

# Measurement of Spurious Resonances of Crystal Units Using Network Analysers with Error Correction

Bernd W. Neubig  
 TELE QUARZ GROUP  
 D-74924 Neckarbischofsheim  
 Germany

## Abstract

### 1. Introduction

The hybrid bridge technique for the measurement of spurious responses as standardized by IEC 283 (1968) [3] has significant limitations in accuracy and reproducibility. Significant progress has been made in the last years applying modern network analysers in connection with error correction methods to the measurement of crystal units [6].

The paper presents a systematic approach to adapt the new techniques to the measurement of spurious responses. The goal is to use the standard test set-up (such as the  $\pi$ -network or a s-parameter test fixture) as it is used for the determination of the crystal parameters of the main mode [5]. In connection with the published method for the measurement of load resonance parameters [7], all crystal parameters can then be measured with one unique set-up configuration.

### 2. Classification of spurious responses

Unwanted or spurious modes have different origin and appearance. They can be

- (1) anharmonic resonances of the same vibration mode (usually shear mode), whose frequency is in general above the relevant main mode (see Fig.1 [8]). These resonances are determined by the equation

$$f_{\text{mnp}} = \frac{1}{2} \sqrt{\frac{1}{\rho}} \sqrt{\frac{c_{66} n^2}{t^2} + \frac{c_{11} m^2}{l_x^2} + \frac{c_{55} p^2}{l_y^2}}$$

- (2) unwanted modes (and overtones thereof) of other vibration modes, which may lie below or above the desired mode.

## Resonant Vibrations of a Quartz Plate

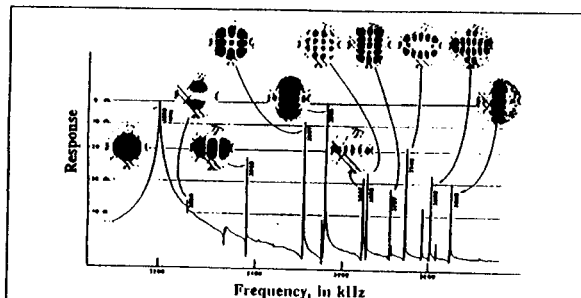


Fig. 1 X-ray topographs ( $2\bar{T}\cdot 0$  plane) of various modes excited during a frequency scan of a fundamental mode, circular, AT-cut resonator. The first peak, at 3.2 MHz, is the main mode; all others are unwanted modes. Dark areas correspond to high amplitudes of displacement.

Spurious modes can be

- (i) isolated from the main mode, or can be
- (ii) mechanically coupled to it, causing non-linear effects which depend on drive level (power dissipation) etc.

Spurious modes can lie

- (i) very close to the main mode (a few kHz), such that the peak of the spur is on the slope of the main response.
- (ii) well apart from the main mode, and therefore can be considered as a separate peak.

Unwanted modes can cause problems

- (a) in filters by distortion of the transfer characteristic in the pass band or the stop band: attenuation "dips" and phase discontinuities in the pass band, deterioration of the attenuation in the stop band
- (b) in oscillators, the working frequency can "jump" to a strong unwanted mode or the frequency response can show discontinuities ("activity dips") at particular operation conditions (temperature, load capacitance/pulling voltage, drive level). VCXOs can show "dips" in the pulling characteristic (frequency vs. pulling voltage) or in the response of the deviation vs. modulation frequency. TCXOs can show "dips" in the frequency vs. temperature characteristics.

In the following we will distinguish phenomenologically four different cases:

- case A: strong spurs well "isolated" from the main mode,
- case B: weak spurs well "isolated" from the main mode,
- case C: strong spurs in the vicinity of the main modes,
- case D: weak spurs in the vicinity of the main mode.

These cases are depicted in Fig. 2.

