Correlation of predicted and real aging behaviour of crystal oscillators using different fitting algorithms

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1. Abstract

This paper reports on the test results determinating the ageing behaviour of quartz oscillators more precisely as it was possible before. Different methods are presented for the calculation and curve fitting of measured ageing data. It was tried to minimize the time necessary for correct predictions of the OCXO in the ageing system.
The OCXO ageing system of TQ allows a simultaneous measurement of up to 1600 OCXOs for any period of time. The measurement data of all OCXOs produced are put into archives since the end of 1995. Thus as data basis one could go back to extensive archives of ageing data in which there is now the data of over 50000 OCXOs. OCXOs were sometimes left in the measuring system for tests during new developments; some of them longer than 250 days. Those data were used for the tests.

2. Introduction

The prediction of long-term ageing behaviour of high frequency OCXO is always connected with the factor of uncertainty whether the measured ageing curves of the oscillator are really kept. For the calculation of the coefficients for aging prediction several functions can be applied:
The exponential function [1]

(1) $\Delta f / f_0 = a_0 + a_1\left(1 - \exp\left(-a_2 t\right)\right)$

The polynomial function [2]

(2) $\Delta f / f_0 = a_0 + a_1 t + \sqrt{a_2 t}$

The pure logarithmic function [1]

(3) $\Delta f / f_0 = a_0 + a_1 \log t$ or

(4) $\Delta f / f_0 = a_0 + a_1 \ln t$ with $t \geq 1$

and the modified logarithmic function [3],[4]

(5) $\Delta f / f_0 = a_0 + a_1 \ln\left(a_2 t + 1\right)$

This function, proposed in MIL-O-55310 [3] is the most recently used function for aging estimations. [4].
TELE QUARZ is using the modified logarithmic function (3) and the polynomial function (2) depending on which of the two functions describes the measured frequency points better. The coefficients for the aging functions result from [3] through the curve fitting of the measurement data of the OCXOs collected over 30 days. After this relatively short time related to the period of prediction the coefficients were adapted to the measuring data with least square algorithms. The ageing values for the prediction are now derived from these coefficients.

3. Problem description

At present TELE QUARZ is adapting the coefficients of the logarithm function to the measuring data through the nonlinear least square fitting algorithm proposed by Filler [5]. The measuring time is 10 ... 20 days depending on the OCXO type. The coefficients obtained are directly used for predicting the ageing behaviour. Long series of measurement revealed that in almost 80% of all cases the combination of modified log function and Filler algorithm provide too bad results. That means that the real ageing rates of over 80% of all measured OCXOs are better than the prediction. Figure 1 shows a typical measuring curve with the corresponding fitted curve. The real ageing curve has a clearly flatter course than the fitted function. Thus the predicted values are higher than the actual expected ones.

