# A new approach to modeling short conductor wires in high-frequency circuits

## Introduction

Anyone who has ever developed a high-frequency circuit and simulated it before construction is usually quite helpless as soon as the question arises, which value of inductance would best represent the infuence of the short leads between the components in the equivalent circuit diagram.

These connections can be quite critical already in the 70 cm band. Or do you know, dear reader, the inductance of a connection wire of 4.5 mm diameter and 9 mm in length? Decide spontaneously on: a) 9 nH, b) 6 nH, c) 3 nH or d) I don't know. If you have chosen d) or want to quickly check [1], you should continue reading this evaluation.

This question has been the starting point of an investigation that took me a total of 5 years and the goal of which was only to disprove once and for all the common belief that answer a) is correct.

The final result is a lot more:

1. a new way of effectively and correctly computing and modeling short connections in RF circuits by transmission lines, where all types of lines are calculated using a single formula in which a single structural factor fully describes the configuration,

2. the use of line theory for the calculation of short connections and presentation of 9 models with which these connections can be described,

3. the consideration of asymmetric structures by adapted interpolation formulas and without having to model the structures individually by means of field computation programs,

4. The extension of the usual approximation formulas for the calculation of the characteristic impedance on structures with - regarding the ratio - thick inner conductor, where all previous formulas have failed, even in the most important application: the circular inner conductor in a square outer conductor, where as far as I know, not a single reliable calculation method exists, and finally

5. a universally applicable program for the calculation of all kinds of transmission lines, which runs without any adaptation on every operating system!

## 1. State of the Art

The most common finding in the literature is the answer a) 10 nH per cm of wire length. There is no distinction between short or long wires and the diameter of the wire apparently does not matter. The problem has been known and investigated for over a hundred years. In 1908, Edward B. Rosa [2] derived a first formula under strong simplifications, which Tim Healy [3] examined in more detail and found two variants:

$$L = \frac{\mu_0}{2\pi} * l * \left[ \ln\left(\frac{4l}{d}\right) - 1 \right] \quad (1) \qquad \text{and} \qquad L = \frac{\mu_0}{2\pi} * l * \left[ \ln\left(\frac{4l}{d}\right) - 0.75 \right] \quad (2)$$

Formula (2) applies to low frequencies where the internal inductance is still present. For higher frequencies, e.g. already from the long and medium wave range upwards, where the skin effect hits fully, formula (1) applies.

Unfortunately, both expressions are unsuitable for practical application, as David W. Knight, G3YNH, makes clear in a highly interesting article [4], because they are only valid if

- l >> d, so the length of the wire is much larger than the wire diameter and

- if the magnetic field around the wire is not affected by the current flowing back, that means: the wire is far away from other current-carrying or even conductive parts.

**Both** conditions are usually **not** fulfilled in typical high-frequency circuits, so that the reliabiliy of the results is often very bad. Knight states: The environment of the short conductor has to be included into the considerations. In other words, the round trip of the electrical current has to be taken into account in any calculation of whatever kind.

His proposal: assume the lead to be modeled as part of a more or less circular current loop for which Thompson [5] gives a formula. This formula unfortunately only applies if further conductive structures are far away and amplitude and phase of the loop current are identical in all parts of the path. Since this is almost never the case, only a new approach will help, where the 'round trip' of the current is always considered simultaneously for both directions.


## 1.1  Small excursion into the theory of transmission lines

This approach is fulfilled by transmission line theory, which describes the transport of energy in the form of an electromagnetic wave. The propagation in free space is common to every amateur radio operator. On the other hand the behavior of guiding the wave in or on a given conductive structure is rather unknown.

So, why not regard a short piece of conductor including its return path, e.g. as a short coaxial cable, as a two-wire line or as "one half" of a two-wire line (wire along a conductive wall)?  For these arrangements, there are formulas which calculate the respective characteristic impedance. If the analysis program allows such a model, you may even model the conductor piece directly as a cable. Or: starting with the characteristic impedance Z it is possible to extract the inductance per unit length and multiply this by the actual length of the wire. Yes, one can also determine the distributed capacitance to finally get the parallel capacitance C against ground. If necessary - both elements could be included in the equivalent circuit diagram. There are many solutions with improved accuracy using this approach.


## 1.2  Practical approach

First, two formulas are required, which will calculate the inductance and capacitance per unit length from the characteristic impedance Z of a line and eventually a reduction factor $k_v = 1/\sqrt{\varepsilon_r}$ due to dielectric material involved. Multiplying these values by the actual wire length will yield the values of the inductor and / or capacitor for the complete equivalent circuit diagram. The following relationships between the characteristic impedance Z and the propagation velocity v exist

$$Z = \sqrt{\frac{L'}{C'}} \quad (5) \qquad \text{and} \qquad v = \frac{1}{\sqrt{L' * C'}} = \frac{c}{\sqrt{\varepsilon_r}} \quad (6)$$

where c = 300000 km/s (speed of light, the exact value is 299792.458 km/s), the relative permittivity $\varepsilon_r$ of the insulator (in the case of air $\varepsilon_r$ =1) and the inductance and capacitance per unit length L' and C'. L' and C' are the values of L and C distributed over the cable length.

From (5) and (6) the desired relations are obtained by multiplication or division

$$L' = \frac{Z * \sqrt{\varepsilon_r}}{c} \quad (7) \qquad \text{and} \qquad C' = \frac{\sqrt{\varepsilon_r}}{Z * c} \quad (8)$$

and from this with **length l in millimeters** the size equations for L and C

$$L = L' * l = l * \frac{\sqrt{\varepsilon_r} * Z}{300} \quad \text{[nH]} \quad (7a) \qquad\qquad C = C' * l = l * \frac{\sqrt{\varepsilon_r}}{Z * 0{,}3} \quad \text{[pF]} \quad (8a)$$

**But attention**: In both equations (7a) and (8a), $l$ is always the actual geometric extension and not the electrically effective length.

The interested reader may also be happy to know about the following relationships

$$c^2 = \frac{1}{\mu_0 * \varepsilon_0} \qquad \text{and} \qquad Z_0 = 120\pi = c * \mu_0 = \frac{1}{c * \varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

If the shortening factor $k_v$ or $\varepsilon_r$ of a cable is unknown but the capacity in pF/m is known from the datasheet, calculations of C' should start with (8) and a cable having the same Z but $\varepsilon_r$ = 1 instead of the unknown value. Then, the ratio of this calculated C' in relation to the data sheet specification of C' gives the shortening factor $k_v$. And from this we get $\varepsilon_r = 1/k_v^2$ for the final calculations.

