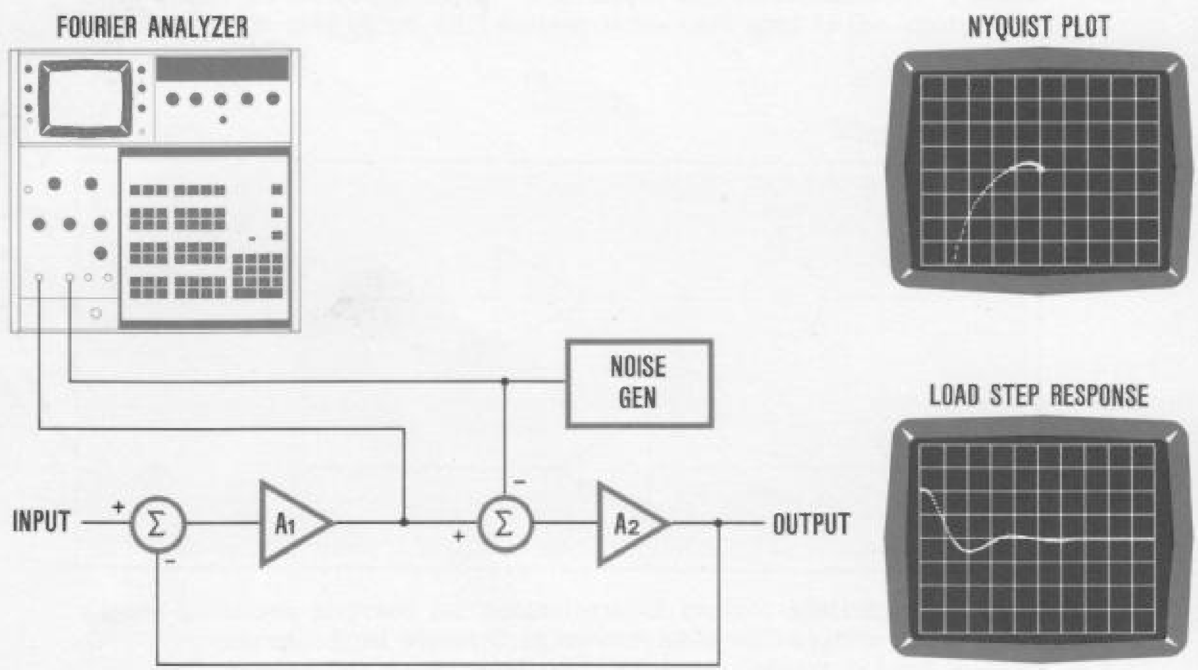
APPLICATION NOTE 140-2

Feedback Loop and Servomechanism Measurements Using HP Fourier Analyzers

Summary of this Note

This note describes the theory and practice of feedback loop measurements using HP Fourier Analyzers. The techniques described apply equally to servomechanisms, process control, and feedback amplifiers. Details are given on the use of a small amount of random noise added to the feedback loop to measure the loop gain and phase as well as other important loop parameters. A method is developed for making these measurements on-line without disturbing the normal system operation. An example of these procedures applied to system identification in a feedback temperature control is included.



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FEEDBACK LOOP AND SERVOMECHANISM MEASUREMENTS

I. INTRODUCTION

The measurement of control system parameters can be a slow and difficult process. It is slow because of the long time constants typically encountered, difficult because of the limited range over which control systems can be expected to operate in a linear mode. The Fourier Analyzer offers solutions to both of these problems. Because it makes a measurement at all frequencies of interest in the amount of time required to make a conventional sinusoidal test at one frequency, it can speed up a measurement by as much as two orders of magnitude. Secondly, by making the measurement on the control system while it is in an on-line operating condition, the Fourier Analyzer can obtain data which is representative of the control system's true operating parameters. The technique described in this note uses a small amount of noise added to the control signals normally present in the feedback loop. By so perturbing the control process it is possible to measure the system response around its normal operating point and to obtain a good estimate of its true parameters. Before discussing the technique let us consider what measurements will best describe a control loop.

Figure 1 gives a block diagram of a conventional control loop. The input $C(j\omega)$* controls an output $R(j\omega)$. The output is contaminated by a noise term $N(j\omega)$. N may be internal system noise unrelated to the input, or it

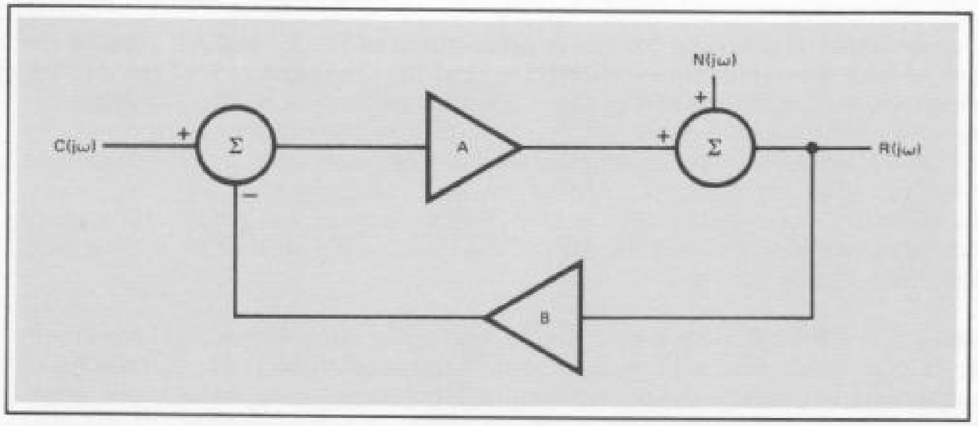


Figure 1. Block diagram for basic form of control system to be measured. Input signal C is uncorrelated with system noise which may be due to internal noise or to variations in load on controlled variable R.

*All functions in this note are written as functions along the real frequency axis where $j\omega = j2\pi f$.

may be a variation in the load which affects the controlled variable R in a way uncorrelated with the input C. The following equations are conventionally written for such a system:

$$R(j\omega) = -R(j\omega)\ AB + C(j\omega)A + N(j\omega) \tag{1}$$

From equation (1) we can derive the following:

$$R(j\omega) = \frac{AC(j\omega)}{1 + AB} + \frac{N(j\omega)}{1 + AB}$$

$$R(j\omega) = GC(j\omega) + FN(j\omega) \tag{2}$$

where

$$G(j\omega) = \frac{A}{1 + AB} \tag{3}$$

$$F(j\omega) = \frac{1}{1 + AB} \tag{4}$$

The function G is the system transfer function and reflects the change in the output due to the input. The factor F, on the other hand, is the gain between either noise fluctuations (or changes in load) and the controlled variable R. While the functions G and F evaluate control system performance relative to inputs, the term AB reflects another aspect of a control system's performance, its stability. If AB, when plotted as a function of frequency on the complex plane, encloses the point (-1, 0) the system will be unstable. How close an approach is made to this critical point determines the stability margin of the loop.

Conventional techniques for the measurement of G, F, and AB involve the use of sine wave signals as excitation, and the measurement of the system response in magnitude and phase. The determination of the resulting response amplitude and phase is often complicated by the long time constants and the effects of noise and non-linearity. The necessary signal integration required to eliminate noise will often make measurement times even longer. In addition, the non-linearities that are generated for sinusoidal signals are harmonics of the test signal and may not be eliminated even with very long integration times.

