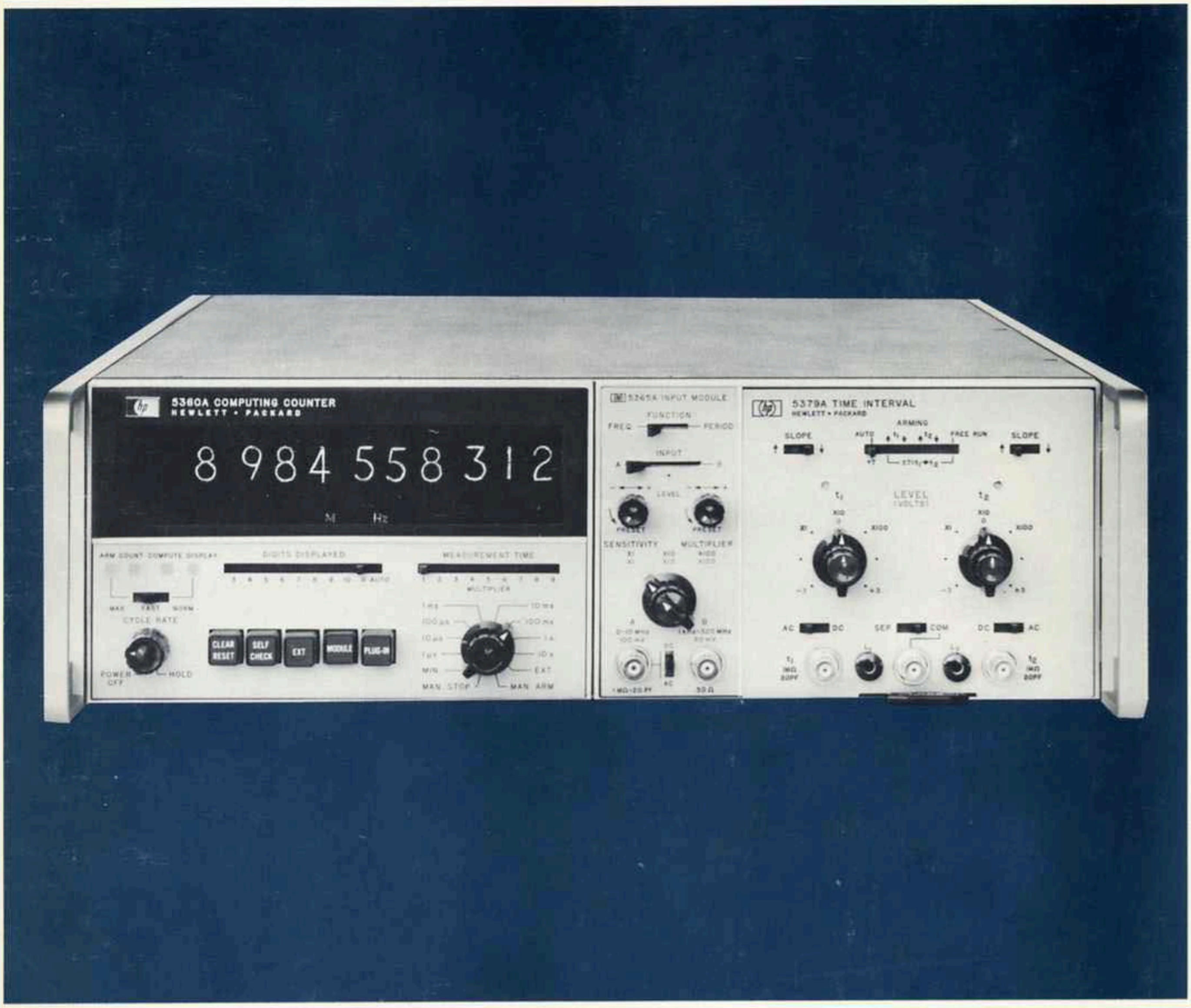
APPLICATION NOTE 116

PRECISION FREQUENCY MEASUREMENTS



HEWLETT hp PACKARD

PRECISION FREQUENCY MEASUREMENTS

Application Note 116

MODEL 5360A

5952-0512

HEWLETT hp PACKARD

JULY 1969

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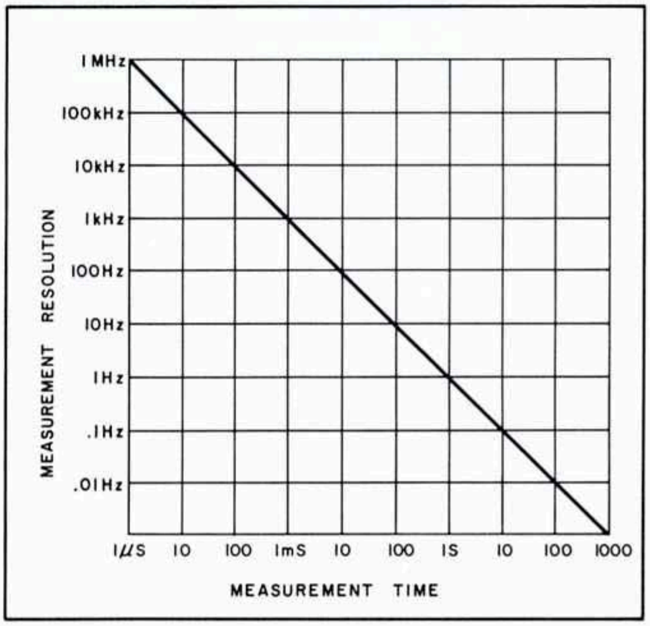
SECTION I

INTRODUCTION

This application note discusses the use of the 5360A Computing Counter in making precise frequency measurements. The 5360A capabilities are compared with conventional counter measurements. Examples of various frequency measurements are shown.

For many years conventional digital frequency counters have provided a convenient, easily used method of obtaining relatively high resolution, accurate frequency measurements. Many instances arise, however, particularly in crystal oscillator work, when the resolution required can only be obtained from the conventional counter by using extremely long measurement times. Resolution obtained from any conventional counter as a function of the measurement time is shown in Figure 1. For example: A 1 Hz resolution is achieved in one second, but 1000 seconds are needed to resolve .001 Hz (a frequent need).

