Two Op-amp Oscillator.

Gary ZL1AN: garyzl1an@gmail.com

1 The Problem:

In the January/February 2011 "*Morseman*" column I set this problem:

"See figure 1, left. What does this circuit do? What waveforms will be observed at each of the op-amp outputs? Can you find an analytic expression describing the frequency of these waveforms?"

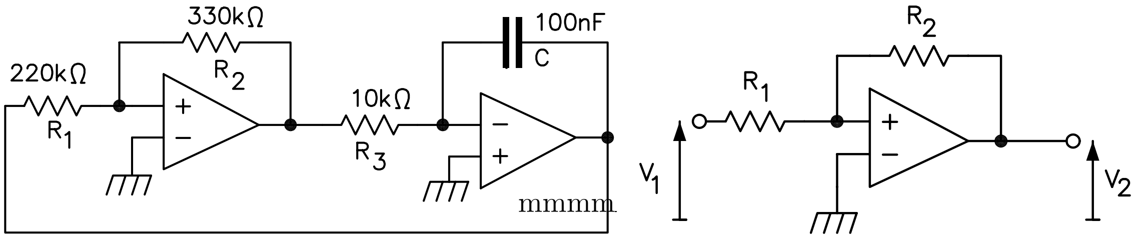

Figure 1: Left: The circuit as given. Right: The first stage, a Schmitt trigger.

2 Solution:

Since the first stage has *positive* feedback (applied to the non-inverting input), it's a Schmitt trigger, a non-linear circuit. The output has two stable states, which can be flipped from one to the other. The second stage is an operational integrator, a linear circuit.

The oscillation frequency of such circuits cannot be found by invoking the linear oscillator condition $A\beta = 1$. The trick in analysing them is to find a point at which the non-linear element changes state, and then deduce the time it will take for it to change state again. Since the Schmitt trigger has symmetrical state-change input voltages, this time will be *half* of the total oscillator period.

Assume ideal op-amps, operated from symmetrical positive and negative power supplies, where the outputs can swing between $+V_s$ and $-V_s$, $\pm V_s$, where V_s is typically about a volt less than the power supply voltages. T

The right-hand figure shows the first stage stage in isolation. Assume that the output voltage has assumed the positive stable state $V_2 = +V_s$. V_2 will be triggered into the negative stable state, $V_2 = -V_s$, if V_1 goes sufficiently negative to force the non-inverting input negative. This will start to happen when the voltage at this input (the "+" terminal on the op-amp symbol) goes from a positive voltage through zero voltage. We can find the voltage V_1 causing this by applying Millman's theorem:

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 0 \tag{1}$$

$$\text{whence } V_1 = -\frac{R_1}{R_2} V_s = -\frac{2}{3} V_s \tag{2}$$

$$\text{and } V_1 < -\frac{2}{3} V_s \quad \text{forces the negative state.} \tag{3}$$

Similarly, if the initial state is $V_2 = -V_s$,

$$\text{then } V_1 > +\frac{2}{3} V_s \quad \text{forces the positive state.} \tag{4}$$

Values of V_1 in between these limits will cause no state change.

Assume that the output of the first stage (Schmitt trigger) has just been caused to flip from the positive to the negative stable state. Then to force it to flip back again, the voltage V_2 must change by an amount

$$\Delta V_2 = 2\frac{R_1}{R_2}V_s = \frac{4}{3}V_s \tag{5}$$

The second stage is an operational integrator, a linear circuit. The voltage at the output of the Schmitt trigger will be $+V_s$. The non-inverting input of the integrator is at zero potential, a virtual earth, so the current flowing through R_3 towards the input, and on through the capacitor is

$$I_C = \frac{V_s}{R_3} = I_{R3} \quad \text{since no current can flow into the op-amp input.} \tag{6}$$

This constant current will cause the integrator output voltage to change by ΔV.

$$\text{since } V = \frac{Q}{C} \quad \text{(capacitor equation,)} \tag{7}$$

$$\text{a change in voltage of } \Delta V = \frac{\Delta Q}{C} = \frac{I_{R3}\Delta t}{C} \tag{8}$$

$$\text{rearranging, } \Delta t = \frac{C\,\Delta V}{I_{R3}} \tag{9}$$

Substituting from equations 5 and 6,

$$\Delta t = 2\frac{R_1}{R_2}V_s\left(\frac{1}{V_s/R_3}\right) = \frac{2R_1R_3}{R_2}C \tag{10}$$

But Δt is the time required to complete half a cycle of the oscillator waveform, so the waveform period, T, will be twice this, or

$$T = 2\Delta t = \frac{4R_1R_3}{R_2}C \tag{11}$$

$$\text{with corresponding frequency } f = \frac{1}{T} = \frac{R_2}{4R_1R_3C} \tag{12}$$

Substituting the component values from the schematic, we find that

$$f = 375\,\text{Hz} \tag{13}$$

The output of an `LTspice` simulation of the circuit is shown in figure 2.

