a radar operating at a pulse width of 4.5 $\mu$sec and $f_c$ of 1300 *MHz*. An interdigital filter (2-4 *GHz*) was used to trap out the fundamental. The sweep width is 3 *MHz*/Div.



10 *MHz*/Div

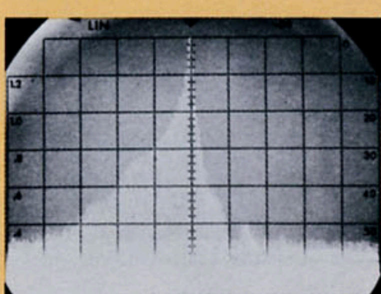


3 *MHz*/Div

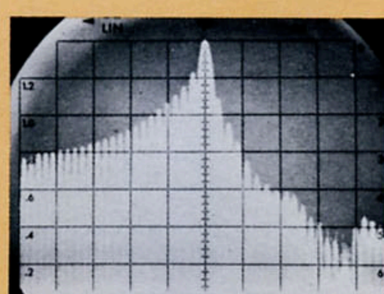


1 *MHz*/Div

Here is a radar where the magnetron is moding rather badly. Note the transients occurring on the high side of the carrier frequency. Only a high speed photographic record would show this.

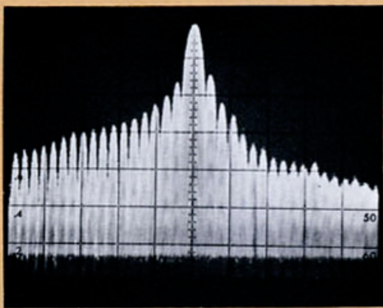


L Band Radar 300 *pps*/4.5 $\mu$sec. 3 *MHz*/ Div spectrum width log display

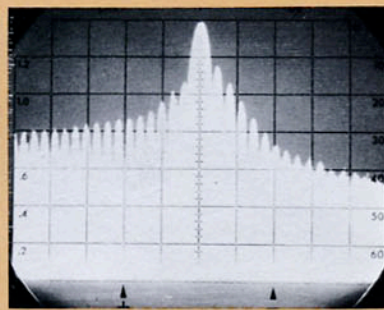


Same conditions as at left except 1 *MHz*/ Div spectrum width

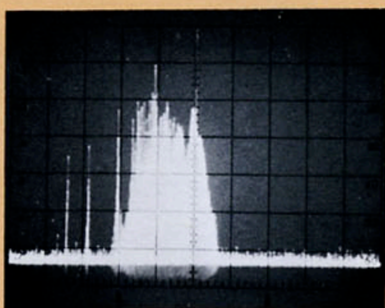
Here is a spectrum display of one of the cleanest radars noted. Note the absence of moding. The FM present seems to be normal for conventional magnetron.



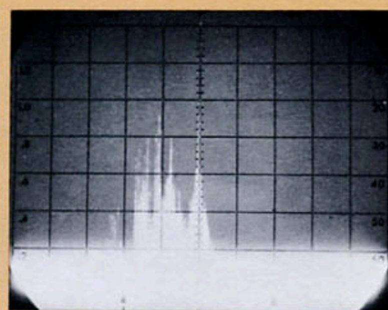
Channel A; 1 *MHz*/Div



Channel B; 1 *MHz*/Div

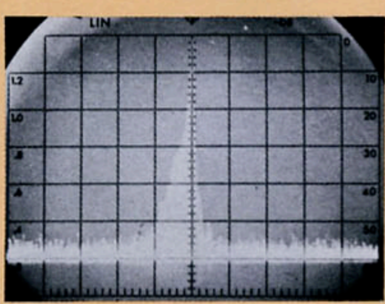


Magnetron output; 100 *MHz*/Div

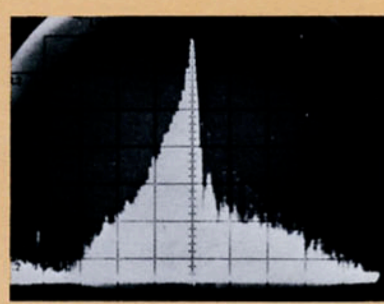


Duplexer output; 100 *MHz*/Div

This is another radar which employed two channels. Channel "*A*" operated normally, but Channel "*B*" had marginal FM. The magnetron output had considerable noise and spikes showing up which did not show up on the duplexer output. Note the one picture when the base-line dimmer was not used. When base-line dimmer is used, true base level remains at zero.

