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GROUND SYSTEMS AS A FACTOR IN ANTENNA EFFICIENCY*

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Summary—Theoretical considerations concerning the losses in ground systems are advanced. These considerations indicate the feasibility of antennas much less than a quarter wave length tall, for low power broadcast use. The desirability of large ground systems is also indicated.

Experimental data are given which show that an eighth-wave antenna is practically as efficient as a quarter-wave antenna. It is also found that a ground system consisting of 120 buried radial wires, each one-half wave long, is desirable. Tests of ground screens show them to be of no importance when adequate ground systems are used.

The experimental data include antenna resistance and reactance, field intensity at one mile, current in the buried wires, and total earth currents, for many combinations of antenna height, number of radial wires, and length of radial wires.

I. INTRODUCTION

I N THE past few years, many investigations have been made of the action of antennas whose heights have been of the order of a half wave length. The chief advantage of an antenna of this height is the antifading property, obtained when the antenna is of the proper shape. For a transmitter of low power, such an antenna is an unwarranted extravagance, since the service area of the station will generally be limited by signal deficiency or by interference from other stations, rather than by fading. For such a station, it has been the practice to use an antenna whose height is about one quarter of a wave length.

For some time, the authors have been of the opinion that much shorter antennas are feasible. This opinion was based on a number of theoretical considerations of antennas and ground systems. It is the purpose of this paper to discuss these considerations and to report on a series of experiments that were made to prove or disprove the validity of the theoretical results.

II. THEORETICAL CONSIDERATIONS

We shall concern ourselves entirely with straight vertical antennas, with a sinusoidal distribution of current on the antenna. The antenna is placed over a flat earth. The following notation will be used:

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a = antenna height

 $\lambda = operating wave length$

G =angular antenna height $= 2\pi a/\lambda$ radians

= $360 \cdot a/\lambda$ degrees (where a and λ are expressed in the same units).

$$a/\lambda = G^{\circ}/360.$$

Another useful relation is

$$a_{ft} = \lambda_m G^{\circ}/110.$$



The expression for the radiation resistance of such an antenna over a perfectly conducting earth is well known and has been published elsewhere.¹

It is convenient to refer this resistance to the loop of antenna current. The radiation resistance referred to the loop current is given on Fig. 1 as a function of antenna height. The resistance at the base of the antenna is obtained from

$$R_r(\text{base}) = R_r(\text{loop})/\sin^2 G. \tag{1}$$

¹ G. H. Brown, "A critical study of the characteristics of broadcast antennas as affected by antenna current distribution," PRoc. I.R.E., vol. 24, p. 52, equation (7); January, (1936).

Then

Fig. 2 shows the radiation resistance referred to the antenna base for $0^{\circ} < G \leq 90^{\circ}$. An approximate expression for this resistance, when G is less than 30 degrees is

$$R_r(\text{base}) \doteq 10 \cdot G^2 \tag{2}$$

where G is expressed in radians. Equation (2) is also plotted on Fig. 2.

The field strength at the surface of the earth, one mile from the antenna is^2

$$F(\text{millivolts per meter}) = 37.25I_0 \frac{1 - \cos G}{\sin G}$$
(3)

where I_0 is the current (amperes) at the base of the antenna.



Fig. 2

For a constant radiated power, (3) becomes

$$F = 37.25 \sqrt{\frac{P}{R_r(\text{base})}} \cdot \frac{1 - \cos G}{\sin G}$$
(4)

where P is the power (watts) fed into the antenna.

Fig. 1 shows the field strength at one mile when the power is 1000 watts. We see that a quarter-wave antenna yields 194.5 millivolts per meter at one mile, while the best antifading antenna (G = 190 degrees) yields 245 millivolts per meter. This represents an increase of 27 per

² H. E. Gihring and G. H. Brown, "General considerations of tower antennas for broadcast use," Proc. I.R.E., vol. 23, p. 345, equation (16); April, (1935). cent over the quarter-wave antenna. This increase in field strength is insignificant compared to the importance of the antifading characteristic.

