9 Electron Diffraction

Apparatus

cathode ray tube (Leybold 555 626)
high-voltage power supply (new Leybold)
100-kΩ resistor with banana-plug connectors
Vernier calipers

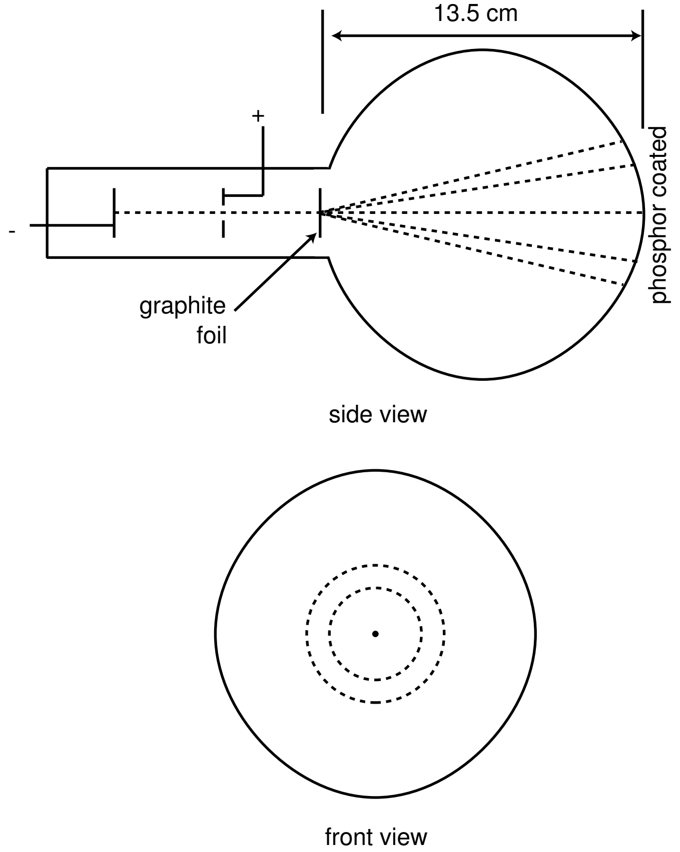
Goals

Observe wave interference patterns (diffraction patterns) of electrons, demonstrating that electrons exhibit wave behavior as well as particle behavior.

Learn what it is that determines the wavelength of an electron.

Introduction

The most momentous discovery of 20th-century physics has been that light and matter are not simply made of waves or particles — the basic building blocks of light and matter are strange entities which display both wave and particle properties at the same time. In our course, we have already learned about the experimental evidence from the photoelectric effect showing that light is made of units called photons, which are both particles and waves. That probably disturbed you less than it might have, since you most likely had no preconceived ideas about whether light was a particle or a wave. In this lab, however, you will see direct evidence that electrons, which you had been completely convinced were particles, also display the wave-like property of interference. Your schooling had probably ingrained the particle interpretation of electrons in you so strongly that you used particle concepts without realizing it. When you wrote symbols for chemical ions such as Cl^- and Ca^{2+}, you understood them to mean a chlorine atom with one excess electron and a calcium atom with two electrons stripped off. By teaching you to count electrons, your teachers were luring you into the assumption that electrons were particles. If this lab's evidence for the wave properties of electrons disturbs you, then you are on your way to a deeper understanding of what an electron really is — both a particle and a wave.



The electron diffraction tube. The distance labeled as 13.5 cm in the figure actually varies from about 12.8 cm to 13.8 cm, even for tubes that otherwise appear identical. This doesn't affect your results, since you're only searching for a proportionality.

Method

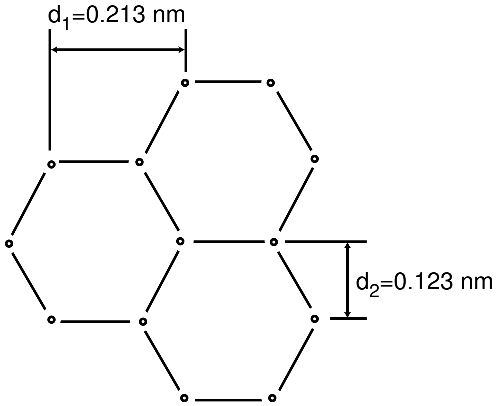
What you are working with is basically the same kind of vacuum tube as the picture tube in your television. As in a TV, electrons are accelerated through a voltage and shot in a beam to the front (big end) of the tube, where they hit a phosphorescent coating and produce a glow. You cannot see the electron beam itself. There is a very thin carbon foil (it looks like a tiny piece of soap bubble) near where the neck joins the spherical part of the tube, and the electrons must pass through the foil before crossing over to the phosphorescent screen.

The purpose of the carbon foil is to provide an ultra-fine diffraction grating — the "grating" consists of the crystal lattice of the carbon atoms themselves! As you will see in this lab, the wavelengths of the electrons are very short (a fraction of a nanometer), which makes a conventional ruled diffraction grating useless — the closest spacing that can be achieved on a conventional grating is on the order of one micrometer. The carbon atoms in graphite are arranged in sheets, each of which consists of a hexagonal pattern of atoms like chicken wire. That means they are not lined up in straight rows, so the diffraction pattern is slightly different from the pattern produced by a ruled grating.

