Beyond the horizon and further
The theory of HF propagation

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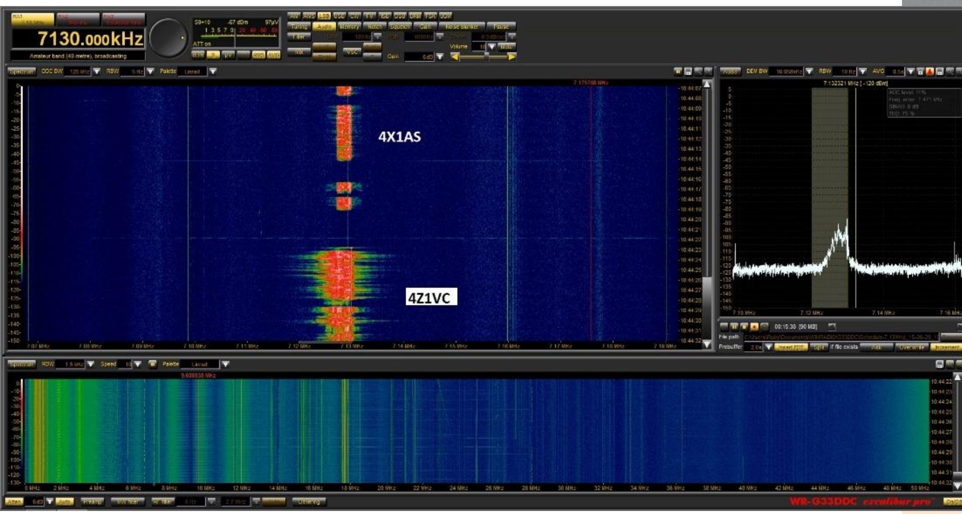
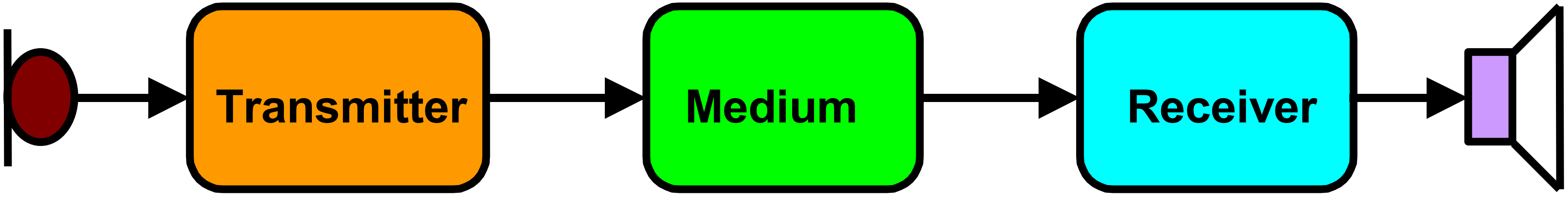
ט"ז סיון תשפ"ה (12.06.2025)

My homebrew solid-state transceiver



My homebrew hybrid transmitter

Communication channel



The electromagnetic spectrum

BAND		IEEE	FREQUENCY	WAVELENGTH
Extremely Low Frequency	ELF		3 – 30Hz	
Super Low Frequency	SLF		30 – 300Hz	
Ultra Low Frequency	ULF		300 - 3,000 Hz	1,000 - 100 Km
Very Low Frequency	VLF		3 - 30 KHz	100 – 10 Km
Low Frequency	LF		30 - 300 KHz	10 - 1 Km
Medium Frequency	MF		300 - 3,000 KHz	1 - 0.1 Km
High Frequency	HF		3 - 30 MHz	100 - 10 m
Very High Frequency	VHF		30 - 300 MHz	10 – 1 m
Ultra High Frequency	UHF		300 - 3,000 MHz	1 - 0.1 m
		L	1 - 2 GHz	
		S	2 - 4 GHz	
Super High Frequency	SHF		3 - 30 GHz	10 - 1 cm
		C	4 - 8 GHz	
		X	8 - 12 GHz	
		Ku	12 - 18 GHz	
		K	18 - 26.5 GHz	
		Ka	26.5 - 40 GHz	
Extremely High Frequency	EHF		30 - 300 GHz	1 - 0.1 cm
		V	40 - 75 GHz	
		W	75 - 110 GHz	
Sub-millimeter (TeraHertz)	FIR		300 - 3,000 GHz	1 – 0.1 mm
Mid infra-red	MIR		3 – 30 THz	100 – 10 μm
Near infra-red	NIR		30 – 300 THz	10 – 1 μm

Industrial, Scientific, Medical (ISM) frequencies

- 6.765-6.795 MHz (centre frequency 6.780 MHz)
- 13.553-13.567 MHz (centre frequency 13.560 MHz)
- 26.957-27.283 MHz (centre frequency 27.120 MHz)
- 40.66-40.70 MHz(centre frequency 40.68 MHz)
- 433.05-434.79 MHz (centre frequency 433.92 MHz) in Region 1
- 902-928 MHz (centre frequency 915 MHz) in Region 2
- 2.400-2.500 GHz (centre frequency 2.450 GHz)
- 5.725-5.875 GHz (centre frequency 5.800 GHz)
- 24-24.25(centre frequency 24.125 GHz)
- 61-61.5 GHz (centre frequency 61.25 GHz)
- 122-123 GHz (centre frequency 122.5 GHz)
- 244-246 GHz (centre frequency 245 GHz)

Electromagnetic wave propagation phenomena

- בליעה (Absorption)
- החזרה (Reflection)
- שבירה (Refraction)
- עקיפה (Diffraction)
- פיזור (Scattering)
- התאבכות (Interference)
- נפיצה (Dispersion)

Electromagnetic waves propagation

$$\tilde{\mathbf{E}}(r,\theta,\phi) = \tilde{\mathbf{V}}(\theta,\phi)\cdot\frac{e^{-j\mathbf{k}\cdot\mathbf{r}}}{r}$$

Propagation factor:

$$k(f) = \frac{2\pi f}{c}\overbrace{\sqrt{\varepsilon_r(f)}}^{n(f)} = \frac{2\pi}{\lambda}\cdot n(f)$$

Refraction index: $$n(f) = \sqrt{\varepsilon_r(f)} = \mathrm{n}'(f) - j\,\mathrm{n}''(f)$$

Real Imaginary

Complex propagation factor

$$k(f) = \frac{2\pi f}{c} \cdot n(f) = \frac{2\pi f}{c} \cdot \underbrace{\left[\mathrm{n}'(f) - j\,\mathrm{n}''(f) \right]}_{n(f)} =$$

$$= -j \underbrace{\frac{2\pi f}{c} \cdot n''(f)}_{\alpha(f)} + \underbrace{\frac{2\pi f}{c} \cdot n'(f)}_{\beta(f)} = -j\alpha(f) + \beta(f)$$

$$\tilde{\mathbf{E}}_{out}(d) = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-jk(f) \cdot d} = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-j \overbrace{\left[-j\alpha(f) + \beta(f) \right]}^{k(f)} \cdot d} =$$

$$= \tilde{\mathbf{E}}_{in}(0) \cdot e^{-\left[\alpha(f) + j\beta(f) \right] \cdot d} = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-\alpha(f) \cdot d} \cdot e^{-j\beta(f) \cdot d}$$

Power attenuation

$$P_{out} \propto \left|\tilde{\mathbf{E}}_{out}\left(d\right)\right|^2 = \tilde{\mathbf{E}}_{out}\left(d\right)\cdot\tilde{\mathbf{E}}_{out}{}^{*}\left(d\right) = \underbrace{\left|\tilde{\mathbf{E}}_{in}\left(0\right)\right|^2}_{\sim P_{in}}\cdot e^{-2\alpha(f)\cdot d}$$

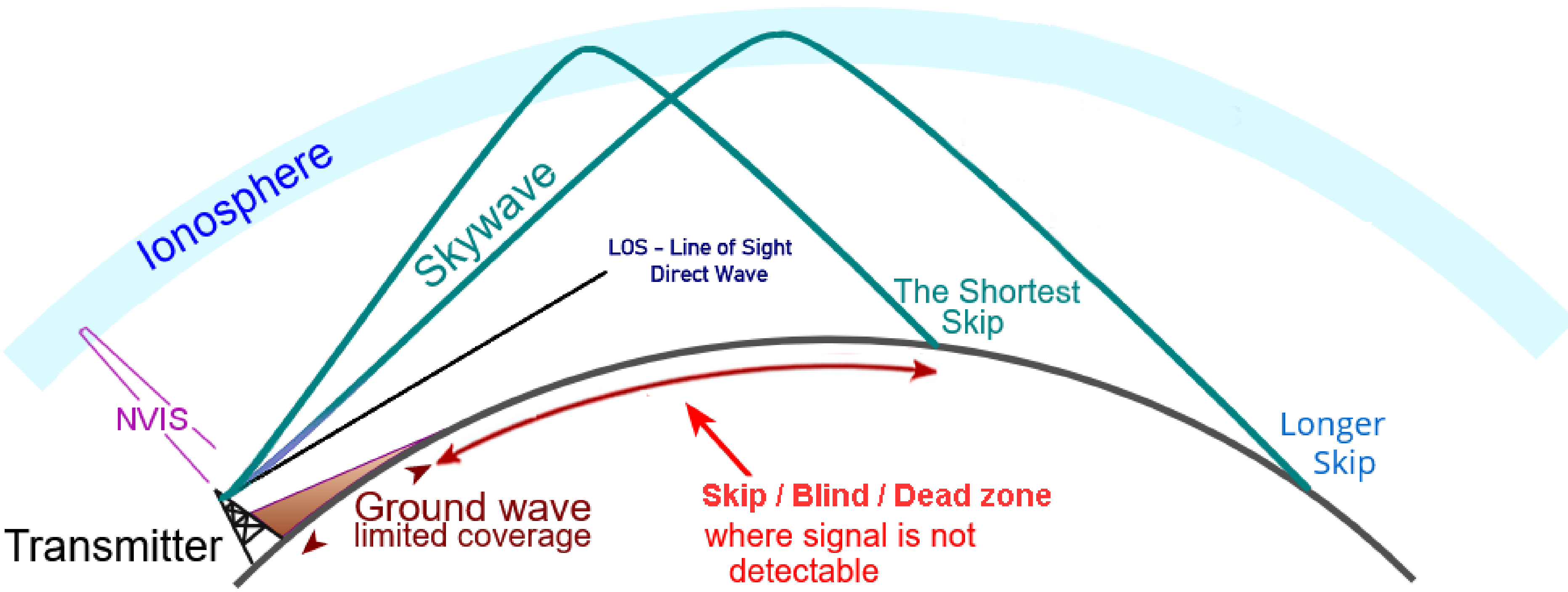
$$\frac{P_{out}}{P_{in}} = e^{-2\alpha(f)\cdot d}$$

$$\text{Attenuation [dB/m]} = 10\cdot\log_{10}\left[e^{+2\alpha(f)\cdot 1}\right] =$$

$$= 20\cdot\log_{10}\left(e\right)\cdot\alpha\left(f\right) = 20\cdot\log_{10}\left(e\right)\cdot\underbrace{\left[\frac{2\pi f}{c}\cdot n''\left(f\right)\right]}_{\alpha(f)}$$

Propagation types

- Direct wave - Line of Sight – LOS
- Ground wave – Surface wave
- Sky wave
- Near Vertical Incidence (NVIS)



Understanding HF Propagation: Forecast, Practice, Tutorial, Apps, Tools

Understanding HF Skywave Propagation
A Guide for Radio Hams

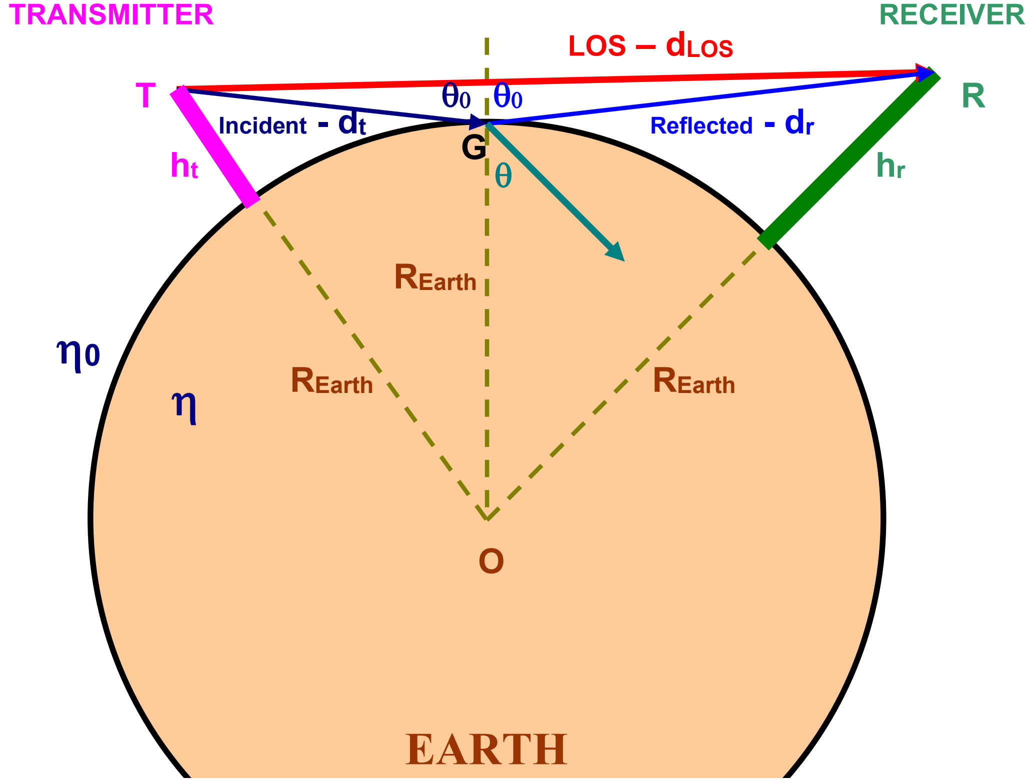
Doron Tal 4X4XM

Understanding HF Propagation: Forecast, Practice, Tutorial, Apps, Tools

https://www.qsl.net/4x4xm/HF-Propagation.htm#WNHFF

Direct wave - Line of Sight – LOS

Line of Sight - LOS



$$TGR:$$

$$d_{LOS}^{2} = d_t^{\ 2} + d_r^{\ 2} - 2 d_t d_r \cos\left(2\theta_0\right) = \left(d_t + d_r\right)^2 \left[1 - \frac{4 d_t d_r}{\left(d_t + d_r\right)^2} \cos^2\left(\theta_0\right)\right]$$

