Parabolic Paradox

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Since its invention by Heinrich Rudolph Hertz nearly a century ago, the parabolic reflector has emerged as one of the most important microwave antennas. But microwave hams are frequently confused by the complex nomographs and elaborate equations used to determine parabolic antenna gain and beamwidth. In this article, Professor Shuch derives simplified rules of thumb for predicting parabolic antenna performance.

Have you ever tried to determine the gain and beamwidth of a parabolic reflector antenna using the equations published in the standard engineering handbooks? No doubt the plethora of units, dimensions and fudge-factors you encountered was enough to make you seek refuge in a ones-and-zeroes factory. After you've plugged in antenna diameter in feet, frequency in megahertz, illumination efficiency in percent and focal length in inches, you'll no doubt have to throw in a correction for the phase of the moon before receiving results that not only bear scant resemblance to reality, but defy intuitive interpretation entirely.

Graphical solutions are no better. If you draw a straight line on the appropriate nomograph, between the vertical scale on the right (representing antenna size) and that on the left (corresponding to operating frequency), chances are your line doesn't even intersect the scales in the middle—the ones that are supposed to show gain and beamwidth. There has to be a better way!

Wherefore Gain?

If you're accustomed to working with amplifiers, you may be wondering how an antenna (obviously a passive device) can exhibit gain in the first place. There's just no way its power out can exceed its power in—in either direction. (Actually, by the "power out over power in" definition, even attenuators have gain. It's just that their gain will likely be less than unity.) When we talk antenna gain, we are really comparing the performance of a particular antenna to that of a specified reference, usually an isotropic point source.

In receiving service, an antenna's gain is related to its ability to scoop up more energy than the reference antenna; hence, gain is related to the antenna's surface area (or more properly, its effective aperture). In transmitting service, gain relates to an antenna's ability to focus more energy *in a given direction* than the reference, and this focusing action also depends on aperture. Antennas are bilateral, so given a proper impedance match, an antenna's gain in transmitting and receiving service will be the same. And that gain relates to the size of the antenna. The bigger the better (until it falls over).

> ***"As soon as I had succeeded in proving that the action of an electric oscillation spreads out as a wave in space, I planned experiments with the object of concentrating this action and making it perceptible at greater distances by putting the primary conductor (ie, dipole) in the focal line of a large concave parabolic mirror."***
> ***— Heinrich R. Hertz (circa 1890)***

How Wide the Beam?

Since any antenna achieves its gain by focusing energy in a narrow, directed beam, there will obviously be an inverse correlation between beamwidth and gain. Beamwidth is traditionally defined as the angle, in degrees or radians, measured between the two points on either side of the antenna's main lobe (or bore sight) at which the received signal power at a fixed distance is exactly half (3 dB less than) the maximum received power. You might think that an isotropic radiator would exhibit a beamwidth of 360 degrees. But remember, if it's truly omnidirectional, it has *no* −3 dB points, thus no definable beamwidth.

Another thing to watch out for is that some antenna users find it more convenient to measure the angle between bore sight and one 3-dB point, on one side of the antenna's pattern. This angle is properly referred to as half-power *half* beamwidth (not to be confused with half-power beamwidth).

The main point of all this is the relationship between beamwidth and gain. Since large antennas exhibit high gains, they exhibit narrower beamwidths than smaller antennas, all else being equal.

What About Wavelength?

But what, exactly, do we mean by "large" antennas? An antenna's electrical size may bear little relation to its physical size, and what constitutes a "large" (hence narrow-beam and high-gain) antenna at microwaves may prove a "small" (wide-beam, low-gain) antenna at VHF. High-gain, narrow-beam antennas tend to be large *relative to their operating wavelength*. Obviously, as you go higher in frequency, an electrically "large" antenna can be physically smaller. I find it easiest to define a parabolic reflector's size, for example, in terms of its diameter, measured not in feet, meters, inches or angstroms, but rather in *wavelengths* at the intended operating frequency.

Deriving an Equation

Let's start by defining an antenna's *voltage gain* (A_v) as the EMF induced into a particular antenna, measured at a given distance from a specified source, as compared to the EMF induced into an isotropic reference matched to the same impedance. For reasons that I'll not bother to derive here (we academics say it's "beyond the scope of this course"), the voltage gain of a perfectly matched, uniformly illuminated, ideal parabolic dish exactly equals its circumference, measured in *wavelengths*. That is

$$A_v = \frac{\pi D}{\lambda} \qquad \text{(Eq 1)}$$

where

A_v = voltage gain as defined above (a unitless ratio)
D = the antenna's diameter (πD is the antenna's circumference)
λ = the operating wavelength.

