# Innovative demodulation method for SSB technique

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Abstract: A single sideband (SSB) demodulation method is described, in which frequency mixing is not required. The proposed method is based on the phase shift method, but no local oscillator is employed. The IQ demodulator is replaced with an envelope detector and an FM demodulator. It can replace the synchronous demodulation method, which requires an expensive phase-locked loop for carrier recovery. Not only is less interference encountered in the proposed circuit, but the production costs of the SSB and independent sideband receiver are low. Furthermore, the Doppler effect is not a serious problem since the carrier transmitted by the transmitter is used for demodulation.

# 1 Introduction

In 1987, WARC decided on a transition from double sideband to single sideband (SSB) emissions in HF broadcasting. All double sideband broadcasting would have to cease by 31 December 2015. Within the transition period, some designers have introduced the compatible SSB transmitter to preserve existing low-cost AM radio receivers [1, 2]. Concerning the receiving side, synchronous demodulation of SSB has been introduced [3]. However, the manufacturing cost of the receivers is expensive if synchronous detection is used with a phase-locked loop and a high Q bandpass filter. Furthermore, if independent sideband (ISB) broadcasting [4] is promoted, then existing AM radio receivers have to be phased out.

In this paper low production cost SSB/ISB demodulator is introduced, in which no additional local oscillator (LO) is required, to simplify the circuit and overcome the problem of the high production cost. This demodulator can also be used to demodulate existing double sideband AM signals.



# Fig.1 Spectrum of the SSB signals with common carrier

## 2 SSB waveform

The spectrum of the SSB signals with a common carrier frequency is shown in Fig. 1. The waveform expression of the SSB signals is given in the following:

$$f(t) = A\cos(\omega_C t) + \frac{m_U A}{2}\cos(\omega_C t + \omega_U t + \theta_U)$$

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$$+\frac{m_L A}{2}\cos(\omega_C t - \omega_L t + \theta_L) \tag{1}$$

 $A \cos(\omega_C t)$  is the common carrier;

where

 $m_{II}$  is the upper sideband modulation index;

 $m_U A/2 \cos(\omega_C t + \omega_U t + \theta_U)$  is the upper sideband signal;

 $\theta_U$  is the upper sideband phase difference relative to the carrier;

 $m_L$  is the lower sideband modulation index;

 $m_L A/2 \cos(\omega_C t + \omega_L t + \theta_L)$  is the lower sideband signal; and

 $\theta_L$  is the lower sideband phase difference relative to the carrier.

The expression of the envelope is shown below:

$$f(t) = AB\cos(\omega_C t - \phi) \tag{2}$$

where

$$B = \left\{ \left[ 1 + \frac{m_U}{2} \cos(\omega_U t + \theta_U) + \frac{m_L}{2} \cos(\omega_L t - \theta_L) \right]^2 + \left[ \frac{m_L}{2} \sin(\omega_L t - \theta_L) - \frac{m_U}{2} \sin(\omega_U t + \theta_U) \right]^2 \right\}^{\frac{1}{2}} \tan^{-1}\phi = \frac{\frac{m_L}{2} \sin(\omega_L t - \theta_L) - \frac{m_U}{2} \sin(\omega_U t + \theta_U)}{1 + \frac{m_U}{2} \cos(\omega_U t + \theta_U) + \frac{m_L}{2} \cos(\omega_L t - \theta_L)}$$

When  $m_U \ll 1$  and  $m_L \ll 1$ 

$$f(t) \approx A \left[ 1 + \frac{m_U}{2} \cos(\omega_U t + \theta_U) + \frac{m_L}{2} \cos(\omega_L t - \theta_L) \right]$$
$$\cdot \cos \left[ \omega_C t + \frac{m_U}{2} \sin(\omega_U t + \theta_U) - \frac{m_L}{2} \sin(\omega_L t - \theta_L) \right]$$
(3)

The mathematical derivation of eqns. 1-3 is presented in the Appendix.

# 3 Conventional methods of SSB demodulation

There are three classical methods for SSB demodulation.

### 3.1 Filter method

As shown in Fig. 2, bandpass filters are used as the sideband filter before synchronous demodulation. The filter

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first suppresses one sideband and lets the other sideband be demodulated using a product detector. For example, the lower sideband is suppressed if demodulation of the upper sideband is desired. The performance of the demodulator is very dependent on the selectivity of the sideband filter.