Since the HP 5450 series of Fourier analyzers computes a total spectrum from one "period" of any signal, much faster and more accurate measurements can be made. Faster because all frequency components are computed from a time record of a length which is the reciprocal of the frequency resolution required. In general, this period is four or five times the longest significant time constant in the system response. Thus, in the time needed to measure one magnitude and phase using sinusoidal excitation, the Fourier Analyzer can measure the data needed to determine the magnitude and phase at all frequencies. To implement a simultaneous measurement at all frequencies this technique uses wide-band random noise as an excitation. Random noise not only allows for faster measurements, but it also makes for more accurate results. By measuring spectrum simultaneously at the system input and output the Fourier Analyzer discriminates against signals that are not phase coherent with the input at each frequency. Random noise provides more accurate results than sinusoidal test signals in this case because it correlates least of all with any other signal. In fact, unlike sinusoids, it is not even coherent with its own distortion products. Thus, the use of random noise allows for making an estimate of a good linear approximation to the transfer function of a nonlinear system.

To implement the use of a random stimulus in an operating control loop the setup shown in Figure 2 can be used. Here the feedback loop is opened and a new summing node is inserted. A noise signal $X(j\omega)$ is connected to one branch. The second input branch closes the loop and allows for normal system operation. The noise signal $X(j\omega)$ is propagated around the loop, and at those frequencies where the control loop gain is high it returns as $Y(j\omega)$ at an amplitude and phase nearly equal to $X(j\omega)$. Thus, the test signal is almost cancelled at the output of the summing node. This is easily shown by writing the equations for Figure 2.

$$Y(j\omega) = A_2BA_1[X(j\omega) - Y(j\omega)] + BA_1N(j\omega) + A_1C(j\omega)$$

where $A_1A_2 = A$

$$Y(j\omega) = \frac{AB}{1+AB}X(j\omega) + \frac{BA_1\,N(j\omega)}{1+AB} + \frac{A_1\,C(j\omega)}{1+AB} \qquad (5)$$

The point of this measurement procedure is to determine the parameters of the control system. It can be shown that this is easily done by determining the magnitude and phase of the term relating $X(j\omega)$ to $Y(j\omega)$. We shall call this term the test ratio T where

$$T = \frac{Y(j\omega)}{X(j\omega)}$$

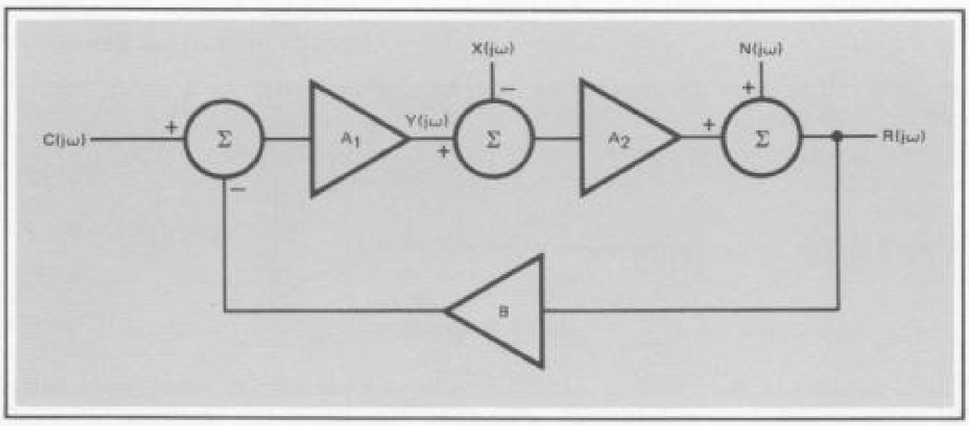


Figure 2. This is the basic configuration for adding random noise at X, uncorrelated with C or N, to a control system. The transfer function between the added noise $X(j\omega)$ and the control system return $Y(j\omega)$ is the test ratio, T. From T the system loop gain, transfer function, and other parameters may be computed. The only requirements on the summing node are that the output is Y-X and X and Y are isolated from each other.

and

$$T = \frac{AB}{1 + AB} \tag{6}$$

T is a relatively convenient parameter to measure for a number of reasons. First of all, over the significant range where AB is large, T is nearly 1 and the signal Y is nearly equal to X as we stated above. Thus, the instrument used to measure the ratio of X to Y, does not require a large dynamic range. This cannot be said of a direct measurement of the system in an open loop configuration. Secondly, since the output of the test summing node is nearly 0, placing it close to the input of the most sensitive system element will minimize the disturbance. For example, if the controlled variable R is critical, placing the test point near the output will result in little or no effect on R even if a fairly large test level for X is used. Last of all, since the term relating X to Y in equation (5) has the highest gain, the effect of $C(j\omega)$ and $N(j\omega)$ on the measurement of T is minimized. In many cases, C and N are not large wide band signals and it is a fairly easy matter for a measurement of T using the Fourier Analyzer to discriminate against them.

T, of course, does not directly measure any important parameter of the control system under test. It is a simple matter, however, to use a measurement of T to yield several significant parameters of the control system. The total loop gain for the system is

$$AB(j\omega) = \frac{T}{1 - T} \tag{7}$$

Here the open loop gain is found from data measured while the loop is closed and operating. The frequency response to noise or load variation at the system output is

$$F(j\omega) = 1 - T \tag{8}$$

In many situations the control system can be modeled as a unity return system. In this case

$$H(j\omega) = T \qquad \text{for } B = 1. \tag{9}$$

When B is not 1, but is known

$$H(j\omega) = \frac{T}{B} \tag{10}$$

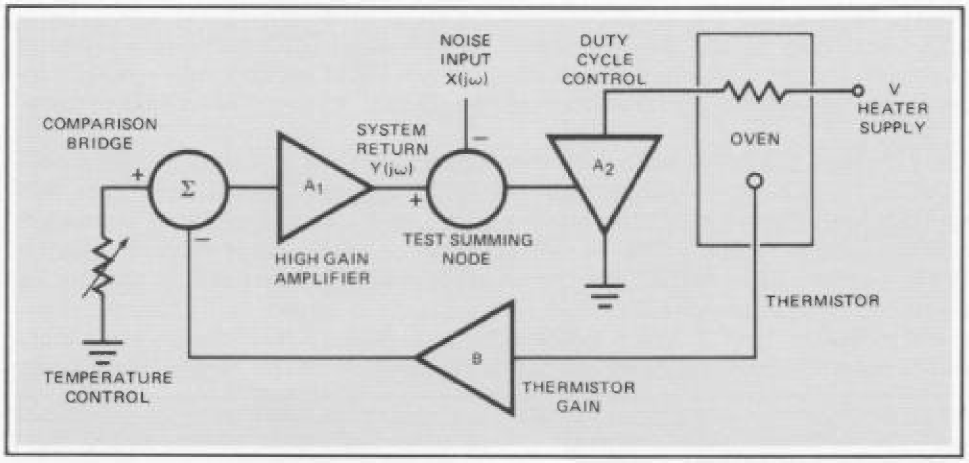
The flexibility of the Fourier Analyzer allows a number of other very useful parameters to be calculated. For example, since the transfer function of the noise or load variations at the output to variations in the controlled variable R is given by $F(j\omega)$, the inverse Fourier transform of F is the impulse response of R to N.

$$f(t) = \mathcal{F}^{-1}\left[F(j\omega)\right]$$

If the integral of f is taken, the response of the system output to a step change in load can be determined. As we shall see these are all straightforward operations with the Fourier Analyzer.