Fig. 1. Conventional counter measurement resolution vs measurement time

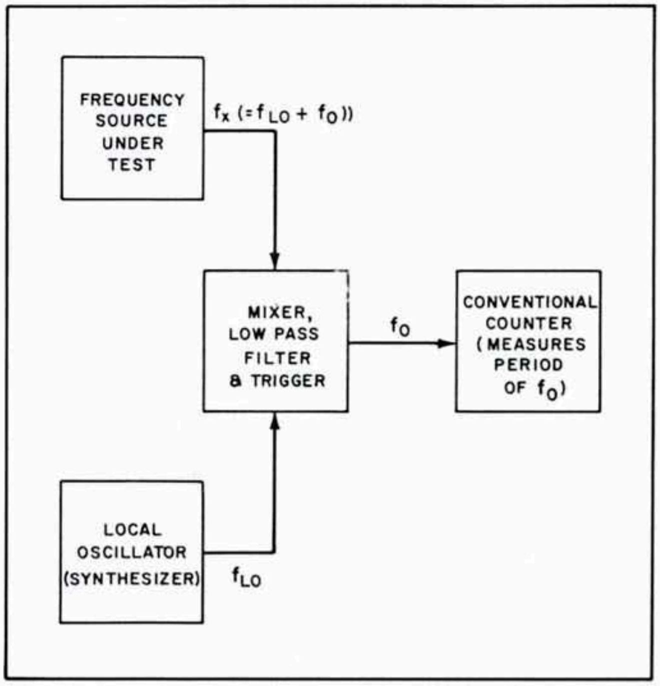


Techniques were developed in the past to overcome this limitation. The general method used (see Figure 2) is to mix the unknown frequency (usually 100 kHz to 10 MHz, if from a crystal) with a local oscillator frequency to obtain a lower frequency (100 Hz or lower). To eliminate trigger error in the period measurement of f_0, a low noise trigger circuit is usually included with the mixer. The conventional counter then measures the period of the low frequency signal. The measurement resolution is then limited only by the counter's time base frequency. The higher the frequency, the greater the resolution. There are two disadvantages to this technique:

1. The counter indication is in units of time rather than frequency. For this reason, a synthesizer is usually used as the local oscillator. The synthesizer output frequency is varied until the period measurement indicated by the counter is that of an easily recognized frequency. Adding this frequency and the synthesizer frequency determines the unknown frequency.

2. The cost for incremental increases in resolution is relatively large.

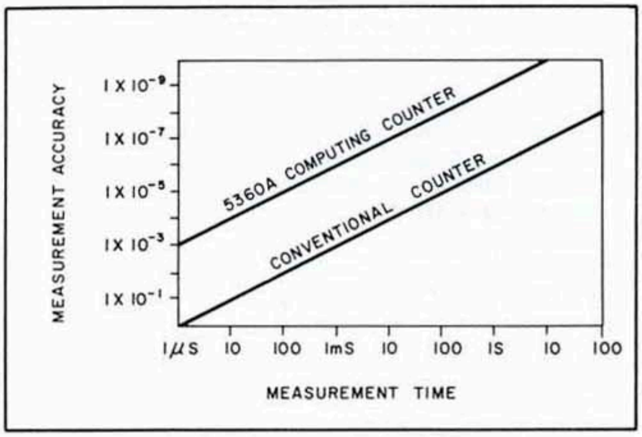
Fig. 2. General method used to increase resolution of a frequency measurement, over that obtainable from the conventional frequency counter.



These disadvantages have largely been overcome with the introduction of the HP 5360A Computing Counter. This instrument can measure in one second any frequency, in the frequency range of 1.0 Hz to 320 MHz, to an accuracy of 1 part in 10^9. For example, a 1 MHz input can be measured, in one second, to an absolute accuracy of .001 Hz.

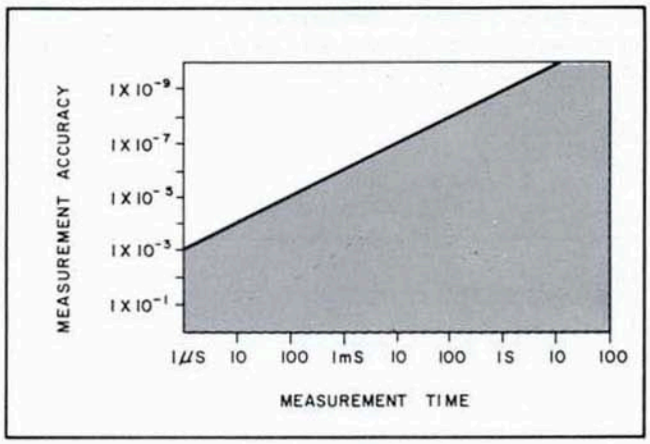
Figure 3 shows comparison of computing counter measurement accuracy with that of a conventional counter.

Fig. 3. Measurement accuracy. Comparison between computing counter and conventional counter (1 MHz input).



The relative accuracy of the computing counter as a function of measurement time is shown in Figure 4. Minimum measurement time is one input period to Channel A, 32 input periods to Channel B. The shaded area indicates the region over which direct frequency measurements can be made. For example: If the relative measurement accuracy required is 1 x 10^{-8}, the computing counter can perform the measurement directly, provided the measurement time is 100 msecs or longer. Under most circumstances, then, the high resolving power of the computing counter means that the system described by Figure 2 may be replaced by the computing counter for all except the most exacting requirements. The latter would include high stability crystal oscillators and atomic frequency standards. Frequency measurements of such devices could only be performed by an indirect technique similar to that described in Figure 2. The synthesizer is not required, however, since the accuracy of the computing counter is retained across its whole frequency range.

Fig. 4. Computing counter. Measurement accuracy vs. measurement time. (Applies to all frequencies from .01 Hz to 320 MHz. Also to 18 GHz with heterodyne converter plug-ins.



SECTION II

PRECISION FREQUENCY MEASUREMENTS

The high resolving power of the computing counter is achieved because this instrument measures the input signal period rather than frequency. By measuring the period, the measurement resolution is dependent upon the counted clock frequency rather than the input frequency magnitude. The period counting measurement accuracy is always greater than that of a frequency counter, provided the counted clock frequency is at the top end of the counter's frequency range. By an indirect method of interpolation, the computing counter achieves the absolute accuracy that would be obtained by direct counting of a 1 GHz clock.

The computing counter basic block diagram is shown in Figure 5. The frequency is computed and displayed from digital data taken from the period measurement.