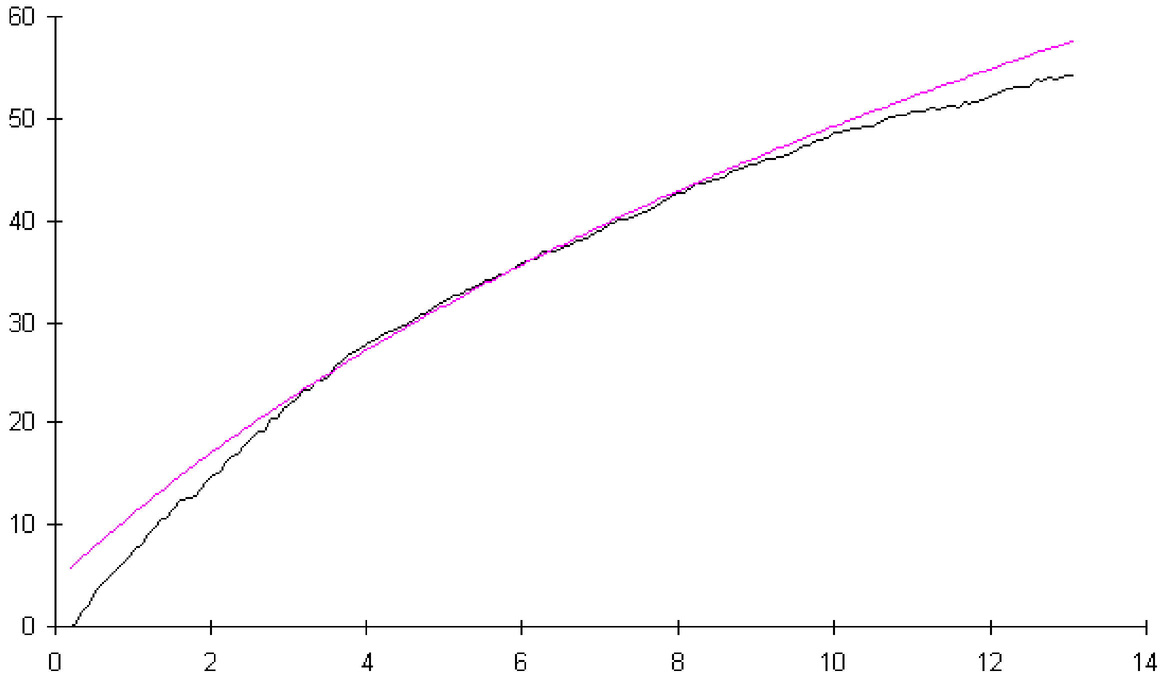

Figure 1 Bad fitting with Filler algorithm

This fact which is at first positive for the customers is again relativated because of higher oscillator prices caused by the internal rejection of good pieces.

All OCXOs used for the tests have to be measured for the period of predictions to be able to compare ageing curves with the curves precalculated. The real ageing curves can now be compared with the precalculated curve which is obtained from the measuring data of the first days

4. Curve discussion

Due to high oscillator frequencies in the MHz range and the essential measuring and processing accuracy the function proposed in MIL Standard MIL-O-55310B for calculating the ageing behaviour

(6) $f(t) = b_0 + b_1 \ln(b_2 t + 1)$

produces high quantities of data. To keep the necessary storage capacity of the measuring computer small it is possible to use a modified formula which gives the ageing values .....

(7) $frel(t) = \frac{f(t) - f(t_{Start})}{f(t_{Start})}$ ==>

(8) $frel(t) = a_0 + a_1 \ln(a_2 t + 1)$

with $a_1 = \frac{b_1}{f(0)}$ $a_2 = b_2$

(9) $a_0 = \frac{f(0) - f(t_{Start})}{f(t_{Start})}$

With this modified aging formula the ageing measuring data can directly be given in parts per billion. Due to the small numerical values the data volume also remains small as the absolute frequency has to be stored only once. Due to the OCXOs of the long-term measurement it can be shown that the modified log function (5) is correlating well with reality (figure 2).
Filler mentions in [5] that the ageing of quartz crystals and oscillators is influenced from many different mechanisms. Some influence factors can be minimized. The temperature dependence of oscillators for example in air-conditioned rooms is only of minor importance. Nevertheless many oscillators show a strange ageing behaviour. Above all in a complex ageing system such as used at TELE QUARZ the OCXOs seem to influence each other through injection locking, thermical influence and mechanical vibration or bumps.
Thus in some cases several OCXOs show frequency jumps at the same time or change their current consumption simultaneously. In these cases a precise estimation of ageing is nearly impossible as the cause usually is not reproducable.
In case of OCXOs with continuous ageing behaviour the prediction accuracy can be considerably improved by suitable actions.
Figure 2 shows a typical aging behaviour of an AT OCXO representative for all measured oscillators. The curve was fitted over the whole measuring time. With over 70% of the measured ageing curves there is a similar good correlation. About 20% of the OCXOs show frequency jumps. They were not considered in this test.