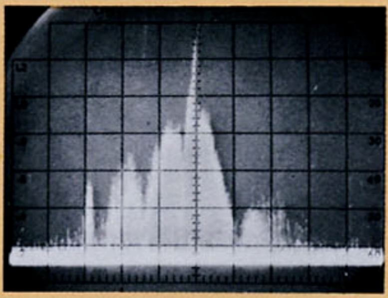


4.5 $\mu$sec



1 $\mu$sec

These are spectra of a radar when the pulse width is changed from 4.5 $\mu$sec to 1.0 $\mu$sec. Note the change in spectrum. An IF bandwidth of 10 *kHz* and sweep width of 10 *MHz*/Div was used. Note the increased transient effects on narrow pulse operations.



This is a display of a 1 megawatt radar at mid-L band showing spurious radiation over 300 *MHz* below the main carrier and interfering in TACAN channels. Horizontal 100 *MHz*/Div, vertical 10 *dB*/Div. Some emitted broadband output is only 20 *dB* down from carrier (10 *kW*).

# APPENDIX A
# TABLE OF IMPORTANT TRANSFORMS

## EXPLANATION OF THE TABLE

The time functions and corresponding frequency functions in this table are related by the following expressions:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \qquad \text{(Direct transform)}$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega \qquad \text{(Inverse transform)}$$

The $1/2\pi$ multiplier in the inverse transform arises merely because the integration is written with respect to $\omega$, rather than cyclic frequency. Otherwise the expressions are identical except for the difference of sign in the exponent. As a result, functions and their transforms can be interchanged with only slight modification. Thus, if $F(\omega)$ is the direct transform of $f(t)$, it is also true that $2\pi f(-\omega)$ is the direct transform of $F(t)$. For example, the spectrum of a $\frac{\sin x}{x}$ pulse is rectangular (pair 6) while the spectrum of a rectangular pulse is of the form $\frac{\sin x}{x}$ (pair 7). Likewise pair 1S is the counterpart of the well-known fact that the spectrum of a constant ($d$–$c$) is a spike at zero frequency.

The frequency functions in the table are in many cases listed both as functions of $\omega$ and also of $p$. This is done merely for convenience. $F(p)$ in all cases is found by substituting $p$ for $i\omega$ in $F(\omega)$. (Not simply $p$ for $\omega$ as the notation would ordinarily indicate. That is, in the usual mathematical convention one would write $F(\omega) = F\left(\frac{p}{i}\right) = G(p)$ where the change in letter indicates the resulting change in functional form. The notation used above has grown through usage and causes no confusion, once understood.) Thus, in the $p$-notation

$$F(p) = \int_{-\infty}^{\infty} f(t)\, e^{-pt}\, dt \qquad f(t) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} F(p)\, e^{pt}\, dp$$

The latter integral is conveniently evaluated as a contour integral in the $p$-plane, letting $p$ assume complex values.

The frequency functions have been plotted on linear amplitude and frequency scales, and where convenient, also on logarithmic scales. The latter scales often bring out characteristics not evident in the linear plot. Thus, many of the spectra are asymptotic to first or second degree hyperbolas on a linear plot. On a log plot these asymptotes become straight lines of slope $-1$ or $-2$ (i.e., $-6$ or $-12$ $dB$/octave).

The time functions in the table have all been normalized to convenient peak amplitudes, areas or slopes. For any other amplitude, multiply both sides by the appropriate factor. Thus, the spectrum of a rectangular pulse 10 volts in amplitude and 2 seconds long is (from pair 7) $20 \frac{\sin \omega}{\omega}$ volt-seconds.

Again, upon multiplication by a constant having appropriate dimensions, the frequency functions become filter transmissions. Thus, if pair 1 is multiplied by $\alpha$, the frequency function represents a simple RC cutoff. A one coulomb impulse (pair 1S) applied to this filter would produce an output (impulse response) with the spectrum $\frac{\alpha}{p+\alpha} \times 1$ coulomb, representing the time function $\alpha e^{-\alpha\tau}$ coulombs (which has the dimensions of amperes). Or a 1 volt step function (pair 2S) would produce the output spectrum $\frac{\alpha}{p+\alpha} \times \frac{1}{p}$ volts, which represents the time function $(1-e^{-\alpha\tau})$ volts (pair 4S).

The entries 1S through 6S in the table are singular functions for which the transforms as defined above exist only as a limit. For example, 1S may be thought of as the limit of pair 7 $\left(\text{multiplied by } \frac{1}{\tau}\right)$ as $\tau \longrightarrow 0$.