Communication range

$TGR:$

$$d_{LOS}^2 = d_t^{\,2} + d_r^{\,2} - 2d_t d_r \cos\left(2\theta_0\right) = \left(d_t + d_r\right)^2 \left[1 - \frac{4d_t d_r}{\left(d_t + d_r\right)^2}\cos^2\left(\theta_0\right)\right]$$

$TGO:$

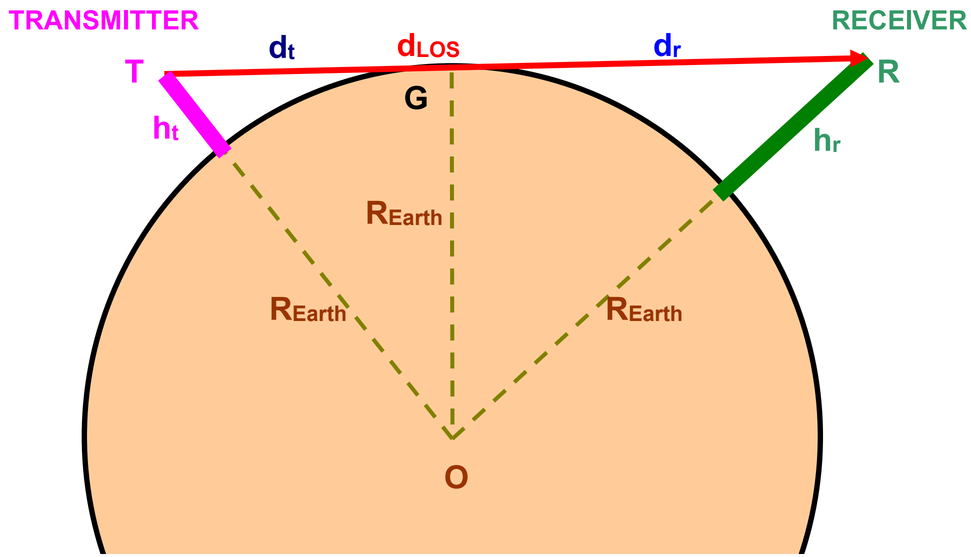
$$\left(h_t + R_{Earth}\right)^2 = d_t^{\,2} + R_{Earth}^{\;\;2} + 2d_t R_{Earth}\cos\left(\theta_0\right)$$

$RGO:$

$$\left(h_r + R_{Earth}\right)^2 = d_r^{\,2} + R_{Earth}^{\;\;2} + 2d_r R_{Earth}\cos\left(\theta_0\right)$$

$R_{Earth} \cong \quad 6{,}378\; Km$

Horizon limit



$$\theta_0 = 90^\circ \longrightarrow d_t + d_r = \sqrt{\left(h_t + R_{Earth}\right)^2 - R_{Earth}{}^2} + \sqrt{\left(h_r + R_{Earth}\right)^2 - R_{Earth}{}^2} = d_{LOS_{\max}}$$

$$h_t, h_r << R_{Earth} \longrightarrow d_{LOS_{\max}} = \left(\sqrt{h_t} + \sqrt{h_r}\right) \cdot \sqrt{2R_{Earth}}$$

Ground wave

Ground wave

- Surface wave
- Below 5MHz
- Quickly attenuated
- Vertically polarized
- Tens of km – up to 100km

Propagation in the Atmosphere

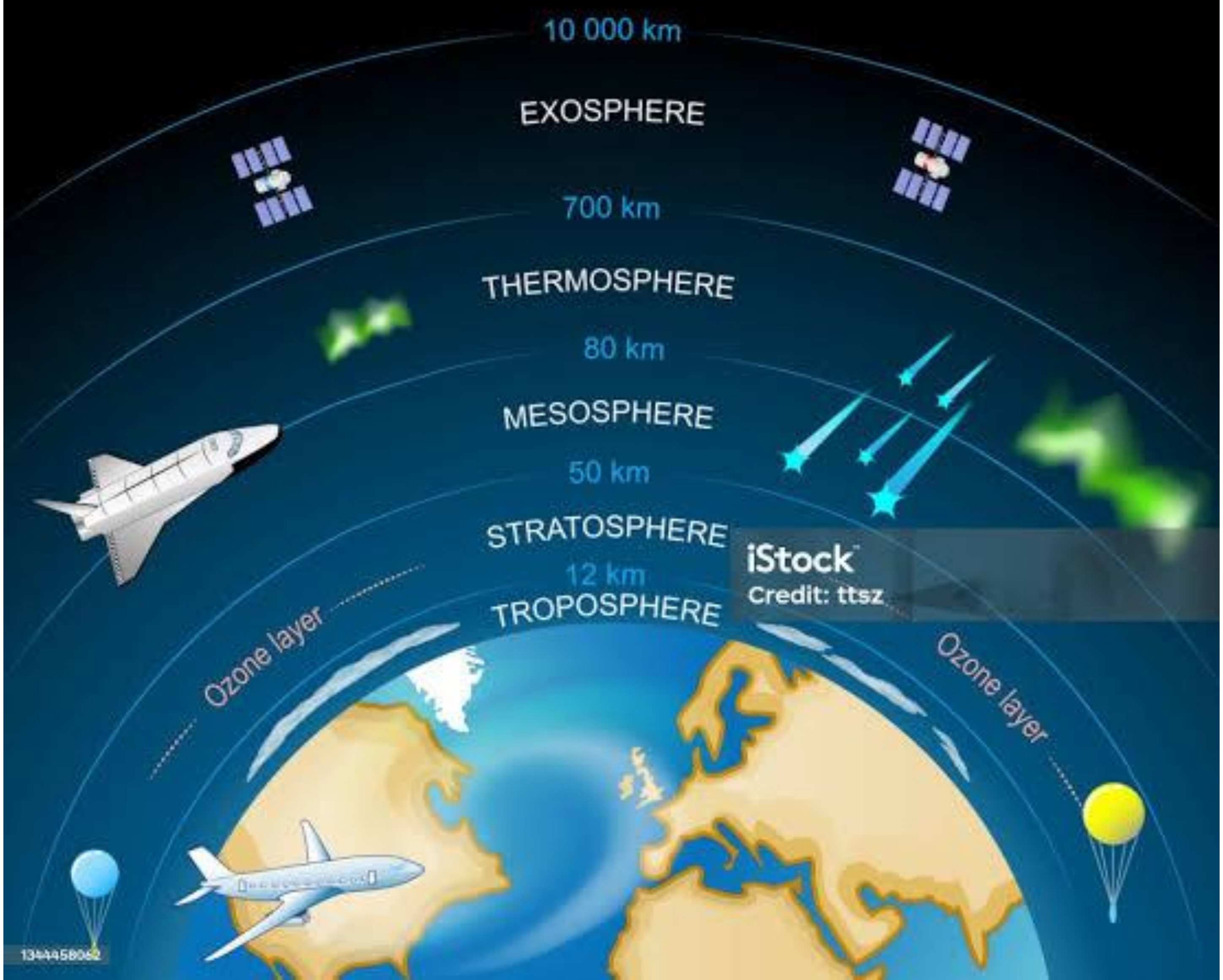
The Atmosphere

- Nitrogen - N_2 - 78.084%
- Oxygen - O_2 - 20.948%
- Argon – Ar - 0.934%
- Carbon dioxide - CO_2 - 0.0314%
- Trace gases - 0.0434%
- Water vapor - H_2O - 1% at sea level

Atmospheric layers

70,000Km	Outer space				Magnetosphere
10,000Km				Plasmasphere	
1,000Km		Exosphere	Ionosphere		
700Km		Thermosphere			
80Km	Atmosphere	Mesosphere			
50Km		Stratosphere			
8-18Km		Troposphere			

The atmosphere of Earth



Refraction index, Refractivity

$$n(f)=\sqrt{\varepsilon_r(f)}=\sqrt{1+\chi_e'(f)-j\chi_e''(f)}\cong$$

$$\cong 1+\underbrace{\frac{1}{2}\chi_e'(f)}_{\left[N_0+N'(f)\right]\cdot 10^{-6}}-j\underbrace{\frac{1}{2}\chi_e''(f)}_{N''(f)\cdot 10^{-6}}=$$

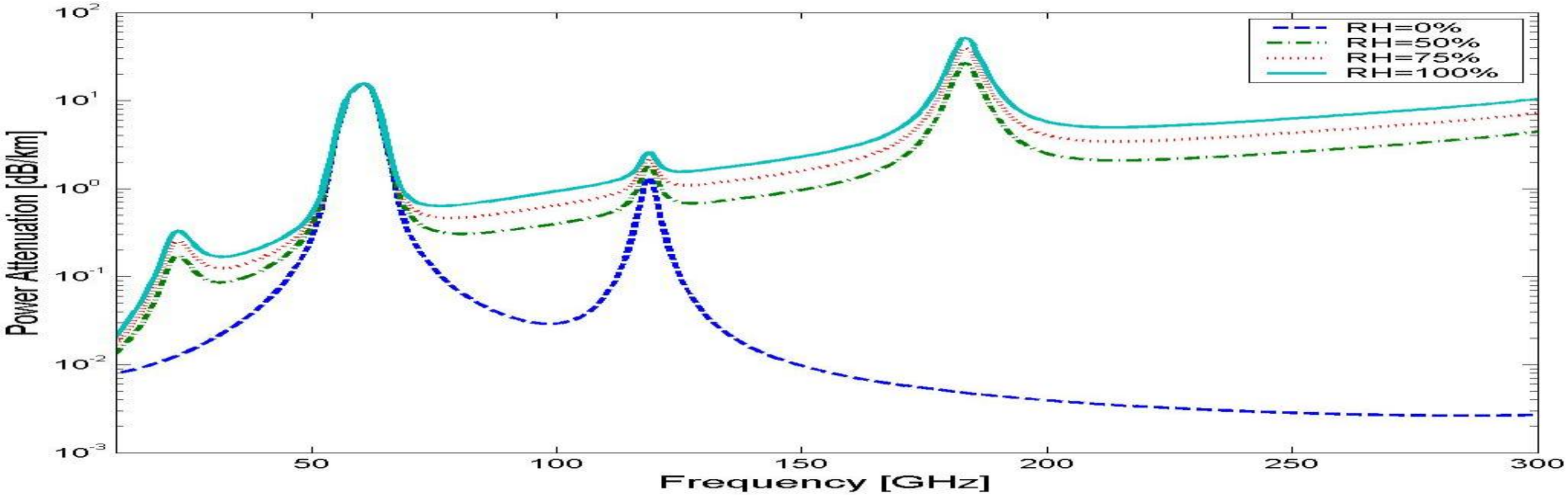
$$=\underbrace{1+\left[N_0+N'(f)\right]\cdot 10^{-6}}_{n'(f)\text{ - Real}}-j\underbrace{N''(f)\cdot 10^{-6}}_{n''(f)\text{ - Imaginary}}$$

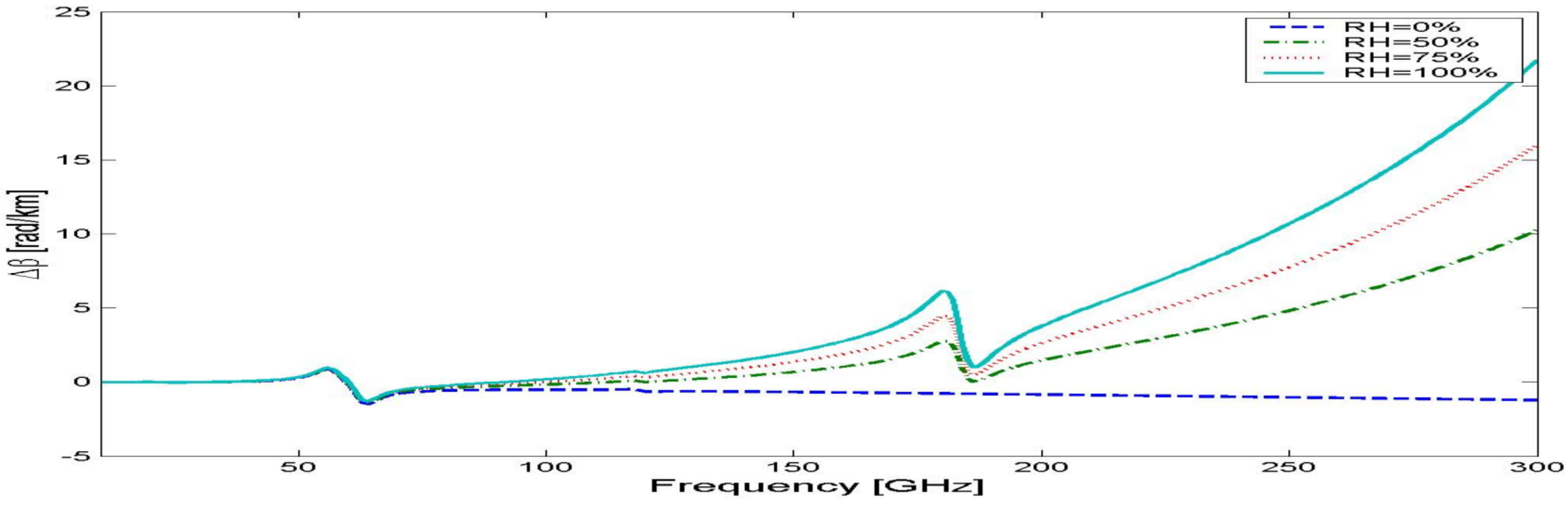
Propagation factor

$$k(f)=\frac{2\pi f}{c}\cdot n(f)=\underbrace{-j\frac{2\pi f}{c}\cdot N''(f)\cdot 10^{-6}}_{\alpha(f)=\frac{\pi f}{c}\chi_e''(f)}+$$

$$+\underbrace{\frac{2\pi f}{c}\cdot\left(1+N_0\cdot 10^{-6}\right)+\overbrace{\frac{2\pi f}{c}\cdot N'(f)\cdot 10^{-6}}^{\Delta\beta(f)}}_{\beta(f)=\frac{2\pi f}{c}+\frac{\pi f}{c}\chi_e'(f)}$$