Fig.2 Filter method



Fig. 3 Phase-shift method

# 3.2 Phase-shift method

As shown in Fig. 3, the SSB signal is first synchronously detected by an IQ mixer. The detected signal is then phase equalised by the  $\alpha$  and  $\beta$  network. Outputs of the phase equalisers maintain a constant phase difference of 90° within the desired frequency band. Summing the two phase-equalised signals gives the upper sideband baseband signal, and subtracting them from one another gives the lower sideband baseband signal.

# 3.3 Weaver method

The Weaver method [5] is shown in Fig. 4. Two IQ mixers are used to avoid the need for a wideband 90° phase shifter. The incoming SSB signal is first converted to zero frequency. This makes the upper audio sideband of one sideband extend over equal positive and negative frequency bandwidths. The unwanted sideband is then removed by a low-frequency filter with a sharp roll-off characteristic. The baseband is finally recovered by the following IQ frequency mixer and voltage summer/subtracter.



Fig.4 Weaver method

# 4 Theory of proposed SSB demodulator

From eqn. 3, when the modulation index of both sidebands is low enough, the envelope of the modulated signal contains the vector sum of the two baseband signals and the phase is modulated by the two basebands of vector subtraction. Using this property, the demodulator illustrated in

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Fig. 5 can be used to retrieve the two baseband signals individually.



Fig.5 Proposed demodulator

The waveforms and operation of the proposed demodulator can be explained by the following expressions:

$$g_1(t) = A \left[ 1 + \frac{m_U}{2} \cos(\omega_U t + \theta_U) + \frac{m_L}{2} \cos(\omega_L t - \theta_L) \right]$$
(4)

$$g_2(t) = \frac{m_U A}{2} \sin(\omega_U t + \theta_U) - \frac{m_L A}{2} \sin(\omega_L t - \theta_L)$$
(5)

$$g_{3}(t) = A \left[ 1 + \frac{m_{U}}{2} \cos(\omega_{U}t + \theta_{U} + \alpha) + \frac{m_{L}}{2} \cos(\omega_{L}t - \theta_{L} + \alpha) \right]$$
(6)

$$g_4(t) = \frac{m_U A}{2} \sin(\omega_U t + \theta_U + \beta) - \frac{m_L A}{2} \sin(\omega_L t - \theta_L + \beta)$$
(7)

$$\begin{split} i(t) &= g_3(t) + g_4(t) \\ &= A + \frac{m_U A}{2} \cdot \sqrt{2 \left[1 - \sin(\alpha - \beta)\right]} \\ &\quad \cdot \cos\left[\omega_U t + \theta_U + \tan^{-1}\left(\frac{\sin\alpha - \cos\beta}{\cos\alpha + \sin\beta}\right)\right] \\ &\quad + \frac{m_L A}{2} \cdot \sqrt{2 \left[1 + \sin(\alpha - \beta)\right]} \\ &\quad \cdot \cos\left[\omega_L t - \theta_L + \tan^{-1}\left(\frac{\sin\alpha + \cos\beta}{\cos\alpha - \sin\beta}\right)\right] \end{split}$$

$$q(t) = g_{3}(t) - g_{4}(t)$$

$$= A + \frac{m_{U}A}{2} \cdot \sqrt{2 \left[1 + \sin(\alpha - \beta)\right]}$$

$$\cdot \cos \left[\omega_{U}t + \theta_{U} + \tan^{-1}\left(\frac{\sin\alpha + \cos\beta}{\cos\alpha - \sin\beta}\right)\right]$$

$$+ \frac{m_{L}A}{2} \cdot \sqrt{2 \left[1 - \sin(\alpha - \beta)\right]}$$

$$\cdot \cos \left[\omega_{L}t - \theta_{L} + \tan^{-1}\left(\frac{\sin\alpha - \cos\beta}{\cos\alpha + \sin\beta}\right)\right]$$
(9)

The phase equaliser  $\alpha$  and  $\beta$  are chosen such that the phase difference  $(\alpha - \beta)$  is 90° in the desired frequency band. Hence, the demodulated signals can be expressed in the following:

$$i(t) = A + m_L A \cdot \cos \left[ \omega_L t - \theta_L - \tan^{-1}(\cot \beta) \right]$$
(10)

$$q(t) = A + m_U A \cdot \cos\left[\omega_U t + \theta_U - \tan^{-1}(\cot\beta)\right]$$
(11)

(8)