Note that if wavelength and diameter are specified in the same units (and they must to be dimensionally consistent), those units cancel, and voltage gain ends up, as it should, a unitless ratio.

Eq 1 is the fundamental relationship from which all of the published equations and nomographs you're familiar with are derived. By getting to the source, perhaps we can strip away some of the mystique.

Generally, we are more interested in an antenna's power gain than its voltage gain. If impedance remains unchanged, power ratio always varies with the square of voltage ratio. Because of this relationship, we can say that the power gain of an ideal parabolic antenna equals the square of its circumference (measured in wavelengths). Mathematically,

$$A_p = \left[\frac{\pi D}{\lambda}\right]^2 \qquad \text{(Eq 2)}$$

where A_p represents power gain, and the other literals are as defined for Eq 1. Note that Eq 2 is sometimes written

$$A_p = \frac{\pi^2 D^2}{\lambda^2}$$

which means the same thing.

So far, we have assumed ideal, perfectly matched feeds and uniformly illuminated parabolic reflectors. Of course, the real world isn't like that, and the power gain of an actual antenna will be something less than we've predicted above. The illumination efficiency of an antenna is a factor between 0 and 1, which we abbreviate η (the Greek letter *eta*). Applying the efficiency factor to Eq 2 gives us

$$A_p = \frac{\eta \pi^2 D^2}{\lambda^2} \qquad \text{(Eq 3)}$$

which is a more realistic indicator of expected antenna gain—if you chose the right η. The actual value of η depends on the dish surface accuracy and material, focal-length-to-diameter ratio and feed system design. Since the 1950s, the industry standard efficiency factor for parabolic antennas feed with flared rectangular waveguide feed horns has been assumed to equal about 55 percent (0.55). Advances in feed-horn technology in recent years have probably raised the typical efficiency of commercial antennas to something closer to 60%.

Finally, I am assuming you wish to express antenna gain in decibels. Since we already know the antenna's power gain, it's a simple matter to take ten times its common logarithm:

$$A_p(\text{dB}) = 10 \log \left[\frac{\eta \pi^2 D^2}{\lambda^2}\right] \qquad \text{(Eq 4)}$$

which is one of the very textbook equations I challenged at the outset. But doesn't it make more sense, now that you know where it came from?

Ditto for Beamwidth

We've already established that beamwidth varies inversely with voltage gain, so it shouldn't be difficult to derive a beamwidth equation. It turns out that the 3-dB beamwidth of a dish, in *radians*, equals the reciprocal of the dish's diameter, again measured in *wavelengths*. (Let's see . . . that's the same as the wavelength measured in diameters, isn't it?) For those of you who like equations

$$\theta(\text{rad}) \approx \frac{\lambda}{D} \qquad \text{(Eq 5)}$$

where wavelength and diameter are measured in the same units.

You say you have trouble with radians? Convert to degrees! See reference 1, or simply multiply by 180, then divide by π.

Scaling from a Rule of Thumb

The equations just derived let you compute the gain and beamwidth of any dish, of any size, at any frequency. Their derivation may help you to make some sense out of what the numbers mean. But if you suffer from acute math anxiety, fear not! Simply scale from a known dimension. The secret is to define the diameter of your particular dish not in inches, feet, meters or miles, but rather in *wavelengths.*

As a starting point, simply memorize this relationship: 40 wavelengths equals 40 dB. Translation: a parabolic reflector 40 wavelengths across at a given frequency, if illuminated at 60% efficiency, exhibits roughly +40 dBi of gain. Check it out using Eq 4, and you'll see this rule of thumb approximates to within better than a quarter of a decibel.

As for beamwidth, the same 40-wavelength dish has a beamwidth of about 1/40 radian, or 25 milliradians—roughly 1.5 degrees. That was easy, wasn't it?

Now, scale away to your heart's content. Dishes only half as large (that is, 20 wavelengths across) have half the voltage gain, hence one quarter the power gain, which is 6 dB less, or +34 dBi. Consequently, our 20-wavelength dish has twice the beamwidth—about 3 degrees —at the half-power points. For larger antennas, the correction goes in the other direction.

Next time you're walking down the street and spot a TVRO dish, you too can amaze your friends by proclaiming, "40 dB of gain, one and a half degrees of beamwidth." When they ask you how you figured it out without even a slide rule, answer "magic." Or hand them this issue of *QEX*.

[1]H. P. Shuch, "Radian Review," *RF Design*, Mar 1986, p 57.

Bits

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