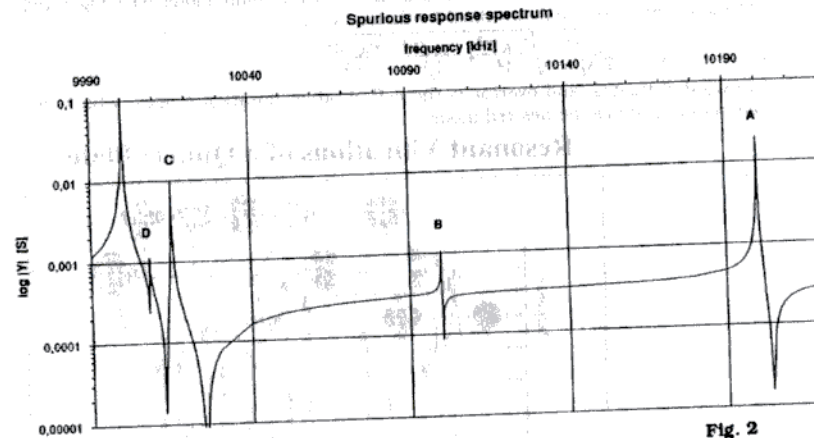


Fig. 2

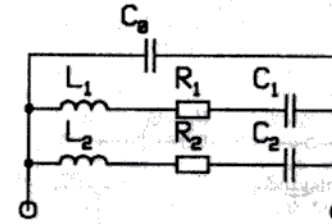
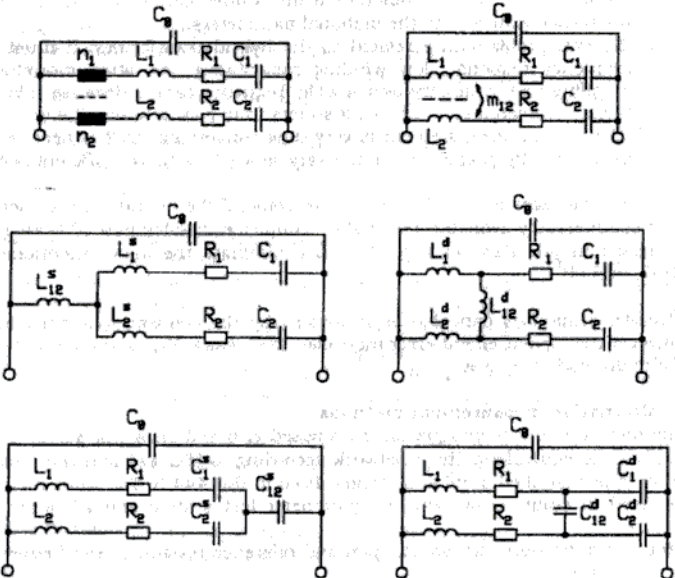


Fig. 3

### 3. Equivalent electrical circuits

Non-coupled resonances can be represented by separate series resonance circuits all connected in parallel to the static capacitance  $C_0$ . In the following we use the symbols  $C_0$  for static capacitance,  $L_1, C_1, R_1$  for the motional parameters of the first (main) mode, and  $L_i, C_i, R_i$  with  $i = 2, 3, 4, \dots$  for the  $i$ -th (spurious) mode (see Fig. 3). Resonances, which are mechanically coupled, must be represented electrically by a coupling circuit such as a transformer or an equivalent coupler e.g. three capacitors or inductors in delta- or star configuration, where one or two of them may have negative element values. Possible circuits are summarized in Fig. 4.

Fig. 4

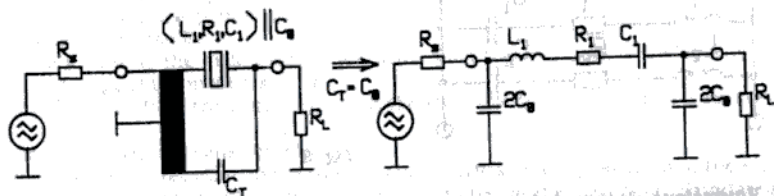


### 4. Classical measurement methods

The classical measurement technique for spurious modes is the hybrid-coil method, described by Horton and Smythe [1] and Priebe [2], which was standardized in IEC 283 [3].

It uses a differential transformer with a variable capacitor  $C_0$  in the second branch to "compensate" the static capacitance  $C_0$  (see Fig. 5).