We see that as the antenna becomes shorter than a quarter wave length, the field strength remains practically constant. Let us consider the case where G is very small. Then the trigonometric terms may be represented by the series expansion

$$\sin G \doteq G$$

$$\cos G \doteq 1 - G^2/2$$

$$1 - \cos G \doteq G^2/2.$$

When these relations, together with (2) are substituted in (4), for a power of 1000 watts, the following result is obtained:

$$F = 37.25 \sqrt{\frac{1000}{10G^2} \cdot \frac{(0.5G^2)}{G}} = 186.25$$
 millivolts per meter.

Thus an antenna of infinitesimal length, subject to no losses, yields a field strength which is only 4.25 per cent less than the field from a quarter-wave antenna.

While the preceding analysis demonstrates that the field of an infinitesimal antenna is practically equal to that of the quarter-wave antenna, the reader may find the following physical argument more satisfying. The distribution of field strength in a vertical plane around a quarter wave antenna is given by $\cos (90^{\circ} \cos \theta)$]/sin θ while the distribution for an infinitesimal antenna is simply sin θ . The angle, θ , is measured from the zenith. A plot of these two equations is shown in Fig. 3. We see that the distribution patterns are very similar. Thus, with a given amount of radiated energy, and two distributions which are alike, the field strength at one mile must be the same in both cases.



The considerations so far presented have been based on an antenna system free from losses, and a constant radiated power. In actual practice, we are interested in a constant power into the antenna. Then, with losses occurring in the system, the radiated power no longer remains constant. It is desirable to keep these losses as small as possible.

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These losses are due to conduction of earth currents through a high resistance earth and to dielectric losses in the base insulator of the antenna. We shall next consider the earth currents flowing toward the antenna.

The earth currents are set up in the following manner. Displacement currents leave the antenna, flow through space, and finally flow into the earth where they become conduction currents. If the earth is homogeneous, the skin effect phenomena keep the current concentrated near the surface of the earth as it flows back to the antenna along radial lines. Where there are radial ground wires present, the earth current consists of two components, part of which flows in the earth itself and the remainder of which flows in the buried wires. As the current flows in toward the antenna, it is continually added to by more displacement currents flowing into the earth. It is not necessarily true that the earth currents will increase because of this additional displacement current, since all the various components differ in phase. Let us now suppose an imaginary cylinder sunk in the earth in such a fashion that the cylinder and the antenna are coaxial. The cylinder is of radius, x. Then we will denote the total earth current flowing radially inward across the surface of the cylinder as \bar{I}_x . If buried wires are present, $\bar{I}_x = \bar{I}_w + \bar{I}_e$, where \bar{I}_w is the component flowing in the wires and \bar{I}_e is the part which actually flows in the earth.

If the earth is perfectly conducting, the absolute value of the total earth current is

$$|I_x| = \frac{I_0}{\sin G} \sqrt{1 + \cos^2 G - 2\cos G \cos k(r_2 - x)}$$
(5)

where,

$$I_0$$
 = current at the base of the antenna
 $r_2 = \sqrt{a^2 + x^2}$
 $k = 2\pi/\lambda$.

For a constant power,

$$|I_x| = \frac{1}{\sin G} \sqrt{\frac{P}{R_r}} \sqrt{1 + \cos^2 G - 2 \cos G \cos k(r_2 - x)}.$$
 (6)

When the distance, x, becomes large compared to the antenna height, r_2 becomes equal to x, and

$$|I_x| = \sqrt{\frac{P}{R_r}} \frac{1 - \cos G}{\sin G}.$$
(7)

Thus we see that the earth current at a distance is proportional to the field strength at one mile. Fig. 4 shows the total earth current for a number of antennas. The radiated power is 1000 watts. The antenna heights given here were chosen to conform to later experimental heights. We see that the earth currents at points more than 0.3 wave length from the antenna are practically the same for all antenna heights. Then the power lost in the earth beyond the 0.3-wave length radius will stay constant as the antenna height is changed, provided the radiated



power is maintained constant. Close to the antenna, the earth currents of a short antenna rise to large values. It would thus appear that the earth within the 0.3-wave length radius should be a very good conductor in order to operate a short antenna efficiently. This situation may be roughly approximated by a buried ground system consisting of many radial wires.