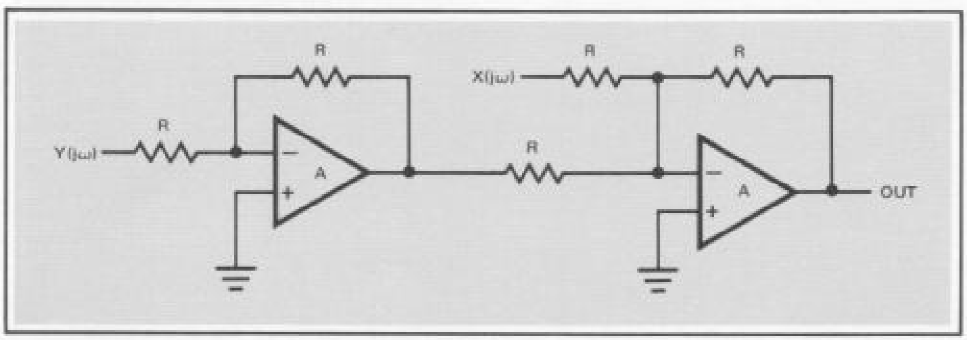
II. EXAMPLE

Before proceeding to give details on the measurement of T and its conversion to AB, G and F, it will be helpful to consider an example. The system measured in this example is a proportional oven control used to stabilize the temperature around an electronic oscillator. A block diagram for the oven control is shown in Figure 3a. The temperature is monitored by a thermistor mounted inside the oven. The thermistor resistance value is compared in a bridge circuit to the desired temperature as set on a potentiometer. The resulting error signal is amplified in A and applied to a voltage-to-duty-cycle converter. The duty cycle of the oven heater and



a. Oven Control Servo

Thermistor senses oven temperature and feeds back to comparison bridge. Error output from the bridge is amplified and applied to heater duty cycle control.



b. Test Summing Node Circuit

Two operational amplifiers are used to implement the test summing node. The output is Y-X. The inputs X and Y are isolated from each other.

Figure 3. Oven Control System

the supply voltage sets the power input to the oven and controls its temperature. The oven supply voltage determines the gain between heater and oven temperature. In this situation it is necessary to examine the effect of heater supply voltage on the stability and performance of the control loop. The oven controller shown here is a position type of servo where the desired temperature is set on the potentiometer and the servo attempts to eliminate environmental or load effects. In this case the direct transfer function from potentiometer input to temperature output is of no real importance. What is important is the relative stability of the loop as indicated by AB and the effect of the environmental load as given by F.

The noise is inserted directly after the error voltage amplifier for several reasons. It is, of course, a point where the loop may be easily opened to insert the summing node circuit shown in Figure 3b. In addition, almost all of the loop gain precedes the point chosen so that the small output of the summing node has almost no effect on oven temperature. Since the voltage-to-temperature and temperature-to-resistance relationships may be non-linear, keeping the signal excursions small at these points can add greatly to the accuracy of the measurement. If the controlled variable were part of some critical process such as an aircraft flight control, the small output would make the disturbance essentially nil during the measurement.

After the test summing node was inserted in the loop and the system stabilized at its normal operating point, a noise signal of about .69V rms was applied to the input X. The return signal, at Y, was about five times the normal system noise. The variation in oven temperature, as monitored by the performance of the electronic components it enclosed, could not be detected. A measurement of the test ratio T is shown in Figure 4. Figure 4 indicates that T has a bandwidth of about 0.15 Hz. To obtain a reasonable set of data, frequencies down to about 0.02 Hz would have been

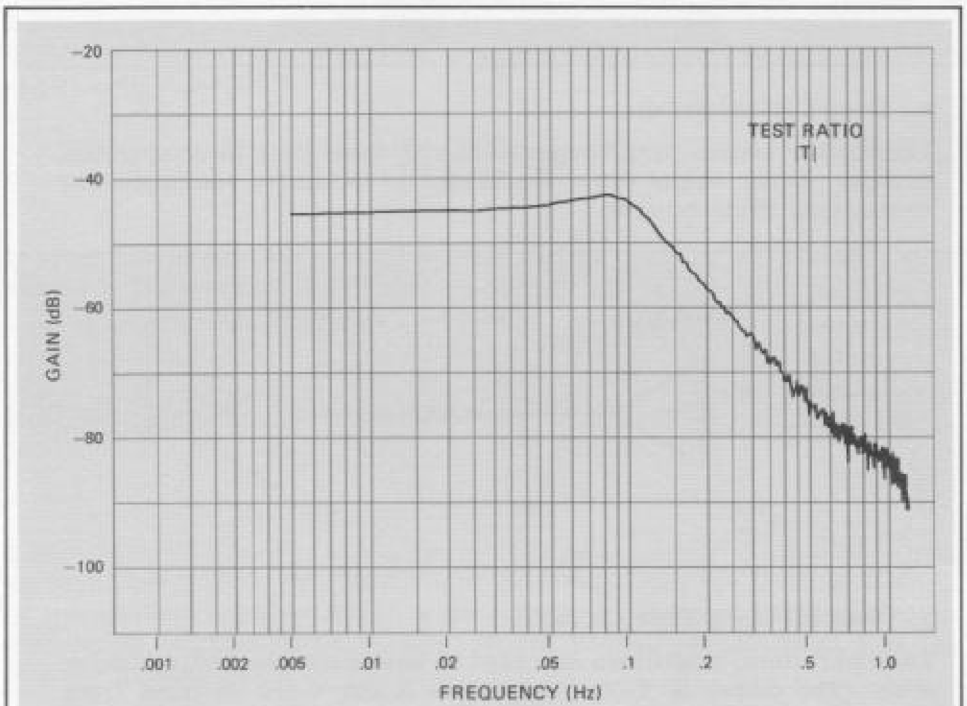


Figure 4. Magnitude of test ratio, T, measured for system of Figure 3. Heater supply voltage set to 24 volts.

adequate. But for the sake of clarity in this example, frequencies down to .005 Hz were measured. In Figure 5, the loop gain AB computed from the measurement of T is shown. It is easy to see that the gain crossover occurs at 0.07 Hz and that there is a phase margin of 45° at this frequency. Phase crossover, on the other hand, occurs at 0.18 Hz where

