A High input Impedance Amplifier

Problem in August/September 2012 *Morseman* column
Gary ZL1AN, garyzl1an@gmail.com

The problem as stated was:

"Figure 1 shows a transistor amplifier which gives a controllable voltage gain with input impedance higher than that of a standard common emitter stage. The voltage measured across R_e is $1\,V$, and V_{be} is $0.7\,V$.

"What is the dc voltage at the collector of the transistor? What is the predicted ac voltage gain, $A_v = V2/V1$? What is the quiescent base current? What is the advantage of biasing the transistor with a resistor from collector to base? Can you estimate the ac input resistance?"

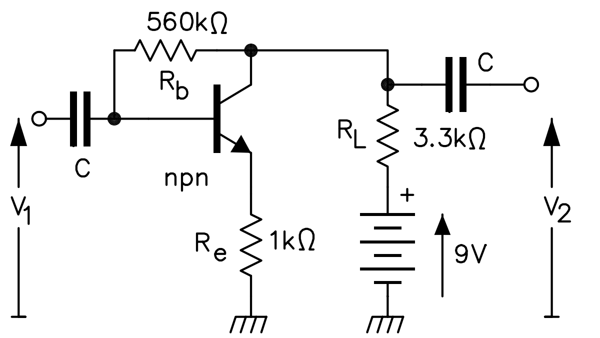


Figure 1: Amplifier problem in August/November 2012 column

Solutions: *dc* Collector voltage:

Let the supply voltage, $V_s = 9\,V$, and the *dc* Emitter and collector currents be I_e and I_c. Then

$$I_e = \frac{V_e}{R_e} = 1 \text{ mA}. \quad (1)$$

$$(I)_{RL} = I_e \quad \text{(since } I_b \text{ flows in both)} \quad (2)$$

$$V_c = V_s - (I)_{RL}R_L = 9 - 3.3 = 5.7\,V \quad (3)$$

The *ac* Voltage gain:

The same current flows in R_L and R_e. $V_{be} = 0.7\,V$ =constant, so if V_{in} increases by $1\,V$, V_{Re} will also increase by $1\,V$. The voltage across R_L will increase proportionately.

$$V_{RL} = V_{be}\frac{R_L}{R_e} = 3.3\,V \quad (4)$$

$$\text{so } A_v = \frac{V_{RL}}{V_{Re}} = -3.3 \quad (5)$$

The sign is negative because as V_1 increases, V_c decreases, since the battery end of R_L is held at a constant voltage. So it's an *inverting amplifier*.

Quiescent *dc* base current:

The collector end of R_b is at $5.7\,V$. The base end is at $V_{Re} + 0.7 = 1.7\,V$. Therefore the base current is given by

$$I_b = \frac{V_c - V_b}{R_b} = \frac{4}{560} = 7.14 \times 10^{-3}\ \text{mA} = 7.14\,\mu\text{A} \qquad (6)$$

The biasing method:

Figure 2 shows three methods of biasing a common-emitter amplifier. That shown in figure (a) is simple and economical of resistors, but not recommended, as R_b must be chosen to match the transistor's β. If another transistor having twice this β is substituted, the transistor will saturate. The design rule is