The actual earth current and the current flowing in the radial wires are given rather accurately by

$$\bar{I}_{e}/\bar{I}_{w} = j\gamma_{e} \cdot 4\pi^{2} \cdot 10^{-9} fc^{2} \left\{ \log \frac{c}{r} - 0.5 \right\}$$
(8)

where,

 γ_{e} = earth conductivity (mhos per centimeter cube) f = frequency (cycles per second) x = distance from antenna (centimeters) n = number of equally spaced radial wires $c = \pi x/n$ r = radius of the wire in the ground system.

From (8) we see that the earth current proper leads the current in the wires by 90 electrical degrees.



Fig. 5

The current in the wires is expressed in terms of the total earth current as

$$\left| \left| \bar{I}_w / \bar{I}_x \right| = \left| \frac{1}{1 + \bar{I}_e / \bar{I}_w} \right| \tag{9}$$

while the current actually flowing in the earth is

$$\left| \bar{I}_{e}/\bar{I}_{x} \right| = \left| \frac{1}{1 + \bar{I}_{w}/\bar{I}_{e}} \right|$$
 (10)

Thus from (8), (9), and (10), together with Fig. 4, we may obtain the actual current in the earth and the current in the wires. It should be remembered that \bar{I}_w is the current in a single wire multiplied by the number of wires.

Fig. 5 shows the current in the wires for the following conditions:

 $G = 88^{\circ}$ $\gamma_e = 0.2 \times 10^{-4}$ mhos per cm cube = 20×10^{-15} e.m.u. f = 1000 kc.

Fig. 6 shows the actual current in the earth for the same conditions. These diagrams show that the ground system consisting of only 15 radial wires need not be more than 0.1 wave length long, while the system consisting of 113 radials is still effective out to 0.5 wave length.



Fig. 6

Since later experimental work was carried out on a frequency of 3000 kilocycles, the following calculations are made on this basis. The current in the wires is shown for the following conditions:

Fig. 7 $\gamma_e = 0.2 \times 10^{-4}$ mhos per cm cube = 20×10^{-15} e.m.u. $G = 88^{\circ}$. Fig. 8 $\gamma_e = 1.0 \times 10^{-4}$ mhos per cm cube = 100×10^{-15} e.m.u. $G = 88^{\circ}$. Fig. 9 $\gamma_e = 0.2 \times 10^{-4}$ mhos per cm cube = 20×10^{-15} e.m.u. $G = 22^{\circ}$. Fig. 10 $\gamma_e = 1.0 \times 10^{-4}$ mhos per cm cube = 100×10^{-15} e.m.u. $G = 22^{\circ}$. The actual earth current, \bar{I}_e , is shown for the same conditions by Figs. 11, 12, 13, and 14. In all cases the radiated power is 1000 watts. These figures show the importance of using a large number of radial wires, of great length. When the earth is of good conductivity, the current leaves the wires and enters the earth closer to the antenna than it does when the earth is a poor conductor. Thus the regions of high current density are subjected to still more current with higher losses in these regions. There seems to be a compensating effect which tends to make the system somewhat independent of earth conductivity, over a limited range.

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Fig. 8







Fig. 10



Fig. 11



Fig. 12







Fig. 14

From (8), we see that the distribution of currents depends on the wire size in a logarithmic fashion. Thus quite a variation in wire size



Fig. 15





may be tolerated. Fig. 15 shows the current in the buried wires for three wire sizes, No. 2, No. 8, and No. 14. Fig. 16 shows the actual earth current for these three wire sizes.