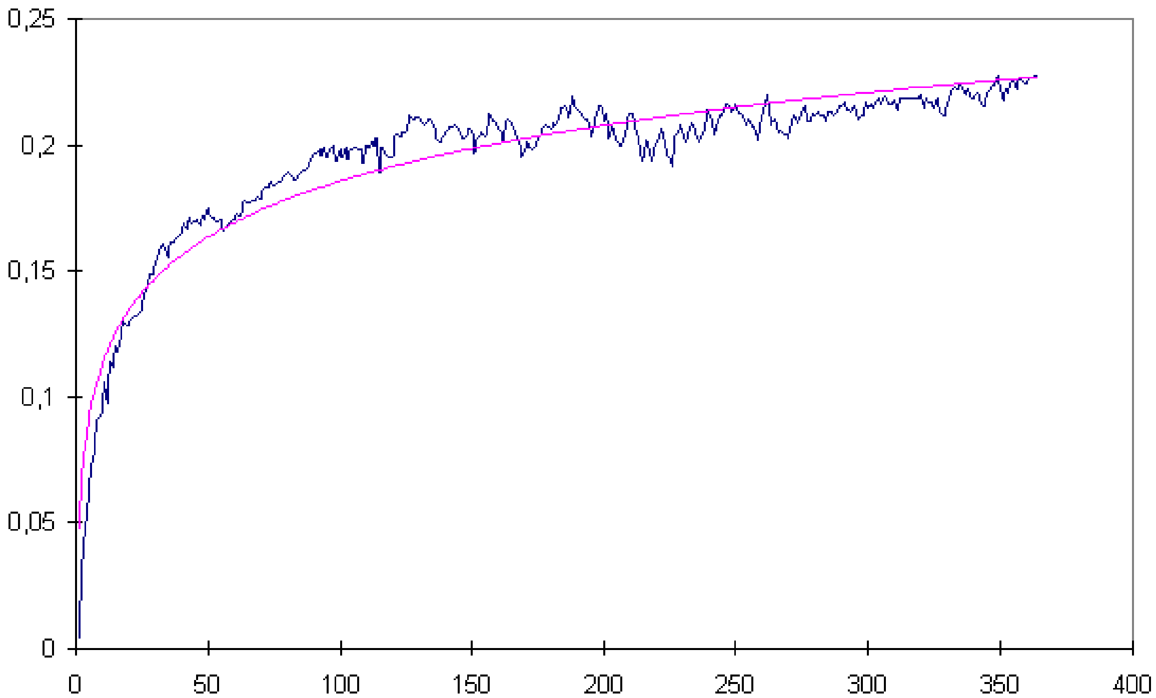

Figure 2 Measured ageing curve with fit over one year
The other 10% are very stable OCXOs with SC cut crystals, where other effects which are stronger than aging (like temperature stability, influence of adjacent oscillators, supply voltage variations, injection locking etc.) influence the ageing measurement. Thus the fitting algorithm can often find no satisfactory solution which partly leads to extreme results. (figure 3)