$$R_b = \frac{\beta(V_s - 0.7)}{R_L} \qquad (7)$$

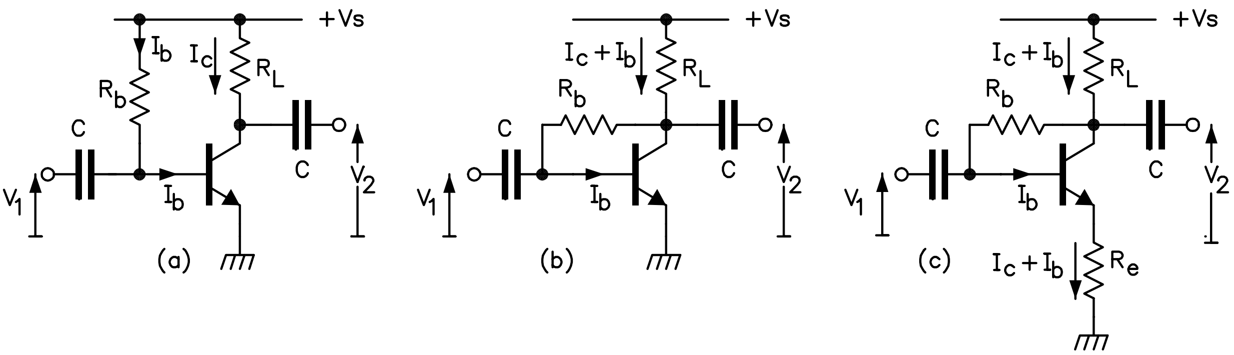


Figure 2: (a) Simplest amplifier. (b) Modified simple amplifier. (c) Modified amplifier with emitter resistance.

Figure (b) shows a variant, where R_b is returned to the collector. This applies negative shunt *dc* feedback, and is much more stable for β variations. The voltage gain is only slightly less than, but the input impedance about half that of circuit (a).[1] An approximate design rule is

$$R_b = \beta R_L. \qquad (8)$$

Figure (c) shows the modified circuit examined here, which adds negative series *dc* feedback with resistor R_e. We will show that this greatly increases the input impedance, and simultaneously reduces the voltage gain. A reasonable design rule is as for (b).

Numerical solutions:

Knowing I_v and I_b, we can calculate the *dc* beta of the transistor in the problem as:

$$\beta = \frac{I_c}{I_b} = \frac{1}{7.14 \times 10^{-3}} = 140. \qquad (9)$$

[1]See *"The simplest Transistor Amplifier"* on this web-page.

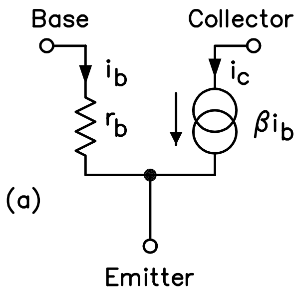
What happens now if another transistor is substituted with *twice* this value, or $\beta = 280$? To find out, equate the power supply voltage to the sum of the voltage drops across the transistor and resistors:

$$\begin{aligned}
V_s - 0.7 &= I_c R_L + I_b R_b + I_e R_e \quad \text{but } I_c = I_e & (10)\\
\text{so } V_s - 0.7 &= I_c \left(R_L + R_e + \frac{R_b}{\beta} \right) & (11)\\
I_c &\rightarrow \frac{V_s - 0.7}{R_L + R_e + \dfrac{R_b}{\beta}} = \frac{9 - 0.7}{3.3 + 1 + \dfrac{560}{280}} = 1.32 \text{ mA} & (12)\\
\text{and now } V_c &\rightarrow 9 - 3.3 \times 1.32 = 4.65\,V \quad \text{so } V_e \rightarrow 1.32\,V & (13)\\
\text{and } V_{ce} &\rightarrow 4.65 - 1.32 = 3.33\,V & (14)
\end{aligned}$$

Thus, instead of saturating, as in the simple biasing circuit, we would still have $3.33\,V$ across the transistor, so it would still be operating in the linear region.

The circuit input Resistance

To find this, we use the approximate small-signal *ac* equivalent circuit of figure 3(a).[2] It can be shown that r_b, the dynamic resistance of the base-emitter junction, is approximately β times the dynamic resistance of a diode conducting current[3]