Let us now examine Fig. 17. The current is flowing toward the antenna through a ring of earth of radius, x, width, dx, and depth, s, where s is the skin thickness of the earth, given by

$$s = \frac{1}{\sqrt{\pi \mu \gamma_e f}}$$
 (cm)

$$\begin{split} & \mu = 4\pi \cdot 10^{-9} \\ & \gamma_e = \text{conductivity of earth (mhos per cm}^3) \\ & f = \text{frequency (cycles per second).} \end{split}$$



Then if no wires are present in the earth, the power lost in a ring of width, dx, is

$$dP = \frac{I_x^2 dx}{2\pi x s \gamma_e} \tag{11}$$

When wires are present, the current will not be distributed the same. We shall assume that the change in distribution is slight, but the current in the earth will now be given by I_{ϵ} , so that the watts lost per centimeter are

$$\frac{dP}{dx} = \frac{I_e^2}{2\pi x s \gamma_e}.$$
(12)

If the distance, x, is measured in meters, the power loss will be given in watts per meter. Fig. 18 shows the distribution of earth loss for G = 22 degrees, and G = 88 degrees, for 15 and 113 radial wires, when the frequency was 3000 kilocycles and the earth conductivity is 0.2×10^{-4} mhos per cm². The area under each curve represents the power lost in the earth. We see that the power lost in a ground system consisting of 113 wires, each 0.4 wave length long, is insignificant. With 15 wires and G = 88 degrees, and a radiated power of 1000 watts, the power lost in the ground system out to 0.4 wave length is 447. watts, while the power lost for a 22-degree antenna is 745. watts.

III. EXPERIMENTAL PROCEDURE AND APPARATUS

In the past, experimental curves similar to the theoretical field intensity curve shown in Fig. 1 have been made by maintaining a fixed antenna height and varying the frequency over a wide range. These curves have generally been flat in the vicinity of quarter-wave antenna heights. However, such results have been questionable, because of the fact that the ground system becomes a different fraction



Fig. 18

of a wave length long each time the frequency is changed, while at the low frequencies, the attenuation in the first mile is less than for the high frequencies. Thus, in planning the experiments about to be described, it was decided that the frequency must remain fixed, while the antenna height itself was adjusted. The frequency of operation was 3000 kilocycles. The following combinations were tested:

1. The radial wires were made 0.411, 0.274, and 0.137 wave length long, (135, 90, and 45 feet, respectively).

2. For each length of radial system, 2, 15, 30, 60, and 113 radial wires were used.



Fig. 19



Fig. 20

3. For each combination of ground wire length and number of wires, the antenna heights were G = 22, 44, 66, 88, and 99 degrees, with intermediate heights where necessary.

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Since the above procedure involved the laying of many miles of wire, the plow shown in Fig. 19 was constructed and used throughout the experiments. The blade cut a narrow furrow, laid a soft No. 8 copper wire in the groove, and partially filled the groove. The wire was thus buried at a depth of approxyimatelix in sches.



Fig. 21



Fig. 22

The antenna consisted of a galvanized iron mast, 2.5 inches in diameter. The mast consisted of four sections, each 20 feet in length, topped by a single section, 10 feet in length. Thus the height could be any multiple of 10 feet, up to 90 feet. The complete mast is shown by Fig. 20. The mast was raised and lowered for adjustment by the arrangement shown in Fig. 21.



Fig. 23



Fig. 24

The base of the mast rested on a hardwood insulator. This may be seen at the left of Fig. 22. The oscillator and measuring equipment may be seen in this same figure. The resistance of the antenna was measured

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by the resistance substitution method. Then the variation of the antenna tuning condenser determined the antenna reactance.

The total earth current as a function of distance from the antenna was measured by a method described elsewhere.² A conventional field intensity measuring set was used for this purpose. (Fig. 23.)

For each antenna height, 0.2 watt of power was fed into the antenna and the field intensity was measured at 0.3 of a mile. This figure was then converted to a basis of a power of 1000 watts and a distance of one mile.