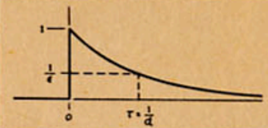
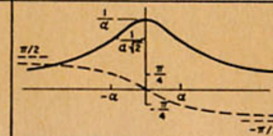
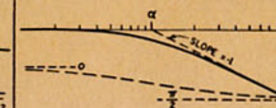
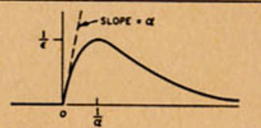
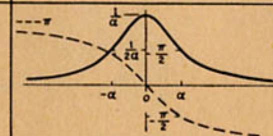
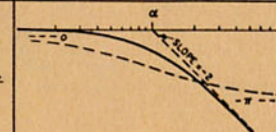
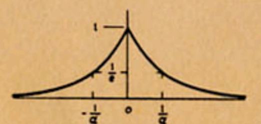
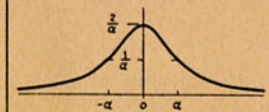
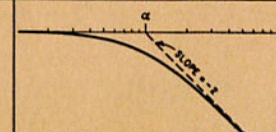
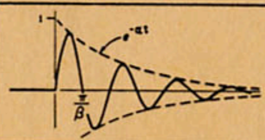
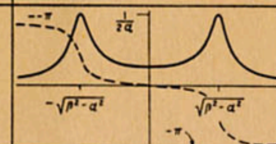
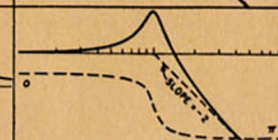
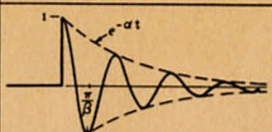
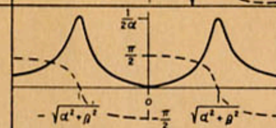
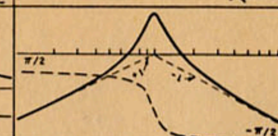
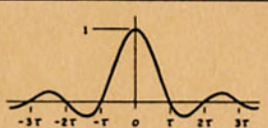
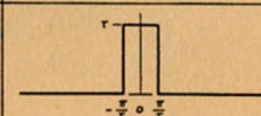
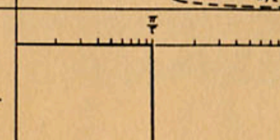
## PROPERTIES OF TRANSFORMS

There are a number of important relations which describe what happens to the transforms of functions when the functions themselves are added, multiplied, convolved, etc. These relations state mathematically many of the operations encountered in communications systems: operations such as linear amplification, mixing, modulation, filtering, sampling, etc. These relations are all readily deducible from the defining equations above; but for ready reference some of the more important ones are listed in the Properties of Transforms on the last page of this appendix.

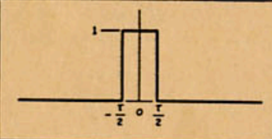
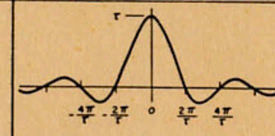
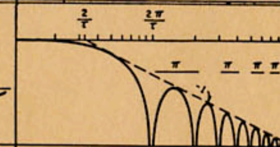
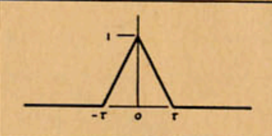
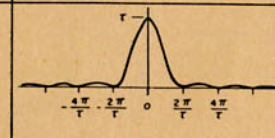
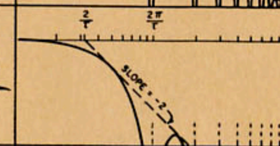
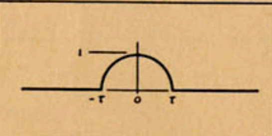
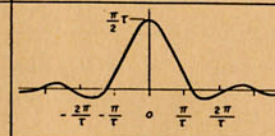
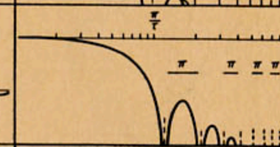
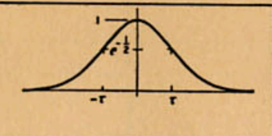
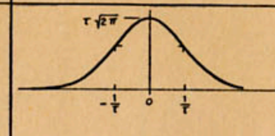
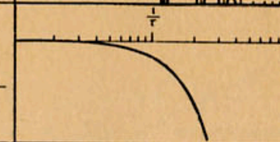
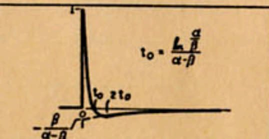
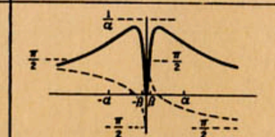
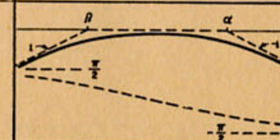
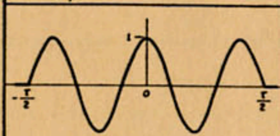
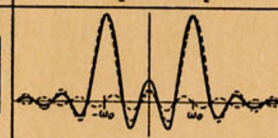
Again, because of the similarity of the direct and inverse transforms, a symmetry exists in these properties. Thus, delaying a function multiplies its spectrum by a complex exponential; while multiplying the function by a complex exponential delays its spectrum. Multiplying any two functions is equivalent to convolving their spectra; multiplying their spectra is equivalent to convolving the functions; etc.