בליעה באטמוספירה בתחום ה- EHF

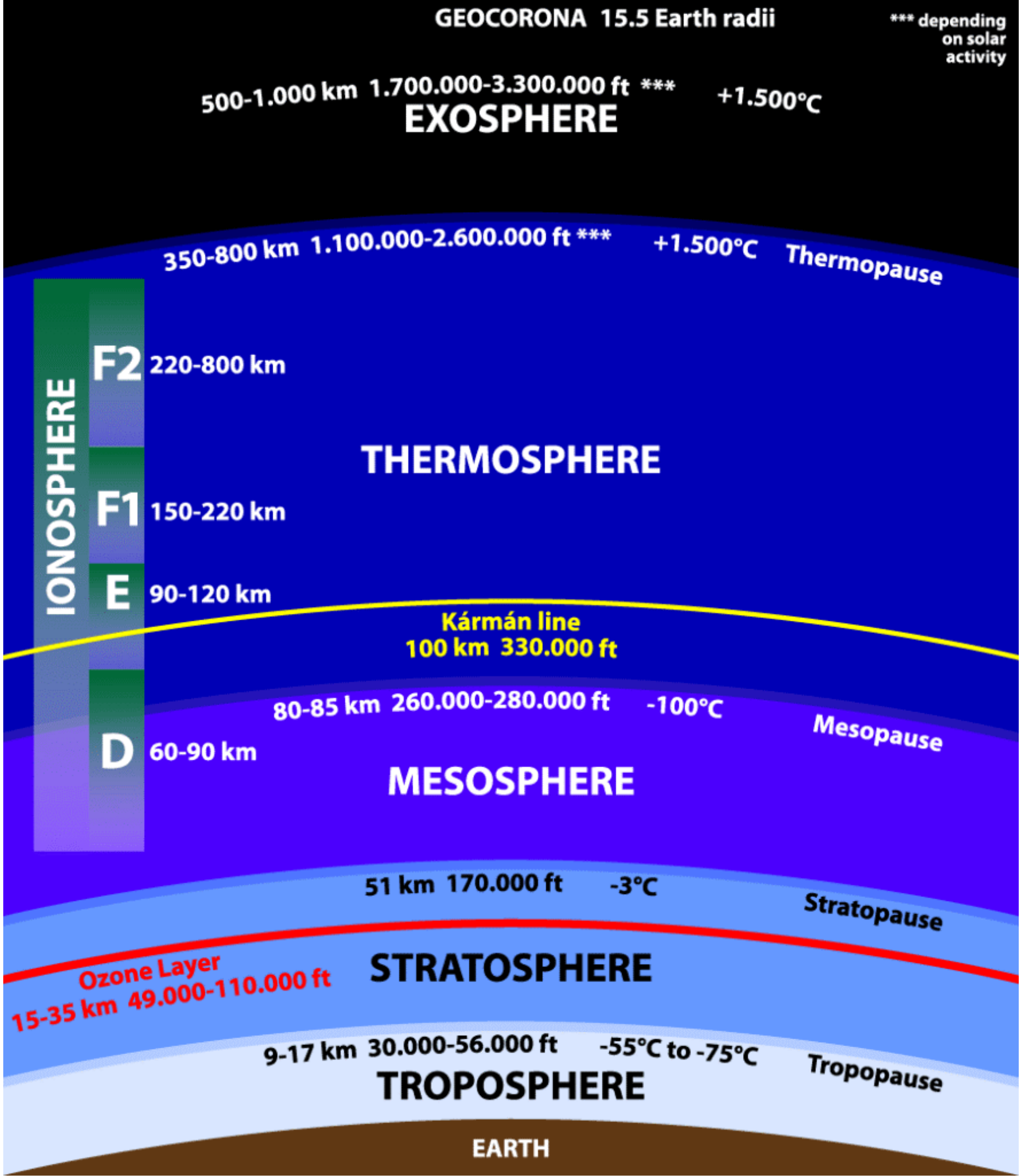




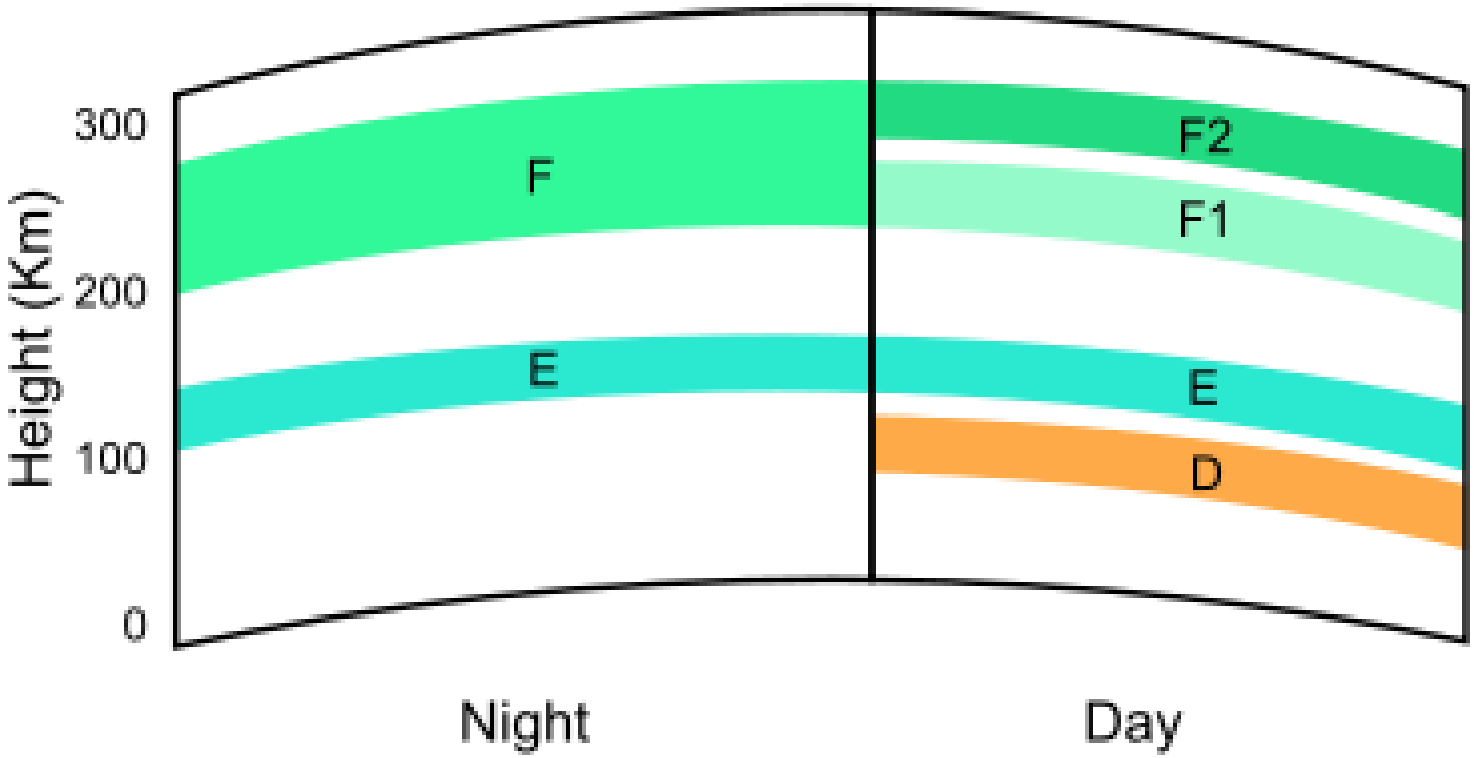
The Ionosphere

Regions

- D layer – Day: 50-80 km – NO^+
- E layer – Day & Night: 90-120 km – O_2^+ , NO^+
- Sporadic E – Summer days: 90-120 km – O_2^+, NO^+
- F1 layer – Day - 130 to 210 km - O_2^+, NO^+, O^+
- F2 layer – Day - over 210 km - O^+ , N^+ , H^+

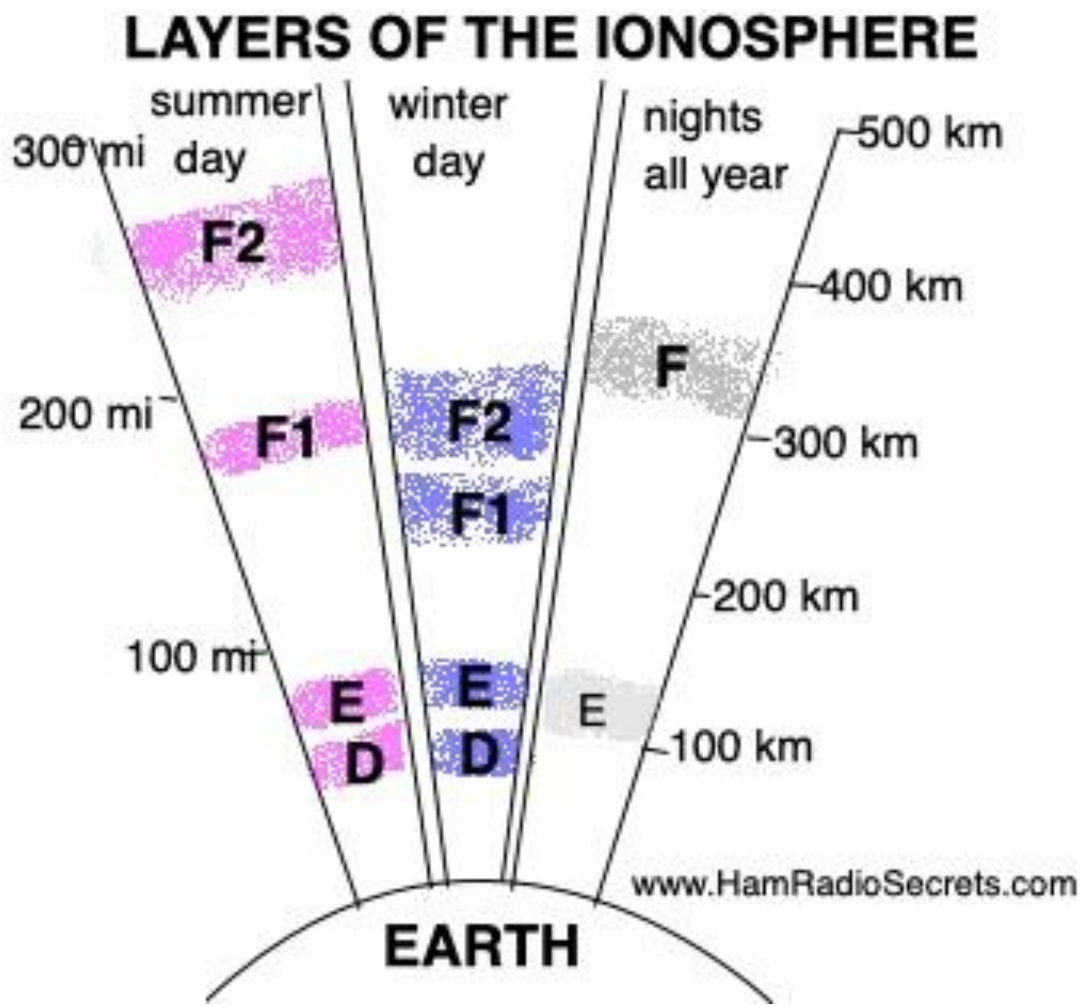

GEOCORONA 15.5 Earth radii
*** depending on solar activity
500-1.000 km 1.700.000-3.300.000 ft *** +1.500°C
EXOSPHERE
350-800 km 1.100.000-2.600.000 ft *** +1.500°C Thermopause
IONOSPHERE
F2 220-800 km
THERMOSPHERE
F1 150-220 km
E 90-120 km
Kármán line
100 km 330.000 ft
80-85 km 260.000-280.000 ft -100°C Mesopause
D 60-90 km
MESOSPHERE
51 km 170.000 ft -3°C Stratopause
STRATOSPHERE
Ozone Layer
15-35 km 49.000-110.000 ft
9-17 km 30.000-56.000 ft -55°C to -75°C Tropopause
TROPOSPHERE
EARTH

Characteristics of different ionospheric layers – Buzztech



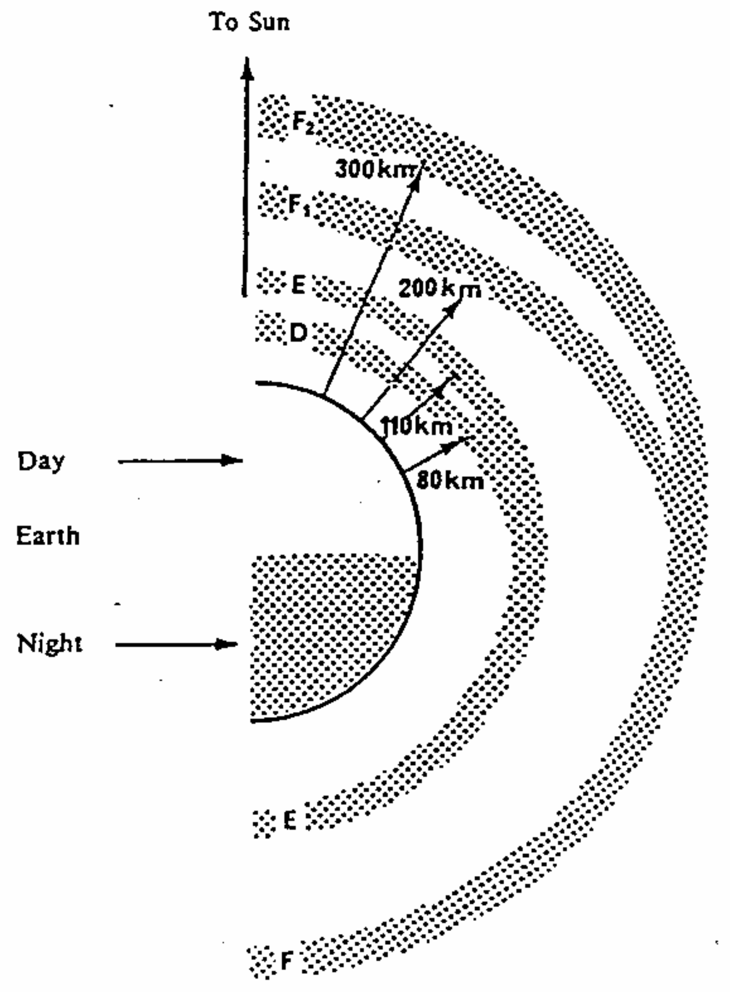
Ionospheric Propagation of Radio Waves Gives Ham Radio Operators "Seven League Boots"!