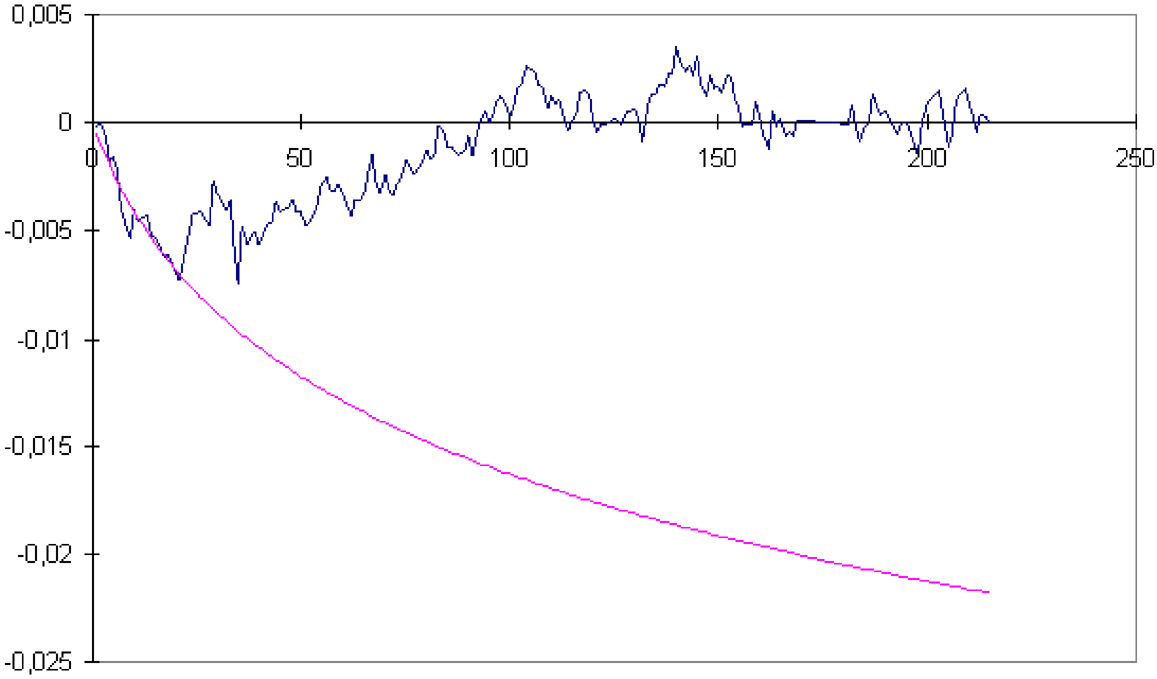

Figure 3 Very stable SC OCXO with bad fitting

The time derivative of the ageing formula is as follows

$$frel'(t) = \frac{a_1 a_2}{a_2 t + 1} \quad (10)$$

for large values of t it is:

$$frel'(t) \approx \frac{a_1}{t} \quad (11)$$

for small values of t it is:

$$frel'(t) \approx a_1 a_2 \quad (12)$$

Therefore, for large t the annual ageing is nearly independent from a_2. It can be described for large values of t in limited time intervals by a straight line with the slope a_1 / t. This is the case after 30 to 250 days depending on the oscillator type.The curve has the biggest slope of $a_1 a_2$ at small values of t. In this time interval usually the curve fitting is done. So for the algorithm the long term coefficient a_1 has nearly the same influence on the least square result than the short term coefficient a_2. Because of this the choice of algorithm to the coefficient extract has great influence on the long term gradient a_1 of the function. Some methods and algorithm are described to increase the accuracy of prediction.

5. Results

The correlation of the fitted measuring curves with the real measured ones can be improved, if the term a_0 in equation (5) is forced to zero. Through this restriction the adaptability of the fit to the measuring values is hardly changing. Figure 4 shows a fitted curve with a_0=15. A second one with a_0=0 was fitted to this curve by least suare algorithm.