The computing counter arithmetic capability has been made available to the user by means of the keyboard shown in Figure 6. This arithmetic capability consists of add, subtract, multiply, divide, and square root. These operations are performed on numbers that result from measurements made by the computing counter. Furthermore, the keyboard has a memory into which these arithmetic operations may be programmed. The computing counter may be programmed to display, in real time, the solution y of the equation $y = f(x)$, where x = measurement or measurements made by the computing counter. When one considers the high resolving capabilities of the computing counter, this real time computing capability becomes a powerful feature.

Fig.5. Basic functional blocks of Computing Counter

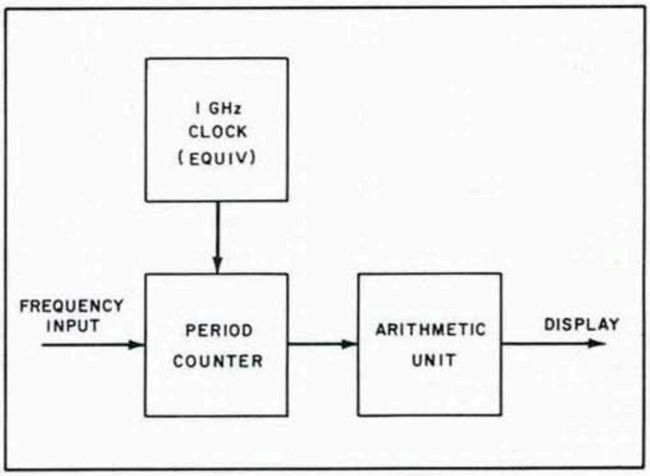


Fig. 6. Layout of 5375A Keyboard

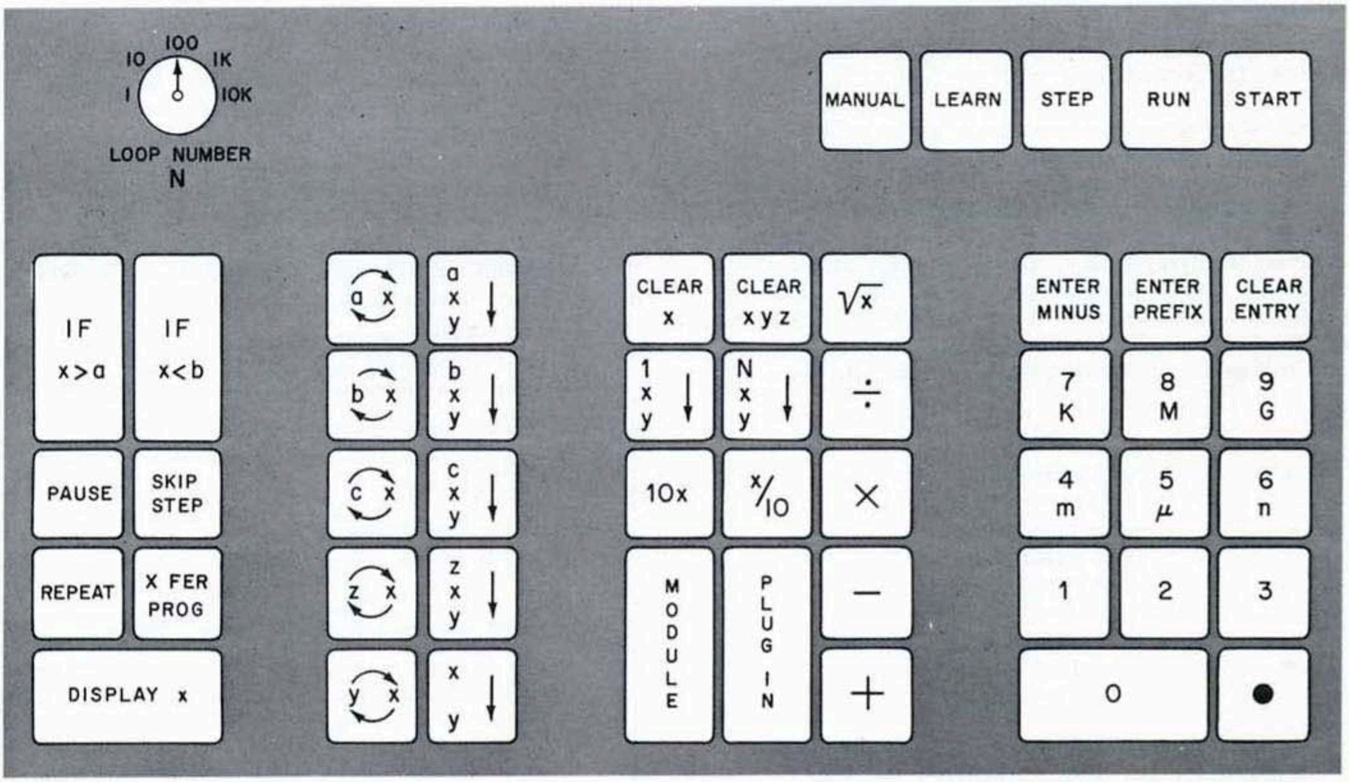
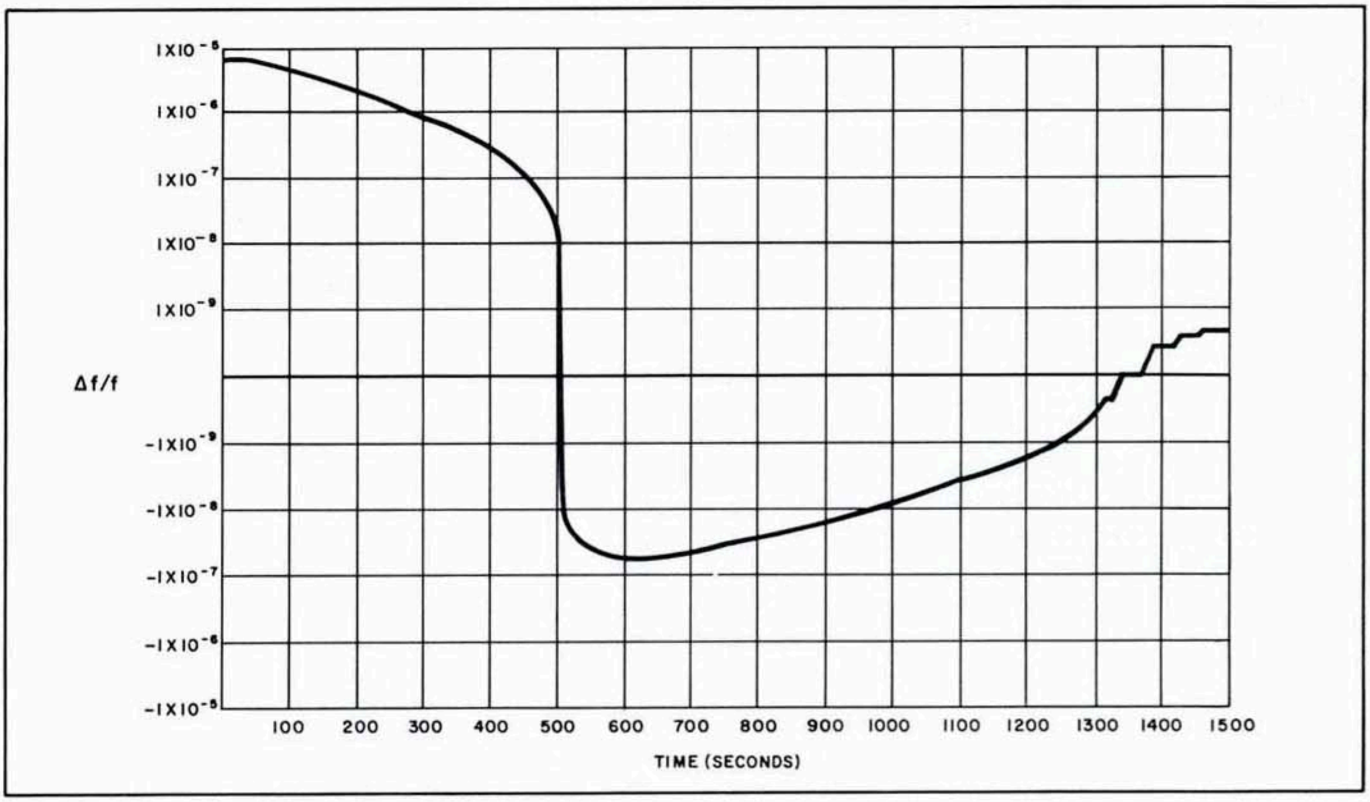