Table 1: Component requirements and characteristics of demodulatio	n
methods	

Method used	Oscillator	Filter	Mixer	Phase shifter
Filter method (Fig. 3)	1	2	2	not required
Weaver method (Fig. 4)	2	2	4	2 single frequency
Phase shift method (Fig. 5)	1	not required	2	2 wideband 1 single frequency
Proposed method (Fig. 6)	not required	not required	not required	2 wideband
Method used	PLL	Vector summer/subtractor		Others
Filter method (Fig. 3)	1	not required		not required
Weaver method (Fig. 4)	2	2		not required
Phase shift method (Fig. 5)	1	2		not required
Proposed method (Fig. 6)	not required	2		envelope, FM detector, integrator

The baseband of the lower sideband signal is retrieved as shown in eqn. 10 and the upper is obtained as in eqn. 11. This results in an additional phase shift of  $-\tan^{-1}(\cot \beta)$  in the baseband signals.

# 5 Comparison of all existing demodulation methods

The advantages of the proposed method are clearly shown in Table 1. In the proposed method, an additional LO is not required, and hence additional interference is prevented in the receiver and the production cost is low. The signalto-noise ratio (SNR) of the demodulated signal is inferior to other methods since it is a non-coherent demodulation technique. However, when the input is of a high SNR, its performance may be competitive with others.

# 6 Experimental verification

The circuit of the proposed demodulator was built according to the block diagram shown in Fig. 5. The broadband phase equalisers,  $\alpha$  and  $\beta$  network was implemented using five-stage all-pass filters. The bandwidth was chosen as 19kHz since the audio frequency ranges from 20Hz to 20kHz. The time constant of the integrator was taken to be 700µs. The maximum modulation index of the modulated signal was set to be 35% [6]. The measured performance of the demodulator is shown in Figs. 6–9.



## 7 Discussion

From the experimental results shown in Figs. 6–9, we can see that the separation at the upper channel is not less than 25dB and that at the lower channel is greater than 10dB. The separation curves rise as frequency increases in the upper channel, but fall as frequency decreases in the lower



channel. This characteristic is mainly caused by the changing of the phase difference between the envelope detector and the frequency demodulator output. Another reason is that the integrator is not ideal at a low frequency. It is pos-

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sible to improve the channel separation if more effort is spent on the refinement of the  $\alpha$  and  $\beta$  phase equalisers.

Furthermore, the separation is degraded with the high modulation index since the harmonic signal generated by the envelope detector cannot be suppressed with this demodulation technique. It is hence not suitable for using in a system with a high modulation index. The modulation index of 35% is a good critical level, as shown in the experimental results. The total harmonic distortion (THD) at both channel output is not greater than 4%. Distortion of the demodulated signals is mainly caused by the nonlinearity of the envelope detector, which is the disadvantage of this demodulation method.

### 8 Conculsions

An innovative demodulation method in which no additional local oscillator is required has been proposed. Except for the additional phase shift and the lower SNR in the recovered baseband signal, performance of the demodulator is similar to that using the phase-shift method when the modulation index does not exceed a certain level. The experimental results also show that it is possible to build the proposed circuit without using additional frequency mixing, so that an additional local oscillator is not required, and it is suitable for use in AM stereo broadcasting using the SSB technique to reduce the production cost.

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### 10 Appendix

$$f(t) = A\cos(\omega_{C}t) + \frac{m_{U}A}{2}\cos(\omega_{C}t + \omega_{U}t + \theta_{U}) + \frac{m_{L}A}{2}\cos(\omega_{C}t - \omega_{L}t + \theta_{L}) = A\cos(\omega_{C}t) + \frac{m_{U}A}{2}\cos(\omega_{C}t)\cos(\omega_{U}t + \theta_{U}) - \frac{m_{U}A}{2}\sin(\omega_{C}t)\sin(\omega_{U}t + \theta_{U}) + \frac{m_{L}A}{2}\cos(\omega_{C}t)\cos(\omega_{L}t - \theta_{L}) + \frac{m_{L}A}{2}\sin(\omega_{C}t)\sin(\omega_{L}t - \theta_{L}) = A\cos(\omega_{C}t) \cdot \left(1 + \frac{m_{U}}{2}\cos(\omega_{C}t)\cos(\omega_{U}t + \theta_{U}) + \frac{m_{L}}{2}\cos(\omega_{L}t - \theta_{L})\right) + A\sin(\omega_{C}t) \cdot \left(\frac{m_{L}}{2}\sin(\omega_{L}t - \theta_{L}) - \frac{m_{U}}{2}\sin(\omega_{U}t + \theta_{U})\right) = AB\cos\phi \cdot \cos(\omega_{C}t) + AB\sin\phi \cdot \sin(\omega_{C}t) = AB\cos(\omega_{C}t - \phi)$$
(12)