But how do we calculate the effective characteristic impedance of the conductor piece to be modeled?

# 2. Characteristic impedances of known arrangements

Radio amateurs know the formulas for calculating the characteristic impedance of a coaxial cable and a two-wire cable from their amateur radio education papers, as e.g. [6] in Germany. But what about other forms of wave guidance?

As a starting point, some structures and associated formulas can be found in Section 11 of the German Handbook of High-Frequency Engineering issued in 1956 [7].

Figure 1 shows a copy from the first edition of 1956. It contains the usual coaxial structure with round outer conductor, the square outer conductor, the round inner conductor with a single surface as outer conductor and the round conductor between two surfaces at equal distance and in a right angled structure. For the determination of Z, there are approximate equations for each structure contained in the text.

However, the given formulas apply only if the distance of the conductor from the nearest conductive wall is at least 1.5 times its diameter. If you look more closely, you may find out, that these approximation equations all look similar and ultimately differ only by a **constant factor** that is **characteristic** to the **respective structure**. So I have called it the **"structure factor" k**.
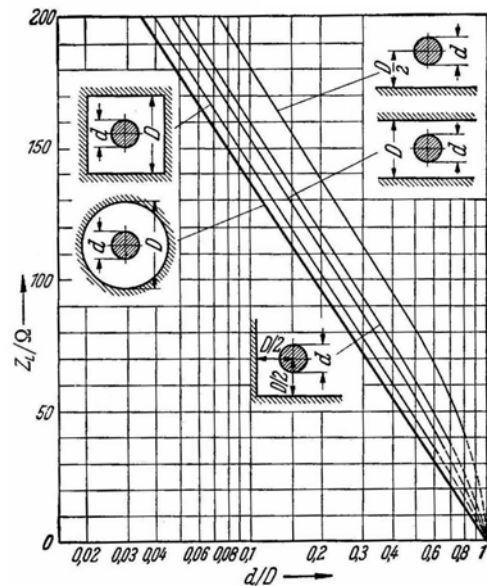
Figure 1: Graphs of the characteristic impedance of a round inner conductor in different outer conductor structures (reprint from the German "Handbook of High Frequency Technology" issued in 1956 [7])

Abb.11.1. Wellenwiderstand von Leitungen mit kreisförmigem Innenleiter und Außenleitern verschiedener Form.

Using this structure factor, I arrive at a uniform equation for the calculation of the characteristic impedance, where the structure factor k lies between 1 (coaxial arrangement) and 2 (single wire against a single flat surface, see top right in Fig. 1). All values of k in between these limits correspond to other structures. Thus the **general approximation equation** for all structures, valid within the range 2a/d>3, is

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln(k\frac{2a}{d}) \qquad (9)$$

Each structure is assigned exactly one value of the structure factor, which usually can be used for identification too.
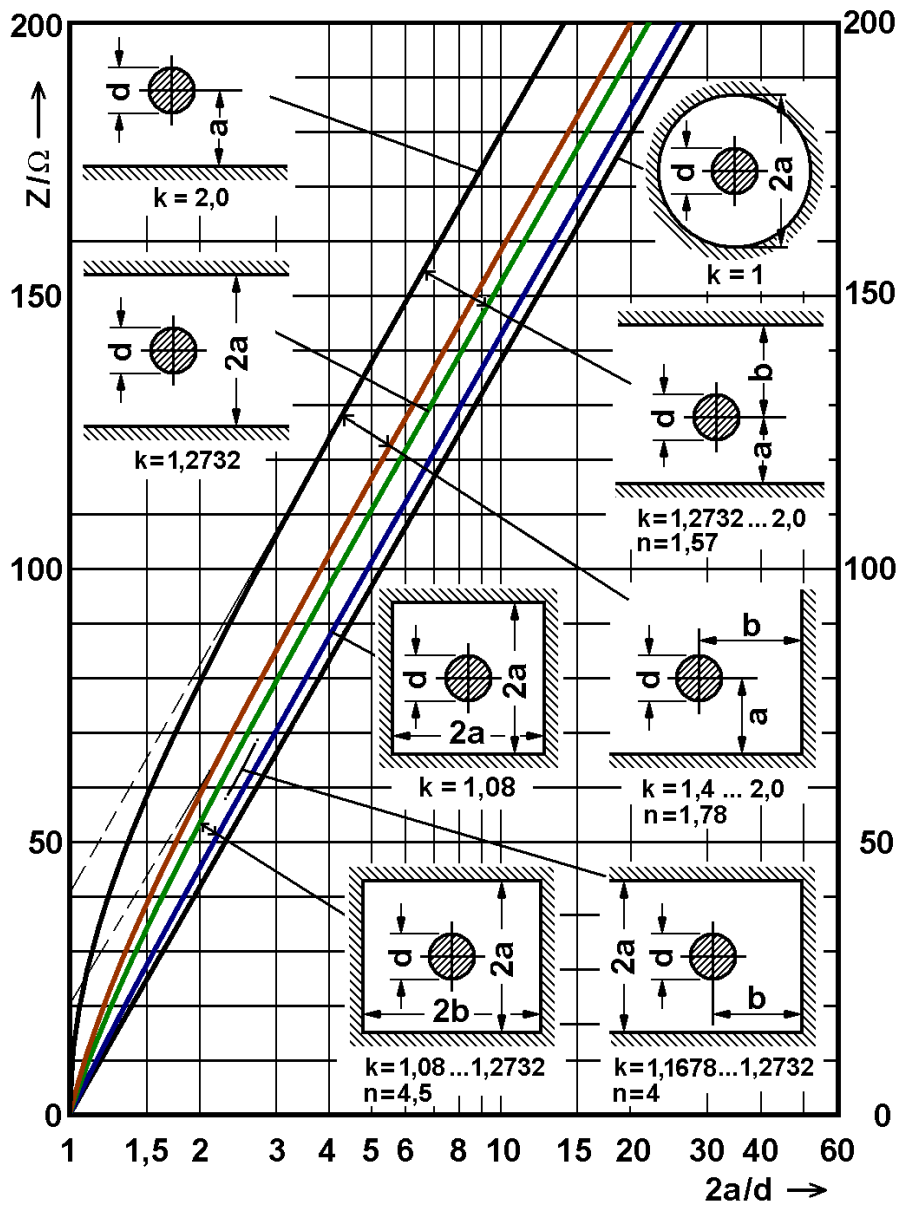
Starting from Figure 1, I investigated the relationship for other structures including unsymmetrical solutions. They are shown in Figure 2. I also changed the scaling: Instead of d/D now the reciprocal 2a/d is used, which does not really change the basic traces. The distance of the inner conductor from the nearest wall is now denoted by a instead of D/2. In the case of asymmetrical structures, the distance b to the more distant wall must also be known.