Fig. 7. Warmup curve of H.P. 105A/B Quartz Oscillator.
1. Oscillator off for one hour prior to measurement
2. Scale for $\Delta f/f$ logarithmic



An example of the use to which this can be put is in the measurement of crystal warm-up characteristics. This characteristic is usually presented as a plot of $\Delta f/f$ against time, where $\Delta f = f_i - f$, f = nominal frequency, f_i = actual frequency.

The computing counter may be programmed from the keyboard to make the measurement f_i, calculate $\Delta f/f$, and display this quantity in real time. The warm-up characteristics of the Hewlett-Packard 105B Quartz Oscillator are shown in Figure 7. The points were entered directly on the graph as the computing counter displayed them. Measurement resolution of 2×10^{-10} (1 second measurement time) caused the effect seen at 1300 seconds after turn-on. Digital recorder print-out for some of these points is listed in Table 1.

Table 1.

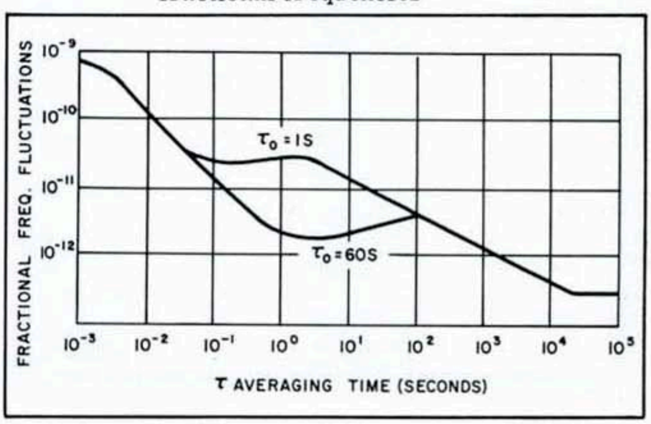
582.37975109n*
641.87972566n*
706.27969813n*
776.17966826n*
852.07963582n*
0.934679600μ*
1.024579562μ*
1.122579520μ*
1.229379474μ*

A portion of the digital recorder-printout.

Short-Term Stability Measurements

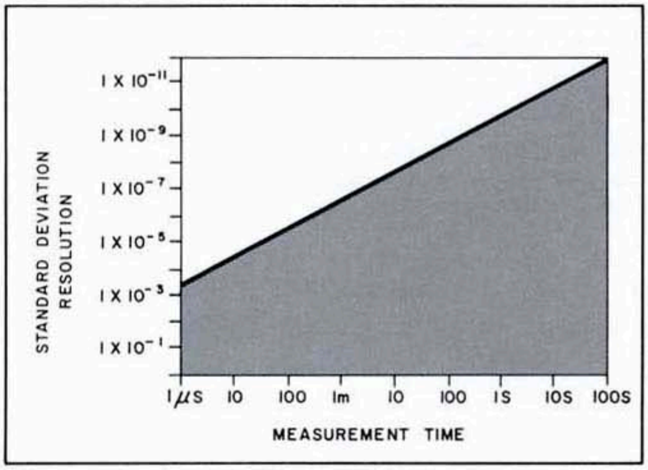
Short-term stability or fractional frequency standard deviation is an important parameter of any frequency source. By means of this parameter, the incidental fm or phase noise inherent in all frequency sources may be characterized. For meaningful characterization, however, the stability must be accompanied by a statement of the averaging time and sample size involved. This point is illustrated by Figure 8, where the typical short-term stability of a Hewlett-Packard 5061A Cesium Beam primary frequency standard is plotted against averaging time.

Fig. 8. 5061A typical standard deviation fractional frequencies



The computing counter may be programmed from the keyboard to directly measure and display standard deviations to within 5×10^{-10} in one second. The limit to which the computing counter can measure standard deviation is a function of measurement time. This limit is given in Figure 9. Note that this is well within the limits of stability of most sources. Moreover, it is a relative figure applying to signals of all frequencies up to its 320 MHz basic frequency range or to 18 GHz with the compatable heterodyne frequency converters (i.e., 5254B, 5255A, 5256A).

