Atomic and nuclear physics

X-ray physics
Attenuation of x-rays

Investigating the wavelength dependency of the coefficient of attenuation

Objects of the experiment

■ To measure the transmittance *T* of a copper foil and a zirconium foil for x-rays as a function of the wavelength λ between 30 and 120 pm.

■ To investigate the wavelength-dependency of the attenuation coefficient μ outside of the absorption edges.

■ To confirm the λ^3 law as a function of the absorption coefficient τ.

Principles

The attenuation of x-rays passing through matter is described by Lambert's law (see experiment P6.3.2.1):

$$R = R_0 \cdot e^{-\mu x} \qquad (I)$$

Here, R_0 is the intensity of the x-ray radiation in front of the attenuator, R is the intensity behind the attenuator, is the linear attenuation coefficient and x is the thickness of the attenuator.

Absorption and scattering both contribute to attenuation. The linear attenuation coefficient μ is thus composed of the linear absorption coefficient τ and the linear scattering coefficient σ.

$$\mu = \tau + \sigma \qquad (II)$$

These coefficients are proportional to the mass and the density ρ of the irradiated material respectively. That is why we often use the so-called mass coefficients

$$\mu_m = \frac{\mu}{\rho}, \ \tau_m = \frac{\tau}{\rho}, \ \sigma_m = \frac{\sigma}{\rho} \qquad (III)$$

or – for the pure metals observed here – the atomic coefficients or cross-sections

$$\mu_a = \mu_m \frac{A}{N_A}, \ \tau_a = \tau_m \frac{A}{N_A}, \ \sigma_a = \sigma_m \frac{A}{N_A} \qquad (IV)$$

A: atomic weight

$N_A = 6.022 \cdot 10^{23} \frac{1}{mol}$: Avogadro's number

Analogously to equation (II), we can say that

$$\mu_m = \tau_m + \sigma_m \qquad (V)$$

and

$$\mu_a = \tau_a + \sigma_a \qquad (VI)$$

The absorption of x-rays is essentially due to the ionization of atoms, which release an electron from an inner shell. The absorption cross-section is thus strongly dependent on the quantum energy $h\nu$ resp. the wavelength λ of the x-ray radiation as well as on the excitation energy of the atoms and thus the atomic number *Z*. For ionization to occur, the quantum energy of the x-rays must be greater than the binding energy *E* of the electrons of the respective shell. The absorption cross-section is thus very small once the quantum energy is just slightly below the binding energy. The limit wavelength at which the quantum energy is just sufficient is called the absorption edge (see Fig. 1).

Outside of the absorption edges, absorption is described to within a close approximation by the relationship
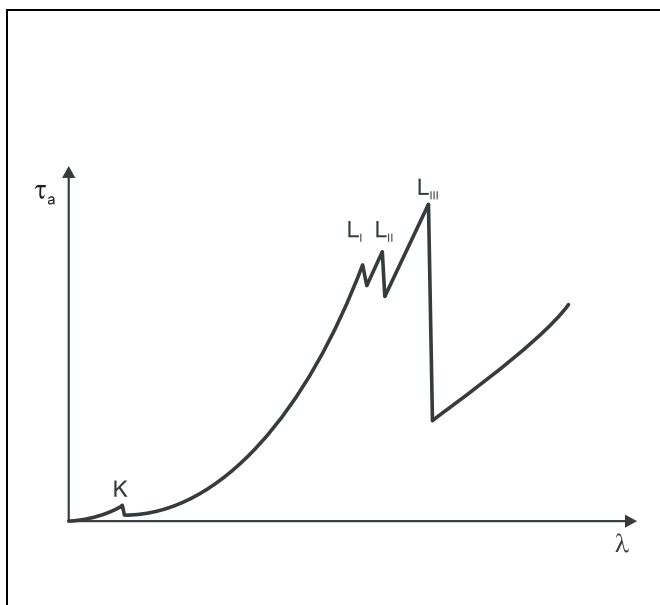
$$\tau_a = C \cdot \lambda^3 \cdot Z^4 \qquad (VII).$$

Fig. 1 Absorption coefficient as a function of the x-ray wavelength (schematic)
K: absorption edge of K shell
L_I, L_{II}, L_{III}: absorption edges of L shell

Apparatus

1 X-ray apparatus 554 811

1 End-window counter
  for α, β, γ and x-ray radiation 559 01

1 Set of absorber foils 554 832

additionally required:

1 PC with Windows 95/98 or Windows NT

This experiment verifies the dependence on the wavelength λ for two different metals, i.e. two different atomic numbers Z.