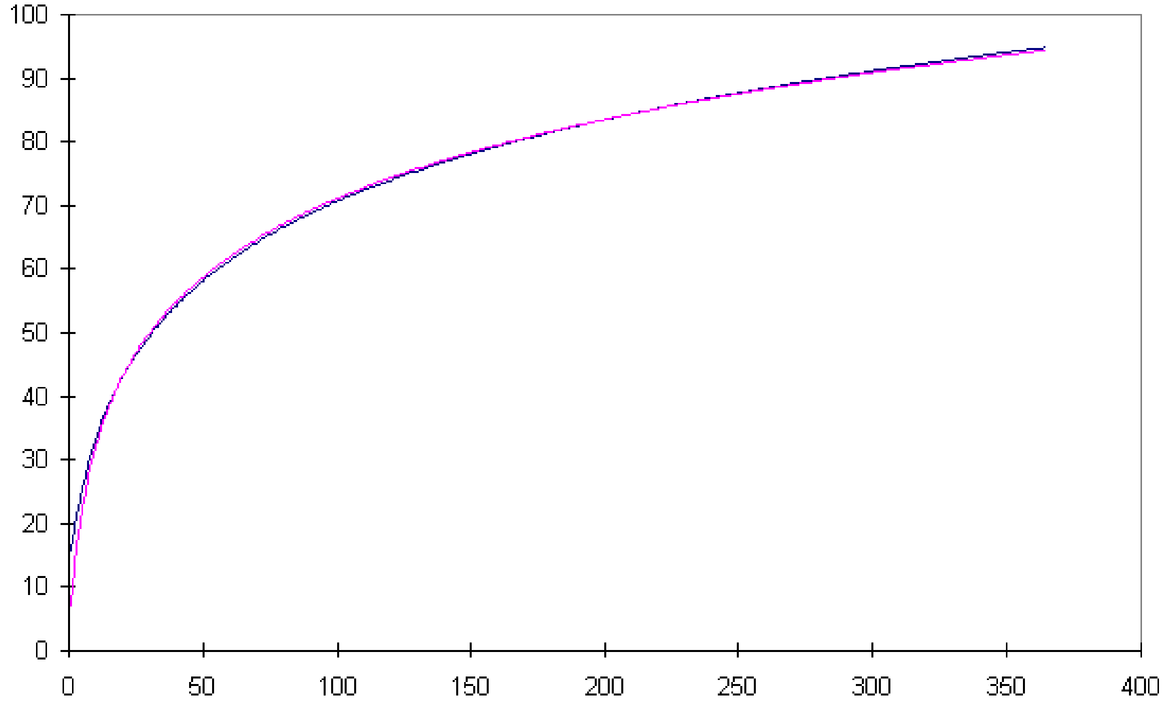

Figure 4 Differences of the curves a_0=0
The diffences are minimal and only important in the first days of operation. The long-term behaviour does not change through this measure as a_0 is only a mathematic offset for the curve. For the fitting algorithm, however, a_0. appears as a series of square sums. Therefore several solutions are possible for the fitting algorithm. So if a_0 is kept at zero during the fitting the algorithm often can provide better solutions (figure 5).

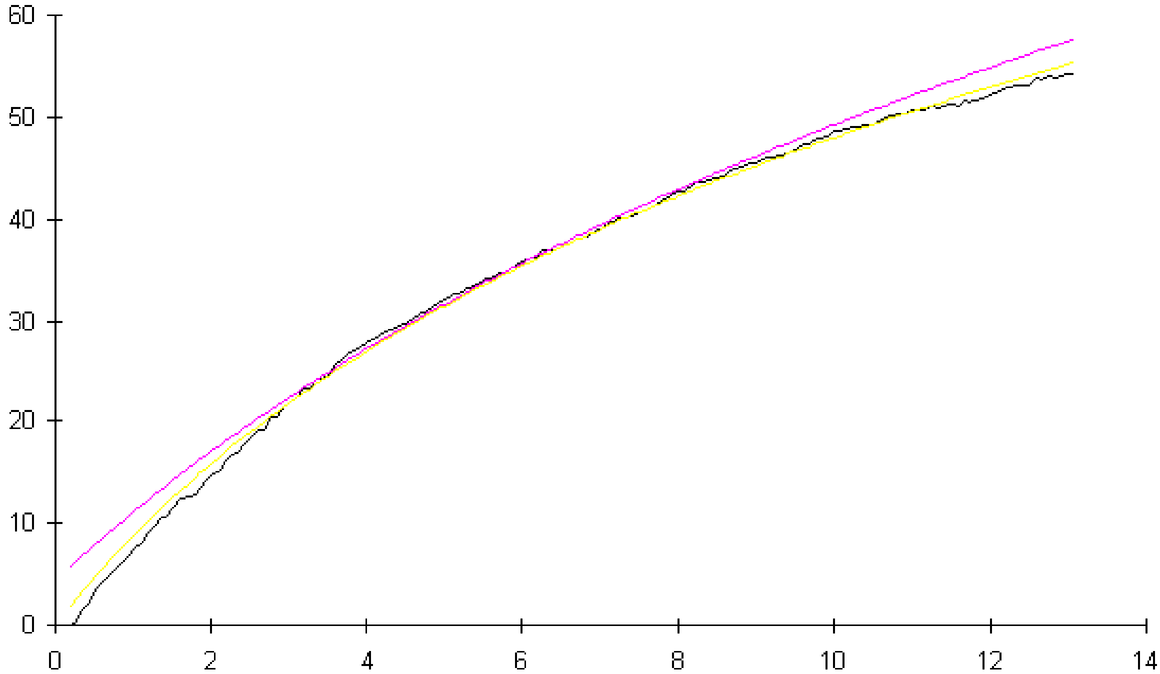

Figure 5 Improvement of the fittings with a_0=0
In the following tests only the offset free Mil Logarithm function was used due to the positive results with a_0=0.
After the first power on OCXOs often show a strong initial aging while the long term aging behaviour shows good values after initial phase. This behaviour seems to depend on the