https://www.hamradiosecrets.com/ionospheric-propagation-of-radio-waves.html



THE IONOSPHERE AND ITS EFFECTS ON RADIOWAVE PROPAGATION

A guide with background to ITU-R procedures for radio planners and users



D - Region

- Mesosphere
- Only daytime
- Altitude: 50 to 80 km
- Average altitude: 70km
- Thickness: 10 km
- NO^+ excited by UVC (121.6nm)
- Low electron density 10^8-10^{10} electrons/m^3
- High recombination rate 10^6 1/sec

D region – interaction with EM waves

- Reflection of LF, VLF, ELF
- $f_e = 1.2 \cdot 10^6$ collisions per second
- Absorbing 160m, 80m, 60m, 40m
- Attenuation of MF and HF below 10MHz (at noon)
- Defines the Lowest Usable Frequency – LUF
- Solar bursts of X-rays (0.1–1nm) causing blackout (minutes to hours)

E - Region

- Thermosphere
- Only daytime
- 90 to 120 km
- Average altitude: 110km
- Thickness: 25 km
- O_2^+, NO^+ excited by EUV (1-10nm)
- Maximum electron density $4 \cdot 10^{11}$ electrons/m^3
- High recombination rate at evening
- Disappears within couple of hours after sunset
- Reflection of frequencies below 10MHz
- Transmitted near vertically (NVIS)

Sporadic –Es – plasma clouds

- Thermosphere
- Only daytime
- Summer days
- 90 to 120 km
- Thin layer
- O_2^+, NO^+
- Temporary high ionization regions (Plasma clouds)
- Electron density above 10^{11} electrons/m^3
- Reflection of frequencies up to 150MHz

F1 - Layer

- Thermosphere
- Only daytime
- Mainly summer days
- 130km to 210km (180km)
- Thickness: 20 km
- O^+, O_2^+, NO^+ excited by EUV (10-100nm)
- Electron density $2 \cdot 10^{11}$ electrons/m^3 at 180km
- High recombination rate at evening
- Reflection of frequencies below tens of MHz
- Summer, often reflects HF at the 30m, 20 bands

F2 - Layer

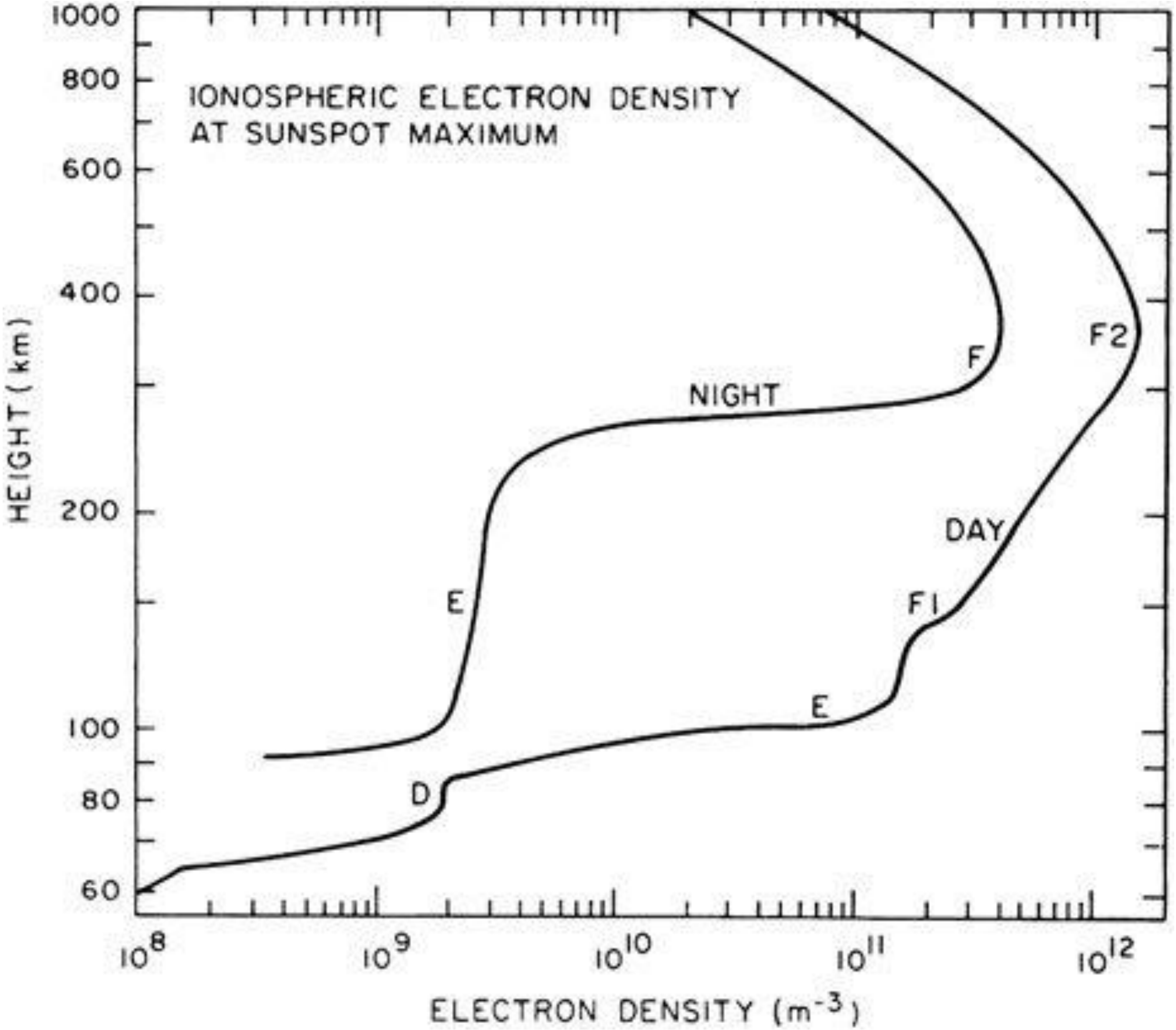
- Thermosphere
- Only daytime
- Summer days
- 250km to 500km
- Average altitude: 325km
- Thickness: 200 km
- O^+ excited by EUV (10-100nm)
- Maximum electron density $1 \cdot 10^{12}$-$4 \cdot 10^{12}$ electrons/m^3
- Average: $2 \cdot 10^{12} m^{-3}$
- High recombination rate at evening
- Reflection of frequencies below tens of MHz

F – Region during the night

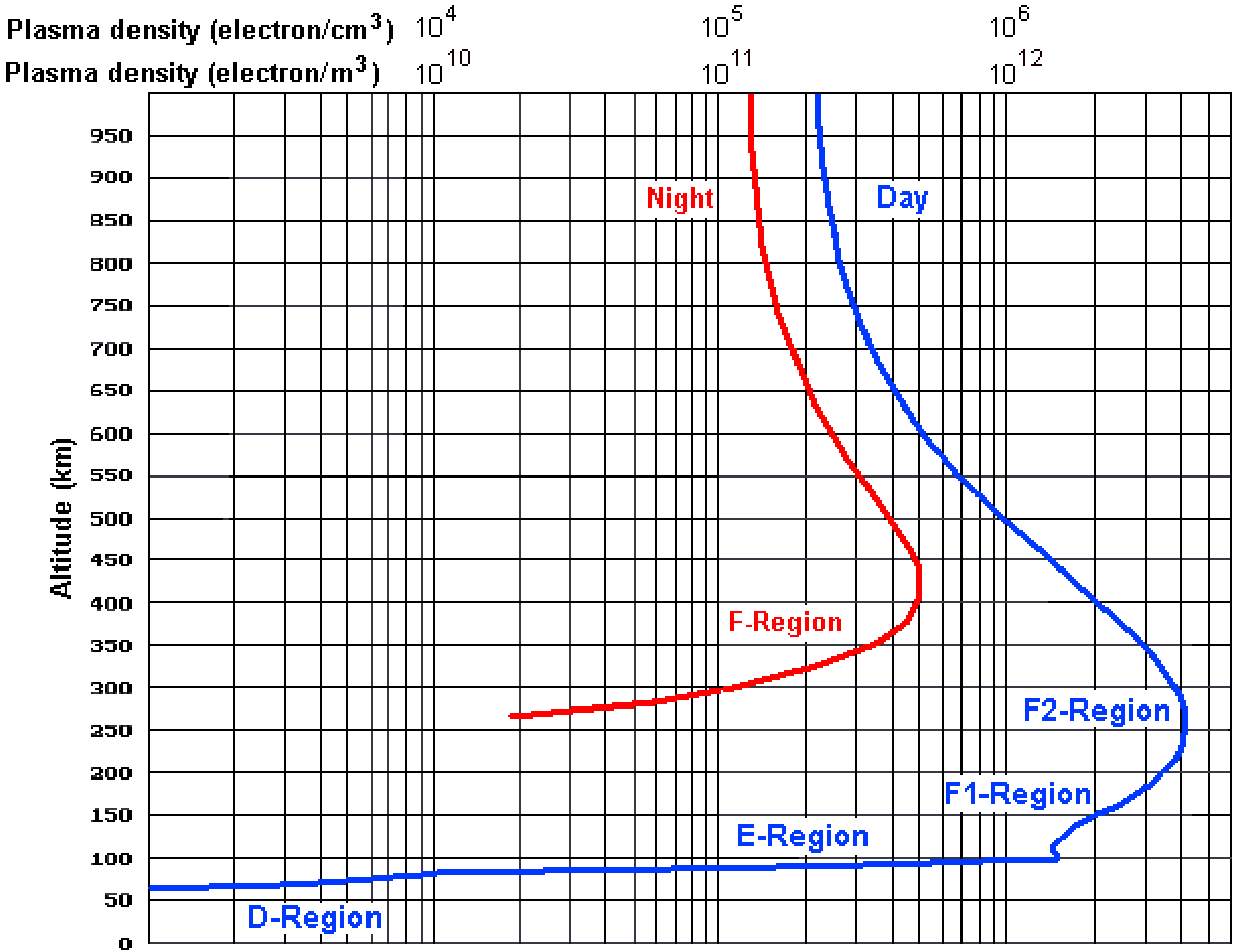
- F1 merges into F2
- Thermosphere
- Night
- All seasons
- 350-400 km
- O_2^+, NO^+, O^+ excited by EUV (10-100nm)
- Electron density reduces to $5 \cdot 10^{10}$ electrons/m^3
- Reflection of frequencies below tens of MHz

Electron density – m^{-3}

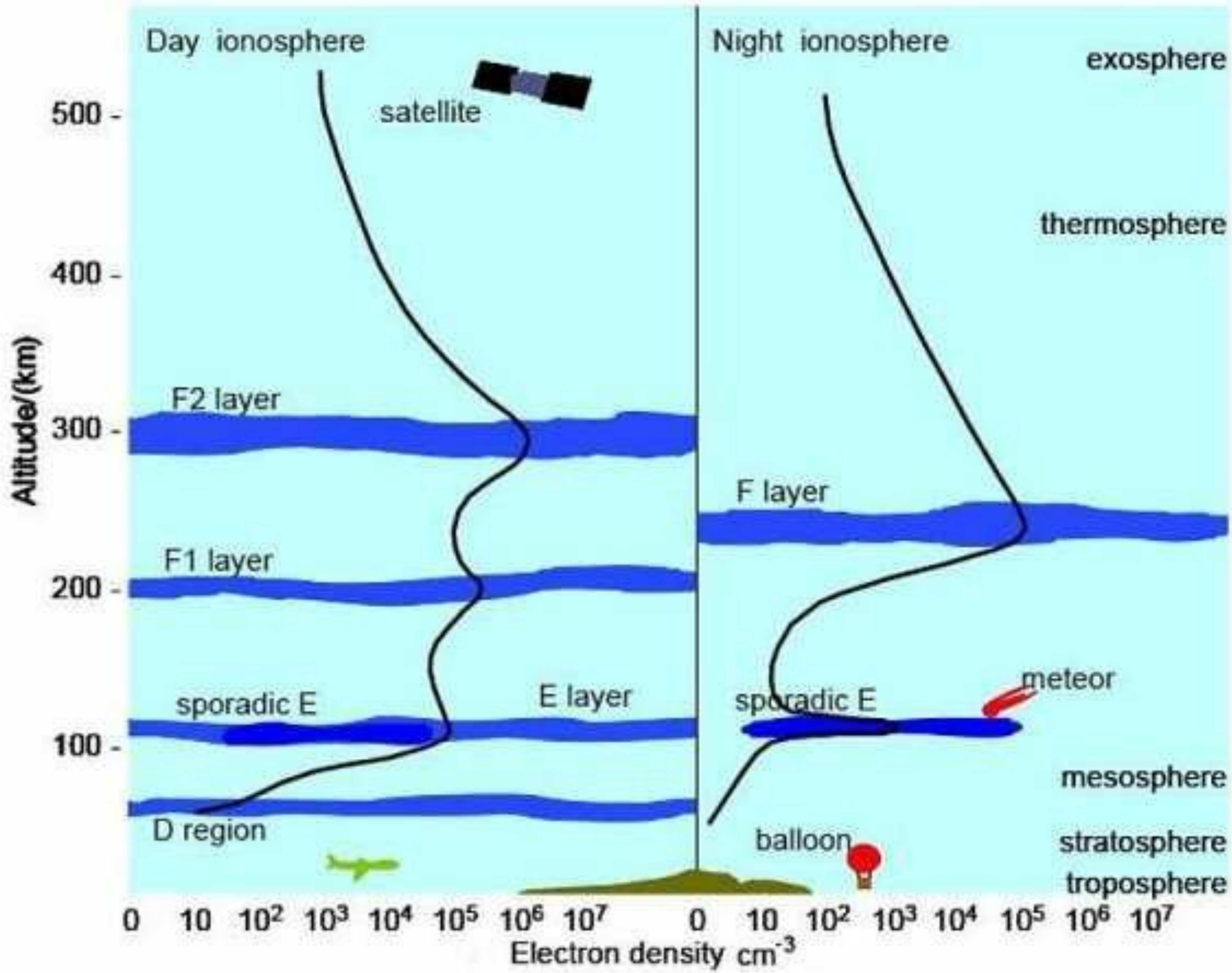
				DAY	NIGHT
1,000Km	Atmosphere	Exosphere	Ionosphere		
700Km		Thermosphere		F2 (O^+, N^+ 10^{12})	F (O^+ $5 \cdot 10^{10}$)
210Km					
200Km				F1 (O^+ $2 \cdot 10^{11}$)	
140Km					
120Km				E (O_2^+ 10^{11})	E (O_2^+ $1.5 \cdot 10^9$)
90Km					
80Km		Mesosphere		D (NO^+ 10^8-10^{10})	
60Km					
50Km		Stratosphere			
8-18Km		Troposphere			