The evaluation exploits the fact that the scattering cross-section σ_a in the wavelength range $\lambda = 35$–100 pm is significantly less than the absorption cross-section and can be estimated approximately using

$$\sigma_a = 0.2 \, \frac{cm^2}{g} \cdot \frac{A}{N_A} \tag{VIII}$$

By applying a series of transformations to equations (III)-(VII), we can calculate the absorption cross-section as:

$$\tau_a = \frac{\mu}{\rho} \cdot \frac{A}{N_A} - 0.2 \, \frac{cm^2}{g} \cdot \frac{A}{N_A} \tag{IX}$$

This experiment measures the transmittance

$$T = \frac{R}{R_0} \tag{X}$$

of the transilluminated material as a function of the wavelength of the x-ray radiation. When we apply Lambert's law of attenuation

$$T = e^{-\mu x} \tag{XI}$$

we can calculate the linear attenuation coefficient μ and, using this value and equation (IX), the absorption cross-section τ_a:

$$\tau_a = \frac{-\ln T}{\rho \cdot x} \cdot \frac{A}{N_A} - 0.2 \, \frac{cm^2}{g} \cdot \frac{A}{N_A} \tag{XII}$$

Safety notes

The x-ray apparatus fulfills all regulations governing an x-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 807/97 Rö).

The built-in protection and screening measures reduce the local dose rate outside of the x-ray apparatus to less than $1 \, \mu$Sv/h, a value which is on the order of magnitude of the natural background radiation.

■ Before putting the x-ray apparatus into operation inspect it for damage and to make sure that the high voltage is shut off when the sliding doors are opened (see Instruction Sheet for x-ray apparatus).

■ Keep the x-ray apparatus secure from access by unauthorized persons.

Do not allow the anode of the x-ray tube Mo to overheat.

■ When switching on the x-ray apparatus, check to make sure that the ventilator in the tube chamber is turning.

The goniometer is positioned solely by electric stepper motors.

■ Do not block the target arm and sensor arm of the goniometer and do not use force to move them.
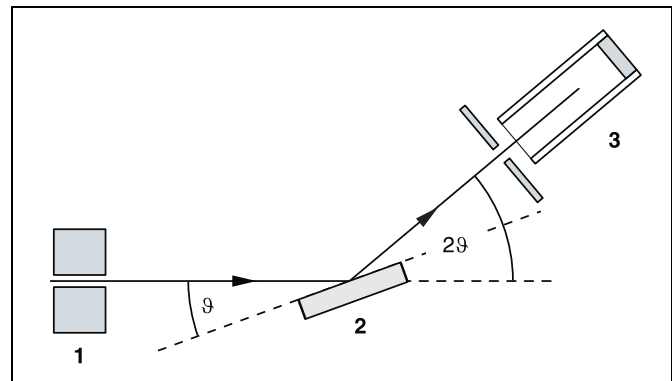

Fig. 2   Diffraction of x-rays at a monocrystal and for 2ϑ coupling between counter-tube angle and scattering angle (glancing angle)
1 collimator, **2** monocrystal, **3** counter tube

A goniometer with NaCl crystal and a Geiger-Müller counter tube in the Bragg configuration are used to record the intensities as a function of the wavelength. The crystal and counter tube are pivoted with respect to the incident x-ray beam in 2ϑ coupling, i.e. the counter tube is turned at an angle twice as large as the crystal (see Fig. 2).

In accordance with Bragg's law of reflection, the scattering angle ϑ in the first order of diffraction corresponds to the wavelength

$$\lambda = 2 \cdot d \cdot \sin \vartheta \tag{XIII}$$
$d = 282.01$ pm: lattice plane spacing of NaCl

Setup

Setup in Bragg configuration:

Set up the experiment as shown in Fig. 3. To do this, proceed as follows (see also the Instruction Sheet for the x-ray apparatus):

– Mount the collimator in the collimator mount **(a)** (note the guide groove).

– Attach the goniometer to guide rods **(d)** so that the distance s_1 between the slit diaphragm of the collimator and the target arm is approx. 5 cm. Connect ribbon cable **(c)** for controlling the goniometer.

– Remove the protective cap of the end-window counter, place the end-window counter in sensor seat **(e)** and connect the counter tube cable to the socket marked GM TUBE.

– By moving the sensor holder **(b)**, set the distance s_2 between the target arm and the slit diaphragm of the sensor receptor to approx. 5 cm.

– Mount the target holder with target stage.

– Loosen knurled screw **(g)**, place the NaCl crystal flat on the target stage **(f)**, carefully raise the target stage with crystal all the way to the stop and carefully tighten the knurled screw (prevent skewing of the crystal by applying a slight pressure).