Fig. 9. Computing counter standard deviation measurement resolution vs measurement time. (Standard deviation measurements can be made-see shaded area. Example: standard deviation 1×10^{-6} can be measured for all sample times greater than 750 μsec.)

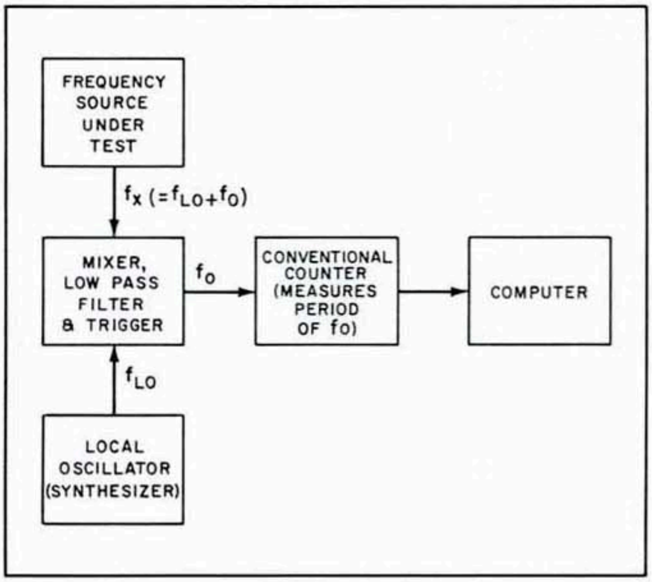


It should be emphasized at this point that the computing counter can be programmed from the keyboard to display, in real time, the standard deviation of the input signal whose frequency it is measuring. The number N of samples or measurements taken is selectable from the keyboard. The standard deviation of the input frequency is displayed after the N samples are taken. The keyboard program to accomplish this measurement is given in Appendix I.

Until the introduction of the computing counter, such measurements could not be performed on line (in real time) without the aid of a complicated system that included a computer. Moreover, the resolution limit of the conventional counter dictates that an indirect method of measurement be used for all except the noisiest of sources. Such a system is shown in Figure 10.

The local oscillator mixer arrangement was required because of the conventional counter's limited resolution. The period of the down converted signal was measured by the conventional counter to obtain the resolution required. The value of f_0 may be chosen to be consistent with the averaging times required. The computer then calculated the standard deviation of the samples taken.

Fig. 10. General method used to measure standard deviation



The computer could have been replaced by a desk calculator, but then the operator would have been faced with calculating, off line, the square root of the sum of the squares of one hundred or more samples. The incremental cost of a small computer is more than offset by the time expended in calculating standard deviation if such measurements were on a repetitive basis.

The alternative to this original system, direct measurement via the computing counter, is shown in Figure 11. The cost saving and ease of operation become obvious, not to mention the measurement time, which takes only as long as the individual samples.

Some sources, such as the best crystal oscillators, rubidium or cesium frequency standards, have short-term stabilities smaller than that directly measurable by the computing counter. In such cases, the source

Fig. 11. An alternative system for standard deviation measurement, provided the limits defined in Figure 9 are not exceeded.

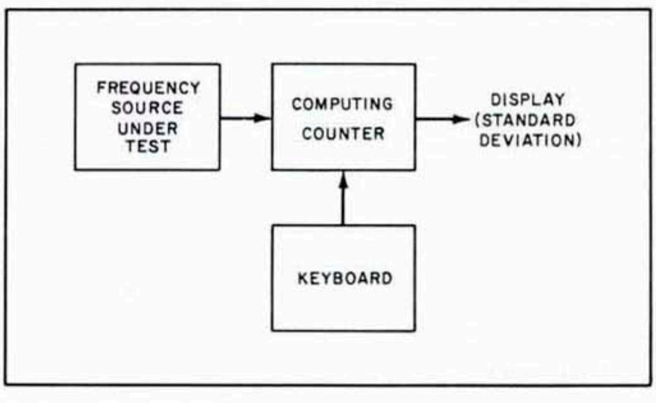
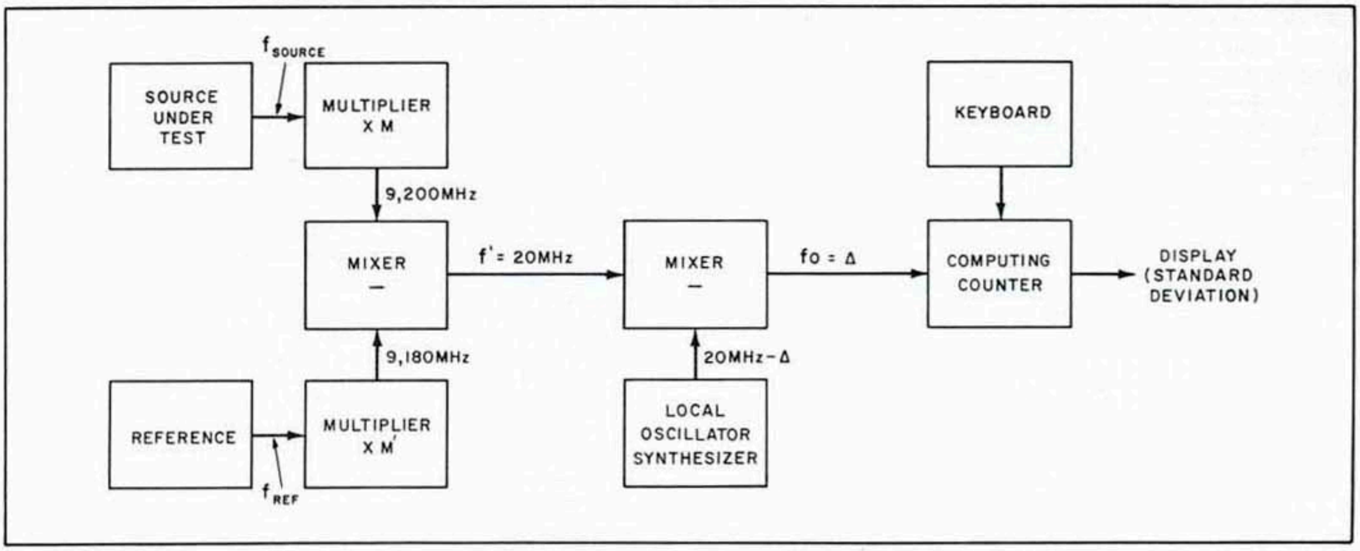


Fig. 12. An indirect system for measuring the standard deviation of a source having a smaller deviation than the computing counters resolution limit.