J. V. Evans and T. Hagfors, Radar Astronomy, 1968. © McGraw-Hill Education



Plasma density (electron/cm^3) 10^4 10^5 10^6
Plasma density (electron/m^3) 10^{10} 10^{11} 10^{12}
Altitude (km)
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
Night
Day
F-Region
F2-Region
F1-Region
E-Region
D-Region



Day ionosphere
Night ionosphere
exosphere
satellite
thermosphere
Altitude/(km)
500
400
300
200
100
F2 layer
F layer
F1 layer
sporadic E
E layer
sporadic E
meteor
mesosphere
D region
balloon
stratosphere
troposphere
0 10 10² 10³ 10⁴ 10⁵ 10⁶ 10⁷
0 10 10² 10³ 10⁴ 10⁵ 10⁶ 10⁷
Electron density cm⁻³

Plasma

Dielectric coefficient

$$\varepsilon_r(f) = 1 - \frac{\left(\frac{f_p}{f}\right)^2}{1 - j\frac{f_e}{f}} = \underbrace{\left[1 - \frac{\left(\frac{f_p}{f}\right)^2}{1+\left(\frac{f_e}{f}\right)^2}\right]}_{\varepsilon'(f)} - j\underbrace{\left[\frac{\left(\frac{f_p}{f}\right)^2}{1+\left(\frac{f_e}{f}\right)^2}\right]}_{\varepsilon''(f)} \cdot \frac{f_e}{f}$$

$$f_p = \frac{1}{2\pi}\sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} = 8.977\sqrt{N_e}$$

$$e \cong 1.6 \cdot 10^{-19}\ C \quad m_e = 9.11 \cdot 10^{-31} Kg \quad \varepsilon_0 = 8.85 \cdot 10^{12}\ F/m$$

Attenuation and Wavenumber

- Attenuation:

$$\alpha(f) = -\mathrm{Im}\left[k(f)\right] = \frac{2\pi f}{c}\sqrt{\frac{\varepsilon'(f)}{2}\left[\sqrt{1+\left(\frac{\varepsilon''(f)}{\varepsilon'(f)}\right)^2}-1\right]}$$

- Wavenumber:

$$\beta(f) = \mathrm{Re}\left[k(f)\right] = \frac{2\pi f}{c}\sqrt{\frac{\varepsilon'(f)}{2}\left[\sqrt{1+\left(\frac{\varepsilon''(f)}{\varepsilon'(f)}\right)^2}+1\right]}$$

Dielectric coefficient

$$f >> f_e \longrightarrow \varepsilon_r\left(f\right) = 1 - \frac{\left(\frac{f_p}{f}\right)^2}{1 - j\frac{f_e}{f}} \approx 1 - \left(\frac{f_p}{f}\right)^2 = \varepsilon'\left(f\right)$$

$$f_p = \frac{1}{2\pi}\sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} = 8.977\sqrt{N_e}$$

$$e \cong 1.6 \cdot 10^{-19}\ C \quad m_e = 9.11 \cdot 10^{-31} Kg \quad \varepsilon_0 = 8.85 \cdot 10^{12}\ F/m$$

Propagation coefficient

Refraction index:

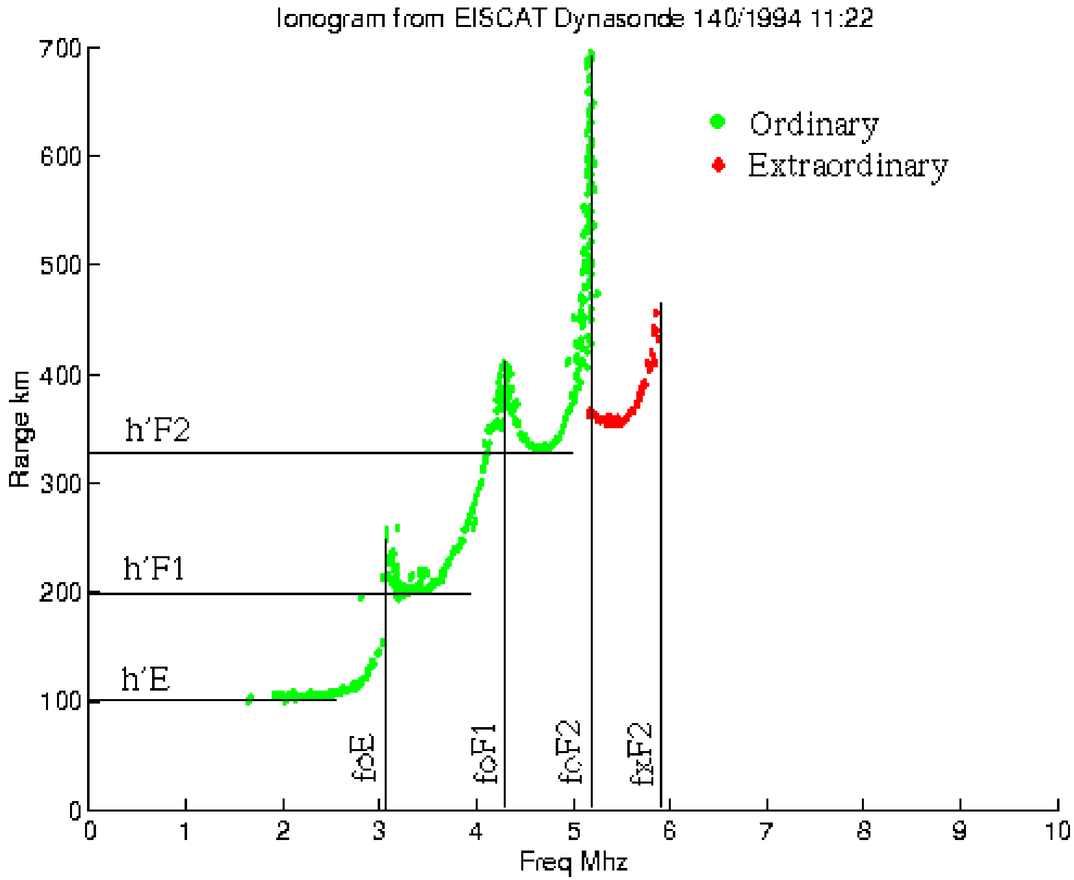
$$n(f) \approx \sqrt{\varepsilon'(f)} = \sqrt{1-\left(\frac{f_p}{f}\right)^2}$$

Wavenumber:

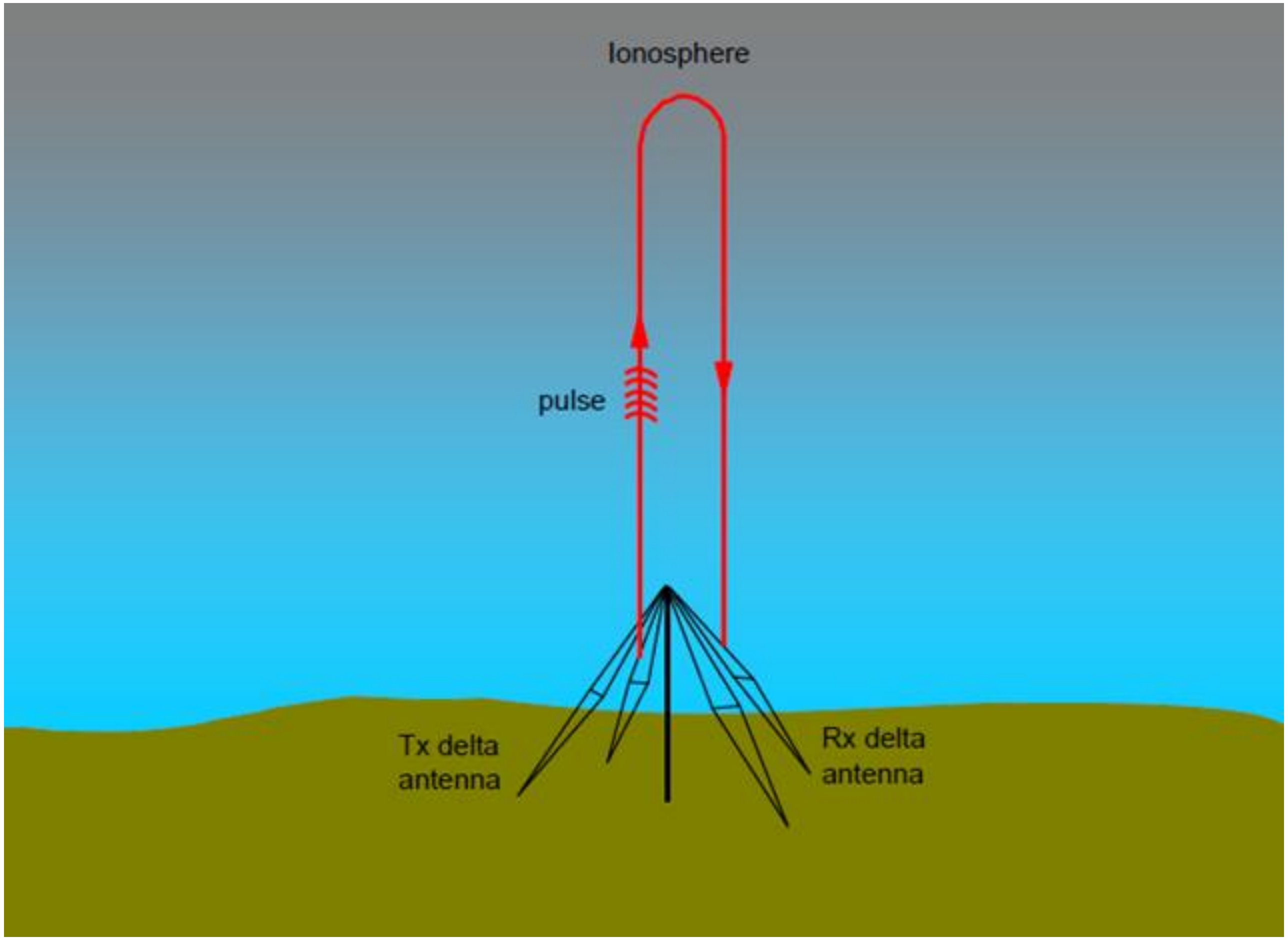
$$\beta(f) = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon'(f)} = \frac{2\pi}{c} \cdot \sqrt{f^2 - f_p^{\ 2}}$$