New to the template is that
- the angular structure has been extended to different distances to the two legs,
- the U-shaped structure (from [8]) was added,
- a rectangular outer conductor has been included and finally the
- calculation of a conductor between two parallel surfaces having different distances has been introduced.

As there are only very few data available for the determination of the characteristic structural factor k for all new models and especially those having different values of a and b, I have examined these relationships through a series of model calculations with the program Arbitrary Transmission Line Calculator atlc2 by Kenneth L. Nist, KQ6QV, [10].

The k values for the limiting cases a=b and b>>a are generally known or can be determined from a single simulation. They are indicated as an area in the small graphics in Figure 2. The more difficult problem is to map the very different behavior of the structure factor as a function of a/b in suitable formulas as accurately as possible.

The graph shows Z/Ω (vertical axis, 0 to 200) versus 2a/d (horizontal axis, logarithmic, 1 to 60) with various conductor arrangement diagrams:

- $k = 2,0$ (round conductor above ground plane)
- $k = 1$ (round conductor in circular outer conductor, coaxial)
- $k = 1,2732$ (round conductor between two ground planes)
- $k = 1,2732 \ldots 2,0$, $n = 1,57$ (round conductor between two planes, offset)
- $k = 1,08$ (round conductor in square outer conductor)
- $k = 1,4 \ldots 2,0$, $n = 1,78$ (round conductor in corner)
- $k = 1,08 \ldots 1,2732$, $n = 4,5$ (round conductor in rectangular outer conductor)
- $k = 1,1678 \ldots 1,2732$, $n = 4$ (round conductor in rectangular outer conductor, offset)

$$Z/\Omega \cong \frac{60}{\sqrt{\varepsilon_r}} \ln\left(k\frac{2a}{d}\right) \quad (\text{Approximation for } 2a/d > 3) \quad (1)$$

$$Z/\Omega = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(k\frac{a}{d} + \sqrt{\left(k\frac{a}{d}\right)^2 - k + 1}\right) \quad (2)$$

$$Z/\Omega = \frac{60}{\sqrt{\varepsilon_r}} \left(\ln\left(\frac{2a}{d}\right) + \frac{\ln(k)}{\ln(2)} \ln\left(1 + \sqrt{1 - \left(\frac{2a}{d}\right)^{-2}}\right)\right) \quad (3)$$

$$k = 1 + m * \left(1 - p\left(\frac{a}{b}\right)^n\right) / \left(1 + p\left(\frac{a}{b}\right)^n\right) \quad (4)$$

$$m = k_{max} - 1; \quad i = k_{min} - 1; \quad p = (m-i)/(m+i)$$

Figure 2: Characteristic impedance of a round conductor in various geometrical arrangements (according to [9]) with additions. The approximate equation (1) applies only to 2a/d>3, while the formulas (2) and (3) cover the entire range of 2a/d. The derivation of these two formulas will be discussed in the next section.

Hints: For the structure factor k = 1.27 in [7], the expression 4/π can be found in [13] and [17], yielding 1.27324. Occasionally, values of 1.078 [12] or 1.079 are found instead of k = 1.08 for the square outer conductor. Formula (4) calculates the value of the structure factor k as a function of the ratio a/b for all four structures in which the distance b occurs. The exponent n and the start and end values are different to each structure. Their values can be found in the small graphics.

To achieve this I have done a series of model calculations with atlc2 for each of the four structures and finally matched the results of k(a/b) with functions that seemed suitable for the problem.

Especially in the case of the rectangular outer conductor, I additionally determined the dependence of the structure factor as a function of a/b at three different ratios of diameter d to distance a. It was found that the fundamental behavior of the factor k(a/b) for b between a and ∞, ie 1>a/b>0, is similar in all three cases, but apparently also an influence of the inner conductor diameter itself is present. To take this into account, neither the amount of data nor the accuracy of atlc2 were sufficient. The selected function can thus not be exact and was chosen so that all dependencies are mapped equally well for thicker inner conductors on average. However, for the intended purpose of modeling short connecting wires, the relationship may be considered sufficiently accurate.

For the other three cases, I limited the calculations to a mean ratio of 2a/d = 2.23, because the function was best for the rectangle, and because this ratio is in a range where the approximate formula (9) is still producing reasonable results.

An interpolation formula for the a/b-dependent structure constant k, which satisfies all four structures containing the distance b, must have a defined slope in the initial region, where b is only slightly larger than a. The amount of this gradient depends on the structure. For large values of b/a (i.e. a/b approaches zero) k must tangentially approach the k-value of the respective boundary structure. A very suitable function for this is

$$k = 1 + m * \frac{1 - p * (a/b)^n}{1 + p * (a/b)^n} \quad (10) \quad \text{using}$$

$$m = k_{max} - 1 \; ; \; i = k_{min} - 1 \text{ and } p = (m - i)/(m + i)$$

$k_{min}$ and $k_{max}$ are the k values of the boundary structures for a=b and b>>a. These are, e.g. for the rectangular outer conductor i) k=1.08 for the square outer conductor and ii) k=4/π=1.27323 for the two parallel surfaces without lateral boundaries.

Thus, only the exponent n determines the initial slope including the further course of the function. The table lists for any of the four structures the start and end values of k, the auxiliary quantities, and the selected exponent n.

| Structure | $k_{min}$ | $k_{max}$ | i | m | p | n |
|---|---|---|---|---|---|---|
| Rectangular outer conductor | 1,08 | 1,2732 | 0,08 | 0,2732 | 0,54705 | 4,5 |
| U-shaped trough | 1,1678 | 1,2732 | 0,1678 | 0,2732 | 0,23920 | 4 |
| L-shaped 90-degree angle | 1,4 | 2 | 0,4 | 1 | 0,42857 | 1,78 |
| 2 planes at unequal distances | 1,2732 | 2 | 0,2732 | 1 | 0,57080 | 1,57 |

Figure 3 gives an impression on the accuracy of the interpolation function. The graph shows the calculated structure factors as a function of a/b for all 4 structures. The points superimposed on the curves are k values which were determined backwards from the characteristic impedances calculated by atlc2. In case of the U-shaped trough they come from an equation found in [13], where the maximum deviation is less than 0.3%, so I could do without the installation of this special function.