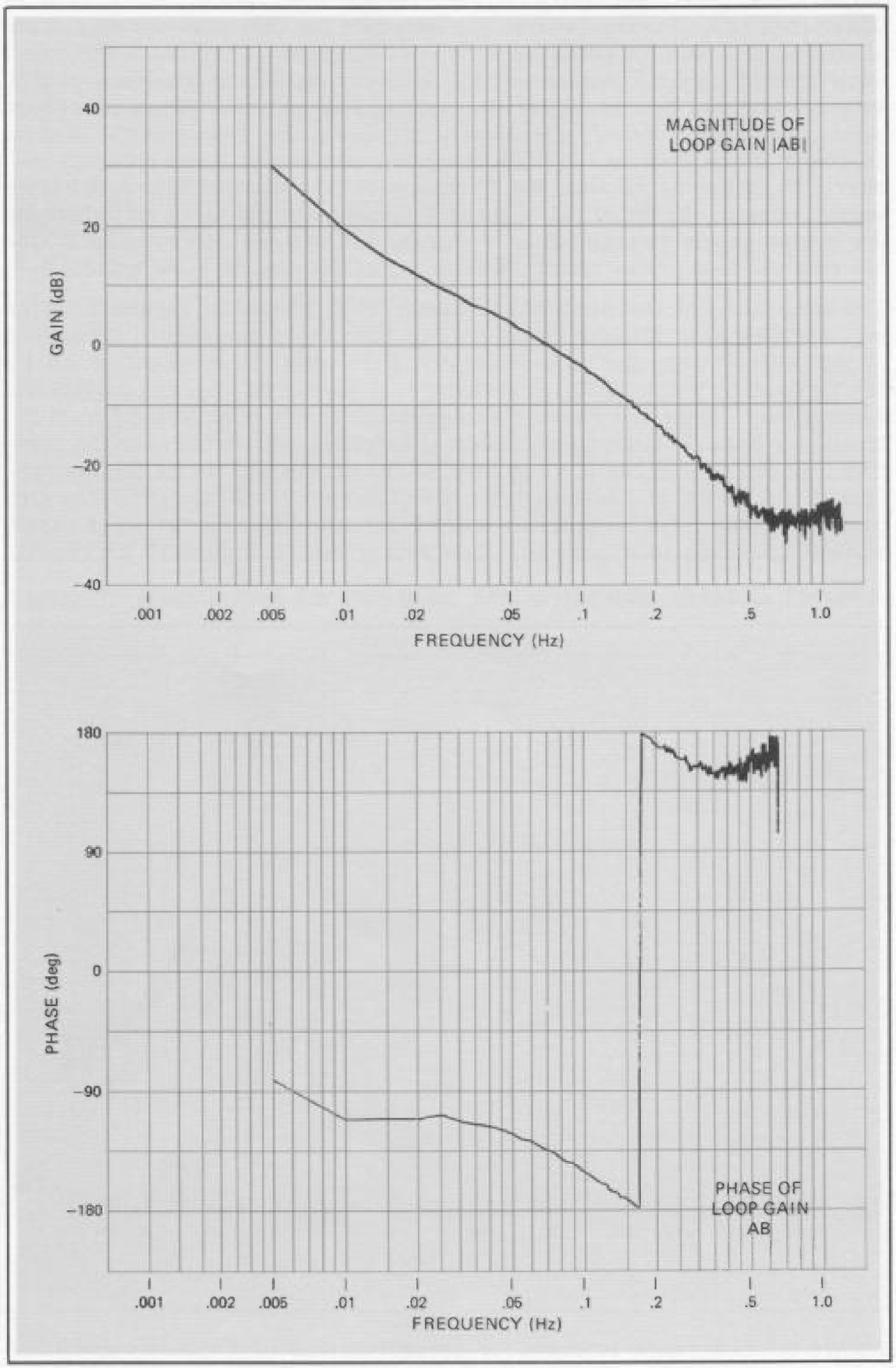


Figure 5. Bode plot for loop gain, AB, computed from data of Figure 4

there is a gain margin of 11 dB. In Figure 6, the coherence function for this measurement is shown. The coherence function is discussed in detail in references 1 and 2. Briefly, this function gives the fraction of power measured at Y which is coherent with the input at X. At frequencies lower than the phase crossover, 0.18 Hz, better than 90% of the power is coherent indicating a very good measurement. Below 0.07 Hz, this increases to better than 98%. Figure 7 shows the loop gain for this same measurement plotted on the complex plane as a Nyquist diagram. (Figures 5 through 7 were plotted directly from the 5450 memory using the standard plotter with no external processing of the data.) The gain and phase crossover points are easy to identify graphically in these plots. Note that the closest approach to the critical -1, 0 point occurs between gain and phase crossover. In Figure 8, we show the Nyquist plot for different values of heater supply voltage. Examination of these Nyquist plots indicates that changing the heater supply voltage not only changes the system loop gain but it also changes the loop phase characteristic, an effect that was not anticipated.

The magnitude of the disturbance factor F is shown in Figure 9. This factor shows what attenuation is applied to oven temperature fluctuations by the control loop: for an oven supply of 24 volts, fluctuations at .01 Hz are reduced by some 22 dB over what would have been the case without the control loop. Since the Fourier Analyzer can easily convert data from the frequency domain to the time domain, it is interesting to observe the function F in the time domain. The inverse transform of F is its impulse response f(t). If f(t) is integrated, we have the step response $f_S(t)$. The step response taken from measurements of F for two different values of supply voltage is shown in Figure 10. The data given in Figure 10 represents

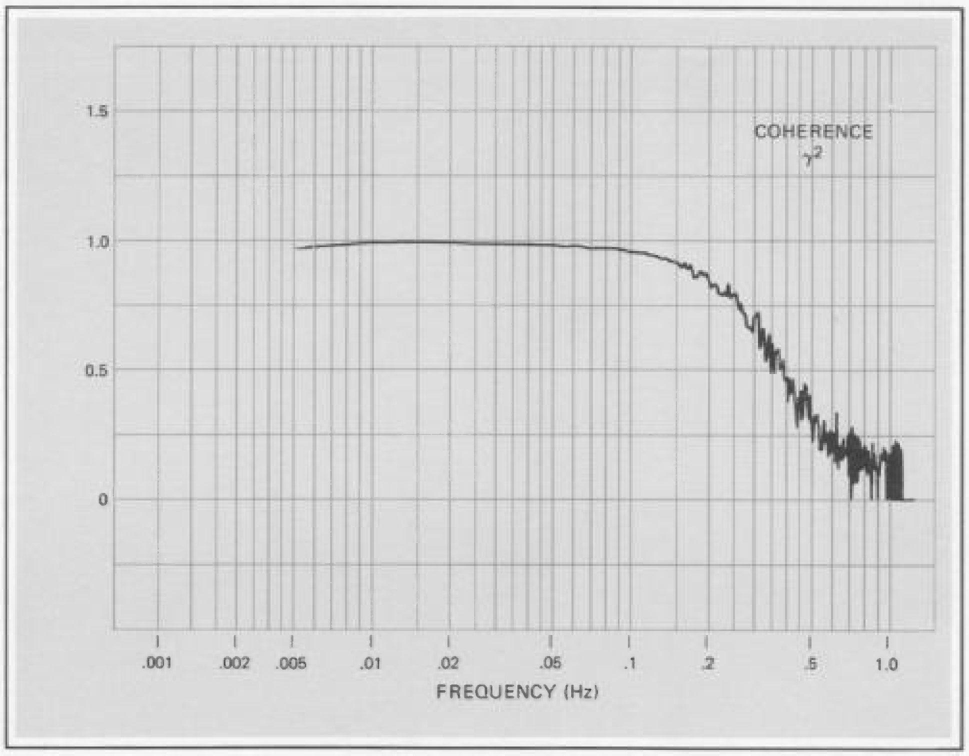


Figure 6. Coherence function for measurements of Figures 4 and 5. A coherence of 1.0 indicates that 100% of all power at the measured point Y was due to the input at X.

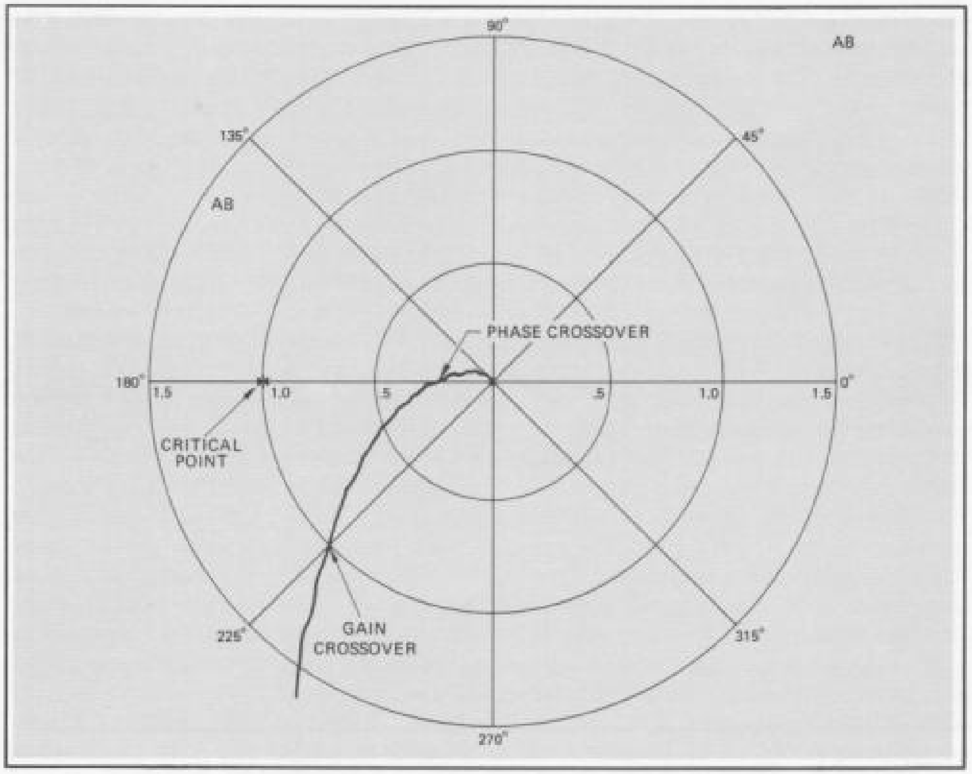


Figure 7. Nyquist plot for loop gain, AB, of the data shown in Figure 5.