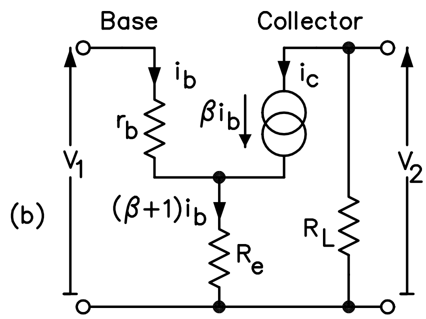


Figure 3: Small-signal model of the amplifier

Figure 3(b) shows this model inserted in the small-signal equivalent of the amplifier considered here.[4] We haven't included the base resistor R_b yet, but will include it below. The mesh equation of the input loop is

$$\begin{aligned}
V_1 &= i_b r_b + (\beta + 1) i_b R_e = i_b \left(r_b + (\beta + 1) R_e \right) & (15)\\
\text{so } r_{in} &= \frac{V_1}{r_b} = r_b + (\beta + 1) R_e & (16)\\
&\approx (\beta + 1) R_e \quad \text{since } r_b << (\beta + 1) R_e & (17)\\
r_{in} &\approx 140\,\text{k}\Omega & (18)
\end{aligned}$$

But this resistance is in parallel with that contributed by the biasing resistor, R_b. This is *not* just the value of R_b, because if the base end increases in voltage by $1\,V$, the collector end changes by $-A_v\,V$,

[2]This circuit is an approximation to the h-parameter circuit, with $h_{12} = h_r = 0$ (typically it's very small, about 10^{-4}) and $h_{22} = h_o = 0$, which assumes that the collector impedance much larger than R_L.

[3]The forward dynamic resistance of a diode conducting current I_d is approximately $1/(40 I_d)$ I_c, or $r_b \approx \dfrac{\beta}{40 I_c} = 3.5\text{k}\,\Omega$. We should really use h_{fe}, the *dynamic* small-signal current gain here, but we approximate it by β.

[4]The small-signal *ac* model grounds all constant voltage connections, since they can have no *ac* variation.

so the total voltage change across the resistor is $(|A_v| + 1)\,V$. The *effective* resistance contributed by R_b is then[5]

$$(R_b)_{eff} \;=\; \frac{R_b}{|A_v|+1} = \frac{560}{4.3} = 130\,\mathrm{k}\Omega \tag{19}$$

And the *total* input resistance is then $140\,\mathrm{k}\Omega$ in parallel with $130\,\mathrm{k}\Omega \approx 67\,\mathrm{k}\Omega$. However, including R_e has still dramatically increased the stage input impedance! For the three circuits of figure 2,

$$\begin{aligned}
(r_{in})_a &\approx 3.5\,\mathrm{k}\Omega && (20)\\
(r_{in})_b &\approx \frac{(r_{in})_a}{2} = 1.7\,\mathrm{k}\Omega && (21)\\
\text{but } (r_{in})_c &\approx 67\,\mathrm{k}\Omega && (22)\\
& && (23)
\end{aligned}$$

Thus, circuit (c) gives a much larger input impedance, but with the trade-off of much-reduced gain.

Measurements on a real circuit

I set up the circuit of figure 4. Since finding a transistor having the same parameters used in the problem was unlikely, I just used a handy BC549.[6] This transistor typically has a higher β, so the predictions will be different. Including R_t enables the input resistance to be measured.

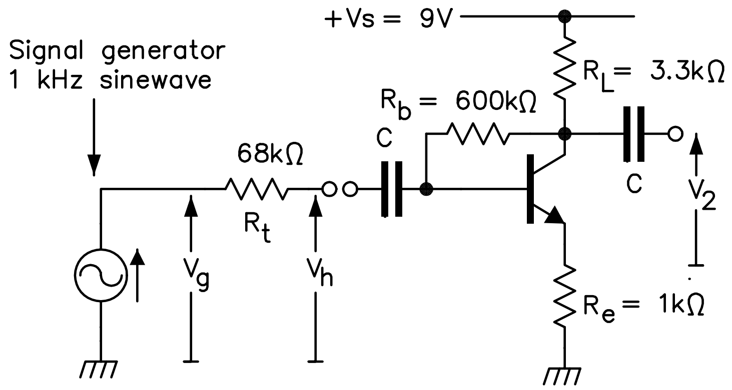


Figure 4: Test circuit used to validate measurements.