For simulations with atlc2, all models of structures which are open on one or two ends had to be terminated at the free ends with a conductive wall due to memory requirements and for reasons of computing time.
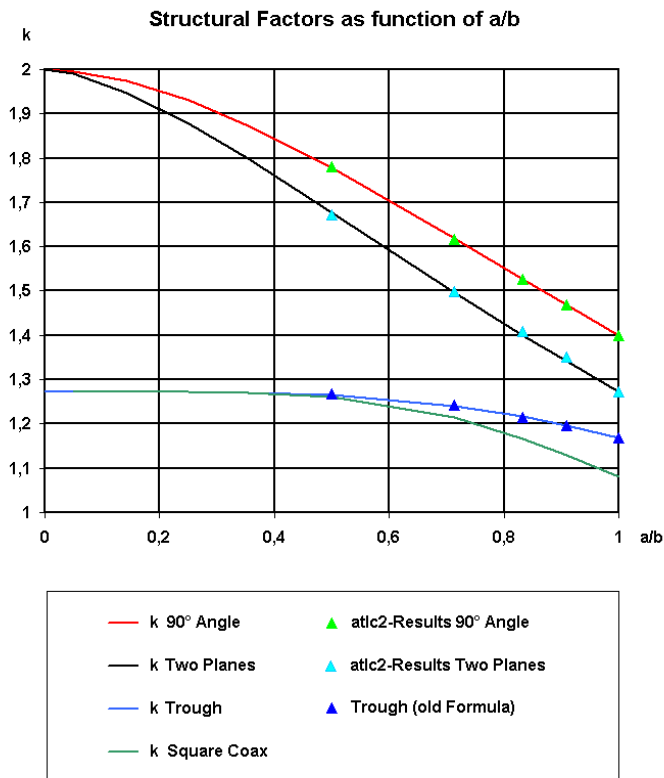
Fig. 3: Gradients of the four interpolation functions

However, these walls were almost at least 4 times as far away in each direction as the distance b. At these distances, I could no longer detect any "flattening" due to the proximity of a wall on the course of the outermost magnetic field lines calculated by atlc2, where the value of this trace was already very low at only 4% of the maximum field strength. Some check calculations with 5 times the distance confirmed that the error caused by the additional wall is negligible because the calculated impedance is only slightly higher in case of 5 times the "safety distance" compared to 4 times. This restriction can be accepted too because, on the one hand the error remains below 1% and, on the other hand there are hardly any applications where there are no influences at all resulting from other "disturbing" conductive parts.

# 3. Equations valid for thick inner conductors or small distances between inner and outer conductors.

All funktions shown in Figure 2 except the coaxial arrangement (k=1) don't follow a straight line below 50 ohms, as described by the approximation formula (9) for all cases, but bend below 2a/d=3 towards zero ohms. This zero-ohm point is achieved independently of k at 2a/d=1, which means that the inner conductor touches the outer conductor. Although this does not occur in practice, it is desirable to have a formula that reliably describes the range not covered by the approximate formula for all values of k, so that, for example, a connection close to a conducting wall can be calculated. And such a formula is long overdue.

When searching for an exact "formula for all cases", I started from two equations which are valid over the entire range of 2a/d. These are the coaxial line (k = 1), for which applies

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln(\frac{2a}{d}) \qquad (11)$$

and the round lead over a conductive surface (case k=2) as indicated in [7]. It differs significantly from the formula of the coaxial conductor and reads:

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * ar\cosh\left(\frac{2a}{d}\right) \qquad (12)$$

In order to check whether this formula matches the approximation formula (9) for 2a/d>3, it is first changed using the equivalence from [11]

$$ar\cosh(x) = \ln(x + \sqrt{x^2 - 1}) \qquad (13)$$

resulting in

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left[\frac{2a}{d} + \sqrt{\left(\frac{2a}{d}\right)^2 - 1}\right] \qquad (14)$$

For large values of 2a/d, the addend -1 below the root can be neglected, so we get the approximation equation as

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln(2\frac{2a}{d})$$

or, using the form factor k = 2, it becomes the known approximation formula (9)

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln(k\frac{2a}{d})$$

What could be more obvious than to try to introduce the structural factor k already in the exact formula (14) ? I started by using the abbreviation p instead of the number 1 under the root. This results in

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(k * \frac{a}{d} + \sqrt{\left(k * \frac{a}{d}\right)^2 - p}\right) \qquad (15)$$

If we now set k=1 in this formula, which is initially valid only for k = 2, we should obtain exactly the formula for the coaxial line, however now p=0 must apply. This means that Eq. (15) would hold for both k = 1 and k = 2 if p is made dependent on k. The simplest relationship that satisfies this requirement is: p=k-1. This yields

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(k * \frac{a}{d} + \sqrt{\left(k * \frac{a}{d}\right)^2 - k + 1}\right) \qquad (16)$$

Should this formula not only apply to k=1 and k=2, but for all other k too, two additional conditions must be fulfilled:

- for large values of 2a/d the equation has to appoach the approximation formula (9) independently of the value of k and
- for 2a/d=1, Z must always be equal to zero regardless of k.

The first condition is satisfied, as can easily be determined, since the term -k+1 is always negligible for larger values of 2a/d. The fact that the second condition is fulfilled can also be shown after inserting 2a/d=1, if one resolves the binomial formula below the root by using the correct sign:

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(\frac{k}{2} + \sqrt{\left(\frac{k}{2}\right)^2 - k + 1}\right) = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(\frac{k}{2} + \sqrt{\left(1 - \frac{k}{2}\right)^2}\right) = \frac{60}{\sqrt{\varepsilon_r}} * \ln(1) = 0$$

So Eq. (16) is a formula that includes all geometries and all distances. To distinguish it from another formula, which will be explained below, I will call this method "k-Interpolation" in the following chapters.


## 3.1  Accuracy of the k-Interpolation and alternative solution

The comparison of the curves calculated in Figure 2 using formula (16) with the dashed parts of the curves of the original drawing (Figure 1) shows no discernible deviation, which at least suggests the validity of the formula determined.

More reliable would be a mathematical comparison, which could be done against two different formulas for the round inner conductor in a square outer conductor (k=1.08). They can be found in two editions of the same Handbook issued 1946 [12] and 1956 [13] respectively.