oscillator type as it usually can be observed at all OCXOs of the same type to nearly the same extent.Curve fitting over the whole measurement time of one year shows that there is good correlation with the logarithm function even in case of strong initial aging. Due to the strong initial drift, the fitted curves show poor correlation when calculated after the first 20 days.
This effect occured at all tested OCXOs of this type. Thus it was supposed that the strong initial drift is not correlated to the long-term stability but other factors. As the long-term behaviour seems to be independent from the initial drift it was tried to give the measured aging data at the end of the measurement interval a higher weight on the result. A simple possiblity of realisation is the use of a weighted fitting procedure where the measurement values are evaluated with a variable significance. The significance of the measurement values has to increase the closer a measurement point is located to the end of the measuring interval. For weighting the measured values the solution of a differential equation of 1rst order was used. In practice this can be achieved relatively simply. The fault squares of the individual diffences are muliplied with the weighting equation before suming up the function. This gives the following expression whose value has to be minimized.

$$lsq = \sum_{t=0}^{t\max}\left[\left(\frac{\Delta f}{f_0}(t) - a_1\ln(a_2 t + 1)\right)^2 * \left(1 - e^{-bt}\right)\right]$$

(13)

Through the variation of the coefficient b the influence of the weighting function can be "tuned" continuously. At high values for b the weighting reaches the asymptotic boundary (=1) even at small time values. Thus the weighting fit has no influence anymore and algorithm works as before. Small values of b, however, are more interesting. Here each series of measurement can be weighted which set priorities at the end of the measurement interval.
The coefficient b, however, is different for each OCXO-type and has to be evaluated separately for each type during the evaluation process. This method provides better results as the simple fit of the measurement data (figure 6). The measured values, however, have to show monotonic behaviour as frequency jumps and other factors are also weighted.

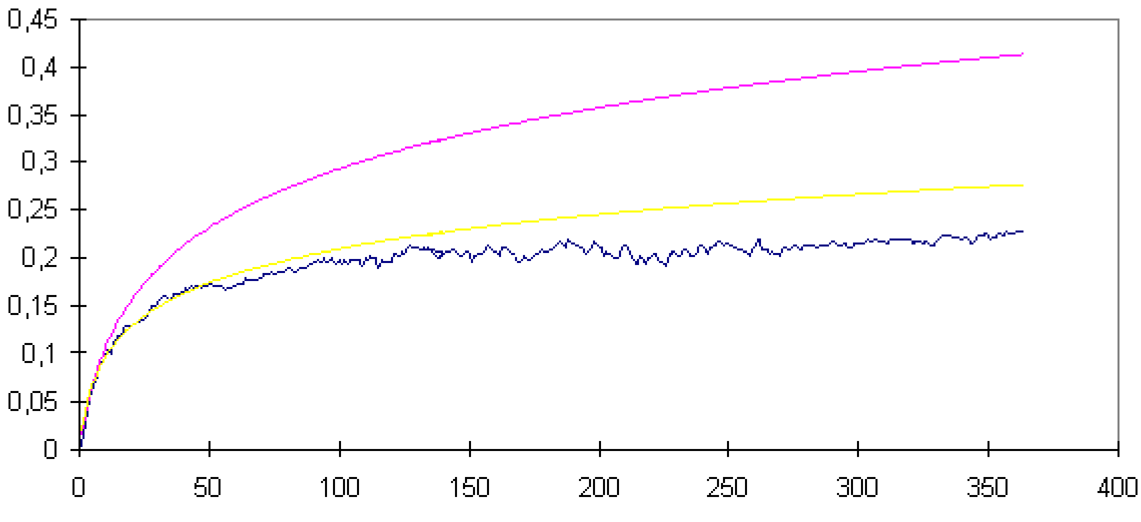

Figure 6 weighted least square fit
A lot of physical processes are balancing procedures which approach the state of equilibrium by an exponential function. It was tried to consider this compensation function in the ageing calculation. Figure 7 shows a measured curve (C) with a fitted modified logarithm function (B) over 365 days. Curve (A) is the conventional fit over 20 days. If the measurement data (C) are subtracted from the fitted curve (B) and the difference of both curves at the time t=0 is added as offset to each point of the result function, an error curve (G) is achieved. Through the calculated points of the error function a function (F) can be fitted which corresponds to the solution formula of a differential equation of first order.