Many of the pairs listed in the Table of Transforms can be obtained from others by using one or more of the rules of manipulation listed in the Properties of Transforms. For example, the time function in pair 8 is $\frac{1}{\tau}$ times the *convolution* of that in pair 7 with itself. The spectrum should therefore be $\frac{1}{\tau}$ times the *product* of that in pair 7 with itself, as it indeed is. Further, by using these properties, many pairs *not* in the table can be obtained from those given. For example, the spectrum of $f(t) = (1 - \alpha\tau) e^{-\alpha\tau}$ is (by the addition property) $F(p) = \frac{1}{p+\alpha} - \frac{\alpha}{(p+\alpha)^2} = \frac{p}{(p+\alpha)^2}$.

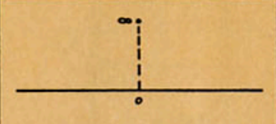
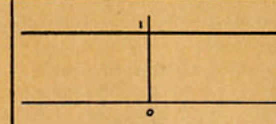
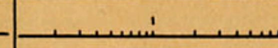
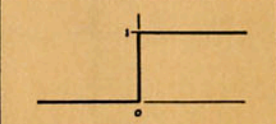
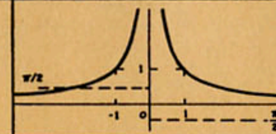
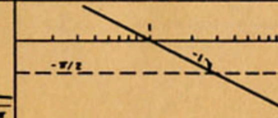
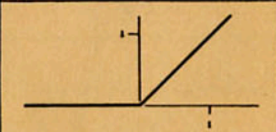
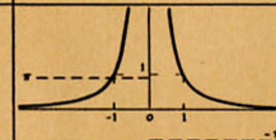
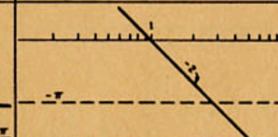
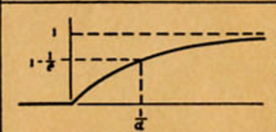
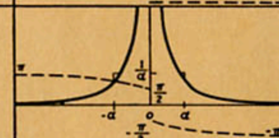
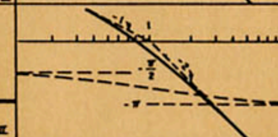
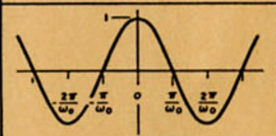
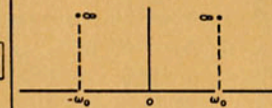
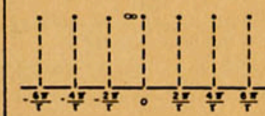
# TABLE OF IMPORTANT TRANSFORMS

| TIME FUNCTIONS | | NO. | FREQUENCY FUNCTIONS | | |
|---|---|---|---|---|---|
| | | | | (LINEAR SCALES) | (LOG AMPL. – LOG FREQ.) |
|  | $f(t)=\begin{cases} 0, & t<0 \\ e^{-\alpha t}, & t>0 \end{cases}$ | 1 | $F(p)=\dfrac{1}{p+\alpha}$ <br> $F(\omega)=\dfrac{1}{\alpha+i\omega}$ |  |  |
|  | $f(t)=\begin{cases} 0, & t<0 \\ \alpha t e^{-\alpha t}, & t>0 \end{cases}$ | 2 | $F(p)=\dfrac{\alpha}{(p+\alpha)^2}$ <br> $F(\omega)=\dfrac{\alpha}{(\alpha+i\omega)^2}$ |  |  |
|  | $f(t)=e^{-\alpha\lvert t\rvert}$ | 3 | $F(p)=\dfrac{2\alpha}{\alpha^2-p^2}$ <br> $F(\omega)=\dfrac{2\alpha}{\alpha^2+\omega^2}$ |  |  |
|  | $f(t)=\begin{cases} 0, & t<0 \\ e^{-\alpha t}\sin\beta t, & t>0 \end{cases}$ | 4 | $F(p)=\dfrac{\beta}{(p+\alpha)^2+\beta^2}$ <br> $F(\omega)=\dfrac{\beta}{(\alpha^2+\beta^2)-\omega^2+i2\alpha\omega}$ |  |  |
|  | $f(t)=\begin{cases} 0, & t<0 \\ e^{-\alpha t}(\cos\beta t-\frac{\alpha}{\beta}\sin\beta t), & t>0 \end{cases}$ | 5 | $F(p)=\dfrac{p}{(p+\alpha)^2+\beta^2}$ <br> $F(\omega)=\dfrac{i\omega}{(\alpha^2+\beta^2)-\omega^2+i2\alpha\omega}$ |  |  |
|  | $f(t)=\dfrac{\sin(\pi\frac{t}{\tau})}{(\pi\frac{t}{\tau})}$ | 6 | $F(\omega)=\begin{cases} \tau, & \lvert\omega\rvert<\frac{\pi}{\tau} \\ 0, & \lvert\omega\rvert>\frac{\pi}{\tau} \end{cases}$ |  |  |