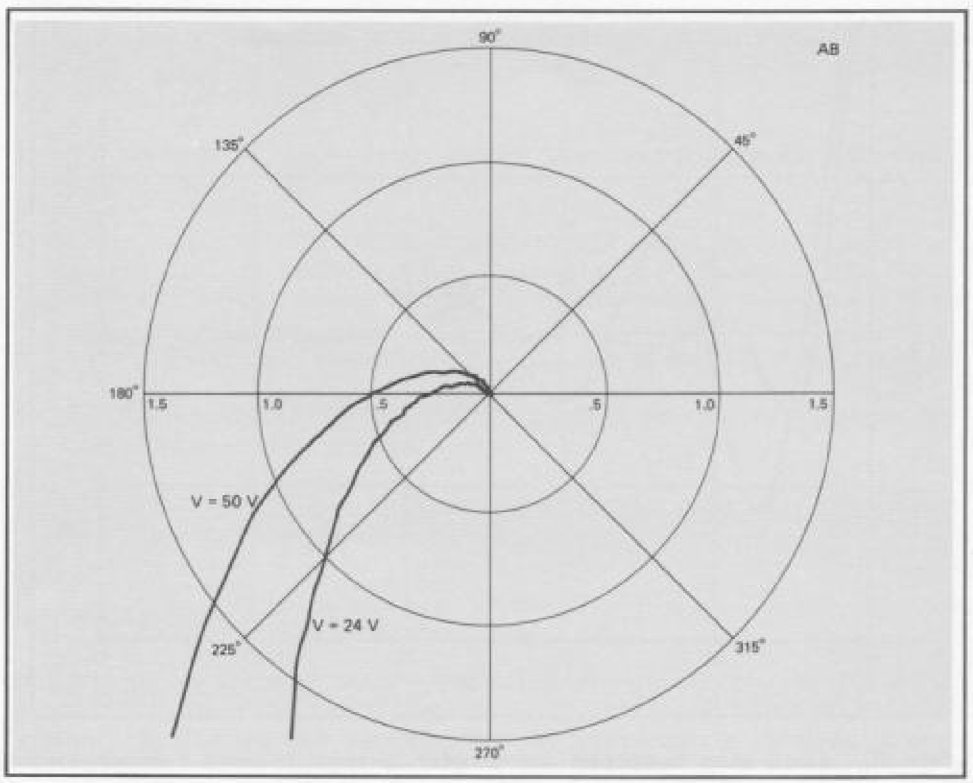


Figure 8. Nyquist plot for AB at two different heater supply voltages.

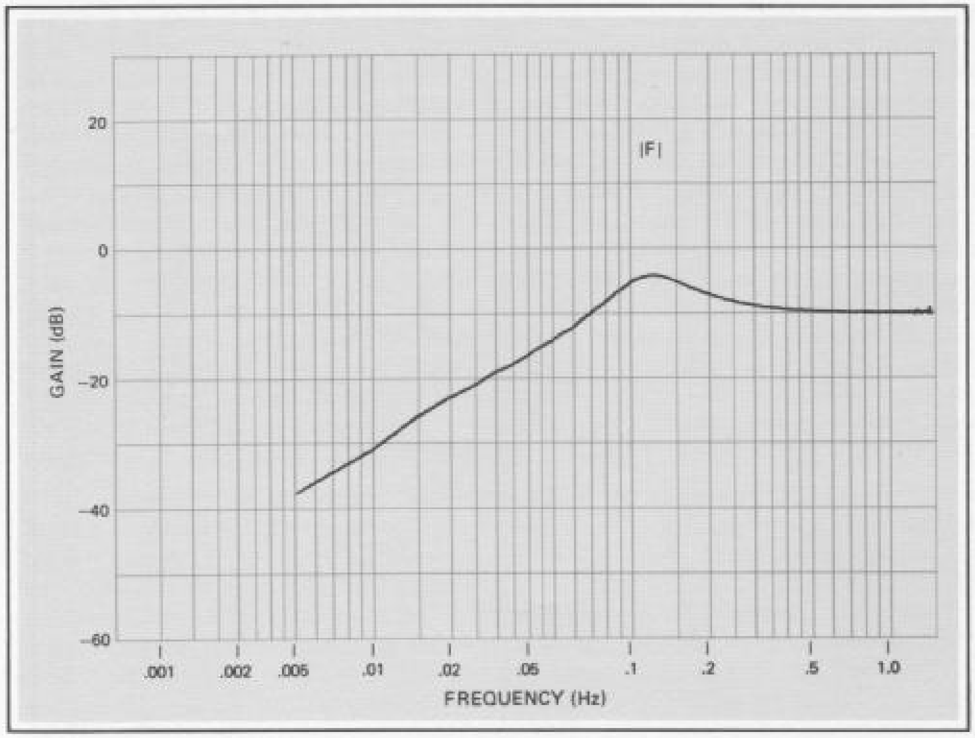


Figure 9. Gain of noise reduction factor F for data of Figure 4.

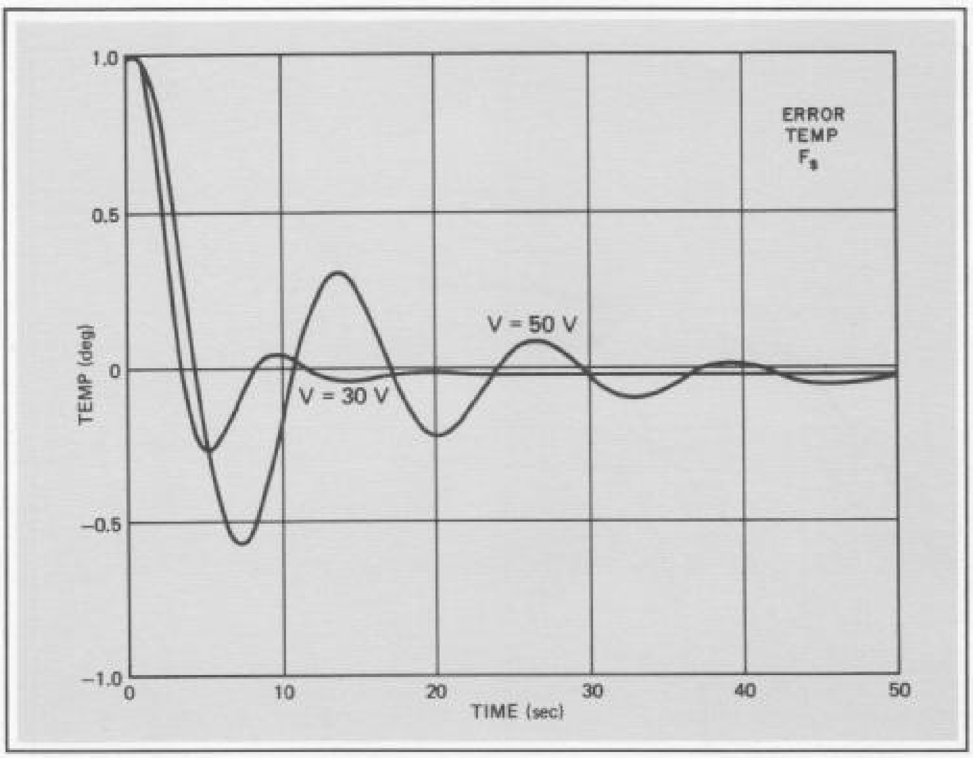


Figure 10. Load step response for 1° step change in oven temperature for two heater supply voltages. Response computed from on-line measurement of AB.

how the system would respond to a step change in temperature inside the oven. It is interesting to note, that as the supply voltage is increased, the step response overshoots more, and oscillates more slowly as it returns to 0. Thus, better performance for the oven is obtained at a lower supply voltage. This very useful performance data would have been difficult to obtain from direct experimental data where the oven temperature was varied. The Fourier Analyzer, however, makes it quite easy to compute this response from an actual measurement of the system in an on-line operating condition. The data in Figure 10 was, as in all previous graphs, plotted directly from the analyzer memory with no additional computation.