Fig. 5



Due to its simplicity in practical application (once the hybrid transformer is made ...), this method is very popular. However it has significant disadvantages:

- (1) Even if  $C_0$  is compensated ( $C_c = C_0$ ), it has not disappeared: It appears with twice its value in parallel to the source and load termination resistors. Therefore the frequency of minimum attenuation is not exactly equal to the series resonance frequency, and the resonance resistance computed by a formula neglecting  $C_0$  - which is the usual practice - is wrong.
- (2) Spurious resonances close to the main response can not be characterized properly. The "dB" or "differential dB" values cannot be directly converted to resistance values or to the motional parameters.
- (3) Several problems are related to the hybrid transformer: It must have a flat frequency response, low winding capacitance, low stray inductance, perfect coupling and symmetry over a wide frequency range. Because this critical part is not commercially available, it suffers from poor reproducibility.
- (4) The manual measurement is very time-consuming, as it is necessary to sweep in the vicinity of each resonance very slowly to achieve sufficient accuracy.

The disadvantages in item (3) can be overcome, if the hybrid transformer is replaced by a commercially available  $180^\circ$  hybrid coupler embedded in a  $50 \Omega$  environment, as proposed in [4]. This however does not eliminate the above mentioned problems (1),(2) and (4).

Stronger responses can also be measured by the conventional  $\pi$ -network method, however the measurement error increases, the weaker the spurs are and the closer to the main mode they are.

### 5. Alternative measurement methods

The measurement technique to be proposed is based on a test set-up with network analyser or equivalent, the  $\pi$ -network according to IEC 444 and the error correction technique described in [5] and standardized in IEC 444-5. The same method can also be used in conjunction with a s-parameter test system and adequate crystal test fixture.

- Step 1: Calibration with short, open and reference resistor in the frequency range of interest
- Step 2: Measurement of the static capacitance according to IEC 444-5
- Step 3: Determination of the resonance frequency  $f_r$  (resp. series resonance frequency  $f_s$ , or load resonance frequency  $f_l$ ) and the equivalent parameters  $R_1$ ,  $C_1$  (and  $L_1$ ,  $Q_1$ ) of the main mode with one of the error-corrected methods described in IEC 444-5. If spurious modes are very close to the main mode (i.e. closer than  $f_s/1000$ ), the values derived herewith may include non-negligible errors, but can be used as starting values for the iterative procedure described later.

- Step 4: Fast sweep (approximately 1 sec) through the frequency range of interest upwards (increasing frequency) and downwards. A minimum of 500 data points is recommended. Subtract the error terms, compute  $Y_{up}(f)$  and  $Y_{down}(f)$ . Subtract the admittance  $j\omega C_0$  from both sets.
- Step 5: Inspect the data and select computationally the peaks with a given minimum (trigger) value  $Y_{min}$ . Average the related peak frequencies of both sweeps. This yields an excellent guess for the resonance frequencies of the spurs. The sensitivity of the peak selection can be improved by mathematical differentiation of the  $Y(f)$  values. Alternatively the network analyser's internal "peak search" function can be used if implemented.
- Step 6: Zoom each spurious response individually and measure  $Y(f)$  with higher frequency resolution.

The following steps depend on the type of spur to be considered (see classification in clause 2).

#### case A (strong spur well "isolated" from the main mode)

Zoomed data points can be selected as in IEC 444-5. Fastest approach is the iterative procedure as described in [5]. From a minimum of two or three test frequencies the  $i$ -th spurious resonance can be characterized by  $f_i$ ,  $R_i$ ,  $C_i$  (and  $L_i, Q_i$ ).

#### case B (weak spur well "isolated" from the main mode)

Method as in case A. To improve sensitivity, the following modifications could be considered:

- averaging of several sweeps
- reduced RF bandwidth and video bandwidth (slower speed)
- use of frequency selective, low noise network analyser. The technique using direct measurement of amplitude and phase is in this respect advantageous over the s-parameter systems.
- increase impedance  $R_T$  "seen" by the crystal in the test fixture. The s-parameter fixture with its  $R_T = 100\Omega$  is advantageous over the IEC 444  $\pi$ -network ( $R_T = 25\Omega$ ).