# TABLE OF IMPORTANT TRANSFORMS

| TIME FUNCTIONS | | NO. | FREQUENCY FUNCTIONS | (LINEAR SCALES) | (LOG AMPL. – LOG FREQ.) |
|---|---|---|---|---|---|
|  | $f(t)=\begin{cases}1, & \lvert t\rvert<\frac{\tau}{2}\\ 0, & \lvert t\rvert>\frac{\tau}{2}\end{cases}$ | 7 | $F(\omega)=\tau\,\dfrac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}$ |  |  |
|  | $f(t)=\begin{cases}1-\frac{\lvert t\rvert}{\tau}, & \lvert t\rvert<\tau\\ 0, & \lvert t\rvert>\tau\end{cases}$ | 8 | $F(\omega)=\tau\,\dfrac{\sin^2\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2}$ |  |  |
|  | $f(t)=\begin{cases}\sqrt{1-\left(\frac{t}{\tau}\right)^2}, & \lvert t\rvert<\tau\\ 0, & \lvert t\rvert>\tau\end{cases}$ | 9 | $F(\omega)=\frac{\pi}{2}\tau\,\dfrac{2J_1(\omega\tau)}{(\omega\tau)}$ |  |  |
|  | $f(t)=e^{-\frac{1}{2}\left(\frac{t}{\tau}\right)^2}$ | 10 | $F(\omega)=\tau\sqrt{2\pi}\,e^{-\frac{1}{2}(\tau\omega)^2}$ |  |  |
|  | $f(t)=\begin{cases}0, & \lvert t\rvert<0\\ \dfrac{\alpha e^{-\alpha t}-\beta e^{-\beta t}}{\alpha-\beta}, & \lvert t\rvert>0\end{cases}$ | 11 | $F(p)=\dfrac{p}{(p+\alpha)(p+\beta)}$ |  |  |
|  | $f(t)=\begin{cases}\cos\omega_0 t, & \lvert t\rvert<\frac{\tau}{2}\\ 0, & \lvert t\rvert>\frac{\tau}{2}\end{cases}$ | 12 | $F(\omega)=\frac{\tau}{2}\left[\dfrac{\sin\left(\frac{\omega-\omega_0}{2}\right)\tau}{\left(\frac{\omega-\omega_0}{2}\right)\tau}+\dfrac{\sin\left(\frac{\omega+\omega_0}{2}\right)\tau}{\left(\frac{\omega+\omega_0}{2}\right)\tau}\right]$ |  | |

# TABLE OF IMPORTANT TRANSFORMS

| TIME FUNCTIONS | | NO. | FREQUENCY FUNCTIONS | (LINEAR SCALES) | (LOG AMPL. – LOG FREQ.) |
|---|---|---|---|---|---|
|  | $f(t)=\lim_{\tau\to 0}\begin{cases}\frac{1}{\tau}, & \|t\|<\frac{\tau}{2}\\ 0, & \|t\|>\frac{\tau}{2}\end{cases}$ $=\delta(t)$ (DELTA FUNCTION) | 1S | $F(p)=F(\omega)=1$ |  |  |
|  | $f(t)=\int_{-\infty}^{t}\delta(\lambda)d\lambda=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$ $=u(t)$ (UNIT STEP) | 2S | $F(p)=\frac{1}{p}$ |  |  |
|  | $f(t)=\int_{-\infty}^{t}u(\lambda)d\lambda=\begin{cases}0, & t<0\\ t, & t>0\end{cases}$ $=s(t)$ (UNIT SLOPE) | 3S | $F(p)=\frac{1}{p^2}$ |  |  |
|  | $f(t)=\begin{cases}0, & t<0\\ 1-e^{-\alpha t}, & t>0\end{cases}$ | 4S | $F(p)=\frac{\alpha}{p(p+\alpha)}$ |  |  |
|  | $f(t)=\cos\omega_0 t$ | 5S | $F(\omega)=\pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$ |  | |
|  | $f(t)=\sum_{-\infty}^{\infty}\delta(t-n\tau)$ | 6S | $F(\omega)=\frac{2\pi}{\tau}\sum_{-\infty}^{\infty}\delta\left(\omega-n\frac{2\pi}{\tau}\right)$ |  | |