Plasma (critical) ionospheric frequency

$$f_p = 8.977\sqrt{N_e}$$



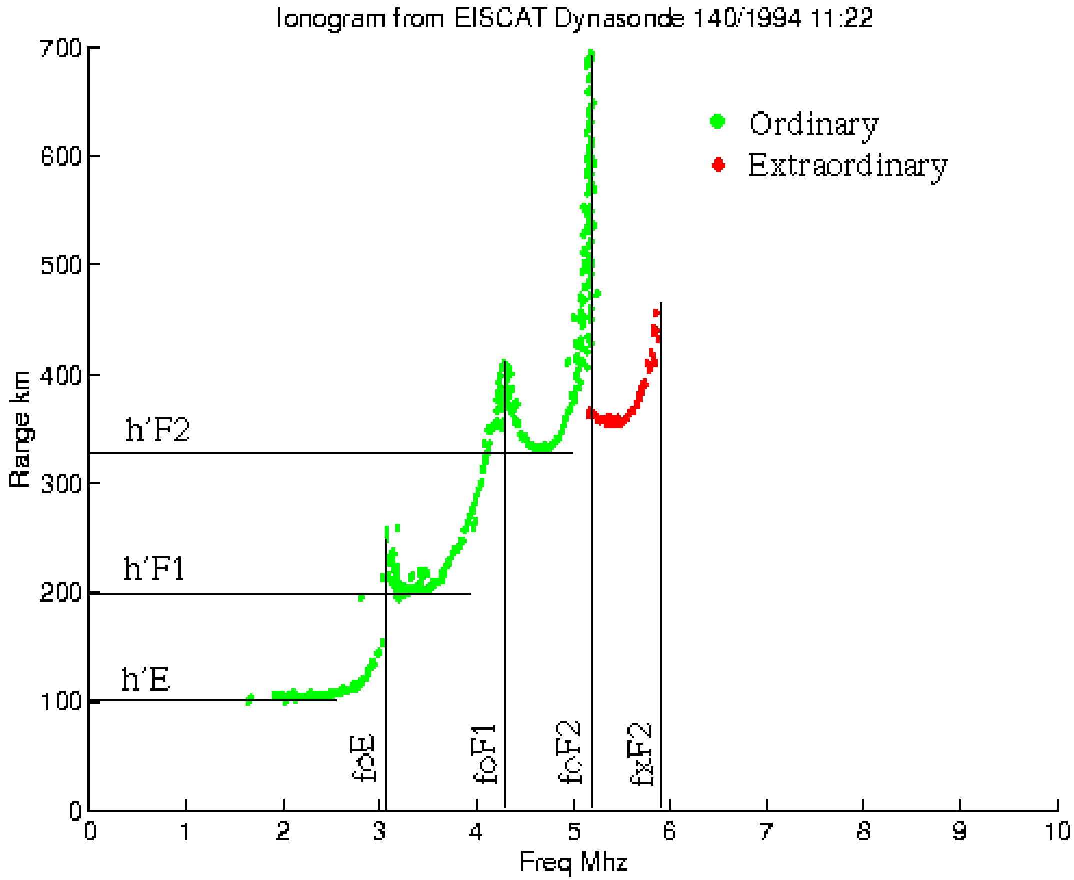
Ionosonde, chirp-sounder



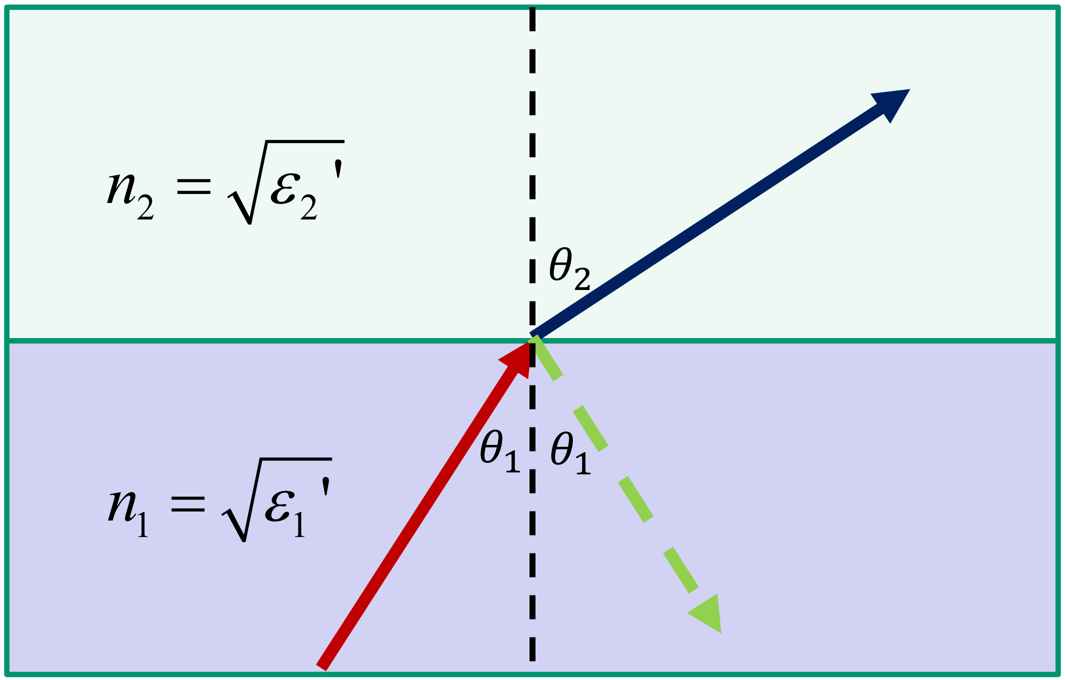
A basic schematic of an ionosonde | The Planetary Society

Critical ionospheric frequency measurment

$$f_p = 8.977\sqrt{N_e}$$



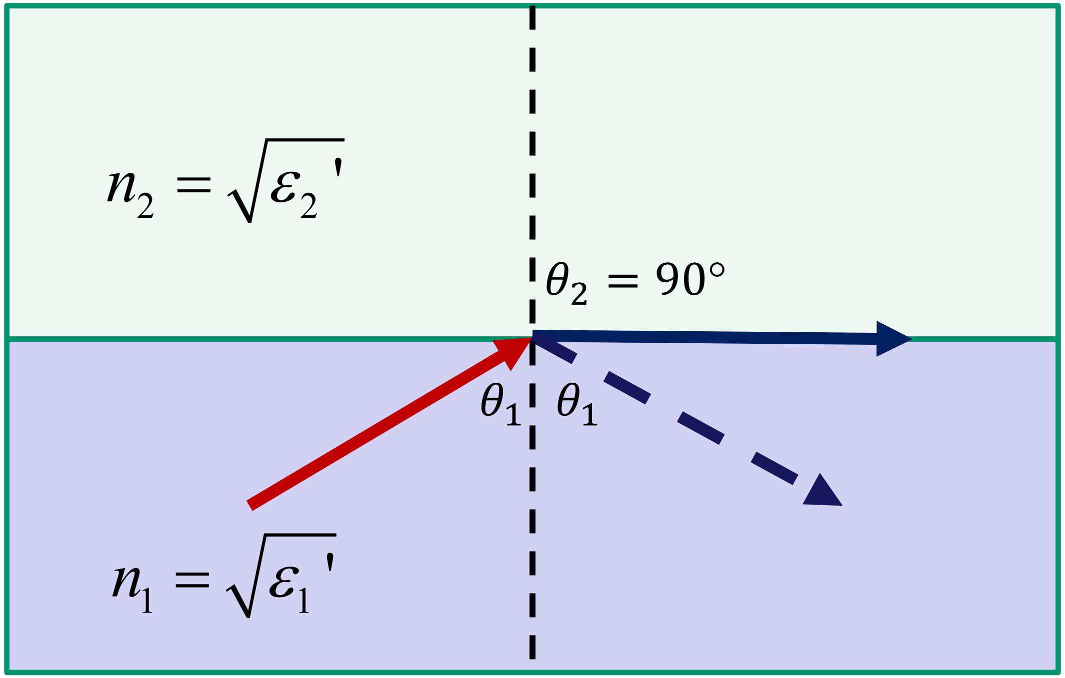
Oblique Refraction



Snell's law: $\underbrace{\sqrt{\varepsilon_1'}}_{n_1} \cdot \sin(\theta_1) = \underbrace{\sqrt{\varepsilon_2'}}_{n_2} \cdot \sin(\theta_2) \longrightarrow \sin(\theta_2) = \frac{n_1}{n_2} \cdot \sin(\theta_1) = \sqrt{\frac{\varepsilon_1'}{\varepsilon_2'}} \cdot \sin(\theta_1)$

$\sqrt{\varepsilon_1'} > \sqrt{\varepsilon_2'} \longrightarrow n_1 > n_2 \longrightarrow \theta_2 > \theta_1$

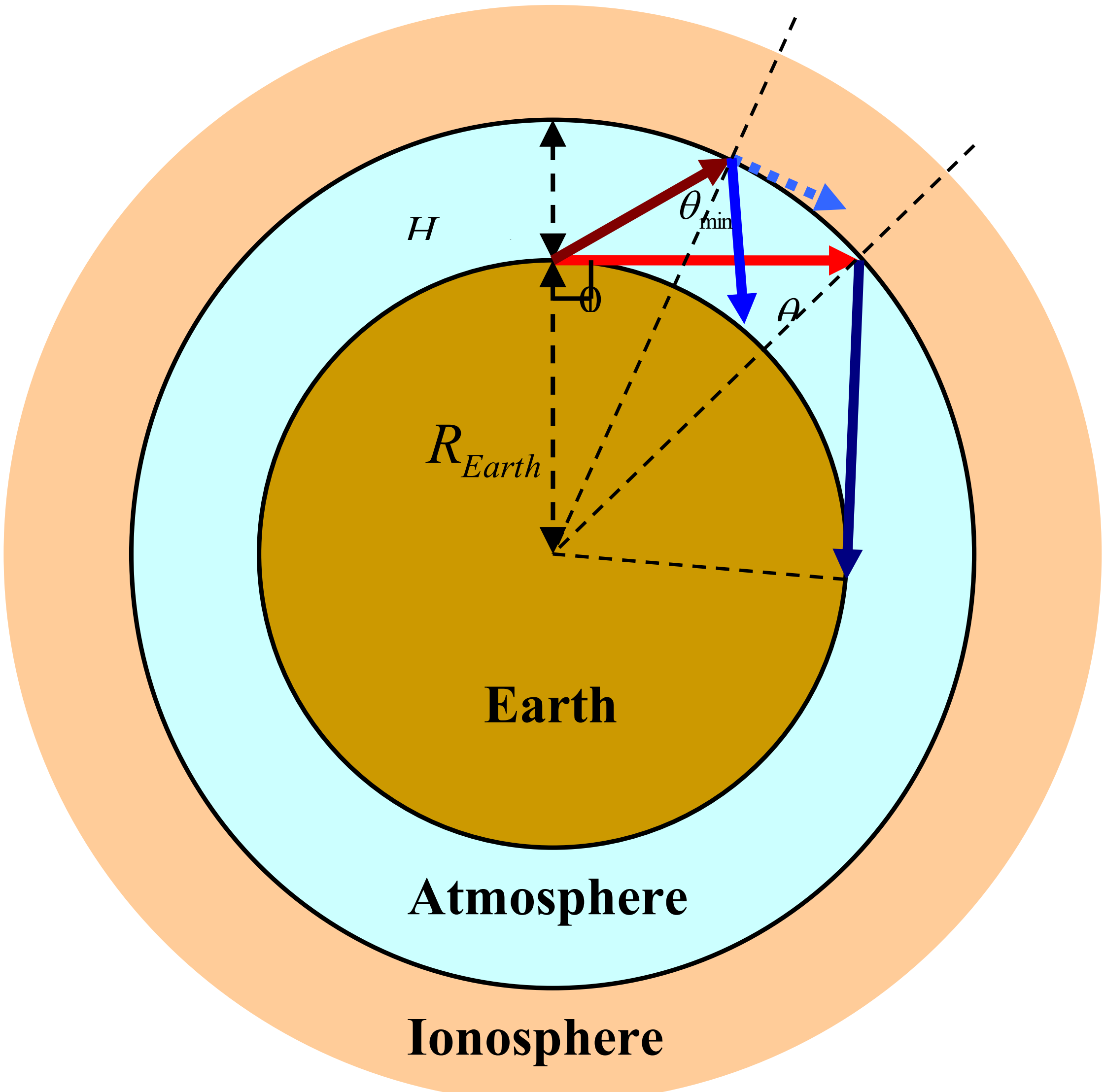
Total Reflection



Total reflection: $\underbrace{\sqrt{\varepsilon_1 '}}^{n_1} \cdot \sin(\theta_1) = \overbrace{\sqrt{\varepsilon_2 '}}^{n_2} \cdot \underbrace{\sin(90^\circ)}_{1} \longrightarrow \sin(\theta_{1_{\min}}) = \sqrt{\frac{\varepsilon_2 '}{\varepsilon_1 '}}$

$$n_1 = \sqrt{\varepsilon_1 '} = 1 \longrightarrow \sin(\theta_{1_{\min}}) = n_2 = \sqrt{\varepsilon_2 '} < 1$$

Reflection from the Ionosphere



Minimum incidence angle for total reflection

Snell's law:

$$1\cdot\sin\left(\theta_{\min}\right)=n\left(f\right)\cdot\sin\left(90^{\circ}\right)\longrightarrow\sin\left(\theta_{\min}\right)=n\left(f\right)=\sqrt{1-\left(\frac{f_p}{f}\right)^2}$$

$$\theta>\theta_{\min}\longrightarrow\sin\left(\theta\right)>\underbrace{\sqrt{1-\left(\frac{f_p}{f}\right)^2}}_{\sin\left(\theta_{\min}\right)}\longrightarrow\underbrace{\sqrt{1-\sin^2\left(\theta\right)}}_{\cos\left(\theta\right)}<\frac{f_p}{f}\longrightarrow f_p<f<\frac{f_p}{\cos\left(\theta\right)}$$

Shorter range: $l\downarrow\longrightarrow\theta\downarrow\longrightarrow\cos\left(\theta\right)\uparrow\longrightarrow f<\frac{f_p}{\cos\left(\theta\right)}\downarrow$