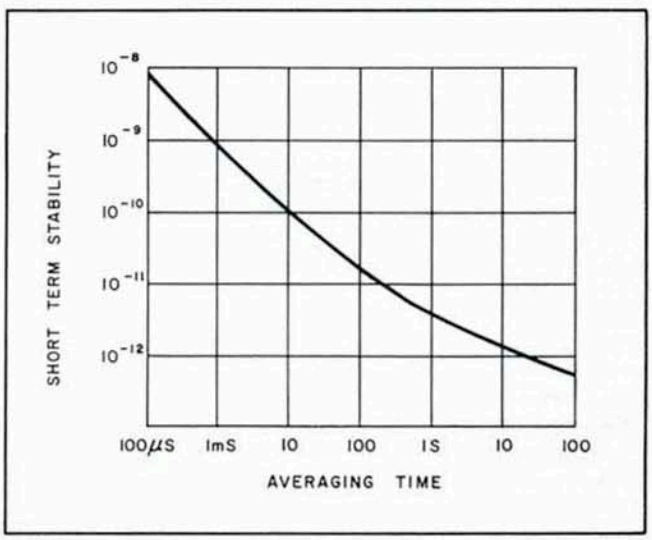


is usually multiplied to a higher frequency and compared against a multiplied reference, usually a similar source. After mixing, the standard deviation of the down converted signal is increased over that of the source by about as must as it is multiplied. This standard deviation is much greater than the synthesizer local oscillator deviations, which further down convert the signal. The eventual output signal measured by the computing counter has a standard deviation magnified many times, but is still related to the input. Such an arrangement is used in the Hewlett-Packard Frequency Standard Laboratory as shown in Figure 12; and it is capable of measuring a standard deviation of 2×10^{-13} in one second, a figure that can be approached only by a hydrogen maser. The synthesizer is used to adjust the down converted output signal to some convenient frequency.

Using the system shown in Figure 12 and the computing counter, the standard deviation of the Hewlett-Packard 5065A Rubidium Frequency Standard was measured for various measurement times from 100 μsec to 90 sec (Figure 13). The computing counter was again programmed to measure and display, in real time, the deviation of the samples for the various measurement times. Since the measurement was not direct, the keyboard program differs slightly from that described in Appendix I. This difference is described and explained in Appendix II.

Another method may be used under certain circumstances in measuring the deviation of crystal oscillators (Figure 14). By offsetting one oscillator from another of similar type, the difference frequency, after mixing, essentially contains the short-term instabilities of both sources. These instabilities are magnified by the ratio of either input frequency to the output difference frequency. The magnitude of the instabilities of this output signal is within the computing counter's direct reading range. A trigger circuit is included in the system to increase the rise time of the difference frequency signal, thereby reducing the measurement error due to trigger uncertainties.

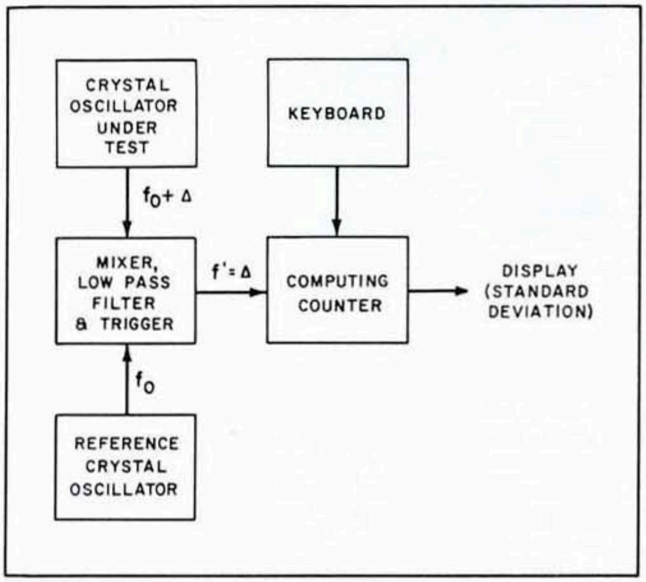
Fig. 13. Hewlett-Packard 5065A rubidium frequency standard short-term stability averaging time. Using measurements obtained with system shown in Figure 12.



As with the other systems discussed, the computing counter may be programmed from the keyboard to measure and display, in real time, the standard deviation of the source for various measurement times. The limitation of this system is that the minimum measurement, or averaging time, is one period of the input signal to the computing counter. This limit is determined by the maximum obtainable offset of the oscillator under test. In practice, this means the shortest averaging time possible is about 0.5 seconds.

The keyboard program required for this measurement differs slightly from that described in Appendix I. These differences are described in Appendix III.

Fig. 14. A simplified system for measuring standard deviation of high stability crystal oscillators



Time Comparison of Frequency Standards

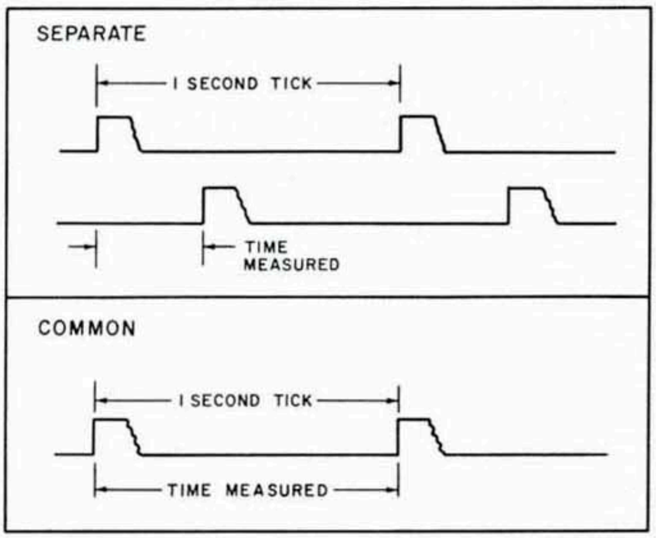
Since the computing counter provides a high resolution time interval capability, signal-to-signal noise (jitter) may be measured to a displayed resolution of 100 psec. (By phase locking the 5360A Standard to the standard under test, a more stable reading may be obtained. Drive the counter with an external 5 MHz.) The following measurements were performed on the HP 5061A Cesium Beam Standard and are indicative of these time comparisons.

