Sine-wave Audio Oscillator.

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1 The Problem:

In the November/December 2010 "*Morseman*" column I set this problem:

"See figure 1. This is a sine-wave audio oscillator, sometimes used for code practice. Unlike most simple oscillators, it has two phase-shifting networks in the feedback path. The right-hand amplifier is a unity-gain voltage follower, giving a low output impedance for driving the speaker, matched with a transformer. What will be the frequency of oscillation? What is the required theoretical gain of the left-hand amplifier, for oscillations to occur?"

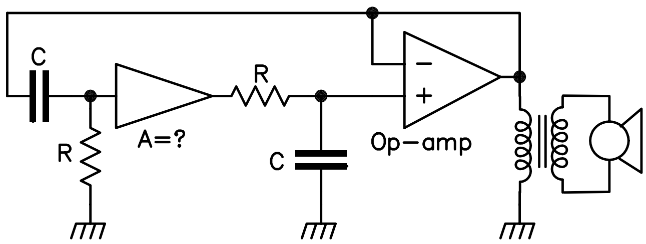

Figure 1: A Sine-wave Audio Oscillator.

2 Solution:

The oscillation condition for any linear circuit is given by equating the closed-loop gain to 1.

$$\text{or } A\beta = 1 \tag{1}$$
$$\text{where } A = \text{total gain of all amplifying elements inside the loop,} \tag{2}$$
$$\beta = \text{total transfer function of all attenuating } RC \text{ elements,} \tag{3}$$

The second op-amp is a unity-gain voltage follower, so all gain must be supplied by the first amplifier. The left and right-hand RC sections have voltage transfer functions given by the voltage division theorem as

$$A_1 = \frac{R}{R+1/sC} = \frac{sRC}{1+sCR} \tag{4}$$
$$A_2 = \frac{1/sC}{R+1/sC} = \frac{1}{1+sCR} \tag{5}$$
$$\text{where for convenience } s = j\omega. \tag{6}$$

The closed loop gain is given by the product of these, and the gain, A of the left-hand amplifier. Hence

$$A\left(\frac{sCR}{1+sCR}\right)\left(\frac{1}{1+sCR}\right) = 1 \tag{7}$$
$$\frac{sCRA}{(1+sCR)^2} = \frac{sCRA}{1+2sCR+s^2C^2R^2} = 1 \tag{8}$$
$$\frac{j\omega RCA}{1+2j\omega RC-\omega^2R^2C^2} = 1 \tag{9}$$
$$\text{where } s^2 = (j\omega)^2 = -\omega^2, \quad \text{since } j^2 = -1 \tag{10}$$

The numerator is imaginary, so the denominator must also be imaginary at the oscillation frequency, denoted by ω_o. This occurs when

$$1 - \omega_o^2 R^2 C^2 = 1 \quad (11)$$

$$\text{or } \omega_o = \frac{1}{RC} \quad (12)$$

$$\text{when the loop gain reduces to } \frac{j\omega_o RCA}{2j\omega_o RC} = \frac{A}{2} = 1 \quad (13)$$

$$\text{whence for oscillation, } A = +2. \quad (14)$$

An `LTspice` simulation of the circuit and its response is shown in figure 2.

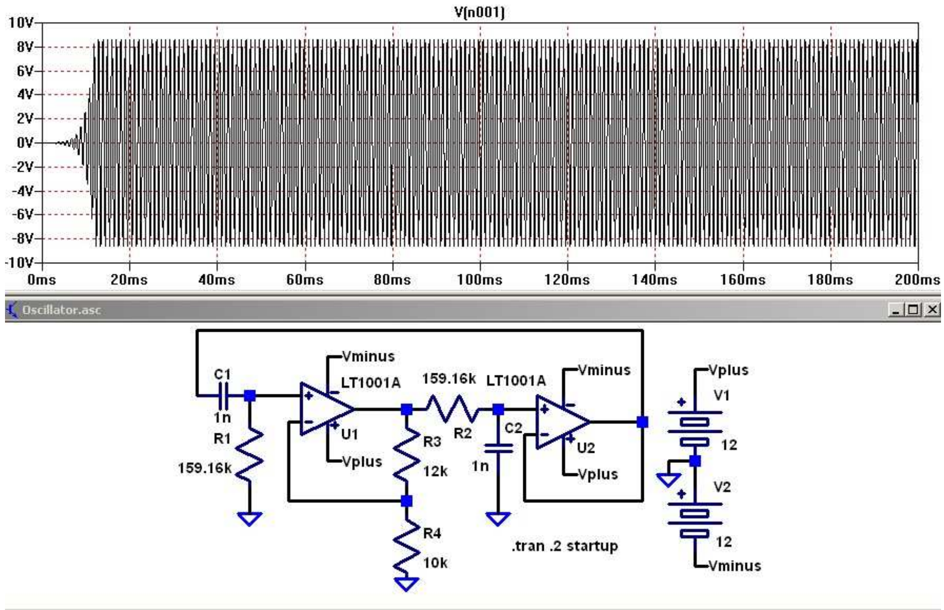

Figure 2: `LTspice` simulation of the oscillator.

Schematic: Both op-amps use the LT1001A model supplied with `LTspice`, with ±12V power supplies. The right-hand one is a voltage follower, the left a positive-gain amplifier, with gain given by

$$A_1 = 1 + \frac{R_3}{R_4} = 2.2 \quad (15)$$

This is somewhat greater than the theoretical value (2) to ensure that the oscillator starts reliably.

The upper window shows the voltage at the output of $U2$ after the power supplies turn on. Oscillation reaches its final magnitude, about 8.6V peak, after about 13 ms. The RC product has been chosen to give a theoretical oscillation frequency of 1 kHz, but inspection of its spectrum using the "FFT" function shows that it is nearer 980 kHz, about 2% low. This is typical of such sine-wave oscillators, where oscillation magnitude is determined by the non-linear characteristics of the op-amps. The second harmonic is about 32 dB down from the fundamental.

Even with this excess of loop gain, the output is still a very good sine wave, illustrating a nice feature of this circuit - it is easy to get a tone of good spectral purity, similar to that heard on an *RF* CW signal. Listening to this is more realistic than that from, for example, a 555 square-wave circuit.

Adding the transformer-coupled speaker to the output of $U2$ has little effect, since the voltage-follower has low output impedance. If a key is inserted in the grounded primary lead of the transformer (see figure 1), the oscillator can run continuously. A capacitor (1 - 100 nF) may be needed across its contacts to reduce clicks.