– If necessary, adjust the mechanical zero position of the goniometer (see Instruction Sheet for x-ray apparatus).

Notes:

NaCl crystals are hygroscopic and extremely fragile.

Store the crystals in a dry place; avoid mechanical stresses on the crystal; handle the crystal by the short faces only.

If the counting rate is too low, you can reduce the distance s_2 between the target and the sensor somewhat. However, the distance should not be too small, as otherwise the angular resolution of the goniometer is no longer sufficient to separate the K_α and K_β lines.

Preparing the PC-based measurement:

– Connect the RS–232 output and the serial interface on your PC (usually COM1 or COM2) using the 9-pin V24 cable (supplied with x-ray apparatus).

– If necessary, install the software "X-ray Apparatus" under Windows 95/98/NT (see Instruction Sheet for x-ray apparatus) and select the desired language.
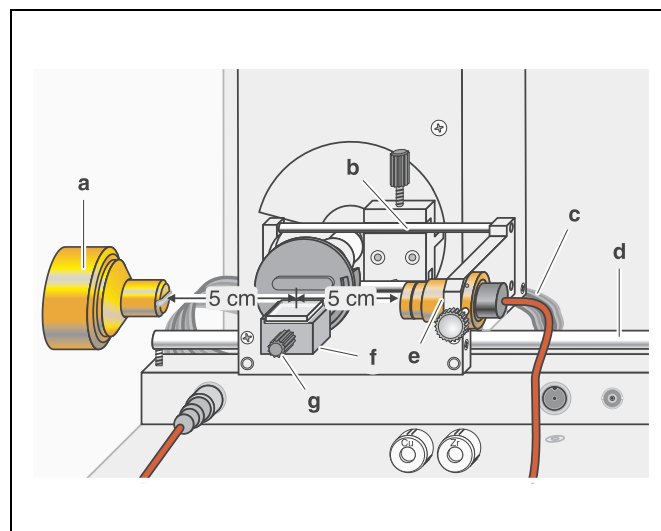

Fig. 3   Experiment for investigating the wavelength-dependency of the attenuation coefficient

Carrying out the experiment

– Start the software "X-ray Apparatus", check to make sure that the apparatus is connected correctly, and clear any existing measurement data using the button ⬚ or the F4 key.

– Set the tube high voltage $U = 30.0$ kV, the emission current $I = 1.00$ mA and the angular step width $\Delta\beta = 0.1°$.

At $U = 30$ kV, the limit wavelength of the bremsstrahlung radiation is $\lambda_{min} = 41.3$ pm (see e. g. experiment P6.3.3.3). This value corresponds to a diffraction angle of 4.19° in the first order of diffraction and 8.42° in the second order of diffraction.

– Press the COUPLED key to activate 2ϑ coupling of target and sensor and set the lower limit of the target angle to 4.2° and the upper limit to 8.3°.

– Set the measuring time per angular step to $\Delta t = 5$ s.

– Start measurement and data transfer to the PC by pressing the SCAN key.

– Once the scan is complete, increase the measuring time to $\Delta t = 10$ s, mount the Cu foil from the set of absorbers (554 832) in the sensor seat **(e)** and start another measurement by pressing SCAN.

– Then replace the Cu foil with a Zr foil and run a third measurement.

– When you have finished measuring, save the measurement series under an appropriate name by pressing the button 🖫 or the F2 key.