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$$B\cos\phi = 1 + \frac{m_U}{2}\cos(\omega_C t)\cos(\omega_U t + \theta_U) + \frac{m_L}{2}\cos(\omega_L t - \theta_L) B\sin\phi = \frac{m_L}{2}\sin(\omega_L t - \theta_L) - \frac{m_U}{2}\sin(\omega_U t + \theta_U) B = \left\{ \left[ 1 + \frac{m_U}{2}\cos(\omega_U t + \theta_U) + \frac{m_L}{2}\cos(\omega_L t - \theta_L) \right]^2 + \left[ \frac{m_L}{2}\sin(\omega_L t - \theta_L) - \frac{m_U}{2}\sin(\omega_U t + \theta_U) \right]^2 \right\}^{\frac{1}{2}} \tan^{-1}\phi = \frac{\frac{m_L}{2}\sin(\omega_L t - \theta_L) - \frac{m_U}{2}\sin(\omega_U t + \theta_U)}{1 + \frac{m_U}{2}\cos(\omega_U t + \theta_U) + \frac{m_L}{2}\cos(\omega_L t - \theta_L)}$$
(13)

 $m_{T}$ 

$$i(t) = g_{3}(t) + g_{4}(t)$$

$$= A + \frac{m_{U}A}{2} \cdot \cos(\omega_{U}t + \theta_{U} + \alpha)$$

$$+ \frac{m_{L}A}{2} \cdot \cos(\omega_{L}t - \theta_{L} + \alpha)$$

$$+ \frac{m_{U}A}{2} \cdot \sin(\omega_{U}t + \theta_{U} + \beta)$$

$$- \frac{m_{L}A}{2} \cdot \sin(\omega_{L}t - \theta_{L} + \beta)$$

$$= A + \frac{m_{U}A}{2} \cdot \cos(\omega_{U}t + \theta_{U}) \cdot (\cos \alpha + \sin \beta)$$

$$+ \frac{m_{U}A}{2} \cdot \sin(\omega_{U}t + \theta_{U}) \cdot (\cos \beta - \sin \alpha)$$

$$+ \frac{m_{L}A}{2} \cdot \cos(\omega_{L}t - \theta_{L}) \cdot (\cos \alpha - \sin \beta)$$

$$- \frac{m_{L}A}{2} \cdot \sin(\omega_{L}t - \theta_{L}) \cdot (\sin \alpha + \cos \beta)$$

$$= A + \frac{m_{U}A}{2} \cdot B' \cdot \cos \phi' \cdot \cos(\omega_{U}t + \theta_{U})$$

$$+ \frac{m_{L}A}{2} \cdot B'' \cdot \sin \phi'' \cdot \sin(\omega_{L}t - \theta_{L})$$

$$- \frac{m_{L}A}{2} \cdot B'' \cdot \sin \phi'' \cdot \sin(\omega_{L}t - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t + \theta_{U} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

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$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

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$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

$$= A + \frac{m_{U}A}{2} \cdot B'' \cdot \cos(\omega_{U}t - \theta_{L} - \theta_{L})$$

Since

$$\begin{aligned} B' &\cdot \cos \phi' = \cos \alpha + \sin \beta \\ B' &\cdot \sin \phi' = \cos \beta - \sin \alpha \\ B'' &\cdot \cos \phi'' = \cos \alpha - \sin \beta \\ B'' &\cdot \sin \phi'' = \sin \alpha + \cos \beta \\ B' &= \sqrt{(\cos \alpha + \sin \beta)^2 + (\cos \beta - \sin \alpha)^2} \\ &= \sqrt{[\cos^2 \alpha + 2\cos \alpha \sin \beta + \sin^2 \beta]} \\ &+ \cos^2 \beta - 2\sin \alpha \cos \beta + \sin^2 \alpha] \\ &= \sqrt{2 + 2(\cos \alpha \sin \beta - \sin \alpha \cos \beta)} \\ &= \sqrt{2 + 2\sin(\beta - \alpha)} \\ \tan^{-1} \phi' &= \frac{\cos \beta - \sin \alpha}{\cos \alpha + \sin \beta} \end{aligned}$$