# PROPERTIES OF TRANSFORMS

| TIME OPERATION | FREQ. OPERATION | SIGNIFICANCE |
|---|---|---|
| **LINEAR ADDITION** $af(t) + bg(t)$ | **LINEAR ADDITION** $aF(\omega) + bG(\omega)$ | Linearity and superposition apply in both domains. The spectrum of a linear sum of functions is the same linear sum of their spectra (if spectra are complex, usual rules of addition of complex quantities apply). Further, any function may be regarded as a sum of component parts and the spectrum is the sum of the component spectra. |
| **SCALE CHANGE** $f(kt)$ | **INVERSE SCALE CHANGE** $\frac{1}{\lvert k \rvert} F\left(\frac{\omega}{k}\right)$ | Time—Bandwidth invariance. Compressing a time function expands its spectrum in frequency and reduces it in amplitude by the same factor. The amplitude reduces because less energy is spread over a greater bandwidth. For same energy pulse as for $k=1$, multiply both functions by $\sqrt{\lvert k \rvert}$. The case where $k=-1$ reverses the function in time. This merely interchanges positive and negative frequencies; so for real time functions, reverses the phase. |
| **EVEN AND ODD PARTITION** $\frac{1}{2}\left[f(t) \pm f(-t)\right]$ | **EVEN AND ODD PARTITION** $\frac{1}{2}\left[F(\omega) \pm F(-\omega)\right]$ | Any real function $f(t)$ may be separated into an even part $\frac{1}{2}\left[f(t) + f(-t)\right]$ and an odd part $\frac{1}{2}\left[f(t) - f(-t)\right]$. The transform of the even part is $\frac{1}{2}\left[F(\omega) + F(-\omega)\right]$ which is purely real and involves only even powers of $\omega$. The transform of the odd part is $\frac{1}{2}\left[F(\omega) - F(-\omega)\right]$ which is purely imaginary and involves only odd powers of $\omega$. Note: for $f(t)$ real, $F(-\omega) = \overline{F(\omega)}$. |
| **DELAY** $f(t-t_o)$ | **LINEAR ADDED PHASE** $e^{-i\omega t_o} F(\omega)$ | Delaying a function by a time $t_o$ multiplies its spectrum by $e^{-i\omega t_o}$, thus adding a linear phase $\theta = -\omega t_o$ to the original phase. Conversely a linear phase filter produces a delay of $-\frac{d\theta}{d\omega} = t_o$. |
| **COMPLEX MODULATION** $e^{i\omega_o t} f(t)$ | **SHIFT OF SPECTRUM** $F(\omega - \omega_o)$ | Multiplying a time function by $e^{i\omega_o t}$ "delays" its spectrum, i.e., shifts it to center about $\omega_o$ rather than zero frequency. Ordinary real modulation—by $\cos \omega_o t$ say—produces the time function $\frac{1}{2}(e^{i\omega_o t} + e^{-i\omega_o t})f(t)$ with the spectrum $\frac{1}{2}\left[F(\omega - \omega_o) + F(\omega + \omega_o)\right]$. |
| **CONVOLUTION** $\int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$ | **MULTIPLICATION (FILTERING)** $F(\omega)G(\omega)$ | The spectrum of the convolution of two time functions is the product of their spectra. In convolution one of the two functions to be convolved is reversed left-to-right and displaced. The integral of the product is then evaluated and is a new function of the displacement. Convolution occurs whenever a signal is obtained which is proportional to the integral of the product of two functions as they slide past each other—in other words, in any *scanning* operation such as in optical or magnetic recording or picture scanning in television. Transform theory states that such scanning is equivalent to filtering the signal with a filter whose transmission is the transform of the scanning function (reversed in time). Conversely, the effect of an electrical filter is equivalent to a convolution of the input with a time function which is the transform of filter characteristic. This function, the so-called "memory curve" of the filter, is identical with the filter impulse response, aside from dimensions. (Note: the convolution of a time function with a unit impulse gives the same function times the dimensions of the impulse.) |
| **MULTIPLICATION** $f(t)g(t)$ | **CONVOLUTION** $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)G(\omega - s)ds$ | The spectrum of the product of two time functions is the convolution of their spectra. This is the more general statement of the modulation property. For example, sampling a signal is equivalent to multiplying it by a regular train of unit area impulses. The spectrum of the sampled signal consists of the original signal spectrum repeated about each component of the (line) spectrum of the train of impulses (see pair 6S). For no overlap, highest frequency in signal to be sampled must be less than half sampling frequency. If this is true original signal spectrum (hence signal) can be recovered by low pass filter (Sampling theorem). |