Maximum incidence angle
Maximum Usable Frequency

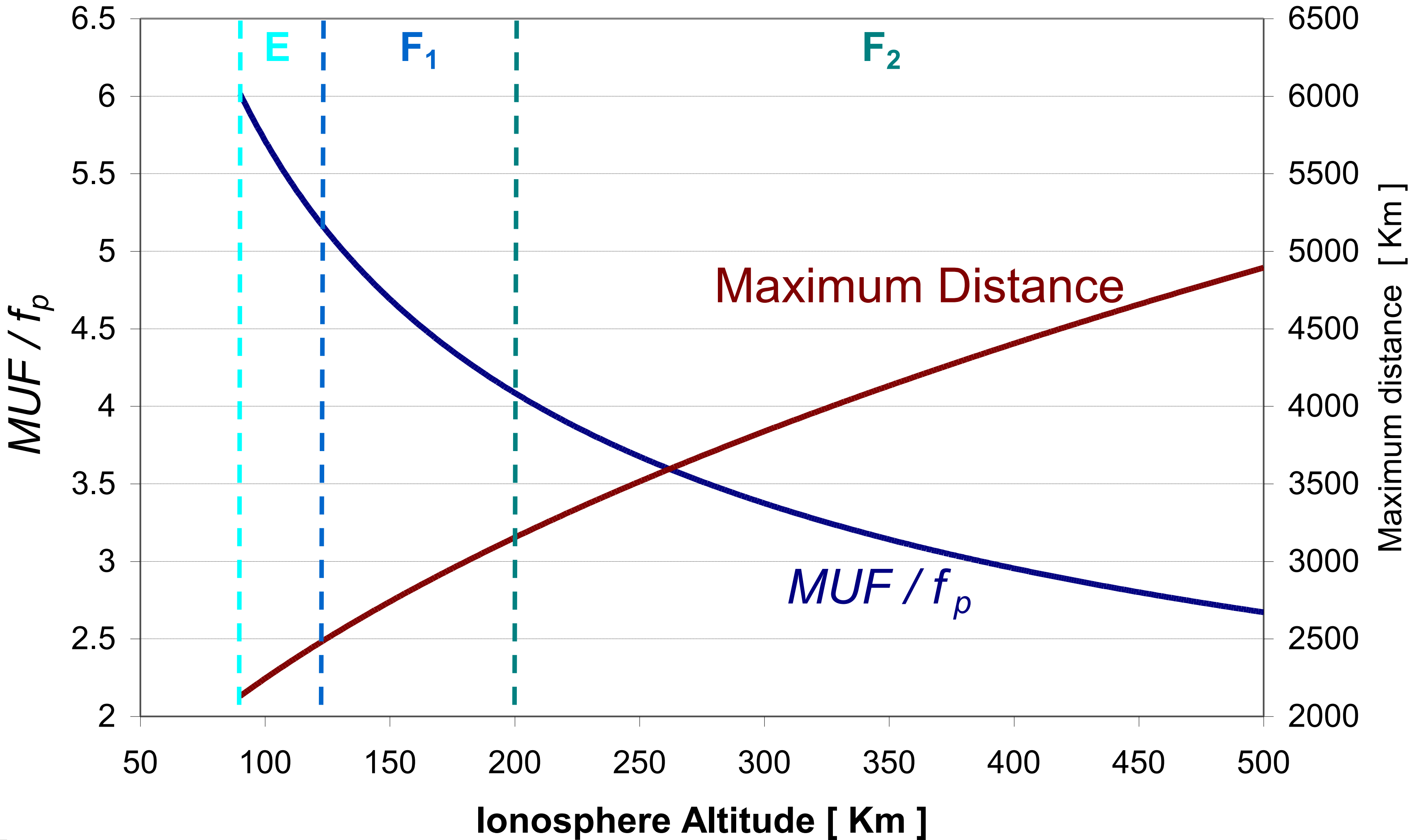
$$\sin\left(\theta_{\max}\right) = \frac{R_{Earth}}{R_{Earth} + H_{\text{Ionosphere}}}$$

$$MUF = \frac{f_p}{\cos\left(\theta_{\max}\right)} = \frac{f_p}{\sqrt{1-\sin^2\left(\theta_{\max}\right)}} = \frac{f_p}{\sqrt{1-\left(\dfrac{R_{Earth}}{R_{Earth} + H_{\text{Ionosphere}}}\right)^2}}$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin\left(\theta_{\max}\right) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos\left(\theta_{\max}\right) \uparrow \longrightarrow MUF \downarrow$$

Above the MUF – no reflection from the Ionosphere !

MUF vs. Altitude



Important notes !

$$f_p \uparrow \longrightarrow MUF = \frac{f_p}{\cos(\theta_{\max})} \uparrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos(\theta_{\max}) \uparrow \longrightarrow MUF \downarrow$$

Lowest Usable Frequency

- Below the LUF – high absorption in the D region !
- Only during day.
- Rises during solar flare bursts.
- May creates blackouts for minutes to hours.

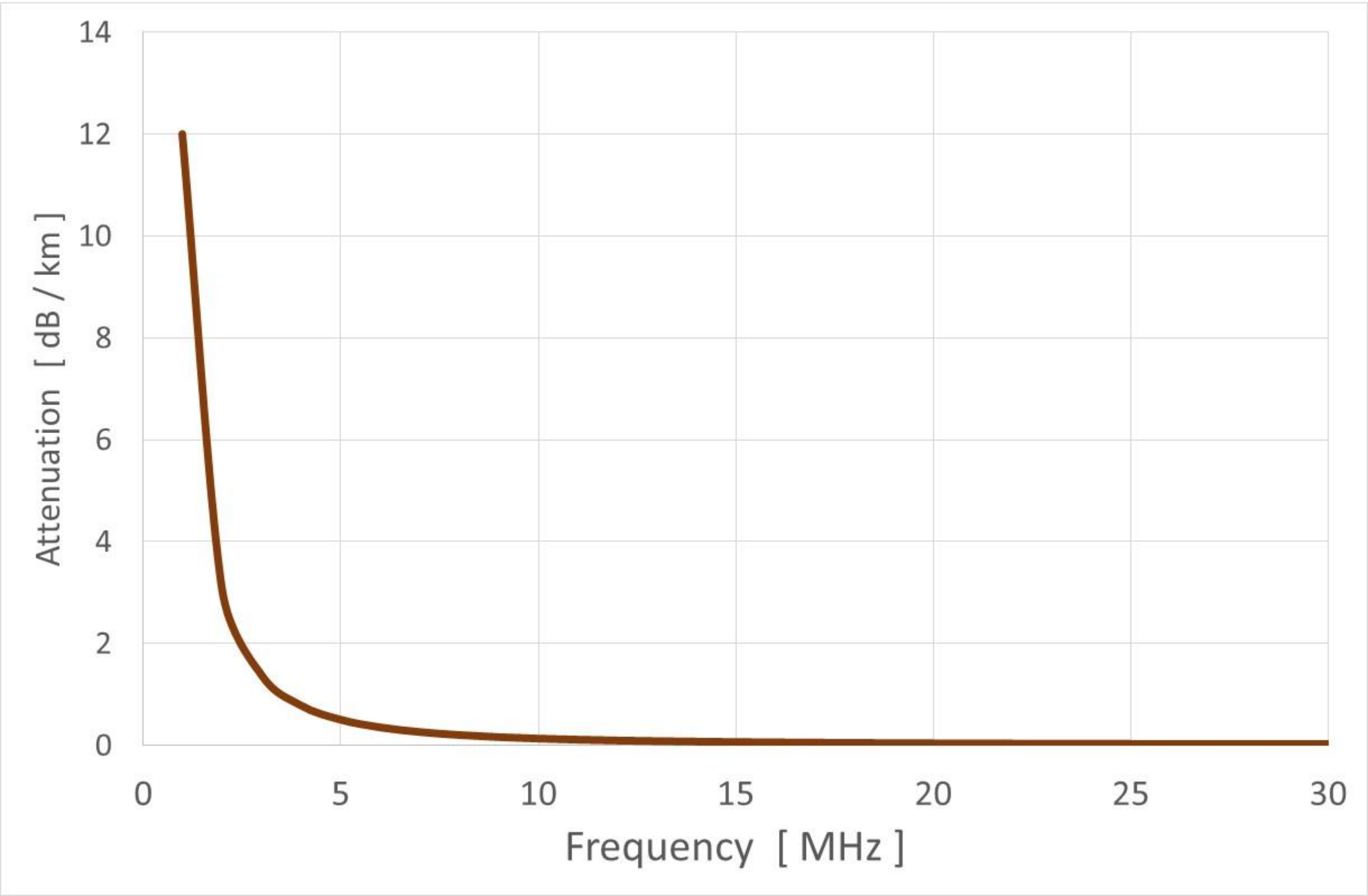
D – layer characteristics

Electron density: $$N_e = 10^9 \frac{electrons}{m^3}$$

Plasma (critical) frequency : $$f_p = 8.977\sqrt{N_e} = 284kHz$$

Collision rate: $$f_e = 1.2 \cdot 10^6 \, collisions\ per\ second$$

D – layer attenuation

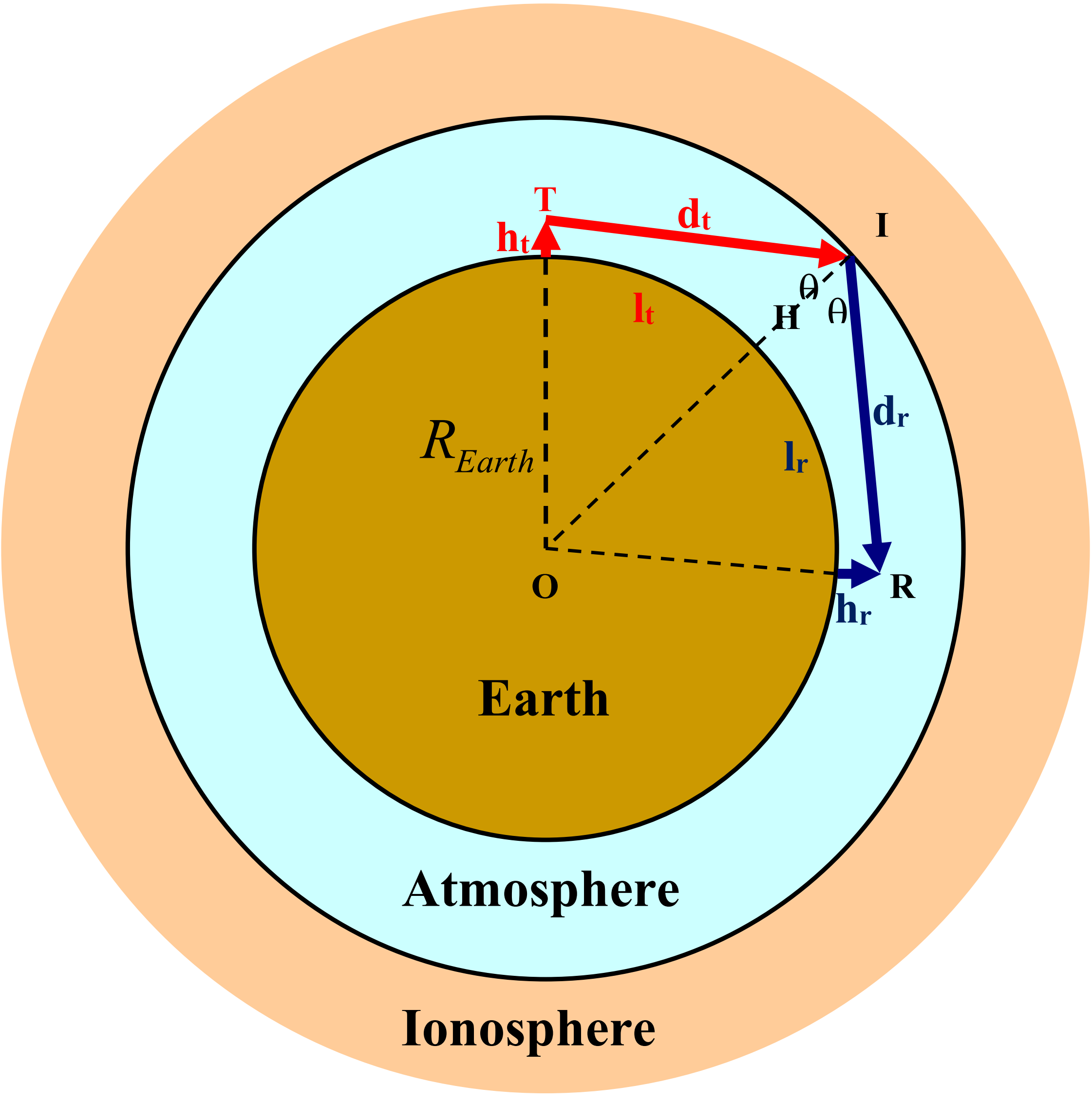


Optimum working frequency (OWF)

Frequency of optimum transmission (FOT):

$$FOT = 0.85 \cdot MUF = 0.85 \cdot \frac{f_p}{\cos\left(\theta_{\max}\right)}$$

Maximum range



Maximum range

$$\left(h_t + R_{Earth}\right)^2 = d_t^{\,2} + \left(H_{\text{Ionosphere}} + R_{Earth}\right)^2 - 2d_t R_{Earth}\cos\left(\theta\right)$$

$$\frac{d_t}{\sin\left(\dfrac{l_t}{R_{Earth}}\right)} = \frac{h_t + R_{Earth}}{\sin\left(\theta\right)}$$

$$\left(h_r + R_{Earth}\right)^2 = d_r^{\,2} + \left(H_{\text{Ionosphere}} + R_{Earth}\right)^2 - 2d_r R_{Earth}\cos\left(\theta\right)$$

$$\frac{d_r}{\sin\left(\dfrac{l_r}{R_{Earth}}\right)} = \frac{h_r + R_{Earth}}{\sin\left(\theta\right)}$$