One may come up with the idea that the formula of 1946, here rewritten using the nomenclature of this article and with k=1.078

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(\frac{2a}{d} * \left(1.078 - 0.078 * \left(\frac{2a}{d}\right)^{-2}\right)\right) \qquad (17)$$

is nothing else but a mathematical transformation of (16), where the root has been replaced by the first two members of the series development of a root [14]. This would be:

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \ln\left(\frac{2a}{d} * \left(k - \frac{k-1}{k} * \left(\frac{2a}{d}\right)^{-2}\right)\right) \qquad (18)$$

Both equations differ only insofar as k is set to 1 in the denominator of the correction term in (18), and the influence of this term in (17) is therefore 8% higher. Fortunately, this results in the desired value Z = 0 for 2a/d=1, which would not be the case with the aborted approximation (18). As can easily be verified, (17) also approaches into the approximation (9) for large values of 2a/d. The differences between the characteristic impedances obtained with (16) and (17) are astonishingly small and are below 0.1 ohms over the whole range of validity. So I could assume that my "k-Interpolation" formula would calculate reliable values.

However, in the fourth edition of the Handbook issued 1956 [13], the 1946 equation was replaced by a markedly complex formula (19), the structure of which deserves a closer look. It reads as follows

$$Z = \frac{60 * \ln(\rho) + 6.48 - 2.34A - 0.48B - 0.12C}{\sqrt{\varepsilon_r}} \qquad (19)$$

where $\quad \rho = 2a/d$

$$A = \left(1 + 0.405\rho^{-4}\right)/\left(1 - 0.405\rho^{-4}\right)$$

$$B = \left(1 + 0.163\rho^{-8}\right)/\left(1 - 0.163\rho^{-8}\right)$$

$$C = \left(1 + 0.067\rho^{-12}\right)/\left(1 - 0.067\rho^{-12}\right)$$

The formula makes use of the property that, for large values of 2a/d, the characteristic impedances or k>1 can be represented by the characteristic impedance of the round coaxial structure (k=1) plus a parallel displacement of 60*ln(k). In Figure 2 this can be seen because all curves above 2a/d=3 are strictly parallel to each other.

The other terms of the formula evidently arose from curve fitting. They consist of a parallel shift of 6.48 ohms reduced by three expressions with higher powers of ρ=2a/d which finally cause Z to approach zero for small values of ρ.

For large values of 2a/d, the results of (19) correspond to the well known approximation formula (9), however only if k=1.061. And that's very unusual since all computed values will be too low by around 1 ohm, even compared to the generally accepted approximation formula. With smaller 2a/d values, the calculated characteristic impedances are higher compared to the "k-Interpolation" (16) using k=1.061. But they deviate significantly compared to the results from (16) using k=1.08.

I wonder why the formula of 1946 [12], which agrees so well with my "k-Interpolation", has been replaced by something that, at least as far as the factor k is concerned, does not seem reasonable.

I expected clarity from the tabular representation of the impedance of the Square Coax over the ratio 2a/d given by Kevin Schmidt W9CF [15]. He calculates the characteristic impedance from the electric field by integrating the Laplace equation. His results, however, are considerably higher than those of all other solutions. Yes, they nearly coincide with the solutions of the - in this area no longer valid - approximation formula with k=1.08, whose values are only 0.1 ohm higher. So I could not be satisfied.

And so I tried to get closer to the truth with the already mentioned program atlc2. However, it turned out that the program did not calculate the round coaxial structure more precisely than 0.2 ohms and that the solutions determined with the square outer conductor did not form a uniform curve.

## 3.2  Results of practical implementations and accompanying simulations

Coincidentally during this time my DARC local group C01 looked for two-way couplers from 2 x 50 Ω to 50 Ω, for the pupose of connecing two identical antennas as a group to the transmitter. For his first solution DL9MEU used a construction proposal from DK7ZB [16], which determines the characteristic impedance by means of the approximation equation (9). Although the result was satisfactory for practical use, the output impedance deviated from the expected 50 ohms to higher values when doing exact measurements in the lab. A second solution according to Bramham [18] with two short transmission lines of about 50 Ω and about 25 Ω, which were calculated with my k-Interpolation, worked just as fine. In this solution however, the impedance at the 50 ohm output to the TRX was – compared to the first implementation – slightly below 50 ohms. These results indicated to me, that the proper impedances have to be in the range between these two formulas.

The backward simulation of the built-up circuits on their measured results gave me the respective electrically effective impedances and lengths. While the latter were expected to fit, the effective characteristic impedances were indeed between the predictions of the approximation formula (9) and the results of the k-Interpolation

(16), which encouraged me to look for a more fitting formula, which I finally called **Z-Interpolation**.

## 3.3 The alternative calculation equation: Z-Interpolation

Encouraged by the idea of parallel displacement in formula (19), I came up with the idea of not interpolating the k values, but of interpolating only between the characteristic impedances of the round coaxial conductor (11) and that of the single conductor with respect to a single surface (12). Both formulas are exact over the whole range of 2a/d, and therefor it should be possible to calculate the Z-values of all other curves in a manner that the relative distance to the two reference curves as computed by the approximation equation (9) for 2a/d>>3 will have the same ratio for any value of 2a/d.

This computes the interpolated Z to

$$Z(k) = \frac{60}{\sqrt{\varepsilon_r}} * \ln(2a/d) + \left[ \frac{60}{\sqrt{\varepsilon_r}} * ar\cosh(2a/d) - \frac{60}{\sqrt{\varepsilon_r}} * \ln(2a/d) \right] * \frac{\ln(k)}{\ln(2)}$$

The first addend is the characteristic impedance of the round structure and the square bracket contains the difference between the two starting formulas (12) – (11) as a function of 2a/d. The last factor calculates the relative proportion for the sought k-value between 1 and 2.

Using only the natural logarithm simplifies this relationship to the formula of the **Z-Interpolation** (20).

$$Z = \frac{60}{\sqrt{\varepsilon_r}} * \left[ \ln(2a/d) + \frac{\ln(k)}{\ln(2)} * \ln\left( 1 + \sqrt{1 - (2a/d)^{-2}} \right) \right] \qquad (20)$$

## 3.4 Comparison of all solutions for the special case of the square outer conductor

The different solutions as well as the measured values for the square outer conductor determined from the practical realizations are summarized in Figure 4. To better discern the differences, all results were reduced by the amount of the round coax. Therfore the displayed value is, in any case, only the part by which the calculated or measured value is higher than the Z of the round coax for the same value of 2a/d.

Even at first glance it can be seen that the three measured values (light blue underlined crosses) and the curve of the Z-Interpolation (20) (yellow triangles) actually harmonize well with each other. In the abscissa area above 2a/d=3, all curves are asymptotically appropaching to the results of the approximation formula (9) (blue curve with white circles in Fig. 4).