(14) $df / f = k\left(1 - e^{-bt}\right)$

The coefficients k und b are again determined through least square algorithms. This equation now includes the influences of the oscillator and the initial quartz crystal drift. If each point of this function is subtracted from the originally measured series of data (C) a new measurement curve (E) is achieved which is reduced by the initial influences. If the modified logarithmic function according to theMIL standard with a_0=0 is now fitted through this modified series of data (curve C) the calculated curve (D) shows a far more better correlation to the real aging response.

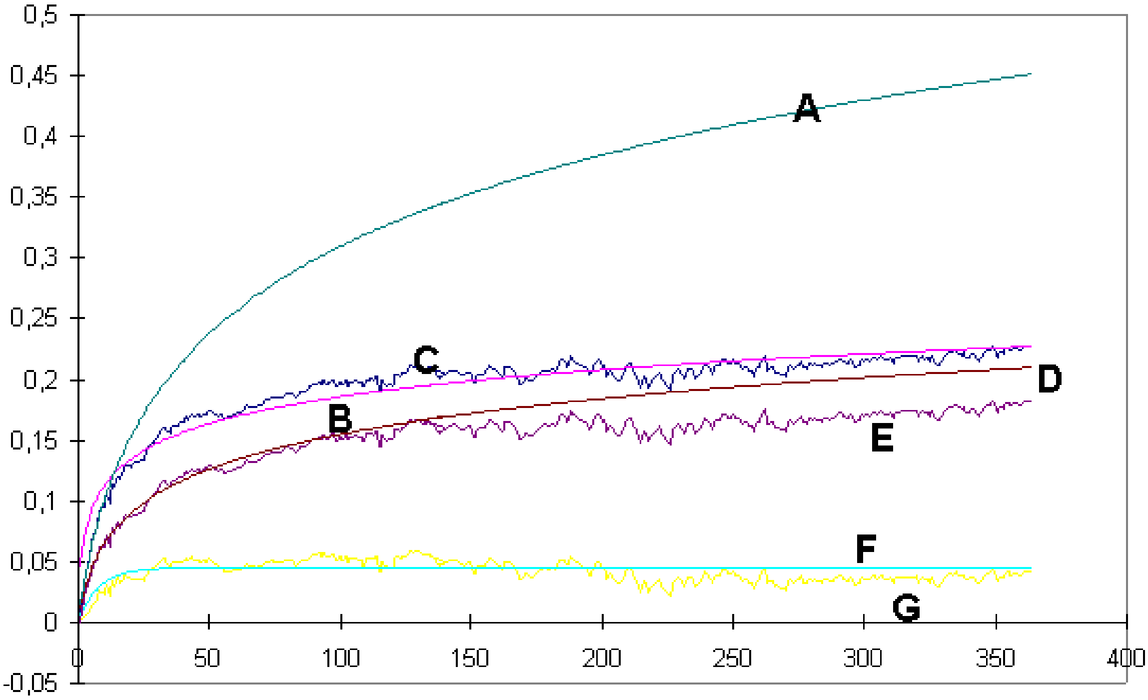

Figure 7 subtracted initial frequency drift

With all tested OCXOs of this type similar coefficients for k and b could be found. With the coefficients once calculated for the error function the prediction accuracy of all oscillators of one type can be improved (figure 8).