The 5061A has a basic 5 MHz output and divided outputs of 1 MHz, 100 kHz, and a 1 Hz "tick" pulse for time keeping purposes. The following measurements may be made from one standard to another or from signal to signal on one standard.

"Tick" to "Tick" Jitter. For separate standards, the two signals are connected to the separate t_1 and t_2 inputs on the 5379A Plug-in. The SLOPE switches are set to (↑) with trigger level set to positive level, and ARMING is set to AUTO. For a single standard, the signal is connected to the COM input with controls set as before, except for the COM-SEP switch (see Figure 15). Note that the positive slope is specified, since the trailing (negative) slope of the "tick" is not as stable in the 5061A as is the leading (positive) slope. For this reason, the 5365A Input Module should not be used to measure the period of a single standard, since the negative edge trigger is used in Channel A.

Previously, the Hewlett-Packard 5061A Cesium Beam has a pulse-to-pulse jitter specification of 20 nsec, because the jitter could not be reliably measured. The computing counter, however, indicates that the magnitude of the jitter is less than 5 nsec! Accordingly, this specification will be changed to conform to what is now known as the standard's actual performance.

Fig. 15. Tick-to-Tick "Jitter" Measurement



<u>5 MHz to Tick Jitter:</u> Tick-to-tick jitter is not a complete specification, since the pulse train could also have a random jitter. This effect would not be apparent on pulse-to-pulse measurements, but it would increase the uncertainty of a time comparison measurement against another standard. The presence of jitter on the pulse train may be detected by determining the occurrence in time of the tick pulse compared to the standard's 5 MHz output.

Since the 5 MHz is the basic signal in the 5061A, it may be considered as the "reference" for the divided signals. The following description applies to the 5 MHz to 100 kHz and 5 MHz to 1 MHz jitter.

Connect the higher (5 MHz) signal to 5379A t_2 (stop channel) jack. Connect the lower (1 MHz, 100 kHz, or 1 second "tick") signal to the 5379A t_1 (start channel) and center the trigger level controls. The ARMING switch must be set to AUTO for these measurements so that a consistent (+ T only) measurement will be displayed (see Figure 16). Computing counter will display signal-to-signal jitter with a 100 psec resolution. The 5 nsec specification previously mentioned will be easily met for the 5061A's outputs.

The AUTO mode in the arming of the 5379A does not allow a t_2 (stop signal) to be accepted until the t_1 (start signal) has been accepted. This t_2 arming takes at least 15 nsec after t_1, and therefore, zero or negative answers are not possible, as is the case with the other arming modes.

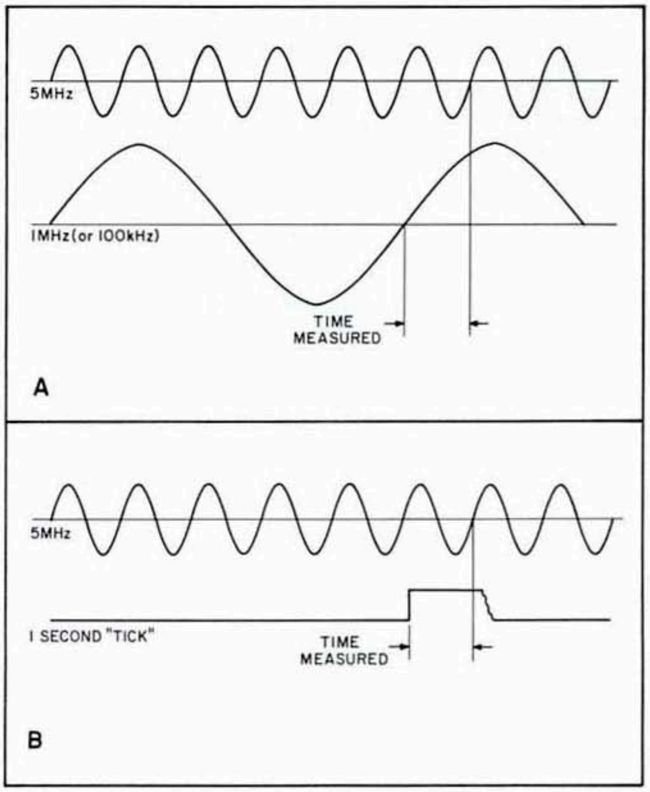


Fig. 16. A. 1 MHz to 5 MHz time measured jitter
B. 1 second tick to 5 MHz time measured jitter

APPENDIX I

MEASURING FRACTIONAL FREQUENCY STANDARD DEVIATIONS

For the particular types of random processes involved in precision frequency sources, the variance of the frequency fluctuations* may be estimated as:

$$\sigma_N^2\left(\frac{\Delta f}{f}\right) = \frac{1}{f_o^2\,(N-1)}\left[\sum_{i=1}^{N}(f_i)^2 - \frac{1}{N}\left(\sum f_i\right)^2\right]$$

For N = 2, this is a particularly simple relation

$$\sigma_2^2\left(\frac{\Delta f}{f}\right) = \frac{(f_1 - f_2)^2}{2f_o^2}$$

In order to improve this estimate, the relation should be averaged over many measurements. Then, the averaged value of the standard deviation is:

$$\left\langle \sigma_2\left(\frac{\Delta f}{f}\right)\right\rangle = \frac{1}{f_o}\sqrt{\frac{1}{2N}\sum_{i=1}^{N}(f_{2i} - f_{2i-1})^2}$$

Where N is the number of pairs of measurements.

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* Statistics of Atomic Frequency Standards
David W. Allan N. B. S.
Proc. IEEE, Vol 54, No. 2, February 1966

This function must be programmed into the computing counter's keyboard to enable this instrument to display, in real time, the standard deviation of the N pairs of samples of the frequency measured. The computing counter then becomes a direct-reading, digital, standard deviation meter.