## PROPERTIES OF TRANSFORMS

| TIME OPERATION | FREQ. OPERATION | SIGNIFICANCE |
|---|---|---|
| **DIFFERENTIATION** $\frac{d^n f(t)}{dt^n}$ | **MULTIPLICATION BY p** $p^n F(p)$ | The spectrum of the nth derivative of a function is $(i\omega)^n$ times the spectrum of the function. A "differentiating network" has (over the appropriate frequency range) a transmission $K \frac{p}{\omega_0}$ where K is dimensionless or has the dimensions of impedance or admittance. Thus the output wave is *proportional* to the derivative of the input. |
| **INTEGRATION** $\underbrace{\int_{-\infty}^{t} \cdots \int_{-\infty}^{t}}_{n} f(\tau)(d\tau)^n$ | **MULTIPLICATION BY** $\frac{1}{p}$ $\frac{1}{p^n} F(p)$ | The spectrum of the nth integral of a function is $(i\omega)^{-n}$ times the spectrum of the function. Thus, the response of any filter to a step function is the integral of its impulse response. An "integrating network" has (over the appropriate frequency range) a transmission $K \frac{\omega_0}{p}$, where K is dimensionless or has the dimensions of impedance or admittance. Thus, the output is *proportional* to the integral of the past of the input. |

# APPENDIX B

## IF AMPLIFIER RESPONSE

Mention was made in the test of the phenomenon of decreased sensitivity and resolution that results when a CW signal is swept by the IF amplifier at a high rate compared to the bandwidth squared. Assuming a Gaussian response for the amplifier, the resulting transient can be determined as follows:

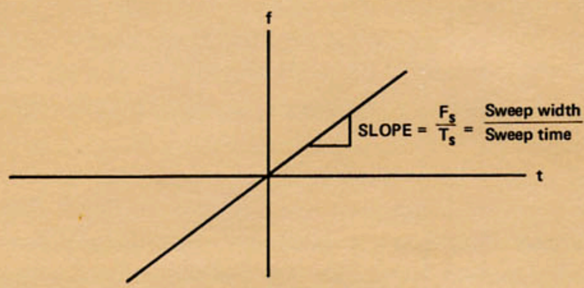



**Figure B-1.**

A sweep frequency signal as illustrated in Figure B-1 can be represented by

$$s(t) = e^{j\pi(F_s/T_s)t^2}, \qquad \text{(B-1)}$$

using pair 10 of Appendix A

$$S(\omega) = \tau\sqrt{2\pi}\; e^{-1/2(\tau\omega)^2} \qquad \text{(B-2)}$$

where $\tau = \sqrt{(jT_s)/(2\pi F_s}$.

If we assume a Gaussian response,

$$H(\omega) = e^{-1/2(\omega/\delta)^2}, \qquad \text{(B-3)}$$

the product of $S(\omega)\; H(\omega)$ gives

$$Y(\omega) = S(\omega)\; H(\omega) = \tau\sqrt{2\pi}\; \exp\left[-\frac{1}{2}\left(\tau^2 + \frac{1}{\delta^2}\right)\omega^2\right] \qquad \text{(B-4)}$$

The output transient is the inverse transform of this function, again using pair 10

$$y(t) = \frac{\tau}{\sqrt{\tau^2 + \frac{1}{\delta^2}}} \exp\left[-\frac{1}{2}\left(\frac{t^2}{\tau^2 + \frac{1}{\delta^2}}\right)\right] \qquad \text{(B-5)}$$

Substituting back for $\tau$ and simplifying

$$y(t) = \frac{1}{\left[1 - j\,\frac{2\pi F_s}{T_s\delta^2}\right]^{1/2}} \exp\left[-\frac{1 - j\,\frac{\delta^2 T_s}{2\pi F_s}}{1 + \left(\frac{T_s\delta^2}{2\pi F_s}\right)^2}\right] \frac{\delta^2 t^2}{2}. \qquad \text{(B-6)}$$