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$$\begin{split} B'' &= \sqrt{(\cos \alpha - \sin \beta)^2 + (\sin \alpha + \cos \beta)^2} \\ &= \sqrt{[\cos^2 \alpha - 2\cos \alpha \sin \beta + \sin^2 \beta]} \\ &+ \sin^2 \alpha + 2\sin \alpha \cos \beta + \cos^2 \beta] \\ &= \sqrt{2 + 2(\sin \alpha \cos \beta - \cos \alpha \sin \beta)} \\ &= \sqrt{2 + 2\sin(\alpha - \beta)} \\ \tan^{-1} \phi'' &= \frac{\sin \alpha + \cos \beta}{\cos \alpha - \sin \beta} \\ q(t) &= g_3(t) - g_4(t) \\ &= A + \frac{m_U A}{2} \cdot \cos(\omega_U t + \theta_U + \alpha) \\ &+ \frac{m_L A}{2} \cdot \cos(\omega_L t - \theta_L + \alpha) \\ &- \frac{m_U A}{2} \cdot \sin(\omega_U t + \theta_U + \beta) \\ &+ \frac{m_L A}{2} \cdot \sin(\omega_U t + \theta_U) \cdot (\cos \alpha - \sin \beta) \\ &- \frac{m_U A}{2} \cdot \sin(\omega_U t + \theta_U) \cdot (\cos \alpha - \sin \beta) \\ &- \frac{m_U A}{2} \cdot \sin(\omega_U t + \theta_U) \cdot (\sin \alpha + \cos \beta) \\ &+ \frac{m_L A}{2} \cdot \cos(\omega_L t - \theta_L) \cdot (\cos \alpha + \sin \beta) \\ &+ \frac{m_L A}{2} \cdot \sin(\omega_L t - \theta_L) \cdot (\cos \beta - \sin \alpha) \\ &= A + \frac{m_U A}{2} \cdot C' \cdot \cos \gamma' \cdot \cos(\omega_U t + \theta_U) \\ &- \frac{m_U A}{2} \cdot C' \cdot \sin \gamma' \cdot \sin(\omega_U t + \theta_U) \\ &+ \frac{m_L A}{2} \cdot C'' \cdot \cos \gamma'' \cdot \cos(\omega_L t - \theta_L) \end{split}$$

$$+ \frac{m_L A}{2} \cdot C'' \cdot \sin \gamma'' \cdot \sin(\omega_L t - \theta_L)$$
  
=  $A + \frac{m_U A}{2} \cdot C' \cdot \cos(\omega_U t + \theta_U + \gamma')$   
+  $\frac{m_L A}{2} \cdot C'' \cdot \cos(\omega_L t - \theta_L - \gamma'')$  (15)

since

$$C' \cdot \cos \gamma' = \cos \alpha - \sin \beta$$

$$C' \cdot \sin \gamma' = \sin \alpha + \cos \beta$$

$$C'' \cdot \cos \gamma'' = \cos \alpha + \sin \beta$$

$$C'' \cdot \sin \gamma'' = \cos \beta - \sin \alpha$$

$$C' = \sqrt{(\cos \alpha - \sin \beta)^2 + (\sin \alpha + \cos \beta)^2}$$

$$= \sqrt{[\cos^2 \alpha - 2\cos \alpha \sin \beta + \sin^2 \beta]}$$

$$+ \sin^2 \alpha + 2\sin \alpha \cos \beta + \cos^2 \beta]$$

$$= \sqrt{2 + 2(\sin \alpha \cos \beta - \cos \alpha \sin \beta)}$$

$$= \sqrt{2 + 2\sin (\alpha - \beta)}$$

$$\tan^{-1} \gamma' = \frac{\sin \alpha + \cos \beta}{\cos \alpha - \sin \beta}$$

$$C'' = \sqrt{(\cos \alpha + \sin \beta)^2 + (\cos \beta - \sin \alpha)^2}$$

$$= \sqrt{[\cos^2 \alpha + 2\sin \beta \cos \alpha + \sin^2 \beta]}$$

$$= \sqrt{2 + 2(\cos \alpha \sin \beta - \sin \alpha \cos \beta)}$$

$$= \sqrt{2 - 2\sin(\alpha - \beta)}$$

$$\tan^{-1} \gamma'' = \frac{\cos \beta - \sin \alpha}{\cos \alpha + \sin \beta}$$
(16)

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