The frequency measured from this simulation is 350 Hz, about 7% lower. This is because

- these are real op-amps, and the Schmitt trigger cannot change states instantaneously. Zooming into the rise and fall portions of the nominal square wave, we see that these actually take about $66\,\mu$ sec, corresponding to a slew rate of about 0.3 V/μ sec.
- There will also be small propagation delays between the input and corresponding output waveforms of the op-amps.

These account for the slightly lower frequency than that theoretically predicted.

Zooming the square wave display, we find that the outputs limit at $V_s \approx \pm 11\,\text{V}$, so the Schmitt trigger outputs limit at about a volt below and above the supply voltage ($\pm\,12\,\text{V}$).

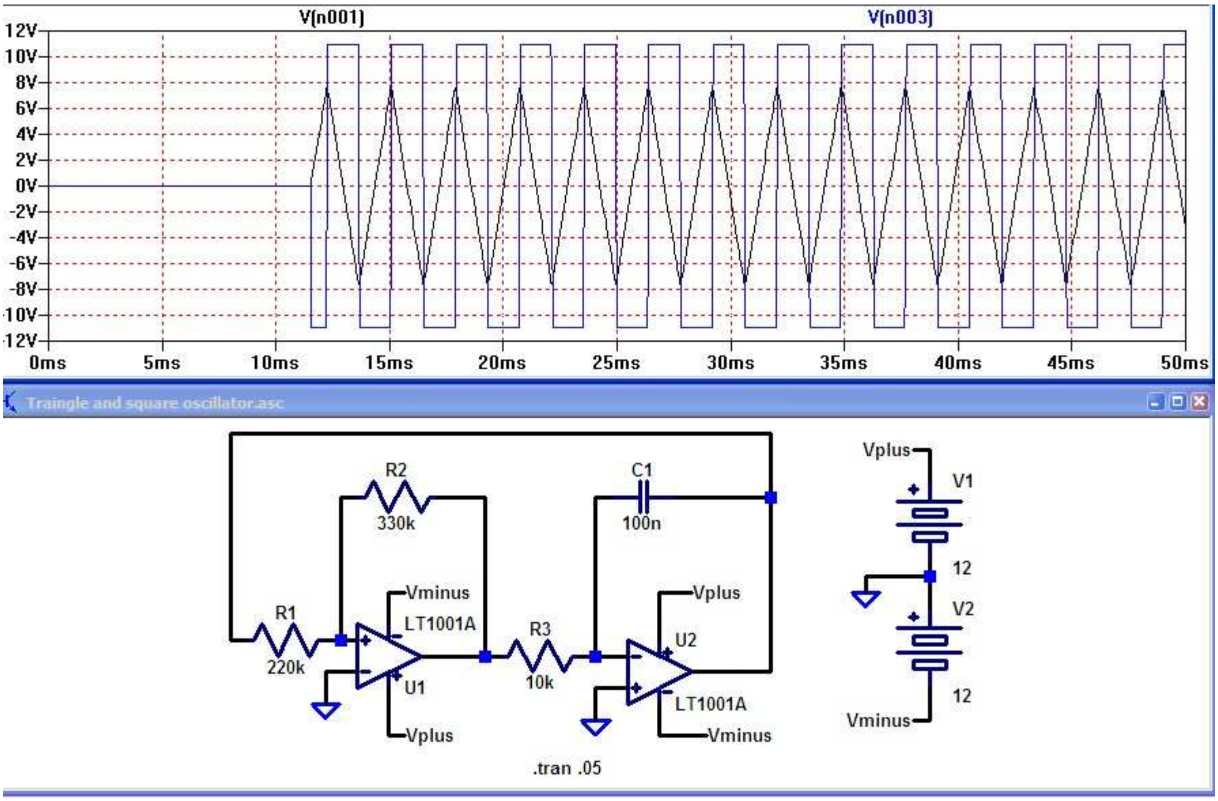

Figure 2: LTspice simulation of circuit. The square and triangle waves are the outputs of the first and second stages respectively.

Zooming the triangle wave display, we find that it limits at about $\pm 7.6\,\mathrm{V}$. Since $V_s \approx \pm 11\,\mathrm{V}$, we predicted that these limits would be

$$V_t \quad \approx \quad \pm \left(\frac{2}{3}\right) 11 = 7.33\,\mathrm{V} \tag{14}$$

The difference is again accounted for by the propagation delay of the second op-amp.

We also note that it takes a finite time for oscillations to start in the simulation. This is because I started it in the “steady state”, with power supply voltages already applied, and the outputs of both op-amps adjusted to zero by the simulation software. This state is also stable, so theoretically, nothing will happen!

In practice, as the simulation proceeds small voltages do build up in the circuit as a result of round-off error in `LTspice`’s software differential equation solvers, and these eventually cause it to transition to the oscillation state. If you start the simulation invoking the `LTspice` condition “Start external DC supply voltages at 0 V”, the simulation starts immediately, because sufficient internal voltage imbalance will occur as the power supplies rise.

With a “real” circuit, such internal voltages will always occur because of thermal Johnson noise in the resistors, so this oscillator will *always* start immediately.