$$l = l_t + l_r$$

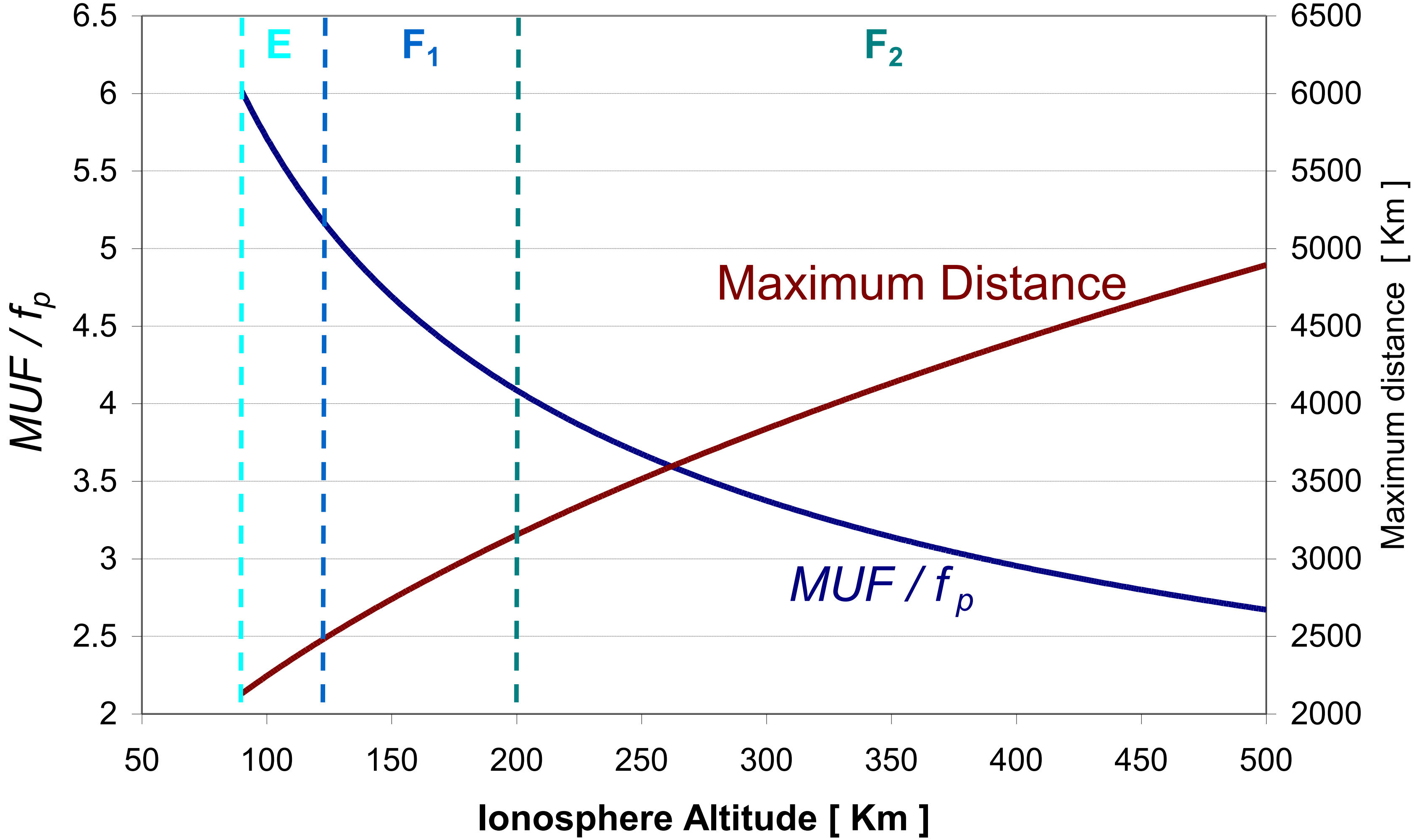
Maximum range

$$h_t = h_r \approx 0$$

$$\sin\left(\theta_{\max}\right) = \frac{R_{Earth}}{R_{Earth} + H_{\text{Ionosphere}}} \longrightarrow \theta_{\max} = \arcsin\left(\frac{R_{Earth}}{R_{Earth} + H_{\text{Ionosphere}}}\right)$$

$$l_{\max} = \left(\pi - 2\theta_{\max}\right) \cdot R_{Earth} = \left[\pi - 2 \cdot \underbrace{\arcsin\left(\frac{R_{Earth}}{R_{Earth} + H_{\text{Ionosphere}}}\right)}_{\theta_{\max}}\right] \cdot R_{Earth}$$

Maximum range vs. Altitude



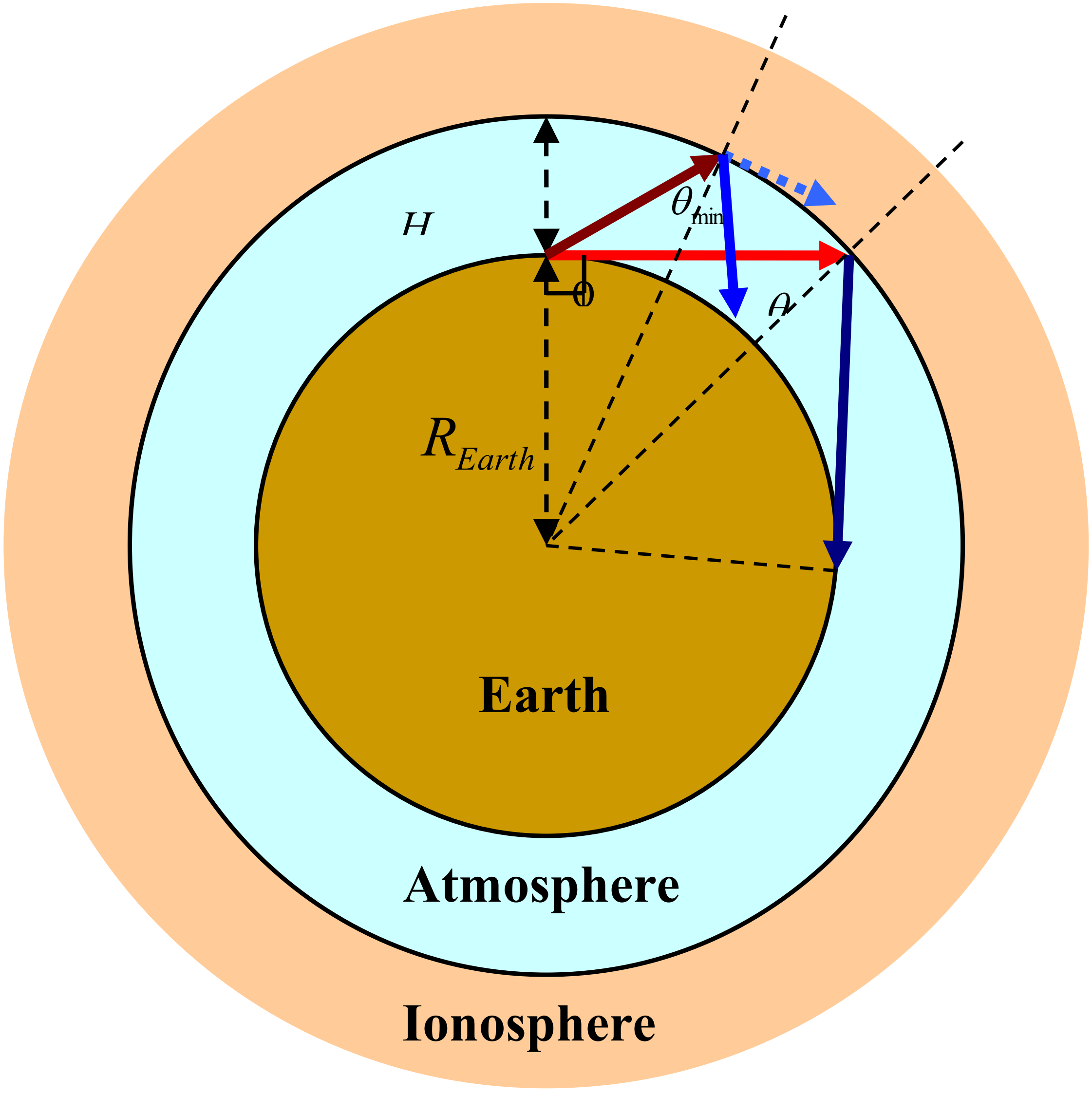
Important notes !

$$f_p \uparrow \longrightarrow MUF = \frac{f_p}{\cos(\theta_{\max})} \uparrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos(\theta_{\max}) \uparrow \longrightarrow MUF \downarrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow l_{\max} \uparrow$$

Minimum range



Minimum range

$$\sin(\theta_{\min}) = \sqrt{1-\left(\frac{f_p}{f}\right)^2} \longrightarrow \theta_{\min} = \arcsin\left[\sqrt{1-\left(\frac{f_p}{f}\right)^2}\right]$$

$$\frac{R_{Earth}}{\sin(\theta_{\min})} = \frac{R_{Earth}+H_{\text{Ionosphere}}}{\sin(\phi)} \longrightarrow$$

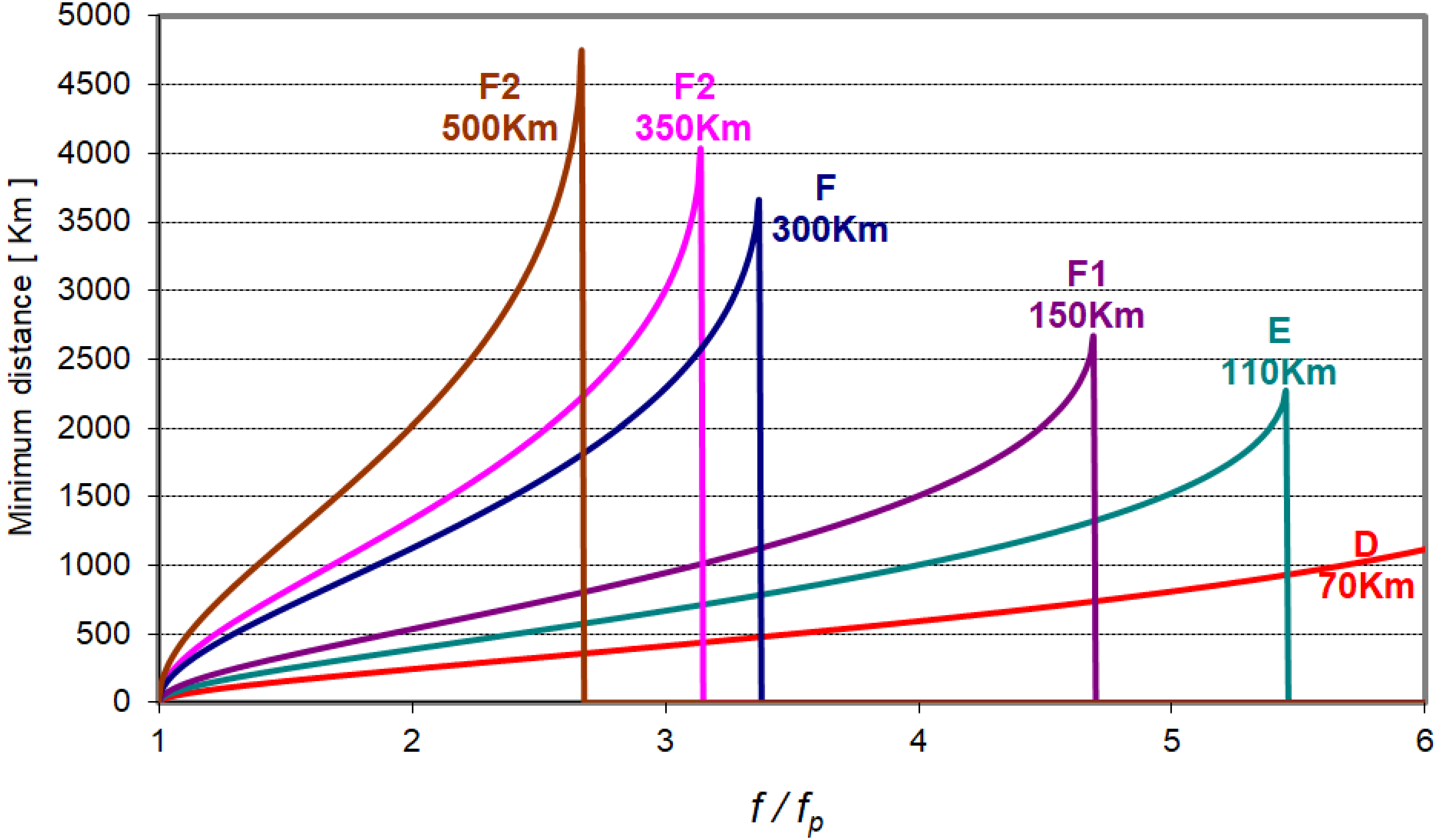
$$\longrightarrow \sin(\phi) = \underbrace{\frac{R_{Earth}+H_{\text{Ionosphere}}}{R_{Earth}}}_{1/\sin(\theta_{\max})} \cdot \sin(\theta_{\min}) = \frac{\sin(\theta_{\min})}{\sin(\theta_{\max})}$$

Skip distance

$$l_{\min} = 2\left(\pi - \theta_{\min} - \phi\right)\cdot R_{Earth} = 2\cdot\left\{\pi - \theta_{\min} - \underbrace{\arcsin\left[\frac{\sin\left(\theta_{\min}\right)}{\sin\left(\theta_{\max}\right)}\right]}_{\phi}\right\}\cdot R_{Earth} =$$

$$= 2\cdot\left\{\pi - \underbrace{\arcsin\left[\sqrt{1-\left(\frac{f_p}{f}\right)^2}\right]}_{\theta_{\min}} - \underbrace{\arcsin\left[\left(1+\frac{H_{\text{Ionosphere}}}{R_{Earth}}\right)\cdot\underbrace{\sqrt{1-\left(\frac{f_p}{f}\right)^2}}_{\sin\left(\theta_{\min}\right)}\right]}_{\phi}\right\}\cdot R_{Earth}$$

Minimum range vs. frequency



Important notes !

$$H_{\text{Ionosphere}} \uparrow \longrightarrow MUF \downarrow$$

$$f \uparrow \longrightarrow l_{\min} \uparrow$$

Summary

Time	Layer	Altitude	Electron density	Plasma (critical) frequency	Maximum Usable Frequency	Minimum distance	Maximum distance
		$H_{\text{Ionosphere}}$	N_e	$f_p = 8.977\sqrt{N_{e_{MAX}}}$	$MUF = \dfrac{f_p}{\cos(\theta_{\max})}$	$l_{\min}$	$l_{\max}$
Day	D	70Km	$4 \cdot 10^8$	180KHz	1.22MHz	518Km@500KHz	377Km
Day	E	110Km	$2 \cdot 10^{11}$	4.01MHz	21.9MHz	518Km@7MHz	321Km
Day	F1	180Km	$2 \cdot 10^{11}$	4.01MHz	17.3MHz	530Km@7MHz 1500Km@14MHz	2995Km
Day	F2	325Km	$2 \cdot 10^{12}$	12.7MHz	41.3MHz	304Km@14MHz 900Km@21MHz 1736Km@28MHz	3988Km
Day	F2	250Km	$4 \cdot 10^{12}$	18MHz	66MHz	306Km@21MHz 616Km@28MHz	3514Km
Night	E	100Km	$1.5 \cdot 10^9$	348KHz	1.99MHz		2244Km
Night	F	300Km	$5 \cdot 10^{10}$	2.01MHz	6.77MHz	904Km@3.5MHz	3838Km
Night	F	450Km	$5 \cdot 10^{11}$	2.54MHz	7.11MHz		4657Km


עם ישראל חי !