The *DC* voltages measured at the transistor terminals were:

$$\begin{aligned}
V_e &= 1.35\,V \quad V_c = 4.44\,V \quad V_b = 2.00\,V && (24)\\
\text{Therefore } I_c &= I_e = 1.35\,\mathrm{mA} && (25)\\
\text{We deduce that } I_b &= \frac{4.44-2}{600} = 4.07\times10^{-3}\,\mathrm{mA} = 4.07\,\mu\mathrm{A} && (26)\\
\text{so } (\beta)_{\text{deduced}} &= \frac{I_c}{I_b} = \frac{1.35}{4.07\times10^{-3}} = 331 && (27)\\
\text{and } (r_{in})_{\text{predicted}|} &= (1+\beta)R_e \parallel \frac{R_b}{|A_v|+1} \quad \text{with this measured beta value} && (28)\\
&= 332\times1 \parallel \frac{600}{4.3} \approx 100\,\mathrm{k}\Omega && (29)
\end{aligned}$$

[5] This is *Miller's Theorem.*

[6] I couldn't find a $560\,\mathrm{k}\Omega$ resistor, so I substituted three $1.8\,\mathrm{M}\Omega$ ones in parallel to make $600\,\mathrm{k}\Omega$.

To measure the gain and input impedance, I applied a 1 kHz sinewave from a signal generator and measured the resulting signal voltages with an oscilloscope. AC signal voltages measured were:

$$V_g = 190\,\mathrm{m}V \quad V_h = 120\,\mathrm{m}V \quad V_2 = 380\,\mathrm{m}V \tag{30}$$

$$\text{Therefore } A_v = \frac{380}{120} = 3.17 \tag{31}$$

This gain is slightly less than estimated from the approximate (but pretty good) expression $A_v = R_L/R_e$. A more accurate theoretical expression deduced using the equivalent circuit of figure 3 is

$$A_v = \frac{\beta R_L}{r_b + (\beta+1)r_b} = -3.23 \tag{32}$$

The input resistance, r_{in}, is deduced using V_g, V_h and R_t:

$$I_b = \frac{V_h - V_g}{R_t} = \frac{V_h}{r_{in}} \tag{33}$$

$$\text{whence } (r_{in})_{\text{measured}} = \frac{R_t}{V_g/V_h - 1} = \frac{68}{190/120 - 1} \approx 116\,\mathrm{k\Omega} \tag{34}$$

Thus summarising, the predicted and experimentally measured parameters for the circuit built were

Parameter	Predicted	Measured
Beta		331
A_v (voltage gain)	-3.3 (-3.23)	-3.17
r_{in}	$\approx 100\,\mathrm{k\Omega}$	$\approx 116\,\mathrm{k\Omega}$

The measured values are in good agreement (oscilloscope measurements are always approximate) with the theoretically estimated values. We see that this circuit gives a much higher input impedance than the "traditional" common-emitter configuration, with the trade-off of reduced voltage gain.

Direct Analysis from Small-signal Equivalent circuit.

The treatment above argues that the high input impedance arises from a combination of two opposing types of negative feedback, one of which decreases, while the other, which turns our to be the most significant, increases it. This may not have been completely convincing, so I'll now show the same results from a direct circuit analysis.

The circuit is shown again in figure 5(a). In (b), the complete small-signal equivalent. I've added the biasing resistor, R_b. Applying Kirchhoff's node and loop laws,

$$i_o = \frac{V_1 - V_2}{R_b} - \beta i_b \tag{35}$$

$$\text{At the input, } V_1 = i_b r_b + (\beta+1)\, i_b R_e \tag{36}$$

$$\text{The output voltage is } V_2 = i_o R_L \tag{37}$$

Voltage Gain:

Again we use the simplified h-parameter equivalent circuit of the transistor. Find the voltage gain, $A_v = V_2/V_1$, from the three simultaneous equations (35) to (37).