It is interesting to consider the saving in measurement time made possible in this example by using random noise as a stimulus and a Fourier Analyzer as the response measuring instrument. In Figures 4 through 9 measurements were made down to .005 Hz for clarity of presentation. In practice, a resolution of .02 Hz would have been adequate. This resolution requires a measurement time of 50 seconds per estimate. Because a periodic noise generator was used (see reference 2 for definition of periodic noise), satisfactory data was obtained with only 5 averages, which is equivalent to an integration time of 250 seconds. The total measurement time of 250 seconds is only 5 periods of the lowest frequency measured. If conventional sine wave techniques were used, at least five periods of the sine wave at each frequency measured would be needed to obtain equivalent results. For the 50 frequencies measured in the example, this optimistic estimate would require 1250 seconds. In practice the time advantage for random noise over sine excitation is usually in the order of 10 to 300 depending on the exact nature of the hardware and the time constant of the system being measured.

The above example illustrates several important advantages of the test node technique when used with a random noise stimulus and a Fourier Analyzer to measure control systems parameters. The major advantages can be summarized as follows:

1. A measurement can be made on-line in an operating mode without disturbing the controlled variable.

2. A large test signal can be used which means a high signal-to-noise ratio for the measurement and a low dynamic range requirement on the instrumentation. At the same time the first advantage can be maintained.

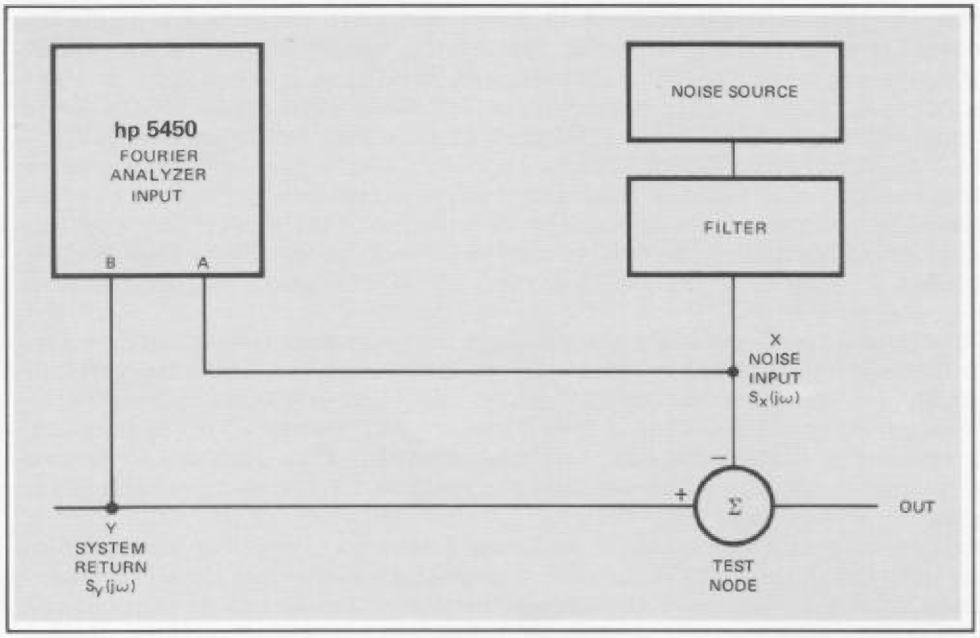
3. A measurement of the stability margin can be directly made from AB.

4. A measurement can be made at any point in the system where the signals are in convenient form.

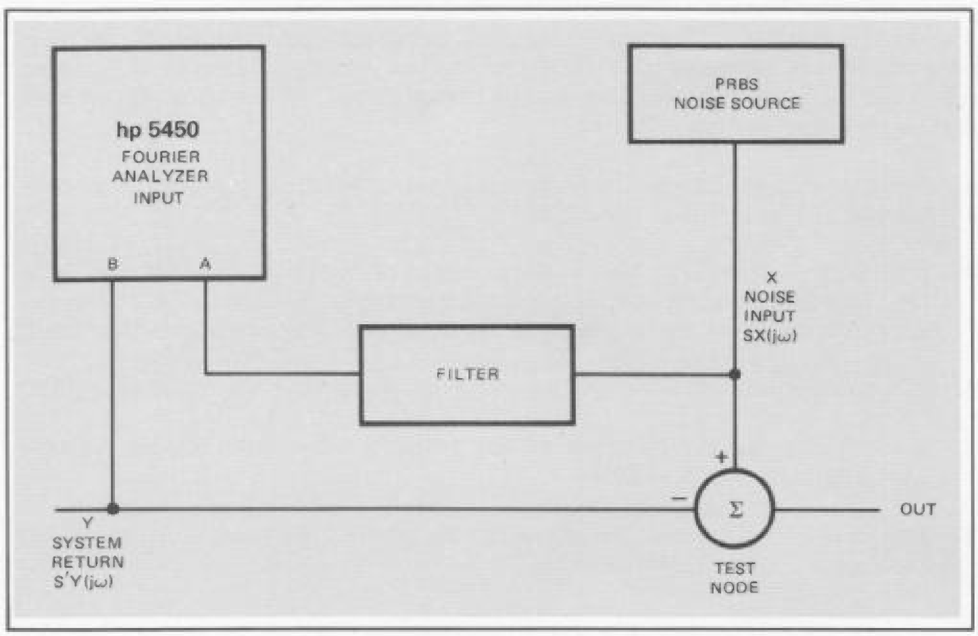
5. The measurement can be made more rapidly than with conventional sine wave instrumentation.

III. MEASURING T

In the previous sections we have indicated how a measurement of T can be a quick and accurate way to determine the characteristics of a control system. In this section we will cover the fundamentals involved in using a Fourier Analyzer to measure T. The setup for this measurement is shown in Figure 11. The input to the test node is measured by channel A of the analyzer while the B channel measures the return. The Fourier



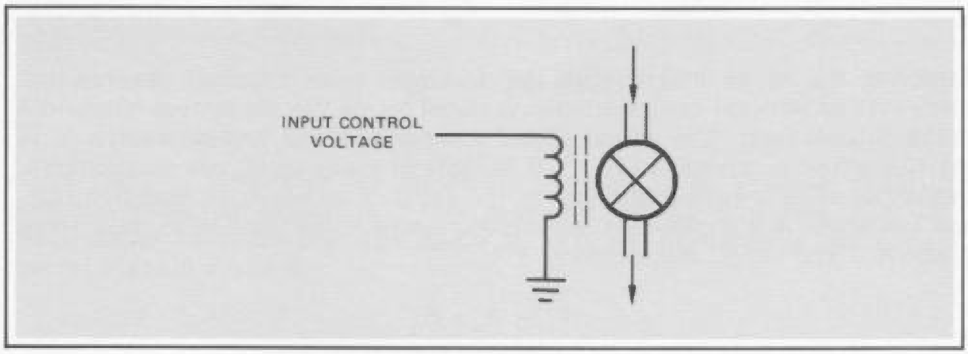
a. Noise source and anti-aliasing filter connected to test node. Note that no anti-aliasing filters are needed on A and B inputs of analyzer because input to system is band limited. Use of filters on A and B inputs will lower measurement accuracy.