– To display the measurement data as a function of the wavelength λ, open the "Settings" dialog with the button ⬚ or F5, and in the tab "Crystal", click on the button "Enter NaCl".
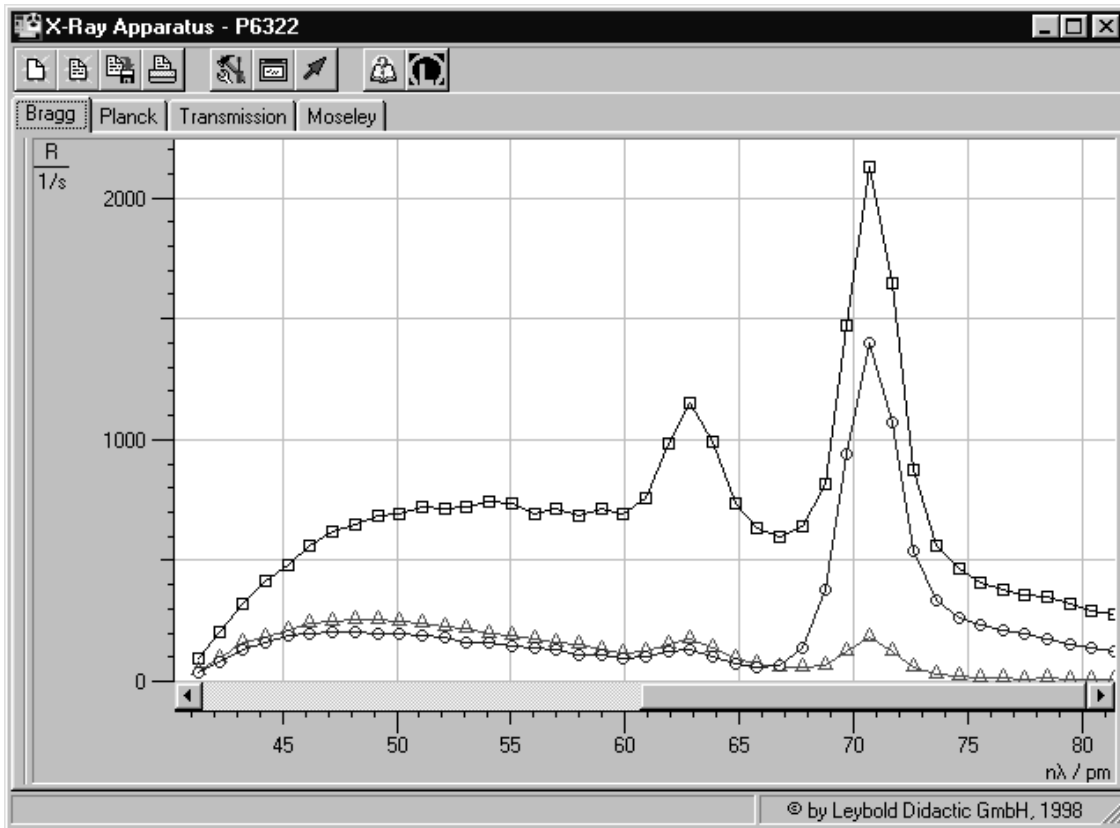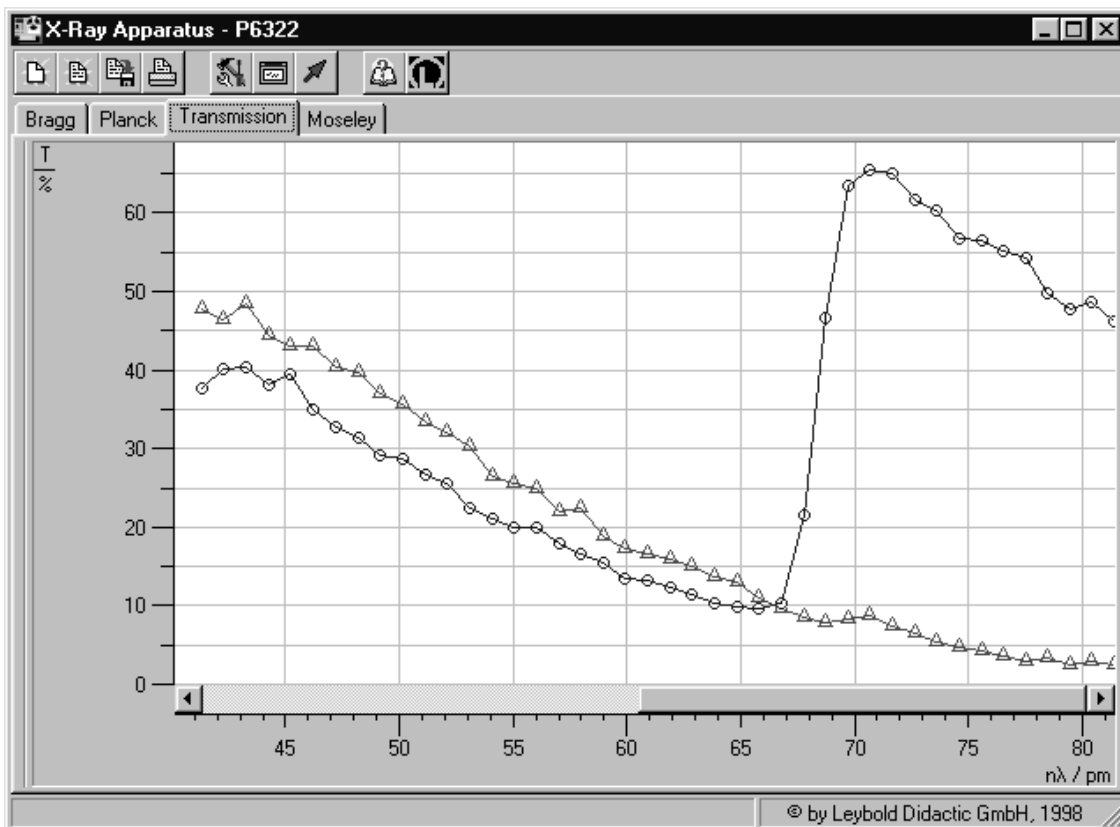
Fig. 4   Diffraction spectra of x-rays with ▲
the characteristic lines of the molybdenum anode
in the wavelength range between 40 and 80 pm
(U = 30 kV)
Squares: not attenuated
Triangles: attenuated with copper foil
Circles: attenuated with zirconium foil