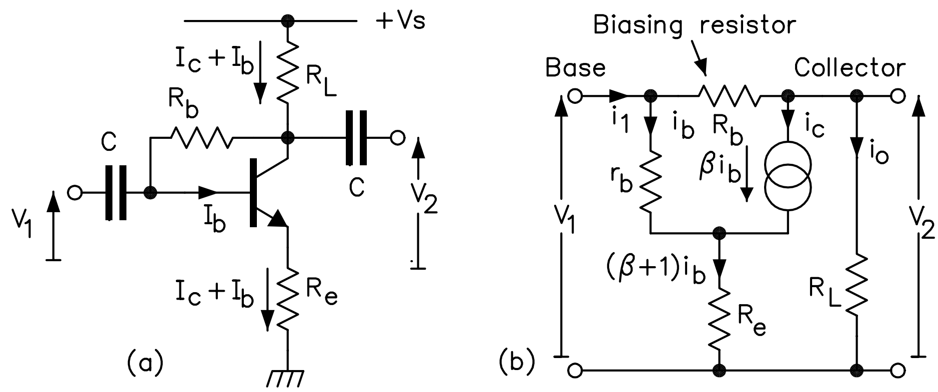


Figure 5: (a) The circuit. (b) Small-signal equivalent.

$$\text{from (36), } i_b = \frac{V_1}{r_b + (\beta+1)R_e} \tag{38}$$

$$\text{using (35), } i_o = \frac{V_1 - V_2}{R_b} - \frac{\beta V_1}{r_b + (\beta+1)R_b} \quad \text{and} \quad i_o = \frac{V_2}{R_L} \tag{39}$$

$$\text{rearranging, } V_2\left[\frac{1}{R_L} + \frac{1}{R_b}\right] = V_1\left[\frac{1}{R_b} - \frac{\beta}{r_b + (\beta+1)R_e}\right] \tag{40}$$

$$V_2\,(R_L + R_b] = V_1\left[R_L - \frac{\beta R_L R_b}{r_b + (\beta+1)R_e}\right] \tag{41}$$

$$\frac{V_2}{V_1} = A_v = \frac{1}{1 + \dfrac{R_b}{R_L}} - \left[\frac{\beta R_L R_b}{R_L + R_b}\right]\left[\frac{1}{r_b + (\beta+1)R_e}\right] \tag{42}$$

Consider the first term on RHS:

$$R_b = 560\,\mathrm{k}\Omega, \quad R_L = 3.3\,\mathrm{k}\Omega \tag{43}$$

$$\text{then } \frac{1}{1 + \dfrac{R_b}{R_L}} \approx 5.9 \times 10^{-3} \quad \rightarrow \quad \text{much smaller than second term, ignore!} \tag{44}$$

$$\text{also } R_b >> R_L, \quad (\beta+1)R_e >> r_b, \quad \text{and } R_e, r_b \text{ are same order of magnitude.} \tag{45}$$

$$\text{then } A_v \approx -\frac{\beta R_L}{(\beta+1)R_e} \quad \text{and } \beta >> 1 \tag{46}$$

$$\text{so } A_v \approx -\frac{R_L}{R_e} \quad \text{as from the simple treatment.} \tag{47}$$

Input Impedance:

$$\text{At the input node, } i_1 = \frac{V_1}{r_b + (\beta+1)R_e} + \frac{V_1 - V_2}{R_b} \tag{48}$$

$$= \frac{V_1}{r_b + (\beta+1)R_e} + \frac{V_1(1 - A_v)}{R_b} \tag{49}$$

$$y_{in} = \frac{V_1}{i_1} = \frac{1}{r_b + (\beta+1)R_e} + \frac{1 - A_v}{R_b} \tag{50}$$

$$\tag{51}$$

This is the admittance of two parallel resistors, R_a and R_b, where

$$R_a = rb + (\beta+1)R_e \quad \text{and} \quad R_b = \frac{1 - A_v}{R_b} \tag{52}$$

$$\text{but } A_v \approx -\frac{R_L}{R_e} \quad \text{and } (\beta+1)R_e >> r_e \tag{53}$$

$$\text{so } r_{in} \approx (\beta+1)R_e \parallel \frac{R_b}{1+R_L/R_b} \quad \text{as before} \tag{54}$$