To finalize the comparison, let's look at the other solutions. Based on the three measured values (blue shaded crosses in Figure 4), it is noticeable that the traces of the 3 formulas where k=1.061 is used (dashed curves in the lower area of Figure 4) are far too low and thus can be disregarded completely. The measured values lie with slight scatter between the solid curves of the k interpolation (16) (green curve) and the table values of W9CF [15] (red curve). The Z-Interpolation (20) (black curve with yellow triangles) harmonises best with the measured values.
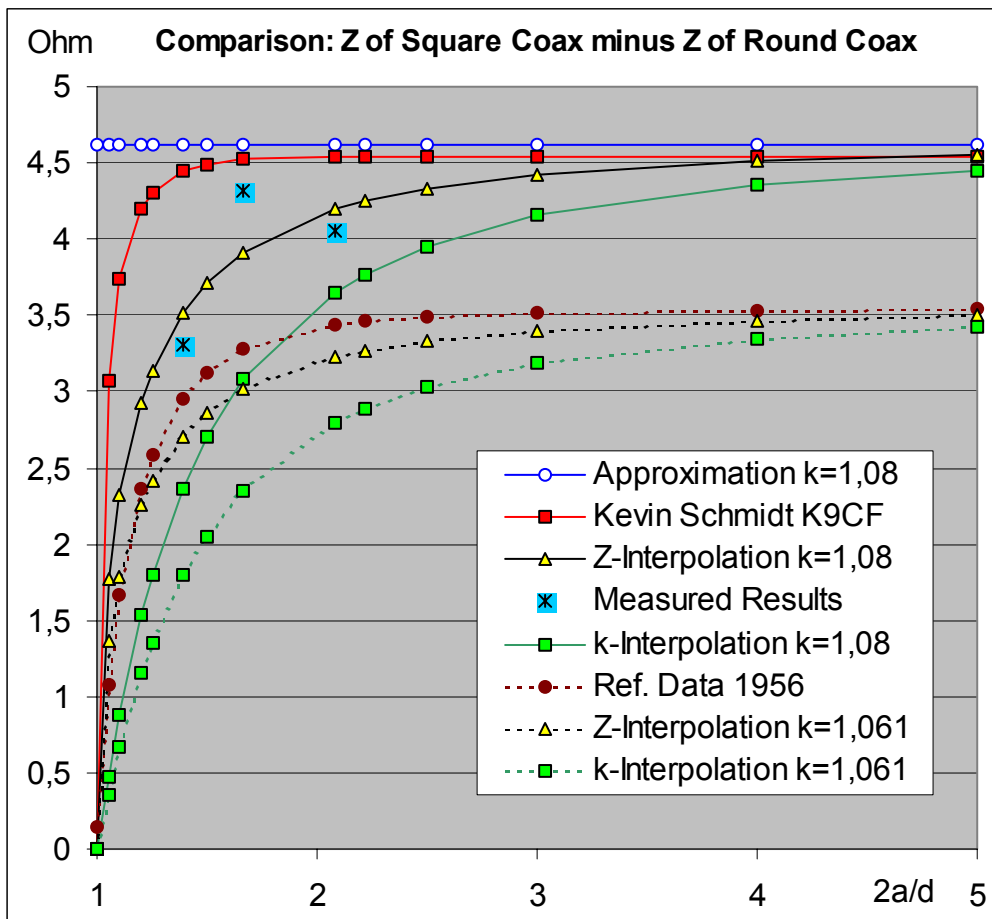
Figure 4: Comparison of different approaches and measured values for the round center conductor in a square outer conductor. Shown is only the difference of the characteristic impedance with respect to the Z of the round coaxial structure where the same ratio of the dimensions applies.

These results make the Z-Interpolation formula to my preferred solution - at least for the coaxial waveguide with square outer conductor. Although its superiority is proven only for the square outer conductor, it is basically used as a pre-selection in the computer program described below.

# 4. The universal program "Transmission Line Calculator"

All well-known computer programs such as e.g. [8], [9], and [17] consider either only individual configurations or are running - except for the two-wire line or the usual round coaxial structure - only the approximate formula. Therefore, I built all my findings into the program **Transmission Line Calculator tlc.htm** with the idea in mind, to simplify calculations significantly.

As a programming language for this universal computer program, I chose JavaScript with a user interface in HTML. Thus, the program will run on any JavaScript-enabled browser, regardless of the operating system and on any computer - whether PC, laptop, tablet or smartphone. The program can thus be used at any time and anywhere without limitation.

Additionally, the program including explanation and help can be downloaded from the German and American homepage of the author [19] [20] and from the ((download page of the FUNKAMATEUR)) [21] and may be tried online already while reading this article. Figure 5 shows a screenshot.
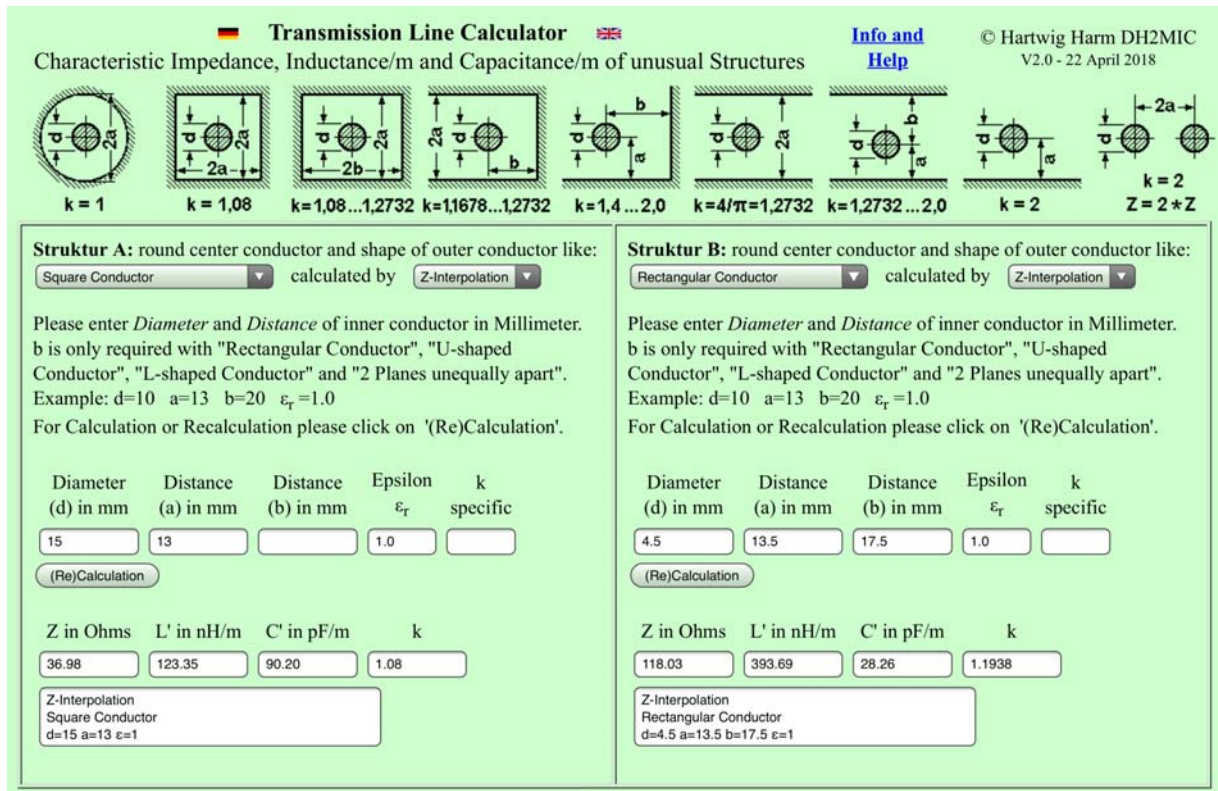
Figure 5: Screenshot of the program tlc.htm. Both worksheets show tasks from practical implementations, which occur in the examples at the end. On the left the calculation of a coaxial transformation line with square outer conductor for λ/4 transformation from 25 to 50 ohms and on the right the determination of characteristic impedance, inductance and capacitance of a 4.5 mm thick conductor in a rectangular outer conductor structure.