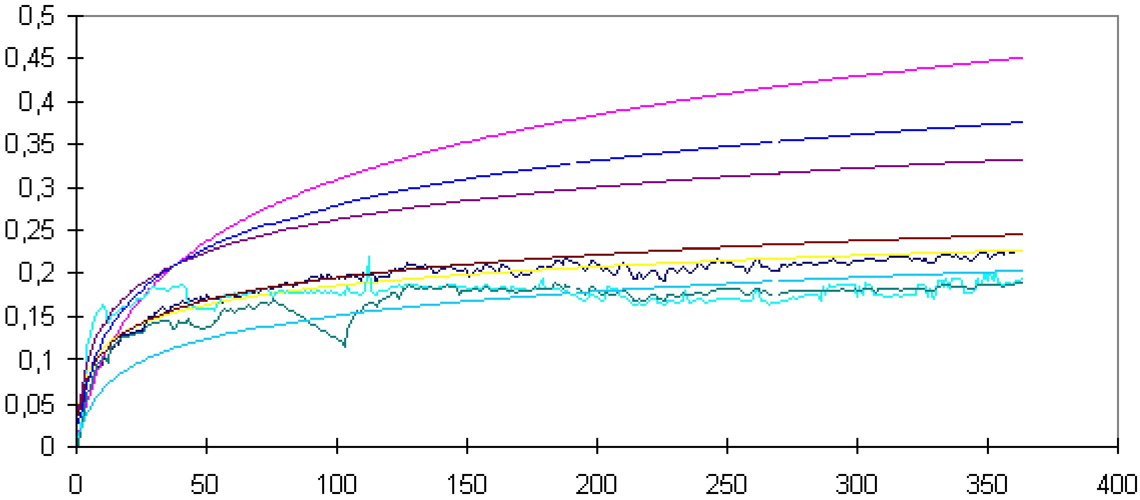

Figure 8 Improvement of fitting accuracy OCXO

Unfortunately this procedure is difficult to integrate in production as the ageing data of each OCXO type is required for the first year. All of the described methods allow an improvement

in prediction accuracy. In series production, however, they can only be used with great expenditure. Due to the enormous variety of TELE QUARZ products this method can be used only in exceptions for high production volumes.
The major disadvantage of the described methods is that they do not eliminate the causes but only includes them in the calculation. The long-term behaviour of OCXOs can be determined earlier but the initial aging drift for the OCXO with tight ageing specification must be well within the measured interval.
In the following figure (9) the same OCXOs are shown as above. The measurement values of the first 20 days were ignored and the measurement value of the 21rst day was taken as reference point for the relative frequency deviation. Thus a 20 days' preaging of the oscillators can be mathematically added before the actual ageing measurement. Figure (9) shows a considerably better correlation of the fits with reality. OCXOs with less initial drift even have a better prediction accuracy.

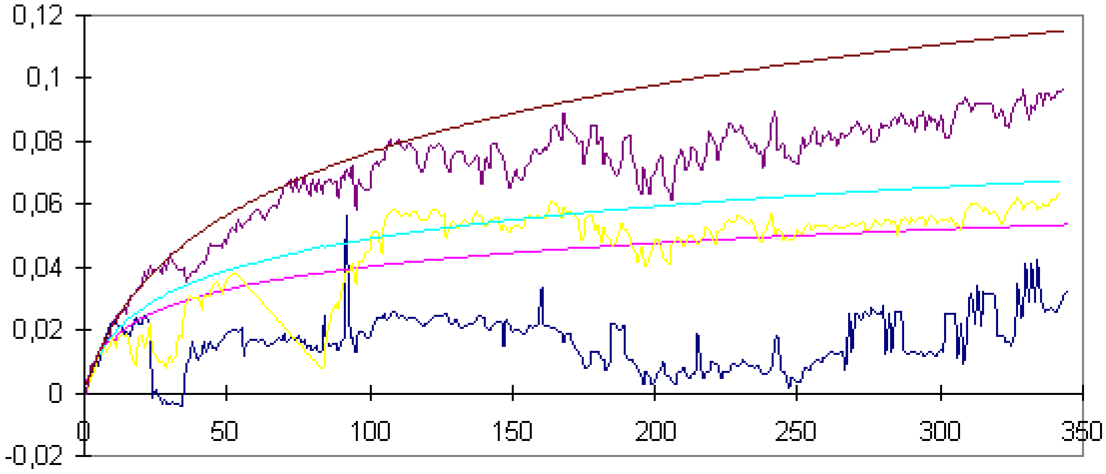

Figure 9 Improvement of fits obtained through preageing

6. Summary

It could be shown that least square algorithms being applied for ageing measuring data provide better results if the term a_0 of the logarithmic function is kept zero. The calculated curves correlate better with the aging observed in reality.
The prediction accuracy of oscillator ageing rates can be considerably increased by mathematic methods which consider the initial aging drift effects.
All mathematic procedures, however, cannot supply better results than simple preageing processes. The longer the oscillators are kept in operation, the better becomes the prediction accuracy.
Especially oscillators with SC cut crystals which show excellent ageing behaviour are significantly influenced by effects like operating voltage fluctuations, injection locking, initial drift, TK etc so that an accurate aging statement is not possible after only 20 days of operation. The aging behaviour of such highly stable OCXOs can only be determined after longer time periods of 30 days and more.

7. Literature

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