#### case C (strong spur in the vicinity of the main mode)

Data are zoomed with high frequency resolution in an interval covering the 3dB-bandwidth of the main mode and of the considered spur (its bandwidth is assumed to be approximately equal to that of the main mode - as is its  $Q$  in most cases). The parameters of both modes are then extracted by a parameter-fitting algorithm.

For uncoupled modes the equivalent circuit diagram of Fig.3 is used, i.e. three variables per resonance have to be determined ( $L_1$ ,  $C_1$ ,  $R_1$  or  $f_1$ ,  $C_1$ ,  $R_1$ ). For coupled modes one circuit of Fig.4 has to be used. For  $N$  resonances  $3 + 4(N-1)$  elements have to be determined.

Rather good starting values for the elements can be used which speeds up the parameter search and reduces the risk of ill-conditioned iterations to wrong local minima.

These are:

- $f_1$ ,  $R_1, C_1$  (and therefore  $L_1$  and  $Q_1$ ) of the main mode from step 3
- $f_2$  from the location of the peak maximum of  $Y(f)$  (after removal of  $j\omega C_0$ )
- $R_2$  from the amplitude of the peak maximum, derived from

$$R_2 = \frac{1}{|Y(f_{peak})|}$$

- $C_2$  from the assumption that the  $Q_2$ -value of the spur is approximately the same as  $Q_1$  of the main mode

$$C_2 = \frac{1}{\omega_2 Q_2 R_2}$$

In the case of coupled modes, the starting value of element which determines the coupling can be arbitrarily set to zero. The iteration procedure minimizes the error function

$$E = \sum w_i |Y_i - Y_i^M|^2 = \sum w_i ((G_i - G_i^M)^2 + (B_i - B_i^M)^2)$$

in which  $Y_i = G_i + j B_i$  denotes the theoretical values from circuit analysis computed at the measurement frequency  $f_i$ , and  $Y_i^M = G_i^M + j B_i^M$  represents the values derived from the measurement at  $f_i$ . The weighting factors  $w_i$  can be selected such, that the data points close to the resonances get the greatest significance, e.g.  $w_i = |Y_i^M|$ . Several searching algorithms for minimization of  $E$  are applicable. One of them (Newton method) is described e.g. in [9]. Another approach is the method of steepest descent, e.g. the Simplex algorithm. The evaluation of the different methods with respect to simplicity and stability is currently under progress.

#### case D (weak spur in the vicinity of the main mode)

The method is identical to case C, except that the starting value of  $R_2$  has to be derived differently, because the amplitude of the spurious peak is too erroneous, as it is located on the slope of the main resonance curve. A more reliable starting value for  $R_2$  is gained as follows: The approximate admittance of the main mode

$$Y_m(f) = \frac{1}{R_1 + j \frac{1}{\omega C_1} (\omega^2 - 1)}$$

is computed at the measurement frequencies  $f_i$  by using the starting values of  $f_1, C_1, R_1$ . It is then subtracted from the measured admittance values  $Y_i^M$  in the vicinity of the main mode and the weak spur. From the peak of this residual  $Y_{spur}$  the starting value of  $R_2$  can be computed as

$$R_2 = \frac{1}{\max(Y_{spur})}$$

It should be noted, that the achievable accuracy for the parameters of very weak spurs is reduced. However, the method proposed herein is superior to the conventional methods used so far.

#### 6. Conclusions

The paper describes a concept for the measurement of spurious resonances, which allows the use of network analysers in conjunction with error correction techniques. The test fixture can be the  $\pi$ -network or the s-parameter test fixture as described in IEC 444-5. The proposal includes the determination and characterization of weak spurs, which may lie close to a strong mode. It considers also the mechanical coupling of modes. The next steps are the optimization of the parameter fitting algorithm. The experimental verification and comparative measurements have to follow. These results will be reported later.

#### 7. Acknowledgements

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#### 8. References

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