The header contains the 9 possible structures with all necessary dimensions. The spreadsheets are available twice to facilitate comparisons between different solutions or to perform two calculations in parallel. Each drop-down menu contains the 9 structures in the same order as in the header. The calculation is performed by default with the Z-Interpolation (Formula (3) in Figure 2). In addition, the approximate equation and the k-Interpolation (Formulas (1) and (2) in Figure 2) can be selected. The approximate equation can be used to verify results originating from other sources. The k-Interpolation (Formula (2) in Figure 2) yields slightly lower impedance values than the Z-Interpolation. However, in the quadratic outer conductor structure, the values of k-Interpolation are almost exactly the same as with a formula from the second edition of the Reference Data for Radio Engineers of 1946 [12], which led me to make that formula available in the program as well.

After entering the dimensions you may hit the "(Re-)Calculation" button to get the results. However, before doing so, there is the possibility to modify the relative dielectric constant, which is preset to 1 or to specify a different k value for the structure factor k in the field "k-specific". This value is deleted in the input field after each calculation but appears in the results window besides the other data. This feature allows to perform computations with structure factors that originate from other sources or considerations, and without having the program to know the appropriate structure.

The results row contains the characteristic impedance Z in ohms as well as the distributed inductance and capacitance as L' in nH/m and C' in pF/m. The actual value of k used for the calculation is displayed too. For most structures, this value is fixed, but in the case of the rectangular outer conductor, the U-shaped and the L-shaped outer conductor as well as the inner conductor between two planes at unequal distance it is calculated from the relationship shown as formula (4) in Figure 2 using the k-values of the two boundary structures and the ratio a/b. The exponent n in formula (4) of Figure 2 has a characteristic value for each of the four structures in which the ratio a/b plays a role. Its magnitude has been derived from model calculations with atlc2 [10]. The respective numerical values can be seen in Figure 2.

Finally, the comment field contains a copy of the the input data. This avoids irritations that may occur if new input data have already been typed in, but the operator has not yet activated the (Re-)Calculation button.

If you are more familiar with - or even rely on - the formulas from [12] or [13] for the square outer conductor, you also try these options. The program "speaks" German and English.

# 5. Examples

This brings me back to the initial question: What is the inductance of a wire 4.5 mm thick and 9 mm long? In the right hand part of Figure 5, the result is calculated for the wire in a rectangular outer structure at distances of 13.5 and 17.5 mm, respectively. It is 0.4 nH/mm and thus multiplied by the length of 9 mm results in 3.6 nH. Just try different distances and structures. It is very instructive, because 1 nH/mm or 1000 nH/m as required for solution a), can only be achieved if the conductor is at least 167 mm away from a single conductive wall. But that is no longer a feeder but almost an antenna. Incidentally, this small example is also included in the following filter.

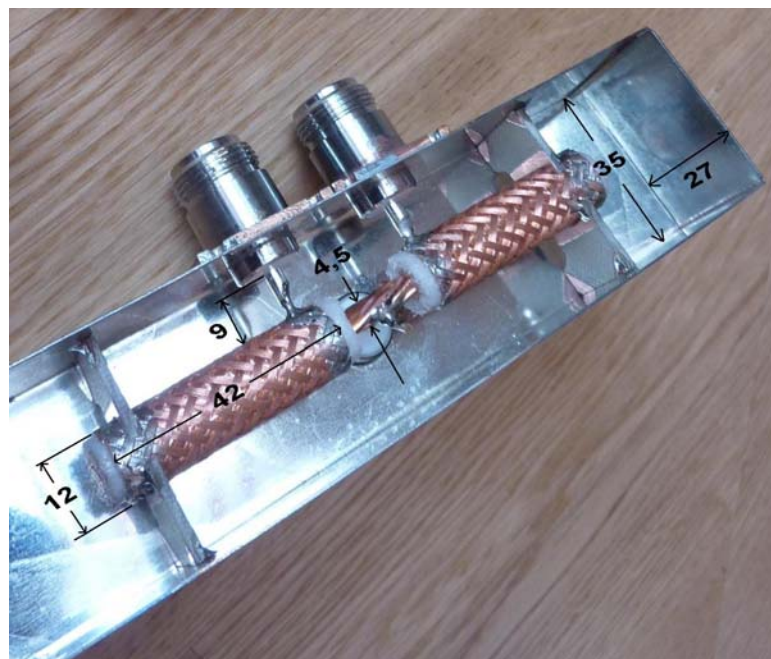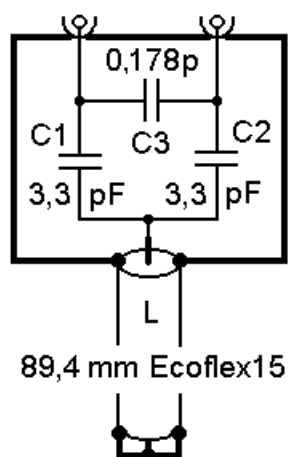**Low pass filter for 2m with deep notch at 432 MHz**



Figure 6: Circuit diagram (top) and first realization (right) of a 2 m low pass filter with deep notch at 432 MHz

A 2-m low-pass filter with deep notch in the 70 cm band is shown as a theoretical circuit diagram and in a practical realization in Figure 6. As can be seen in the photo, the following elements must be considered as "parasitic components" in this realization:

- the connections, 9 mm long, from the center conductors of the N-connectors to the outer pads of capacitors C1 and C2, which were implemented here by a short peace of Ecoflex 15 cable, 42 mm in length,

- the capacities of the outer conductor of C1 and C2 against the housing walls,

- the frequency-dependent input impedance of the small open-ended cable of C1 and C2, instead of a fixed capacitance,

- the connection lines from C1 and C2 to the connection node to which also a short-circuited cable is connected, which acts as an inductance,

- the short connection between the node and the shorted cable,

- and finally the frequency-dependent impedance of the shorted cable itself.