Fig. 25

The current in the buried wires was measured in each case. This was accomplished by placing a coil next to the ground wire at a point where the wire was exposed. The coil was resonated by means of a small shunt condenser. The voltage across this combination was determined with a vacuum tube voltmeter. The combination was calibrated in the laboratory. Fig. 24 shows the procedure in question. This measurement yielded the current in a single wire. To obtain the current flowing in all the buried wires at distance, x, the measured value was multiplied by the number of wires.

IV. EXPERIMENTAL DATA

The first ground system installed consisted of 113 radials, each 135 feet long. The wires were then reduced in number. Fig. 25 shows

² Loc. cit., p. 336.

the measured antenna resistance as a function of antenna height, for the various number of wires used. We see that the resistance is lowered steadily as the number of wires is increased.



Fig. 26





Fig. 26 shows the resistance curves when the ground system was 90 feet in radius. The resistance of this system is slightly higher than that of the system 135 feet in radius, when 113 wires are used. However

when only 15 wires are used, the resistance does not change when the wires are shortened. This is due to the fact that very little of the total



Fig. 28





earth current is flowing in the wires at distances greater than 90 feet, when only 15 wires are used.

When the radial wires were 45 feet long, the measured resistance was practically independent of the number of wires. Evidently, most of the earth loss occurred in regions beyond the periphery of the ground system. Fig. 27 is an average curve obtained for this condition.

When only two radial wires, separated 180 degrees, were used, the resistance was independent of wire length since the current vanished from the wires within a few feet of the antenna. The results of this test are shown in Fig. 28.

The reactance of the antenna was found to vary slightly with the ground system. For all practical purposes, the reactance may be re-



garded as constant for a given antenna height. The antenna reactance as a function of antenna height is given by Fig. 29.

The field intensity at one mile for an antenna power of 1000 watts is given in Fig. 30, when the ground system was 135 feet in radius. It is seen that the ground system consisting of 113 radial wires is very nearly perfect. It was found that the antenna shown in Fig. 31 (G = 22degrees) gave a field strength only 8.5 per cent less than the antenna shown in Fig. 20 (G = 99 degrees). Fig. 32 shows the field intensity when the ground system was 90 feet in radius. The results are somewhat inferior to those obtained with the larger ground system. In Fig. 33, we see that the field strength is nearly independent of the number of



Fig. 31





wires for a ground system only 45 feet in radius, with the reservation that at least 15 wires are used. Two radial wires are much worse.









These data have been replotted in Fig. 34 in a slightly different manner. Here the number of radials is fixed at 113. The three curves of field intensity are for the three lengths of ground system tested. Fig. 35 shows similar curves when 15 radial wires are used. These two figures show the necessity of using many wires in an extended ground system. At the same time, if the ground system consists of only a few radial wires, there is no point in extending the wires to great lengths.





Fig. 36 shows the dependence of field strength and resistance on the number of radials. The antenna height was fixed at 77 degrees and the radial length at 135 feet. When the length was changed to 45 feet, the results shown in Fig. 37 were obtained.

Field intensity and resistance as a function of ground wire length are illustrated in Fig. 38, when 113 radials were used. Fig. 39 shows the same type of curves for 15 radials. These diagrams again illustrate the fact that it is useless to extend a few radial wires, while some gain is realized if a great many wires are extended to great lengths.

The total earth currents were measured, using the apparatus shown in Fig. 23. It was found that the shape of the total earth current curve





Fig. 57







Fig. 39



Fig. 40



was practically independent of ground system for a fixed antenna height. However, the scale factor changed with antenna resistance, since the input power was held constant in all cases. Fig. 40 shows the measured earth currents out to a point which is 0.4 wave length from the antenna, for a number of antenna heights, when the ground system consisted of 113 radial wires, each 135 feet in length. Fig. 41 shows the same results plotted to a more extended scale. Here the distance from





the antenna is expressed in miles. The most remote point, 0.25 mile, is four wave lengths from the antenna.