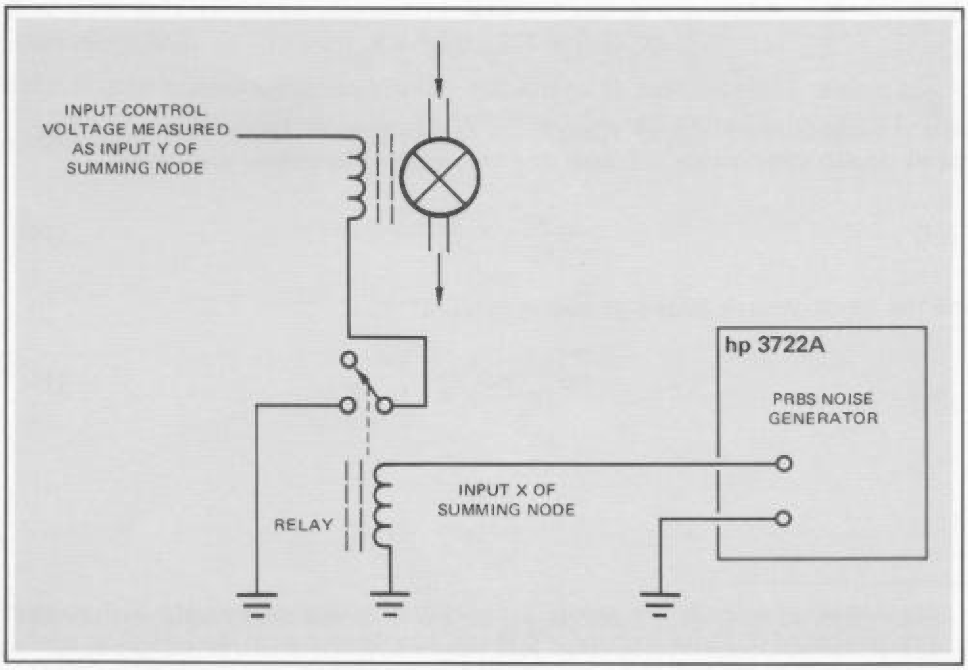
b. When binary sequence is needed for relay operation as in Figure 12, this setup can be used. Anti-aliasing filter on input A lowers accuracy to some extent; a filter on B matched to the A channel filter would be somewhat better.

Figure 11

Analyzer is a digital instrument whose input is an analog-to-digital converter (ADC) that samples signals being measured. Therefore, some precautions must be taken to prevent aliasing error. A filter on the noise source is generally needed to assure that X(jω) has no significant components higher in frequency than 1/2 the sample rate. If the noise source input to the summing node must be a binary sequence to operate an on-off device, as in Figure 12, then the aliasing filter should be placed on the input to the analyzer as in Figure 11b. In general, these filters should have between 20 and 40 dB of attenuation at 1/2 the sample rate. A filter is not usually required at the Y point because the return signal will be low-pass filtered by the system.



a. Normal solenoid control valve in a process control loop



b. Relay added to solenoid valve allows the addition of a PRBS signal to the Y input of the system return. Flow through valve is sum of -X and Y input signals. X signal is isolated from Y signal as required for implementing summing node.

Figure 12. Using a solenoid valve as a test node

The Fourier Analyzer measures the two linear spectrums* $S_y(j\omega)$ and $S_x(j\omega)$ to determine T. Ideally T relates these spectrums by

$$S_y(j\omega) = T S_x(j\omega) \tag{11}$$

These spectrums are computed from a sample record taken over one measurement time period, T_m. T_m is generally made at least four or five times the longest time constant in the system. The frequency resolution of the measurement is the reciprocal of T_m

$$\Delta f = \frac{1}{T_m} \tag{12}$$

Selecting T_m to be longer than the dominant time constant insures that therewill be several frequencies measured inside the narrowest bandwidth in the control loop. The actual signal measured at the system return Y is not that given in equation (11). It is instead made up of two components. One of these is a component due to the input X as given by equation (11), and the other is a component of "measurement noise" due to signals from C and N. The signal measured at Y is

$$S_y' = S_y + S_n$$

where S_n is the composite of all "noise" from N and C. Using equation (11) S_y' is written as**

$$S_y'(j\omega) = T\, S_x(j\omega) + S_n(j\omega) \tag{13}$$

Two very important power spectrums are computed from these two measured linear spectrums. These are the input auto power spectrum

$$G_{xx} = S_x S_x^* \tag{14}$$

and the input-return cross power spectrum***

$$G_{yx}' = S_y' S_x^* \tag{15}$$

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*By linear spectrum we mean a spectrum which is simply a Fourier transform of a time record. Linear spectrums are made up of real, in phase, and imaginary, quadrature components.

**The prime indicates a measured rather than ideal value for a quantity in this Note.

***It may seem that computing the power spectrum is the long way around to get to T when it may be found by simply taking the ratio of S_y'/S_x. However, it is really computationally simpler and more accurate. In addition, using the power spectrum allows the generation of the coherence function, reference (2).

Using equation (13) we see that the measured cross spectrum is

$$G'_{yx} = T(j\omega)\, G_{xx} + S_n S^*_x$$

The measured cross power spectrum, like the measured value for S'_y, contains a term due to T and a "measurement noise" term from S_n. To eliminate the measurement noise, a number of sample records are used to compute an average value for G'_{yx}. When independent samples of G'_{yx} are averaged, the term due to T maintains constant phase from one record to the next. But S_n and S_x are uncorrelated and are not phase coherent. $S_n S^*_x$ will then average to 0. The average value of G'_{yx} will then approach

$$\overline{G'_{yx}} = T(j\omega)\, \overline{G_{xx}}$$

where the $\overline{\quad}$ indicates an ensemble average. The Fourier Analyzer uses the ensemble averages of the measured cross spectrum and auto spectrum to estimate the true value of T in the presence of the normal control signals N and C.

$$T(j\omega) = \frac{\overline{G'_{yx}}}{\overline{G_{xx}}} \qquad (16)$$

How well this is accomplished is determined by the coherence function [reference (2)].

Rather than compute T and then the quantities of real interest, it is a simple matter to use equation (16) in equations (7) through (10) to derive the following:

$$AB(j\omega) = \frac{\overline{G'_{yx}}}{\overline{G_{xx}} - \overline{G'_{yx}}} \qquad (17)$$

$$F(j\omega) = \frac{\overline{G_{xx}} - \overline{G'_{yx}}}{\overline{G_{xx}}} \qquad (18)$$

$$H(j\omega) = \frac{\overline{G'_{yx}}}{\overline{G_{xx}}} \cdot \frac{1}{B} \qquad (19)$$

Thus all the significant parameters of the control loop may be defined directly in terms of the averaged cross and auto power spectrums. A keyboard program to accomplish this measurement is given in Table 1. This program once entered via the keyboard can be automatically executed. When used with the setup shown in Figure 11, the program results in AB, F, and T stored in rectangular coordinates (that is in terms of co and quad spectrum). These quantities may then be displayed as magnitude and phase, Bode plots, Nyquist plots, or they may be inverse transformed to give impulse responses. Integrating the various impulse responses will give appropriate step responses as shown in Figure 10 of the example.