The envelope of $y(t)$ is then

$$y(t) = \frac{1}{\left[1 + \left(\frac{2\pi F_s}{T_s\delta^2}\right)^2\right]^{1/4}} \exp \quad - \frac{\frac{\delta^2 t^2}{2}}{1 + \left(\frac{T_s\delta^2}{2\pi F_s}\right)^2} \tag{B-7}$$

Note that for low sweep rates

$$\frac{T_s}{2\pi F_s} \gg \frac{1}{\delta^2}$$

$$y(t) = \exp\left[-\frac{1}{2}\left(\frac{2\pi F_s}{\delta T_s}\right)^2 t^2\right]. \tag{B-8}$$

This, as was stated earlier, is a plot of the frequency response of the IF amplifier.

## DISTORTION

If the condition on (B-8) is not satisfied, the resulting transient will be altered in both width (time duration) and amplitude. The reduction in amplitude will be

$$\alpha = \frac{1}{\left[1 + \left(\frac{2\pi F_s}{T_s\delta^2}\right)^2\right]^{1/4}}. \tag{B-9}$$

Noting that $\delta = (\pi/\sqrt{1n^2})\, Bf$ where $Bf$ is the 3 $dB$ bandwidth,

$$\alpha = \frac{1}{\left[1 + \left(\frac{21n^2}{\pi}\right)^2 \left(\frac{F_s}{T_s B^2}\right)^2\right]^{1/4}}. \tag{B-10}$$

A plot of this function in $dB$ versus $-F_s/(T_s B^2)$ is included as Figure B-2.

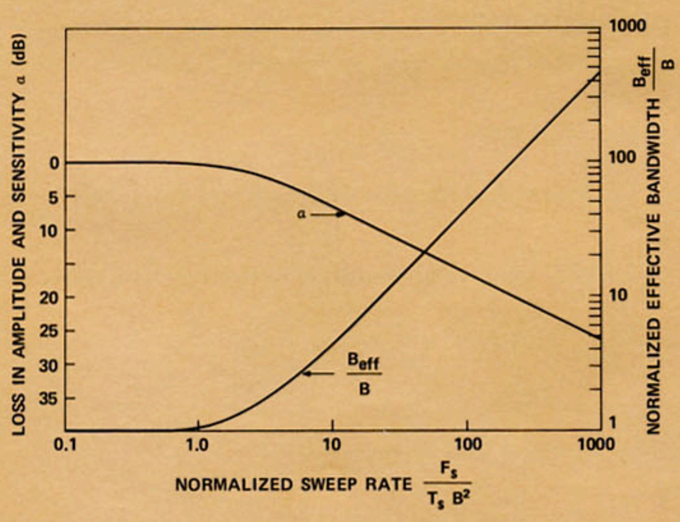



**Figure B-2.** Sensitivity loss and normalized effective bandwidth vs. normalized sweep rate.

If we solve for the 3 $dB$ time duration $\Delta t$ from equation ($B$-8) by setting the function to $1/\sqrt{2}$ and solving for the appropriate $\Delta t$, we get

$$\Delta t = \frac{2\sqrt{1n^2}\ \delta T}{2\pi F_s} \tag{B-11}$$

In a like manner, the 3 $dB$ bandwidth of the function ($B$-7) is

$$\Delta t' = \frac{2\sqrt{1n^2}}{\delta\pi}\left[1 + \left(\frac{T_s\delta}{2\pi F_s}\right)^2\right]^{1/2}. \tag{B-12}$$

The ratio of these times is

$$\frac{\Delta t'}{\Delta t} = \left[1 + \left(\frac{2\pi F_s}{T_s\delta^2}\right)^2\right]^{1/2}. \tag{B-13}$$

This is the ratio of the effective resolving bandwidth of a spectrum analyzer to the bandwidth of the IF amplifier as a function of sweep rate. Rewritten in terms of 3 $dB$ bandwidth $B$.

$$\frac{B_{eff}}{B} = \left[1 + \left(\frac{2\ 1n2}{\pi}\right)^2\left(\frac{F_s}{T_s\Delta f^2}\right)^2\right]^{1/2} \tag{B-14}$$

This function is plotted in Figure B-2.

$F_s$ = Sweep Width
$T_s$ = Sweep Time
$B$ = 3 $dB$ IF Bandwidth
$B_{eff}$ = Effective Bandwidth

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