The program is in two parts:

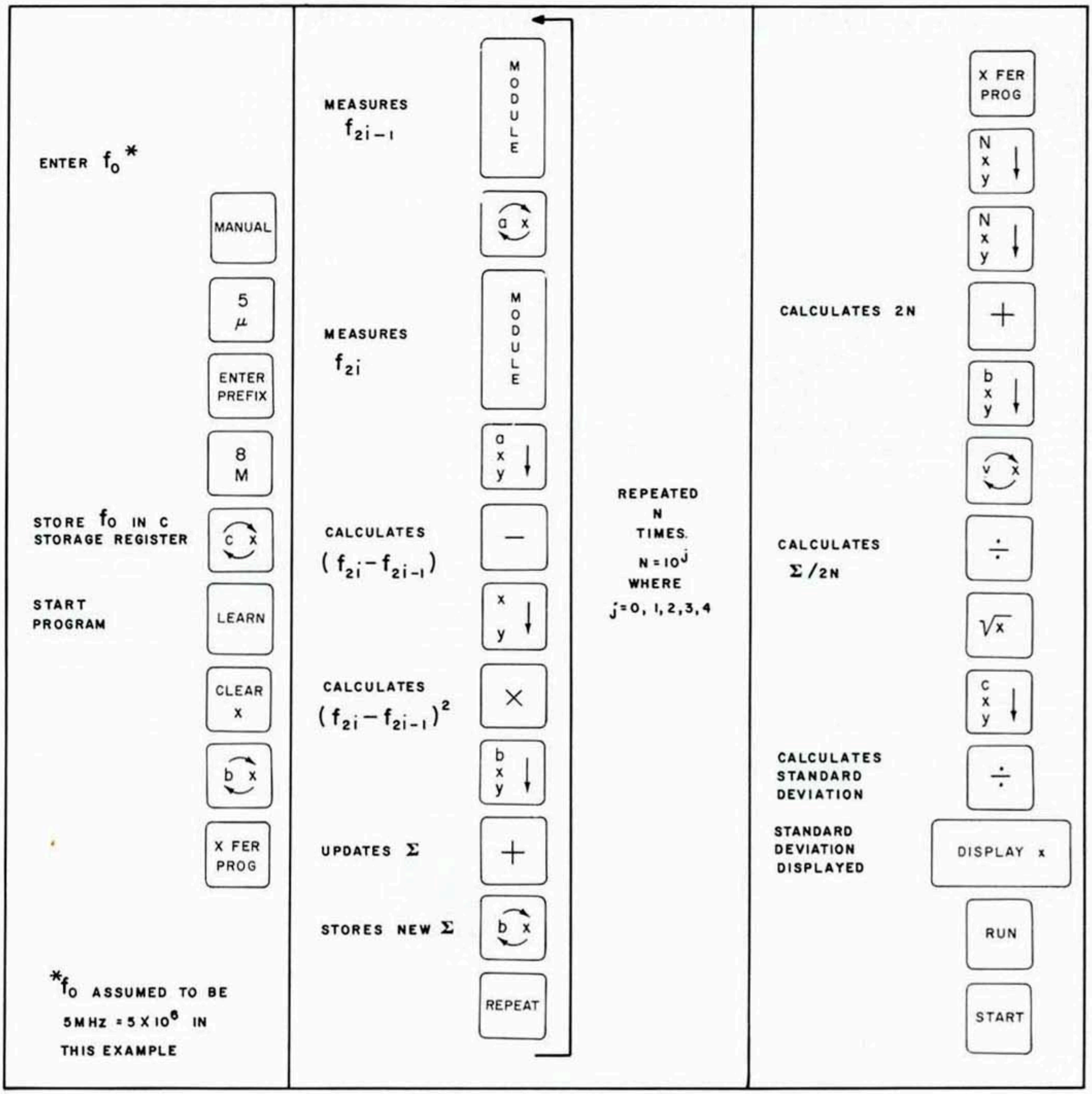
1. A sub-program, which performs the calculation

   $$\sum_{i=1}^{N}(f_{2i} - f_{2i-1})^2$$

   is repeated N times. N is controlled from the keyboard in decade steps from 1 to 10^5. For this type of measurement, at least one hundred samples should be used for a true representation of the deviation.

2. The main program sets the appropriate initial conditions. This involves making and storing the first measurement and clearing the storage register into which the sub-program places the summation. When the sub-program is complete, the main program takes the square root of the sum, recalls the constant f_o (stored prior to initiating the program), and then calculates and displays the standard deviation. Transfer from main to sub-program, and vice-versa is initiated by the "XFER PROG" command. The program is given in Figure 17. Figure 11 shows the method used for making such measurements.

Fig. 17. Program for Fractional Frequency Deviation



APPENDIX II

INDIRECT STANDARD DEVIATION MEASUREMENTS

When the short-term stability of the source is too small to be measured directly by the computing counter, an indirect method of measurement must be used. This technique involves frequency multiplication. If the multipliers are assumed perfect, the absolute value of the noise contributing to the instabilities is increased by the multiplication factor M. A system of the type shown in Figure 12 is used at Hewlett-Packard for these low noise measurements.

The source and references are usually frequency sources of the same type, and the measured noise is assumed to be contributed equally by both. Low noise multipliers multiply the oscillator output frequencies to 9.2 GHz and 9.180 GHz. The difference frequency f' at 20 MHz contains noise due to both oscillators magnified by the amount of multiplication. This noise is retained by the second mixing process, since the synthesizer noise is negligible in comparison. The purpose of mixing a second time is to reduce the nominal carrier frequency, upon which the noise is present, to within the limits of the measuring device's resolution.

The keyboard program used to measure and display the standard deviation is similar to that described in Appendix I. Assuming:

$$f_{ref} = f_{source}$$

$$(\Delta f/f)_{(REF)} = (\Delta f/f)_{(SOURCE)}$$

$$M' \approx M$$

The only change in the program over that described in Appendix I is to initially store in the "C" register the constant $\sqrt{2}\ M\ f_0$ rather than f_0.

APPENDIX III

A SIMPLIFIED INDIRECT STANDARD DEVIATION MEASUREMENT

The system shown in Figure 14 may be used for standard deviation measurements under certain circumstances. The principle of the technique is reducing the carrier frequency, upon which the noise is present, to within computing counter's resolution limit.

The source and reference are usually of similar type and the noise on the output difference frequency f' is assumed to be contributed equally by each oscillator. Since the two oscillators are offset from each other to obtain the difference frequency, this technique is most suitable for crystal oscillators. Atomic standards have only small offset capability and other sources do not have a stability which exceeds the resolving limits of the computing counter. In the latter case, the simpler, direct measurement applies as described in Appendix I.

The keyboard program used to measure and display the source standard deviation is the same as Appendix I except $\sqrt{2}\, f_o$ is stored in the "C" register.

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