IV. CONCLUSION

In this note we have shown how a Fourier Analyzer and a noise source may be used to measure the basic parameters of a control system. The test node measurement scheme what we have introduced allows for a measurement to be made on line without appreciably distorting the control function. This technique also allows for inserting the test signal at a convenient point. If the input and output of the servo are not electrical variables, the test signal may be inserted and measured where the loop variable is one that can be transduced. * While the test summing node could, in principle, be used with sine wave excitation and analog measuring instruments, it is not really practical to do so. The complex computations that are needed to convert T to useful functions requires a digital system if the results are to be rapidly available and accurate. The computational flexibility of a Fourier Analyzer also allows for the use of random noise as a test stimulus. The advantages of a random source are many. The most important of these is the reduction in measurement time which results from the use of wide band noise. Because wide band noise allows all frequencies to be tested simultaneously, the measurement time may be reduced as much as 300 times. Even when very sophisticated and highly optimized analog equipment such as log frequency sweep generators and tracking filters are used, an improvement of better than 10 with the Fourier Analyzer is still to be expected. The proof of these statements is, unfortunately, beyond the scope of this note. The reader is directed to reference 2 for a more complete description of the theory and practice of transfer function analysis.

— Peter Roth
Santa Clara Division
April 1, 1971

REFERENCES

1. Roth, Peter, "Effective Measurements Using Digital Signal Analysis," IEEE Spectrum, April 1971, pp. 62–70.

2. Roth, Peter, "Measurement of Transfer Functions Using Digital Fourier Analysis," Hewlett-Packard, Santa Clara California Division, to be published.

3. Bendat, Julius S. and Piersol, Allan G., Measurement and Analysis of Random Data, John Wiley & Sons, Inc., New York, 1966.

*A direct measurement from input to output could, of course, be made. However, an examination of this case will show that while it measures G, the transfer function, directly, AB cannot be found without knowledge of B. Use of the test summing node on the other hand allows a direct measurement of AB, but requires B to determine the transfer function.

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
LABEL 0 ENTER							Establishes initial label point
CLEAR 1 ENTER		Cleared					Initializes block 1 (removes old data)
CLEAR 2 ENTER			Cleared				Initializes block 2 (removes old data)
CLEAR 3 ENTER				Cleared			Initializes block 3 (removes old data)
LABEL 1 ENTER		Sum of past $G_{YX}(f)$'s. 0 first time.	Sum of past $G_{XX}(f)$'s. 0 first time.	Sum of past $G_{YY}(f)$'s. 0 first time.			Establishes target point for power spectrum summation
ANALOG IN 4 SPACE 5 SPACE 1 ENTER		"	"	"	Current time record from channel A (input, X)	Current time record from channel B (output, Y)	Data input
F 4 SPACE 5 ENTER		"	"	"	Fourier transform of channel A record	Fourier transform of channel B record	Obtain Fourier transform of data
CLEAR 4 SPACE 0 ENTER		"	"	"	Fourier transform of channel A record minus dc value	"	Clears dc value from channel A record
CLEAR 5 SPACE 0 ENTER		"	"	"	"	Fourier transform of channel B record minus dc value	Clears dc value from channel B record
LOAD 5 ENTER	Fourier transform of channel B record minus dc value	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* 4 ENTER	Cross power spectrum ($G_{YX}(f)$) of channel A and B records	"	"	"	"	"	Obtain $G_{YX}(f)$

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
+1 ENTER	Sum of current plus past $G_{YX}(f)$'s	Sum of past $G_{YX}(f)$'s. 0 first time.	Sum of past $G_{XX}(f)$'s. 0 first time.	Sum of past $G_{YY}(f)$'s. 0 first time.	Fourier transform of channel A record minus dc value	Fourier transform of channel B record minus dc value	Add current $G_{YX}(f)$ to sum of past $G_{YX}(f)$'s
STORE 1 ENTER	"	Sum of current plus past $G_{YX}(f)$'s	"	"	"	"	Store results of current pass for next pass
LOAD 4 ENTER	Fourier transform of channel A record minus dc value	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	Auto power spectrum of input ($G_{XX}(f)$)	"	"	"	"	"	Form auto power spectrum of input, $G_{XX}(f)$
+2 ENTER	Sum of current plus past $G_{XX}(f)$'s	"	"	"	"	"	Add current $G_{XX}(f)$ to sum of past $G_{XX}(f)$'s
STORE 2 ENTER	"	"	Sum of current plus past $G_{XX}(f)$'s.	"	"	"	Store results of current pass for next pass
LOAD 5 ENTER	Fourier transform of channel B record (output, Y)	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	Auto power spectrum of output ($G_{YY}(f)$)	"	"	"	"	"	Form auto power spectrum of output, $G_{YY}(f)$
+3 ENTER	Sum of current plus past $G_{YY}(f)$'s	"	"	"	"	"	Add current $G_{YY}(f)$ to sum of past $G_{YY}(f)$'s

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
STORE 3 ENTER	Sum of current plus past $G_{YY}(f)$'s	Sum of current plus past $G_{YX}(f)$'s	Sum of current plus past $G_{XX}(f)$'s.	Sum of current plus past $G_{YY}(f)$'s	Fourier transform of channel A record minus dc value	Fourier transform of channel B record minus dc value	Store results of current pass for next pass
COUNT 1 SPACE N1 ENTER	"	"	"	"	"	"	Loop back to target label 1 N1 times
LOAD 1 ENTER	$\overline{G_{YX}(f)}$	$\overline{G_{YX}(f)}$	$\overline{G_{XX}(f)}$	$\overline{G_{YY}(f)}$	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	$\lvert\overline{G_{YX}(f)}\rvert^2$	"	"	"	"	"	Obtain $\lvert\overline{G_{YX}(f)}\rvert^2$ in block 0
÷ 2 ENTER	$\frac{\lvert\overline{G_{YX}(f)}\rvert^2}{\overline{G_{XX}(f)}}$	"	"	"	"	Fourier transform of channel B record minus dc value	Obtain $\frac{\lvert\overline{G_{YX}(f)}\rvert^2}{\overline{G_{XX}(f)}}$ in block 0
÷ 3 ENTER	Coherence function $\gamma^2 = \frac{\lvert\overline{G_{YX}(f)}\rvert^2}{\overline{G_{XX}(f)\cdot G_{YY}(f)}}$	"	"	"	"	"	Obtain coherence function in block 0
STORE 4 ENTER	"	"	"	"	γ^2	"	Save γ^2 in 4.

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
LOAD 2 ENTER	$\overline{G_{XX}(f)}$	$\overline{G_{YX}(f)}$	$\overline{G_{XX}(f)}$	$\overline{G_{YY}(f)}$	γ^2	Fourier transform of channel B record minus dc value	Prepare to find $\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$.
−1 ENTER	$\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$	"	"	"	"	"	Find $\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$.
STORE 5 ENTER	"	"	"	"	"	$\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$	Save $\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$ in 5.
LOAD 1 ENTER	$\overline{G_{YX}(f)}$	"	"	"	"	"	Prepare to find AB.
÷ 5 ENTER	AB	"	"	"	"	"	Find AB.
STORE 3 ENTER	AB	"	"	AB	γ^2	"	Save AB in 3.
LOAD 5 ENTER	$\overline{G_{XX}(f)} - \overline{G_{YX}(f)}$	"	"	"	"	"	Prepare to find F.
÷ 2 ENTER	F	"	"	"	"	"	Find F.
STORE 5 ENTER	F	"	"	AB	γ^2	F	Save F in 5.
LOAD 1 ENTER	$\overline{G_{YX}(f)}$	"	"	"	"	"	Prepare to find T.
÷ 2 ENTER	T	"	"	"	"	"	Find T.
END	T	$\overline{G_{YX}(f)}$	$\overline{G_{XX}(f)}$	AB	γ^2	F	END

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