Fig. 5   Transmittance T in the wavelength range
between 40 and 80 pm
Triangles: attenuated with copper foil
Circles: attenuated with zirconium foil
▼

Measuring example

See Fig. 4

Evaluation

a) Calculating the transmittance T:

–   In the software "X-ray Apparatus", click on the "Transmission" tab and, using the diffraction spectra (see Fig. 4), generate the transmission spectra for copper and zircon foil (see Fig. 5).

b) Calculating the absorber cross-section τ_a:

–   To copy data to the Clipboard, position the pointer over the table, click the right mouse button and select the menu command "Copy Table".
–   Paste the data from the Clipboard, with the three columns λ, $T(Cu)$ and $T(Zr)$, into a suitable evaluation program (e.g. a spreadsheet).
–   Calculate the absorption cross-section τ_a for the two foils with equation (XII) using the parameters compiled in table 1 (see Fig. 6).

Tab. 1: Parameters of the foils

Element	Z	$\dfrac{\rho}{g \cdot cm^{-3}}$	$\dfrac{x}{cm}$	$\dfrac{A}{g \cdot mol^{-1}}$
Cu	29	8.92	0.007	63.55
Zr	40	6.49	0.005	91.22

c) Verifying the λ^3 law:

–   Calculate the terms $\ln (\tau_a /10^{-24} \, cm^2)$ and $\ln (\_/pm)$ and display the wavelength dependency of the absorption cross-section in the representation $\ln \tau_a = f (\ln \lambda)$ (see Fig. 7).
–   Fit straight lines to the diagram at suitable intervals.

Results

The atomic absorption cross-section of copper increases consistently in the wavelength range between 40 and 80 pm. Zirconium, on the other hand, shows an abrupt change of the absorption cross-section (absorption edge of K-shell). In practice, this property is used for approximately monochromatizing the x-ray output of a molybdenum anode, i.e. to substantially suppress the spectrum of bremsstrahlung and the K_β line (see Fig. 4).

At suitable intervals outside of the absorption edges, the absorption cross-section can be represented in the form

$\ln \tau_a = A \cdot \ln \lambda + B$

The slope of the line A has roughly the value 3, so that the λ^3 law approximately applies:

Outside of the absorption edges the wavelength-dependency of the atomic absorption cross-section is approximately described by the relationship

$\tau_a = C_1 \, \lambda^3$

Fig. 6   Absorption cross-section τ_a in the wavelength range
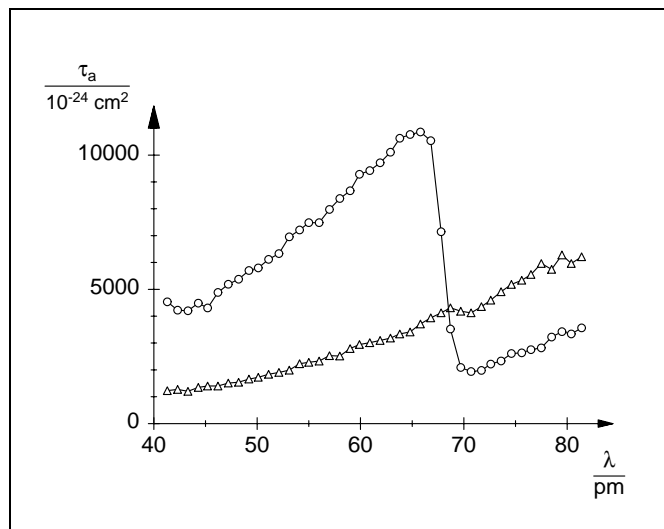between 40 and 80 pm
Triangles: copper (Cu)
Circles: zirconium (Zr)

Fig. 7   Absorption cross-section τ_a shows as $\ln \tau_a$
as a function of λ.
Triangles: copper (Cu)
Circles: zirconium (Zr)
Lines: best-fit straight lines
Slope of line: 2.7 (Cu) and 2.5 (Zr)