For each ground system, the current in the buried wires was measured as shown in Fig. 24. The value measured in a single wire was then multiplied by the number of buried wires. The current in the buried wires for an antenna height of 88 degrees and radial wires 135 feet long is shown in Fig. 42. We see that the current persists in 113 wires much further from the antenna than it does for a smaller number of wires. Fig. 43 shows similar results for the same ground system and a 22-degree antenna. A few tests were made of the action of an earth screen at the base of the antenna. In the first test, the ground system consisted of 113 radial wires, each 135 feet long. The ground screen consisted of a square copper screen, nine feet on a side. Absolutely no difference in field



Fig. 43

strength or antenna resistance could be detected when the screen was removed and the buried ground system used alone.

	Resistance	Field Intensity
113 buried wires; no earth screen 15 buried wires; earth screen 15 buried wires; no earth screen	1.0 1.62 3.24	$1.0 \\ 0.785 \\ 0.555$

The second test was made using 15 buried radial wires and the earth screen. The relative results are shown above. Thus we see that, with a small ground system, the earth screen furnishes a definite improvement. However, the results obtained are not nearly as good as those obtained with the large ground system. Further, when the large ground system is used, the earth screen gave no further improvement.

Another set of measurements was made in which the ground system





consisted of eight radial wires, each 135 feet long. These wires were laid on the surface of the earth. The ends of the wires were terminated in ground rods. Fig. 44 shows the measured antenna resistance for this case, while Fig. 45 gives the measured field strength for the some con-



Fig. 45

ditions. We see that this ground system is about as good as an equal number of buried wires. These data are of interest since this is typical of the portable systems used for testing possible sites for broadcast transmitters.

V. Conclusion

These experiments show that, even with a poor ground system, an eighth-wave antenna performs practically as well as a quarter-wave



Fig. 46

antenna. For any antenna, it is found that a ground system consisting of 120 buried radial wires, each one-half wave long, is desirable.

The economic factor is of great importance. For a station using a nondirectional antenna, the saving due to the use of a short antenna is large. However, when a directional array is used, the amount of money saved by using an eighth-wave antenna in preference to a quarter-wave assumes rather important magnitudes. Fig. 46 shows relative tower costs as a function of tower height. We see that the curve is linear only for heights less than 200 feet. Fig. 47 gives the heights of eighth- and quarter-wave antennas as a function of frequency. Fig. 48 was constructed from Figs. 46 and 47, and shows the ratio of the cost of a quarter-wave tower to the cost of an eighth-wave. Let us now compare the cost of two arrays, operating on a frequency of 900 kilocycles. The first array consists of two quarter-wave towers. The second





consists of four eighth-wave towers. From Fig. 48 we see that the first array would cost 11.0 per cent more than the second array. Further, the use of four towers in the second array would allow a more effective

distribution of energy, with more satisfactory coverage. At 600 kilocycles, the first array would cost 1.575 times the second array.

A study of the properties of coupling systems shows that the short antennas may be fed with good efficiency if low-loss inductances are used in the coupling system. Sufficient money will be available due to the use of the short antenna to allow the use of a slightly more efficient coupling system than would ordinarily be required.





Another factor of some importance is the base insulator voltage. For a given antenna height, the base voltage is

$$E = \sqrt{\frac{\overline{W}}{R}}\sqrt{R^2 + X^2}$$
(13)

where,

E =r-m-s volts for an unmodulated carrier W = carrier power (watts) R = antenna resistance X = antenna reactance. Fig. 49 shows this voltage as a function of antenna height, for a power of 1000 watts and a ground system consisting of 113 radials, each 0.417 wave length long. To obtain the peak volts for a modulation of 100 per cent, these values should be multiplied by 2.828. The base voltage for G = 45 degrees is many times that for G = 90 degrees. Thus the insulators must be somewhat more efficient. It is significant, however, that the wooden base insulator used in the experiments did not noticeably affect the efficiency.

Too much emphasis cannot be given to the fact that, where direct field intensity along the ground is the sole aim, the ground system is of more importance than the antenna itself. Many times in the past, a T antenna and a poor ground system have been replaced by a tall tower antenna and an extensive ground system with a resulting large increase in field intensity which has been attributed to the tower alone.

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