Suggested Circuit design Procedure:

My experience is that the values of R_L and R_e used in the problem give a reasonable combination of high input impedance, with some gain. Including R_L is necessary to give some stability against changes in β.

1. Choose a transistor with a reasonably high β. Find its typical value from data sheets.
2. Using this β value, calculate $R_b = \beta R_L$.
3. The predicted voltage gain, $A_v \approx -\dfrac{R_L}{R_e}$
4. The predicted input impedance is $\quad r_{in} \approx (1+\beta)R_e \parallel \dfrac{R_b}{|A_v|}$

If the value of β assumed is accurate, this design gives approximately equal voltages across R_L and the transistor. If they are too unequal, increase/reduce the value of R_b if the voltage across the transistor is too low/high.

Conclusions

- This circuit gives a much higher input impedance than can be achieved with the "standard base voltage-divider with by-passed emitter resistor" circuit, at the expense of reduced (but accurately determined) voltage gain.
- The circuit is relatively stable with changes in transistor β.
- Theoretical predictions agree well with measurements on a test circuit.

Appendix: Types of Negative Feedback

The classical feedback equation is

$$\begin{aligned} A_f &= \frac{A}{1-A\beta} \\ \text{where } A \text{ and } A_f &= \text{the voltage gains without and with feedback,} \\ \beta &= \text{the feedback fraction.} \end{aligned}$$

Real circuits usually look nothing like the stylized diagram used to derive this equation, and it's usually difficult to deduce what A and β are, see this reference.[7] Neither does this this equation give any information about input and output impedance changes. How these are determined is somewhat confusingly covered with different nomenclature in different references. It turns out that there are 4 possible topologies, summarized in the reference above, which shows sample configurations. You can have either *voltage or current output sampling*, combined with *voltage or current input summing.* See also this Wikipedia reference.[8] Full analysis shows that:

[7] Google on *Inspection analysis of feedback circuits* for a Berkley University reference. There are many others.
[8] Google on *Wikipedia: negative feedback amplifier.*

- Voltage/current output sampling always reduces/increases the output impedance.
- Series voltage input summing/shunt input current summing always increases/reduces the input impedance.

The circuit analysed here is interesting because it simultaneously implements two of these four possible feedback topologies, which change the input impedance in opposite directions.

- The $1\,\mathrm{k}\Omega$ emitter resistor applies *series voltage feedback summing*, which *increases* the input resistance.
- The $560\,\mathrm{k}\Omega$ biasing resistor applies *shunt current feedback summing*, which *decreases* the input resistance.

Here, the effect of the voltage feedback is greater than that of the current feedback, so the net effect is to increase the input impedance, which in a "standard" common-emitter circuit would be only a few kilohm (the input resistance of the transistor) in parallel with the effective resistance of the biasing chain (maybe a few tens of kilohm).

A retrospective note: When I published the solution to this problem in the *Morseman* column, I said that as far as I knew, it was original, since I had not seen it anywhere else.

But John, ZL2AST, drew this reference to my attention.

`http://www.talkingelectronics.com/projects/200TrCcts/200TrCcts.html`

This configuration in used in the "Guitar Fuzz" circuit (search the document for "fuzz") but without any analysis or comment on its properties. John also sent me pages from the 1964 GE Transistor Manual where it's also shown.

The "talkingelectronics" page referenced above is, incidentally, a most useful reference with a great variety of simple circuits. It also contains a nice introduction to transistors, and how to test them. A very good resource if you're teaching a class of electronics beginners!

73, Gary ZL1AN