In the complete equivalent circuit diagram (Figure 7), all parts are included and can therefore be examined more closely on their individual influence by simulation in order to improve the design, if necessary.

In order to determine the values of the parasitic components or their equivalent circuits, one selects from the possible structures that one, which best matches the practical implementation. The program then determines the characteristic impedance Z as well as the inductance and capacitance per unit length. The latter only have to be multiplied by the correct length to be able to enter the component values in Table 1. Whether later only the L, only the C, or the combination of both have to appear in the equivalent circuit, depends on the individual case.

Let's start with the connection from the input or output socket to the outer conductor of the tube capacitors C1 and C2, which are made of 42 mm ECOFLEX 15. They are 9 mm long. The inner conductor of the socket has a diameter of 4 mm and is considered to be constant over its entire length. Its distance to the bottom and to the lid is equal and measures a = 13.5 mm in each case. Since the two ends of the housing are far away, the model "two planes equidistant" with k = 1.2732 and $\varepsilon_r$ = 1 is used. We obtain L = 3.87 nH and C=0.23 pF (see Table 1, first entry).

Next comes the tube capacitor made of ECOFLEX 15. If the analysis program can consider an overall "high" lying cable which is open at the end, we would have finished with Z=50, $\varepsilon_r$ = 1.352, $k_v$ = 1/$\sqrt{\varepsilon_r}$ = 0.86 and l = 42 mm. For RFsim99 [15] we need the equivalent circuit. To do this we do some iteration using the program. We start with the coaxial structure and first set d = 4,5 mm and $\varepsilon_r$ =1,352. Then we play around with the value of a until Z equals exactly 50.00 ohms. This is the case with a = 5.929 mm.

From L' = 193.8 nH/m and C' = 77.6 pF/m, we get a C of 3.26 pF and a series L of 8.14 nH for the 42 mm cable (Table 1, Row 2). Since 42 mm in the 70 cm band can no longer be considered as short, a π-structure of two capacitors of 1.63 pF each and an L of 8.14 nH in between will be used instead.

For the C of the outer conductor of C1 an C2 against the housing, the rectangular cross-section is used. The diameter is d = 12 mm and the distances are a = 13.5 mm (distance to the bottom or lid) and b = 17.5 mm (distance to the side walls), respectively. The result is k = 1.1938, C' = 57.0 pF/m and C = 2.39 pF. In the equivalent circuit diagram, this C appears to be increased by the calculated capacity

of the connecting wire of 0.23 pF and the additional capacitance by the supports of printed circuit board material, which contributes about 0.1 pF. This results in a total of 2.72 pF. The series L can be neglected in this case.

In the center of the circuit, the three lines to the central connection point are still to be considered. The connections of the input and output capacitors have a diameter of 4.5 mm according to the data sheet of ECOFLEX 15 and are effectively 9 mm long (without the central connection node). The distance to the lid and bottom is again 13.5 mm and the side walls are 17.5 mm away, so again the rectangular model with k = 1.1938 is used. We get L' = 393.7 nH/m and C' = 28.3 pF/m and multiplied by 9 mm results in: L = 3.54 nH and C = 0.25 pF.

The stub's inner conductor is also 4.5 mm thick, 9 mm long and 17.5 mm apart from the sidewalls. However, the end faces of the case are far away and therefore the model with k = 1.2732 is used. Result: 4.12 nH and 0.22 pF.

Whether these three short connections are only modeled as an inductance neglecting the capacity or as a short line depends on the individual case. Here it turned out that the three tiny capacities together have a significant influence. After all, the "missing" 0.72 pF would shift the 433 MHz resonance frequency by around 20 MHz and thus would predict the stub length by 7 mm too long!

| d/mm | a/mm | b/mm | k | l/mm | $\varepsilon_r$ | Z | L'/nH/m | C'/pF/m | L/nH | C/pF |
|------|------|------|--------|------|-------|-------|---------|---------|-------|------|
| 4 | 13,5 | - | 1,2732 | 9 | 1 | 129,0 | 430 | 25,9 | 3,87 | 0,23 |
| 4,5 | 5,925 | - | 1 | 42 | 1,352 | 50 | 193,8 | 77,5 | 8,14 | 3,26 |
| 12 | 13,5 | 17,5 | 1,1938 | 42 | 1 | 58,5 | 195 | 57,0 | (8,1) | 2,39 |
| 4,5 | 13,5 | 17,5 | 1,1938 | 9 | 1 | 118,0 | 393,7 | 28,3 | 3,54 | 0,25 |
| 4,5 | 17,5 | - | 1,2732 | 9 | 1 | 137,5 | 459 | 24,3 | 4,12 | 0,22 |

Table 1: Compilation of the values for the equivalent circuit diagram



Figure 7: Equivalent circuit diagram of the small circuit in Figure 6.
The values of the bridging C and the length of the stub have been
obtained by optimization to the desired parameters.

The circuit to be simulated on the basis of these data is shown in Figure 7. The length of the shorted cable found by optimization to the target data corresponds exactly with practice.

The deviation of the length of the stub from the ideal circuit is caused mainly by the parallel-C of 0.72pF and the series L of 4.12 nH, since both are electrically elongating. Assuming the commonly used 1 nH/mm for all five connections, the calculated length would have been 52.7 mm - and thus 20 mm too short (!). And as previously stated, that's far away from reality!

It can thus be seen that the transmission line theory allows for simple, clear and reliable modeling of the unavoidable connecting wires. And that their influnce is often not restricted to the inductors, but also the stray capacities can play a major role. An advantage which only a simulation using the transmission line theory can offer.

And as the most important "by-product" of the investigation, there is finally a **reliable formula** for the **round inner conductor** in a **square outer conductor**, which applies not only to thin inner conductors but any relationship of its diameter to the distance to the outer conductor. I hope that my formula soon finds its way into amateur radio and literature.


Hartwig Harm, DH2MIC, http://dh2mic.darc.de/tlc/ and http://www.qsl.net/dh2mic/tlc/

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