Also, the carbon foil consists of many tiny graphite crystals, each with a random orientation of its crystal lattice. The net result is that you will see a bright spot surrounded by two faint circles. The two circles represent cones of electrons that intersect the phosphor. Each cone makes an angle θ with respect to the central axis of the tube, and just as with a ruled grating, the angle is given by

$$\sin\theta = \lambda/d$$

where λ is the wavelength of the wave. For a ruled grating, d would be the spacing between the lines. In this case, we will have two different cones with two different θ's, θ_1 and θ_2, corresponding to two different $d's$, d_1 and d_2. Their geometrical meaning is shown below.[1]



The carbon atoms in the graphite crystal are arranged hexagonally.

Safety

This lab involves the use of voltages of up to 6000 V. Do not be afraid of the equipment, however; there is a fuse in the high-voltage supply that limits the amount of current that it can produce, so it is not particularly dangerous. Read the safety checklist on high voltage in Appendix 6. Before beginning the lab, make sure you understand the safety rules, initial them, and show your safety checklist to your instructor. If you don't understand something, ask your instructor for clarification.

In addition to the high-voltage safety precautions, please observe the following rules to avoid damaging the apparatus:

______ The tubes cost $1000. Please treat them with respect! Don't drop them! Dropping them would also be a safety hazard, since they're vacuum tubes, so they'll implode violently if they break.

______ Do not turn on anything until your instructor has checked your circuit.

______ Don't operate the tube continuously at the highest voltage values (5000-6000 V). It produces x-rays when used at these voltages, and the strong beam also decreases the life of the tube. You can use the circuit on the right side of the HV supply's panel, which limits its own voltage to 5000 V. Don't leave the tube's heater on when you're not actually taking data, because it will decrease the life of the tube.

Setup

You setup will consist of two circuits, a heater circuit and the high-voltage circuit.

The heater circuit is to heat the cathode, increasing the velocity with which the electrons move in the metal and making it easier for some of them to escape from the cathode. This will produce the friendly and nostalgia-producing yellow glow which is characteristic of all vacuum-tube equipment. The heater is simply a thin piece of wire, which acts as a resistor when a small voltage is placed across it, producing heat. Connect the heater connections, labeled F1 and F2, to the 6-V AC outlet at the back of the HV supply.

The high-voltage circuit's job is to accelerate the electrons up to the desired speed. An electron that happens to jump out of the cathode will head "downhill" to the anode. (The anode is at a *higher* voltage than the cathode, which would make it seem like it would be uphill from the cathode to the anode. However, electrons have negative charge, so they're like negative-mass water that flows uphill.) The high voltage power supply is actually two different power supplies in one housing, with a left-hand panel for

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[1] See `http://bit.ly/XxoEYr` for more information.

one and a right-hand panel for the other. Connect the anode (A) and cathode (C) to the right-hand panel of the HV supply, and switch the switch on the HV supply to the right, so it knows you're using the right-hand panel.

The following connections are specified in the documentation, although I don't entirely understand what they're for. First, connect the electrode X to the same plug as the cathode.[2] Also, connect F1 to C with the wire that has the 100-kΩ resistor spliced into it. The circuit diagram on page 33 summarizes all this.

Check your circuit with your instructor before turning it on!

Observations

You are now ready to see for yourself the evidence of the wave nature of electrons, observe the diffraction pattern for various values of the high voltage, and figure out what determines the wavelength of the electrons. You will need to do your measurements in the dark.

You will measure the θ's, and thus determine the wavelength, λ, for several different voltages. Each voltage will produce electrons with a different velocity, momentum, and energy.

Hints:

> While measuring the diffraction pattern, don't touch the vacuum tube — the static electric fields of one's body seem to be able to perturb the pattern.
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> It is easiest to take measurements at the highest voltages, where the electrons pack a wallop and make nice bright rings on the phosphor. Start with the highest voltages and take data at lower and lower voltages until you can't see the rings well enough to take precise data. To get unambiguous results, you'll need to take data with the widest possible range of voltages.
>
> In order to reach a definite conclusion about what λ is proportional to, you will need accurate data. Do your best to get good measurements. Pay attention to possible problems incurred by viewing the diffraction patterns from different angles on different occasions. Try repeating a measurement more than once, and seeing how big your random errors are.
>
> You need to get data down to about 2 or 3 kV in order to get conclusive results from this experiment. The tubes are not quite identical, and were not designed to operate at such low voltages, so they haven't been tested under those conditions. Experience has shown that some of the tubes work at lower voltages than others. The group that has the tube that works the best at low voltages can share their low-voltage data with the other groups.

Analysis

Once you have your data, plot λ as a function of $1/KE$ and $1/p$. Is one graph more of a straight line than the other? If the graph is a straight line through the origin, then the experiment supports the hypothesis that the wavelength is proportional to that quantity. You can simplify your analysis by leaving out constant factors.

What does λ seem to be proportional to? Your data may cover a small enough range of voltage that both graphs may look linear. However, only one will be consistent with a line that passes *through the origin*, as it must for a proportionality. This is why it is important to have your graph include the origin.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

The week before you are to do the lab, briefly familiarize yourself visually with the apparatus.

Read the high voltage safety checklist.

P1 The figure shows the vacuum tube as having a particular shape, which is a sphere with the foil and phosphor at opposite ends of a diamater. In reality, the tubes we're using now are not quite that shape. To me, they look like they may have been shaped so that the phosphor surface is a piece of a sphere centered on the foil. Therefore the arc lengths across the phosphor can be connected to diffraction angles very simply via the definition of radian mea-

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[2] If you look inside the tube, you can see that X is an extra electrode sandwiched in between the anode and the cathode. I think it's meant to help produce a focused beam.

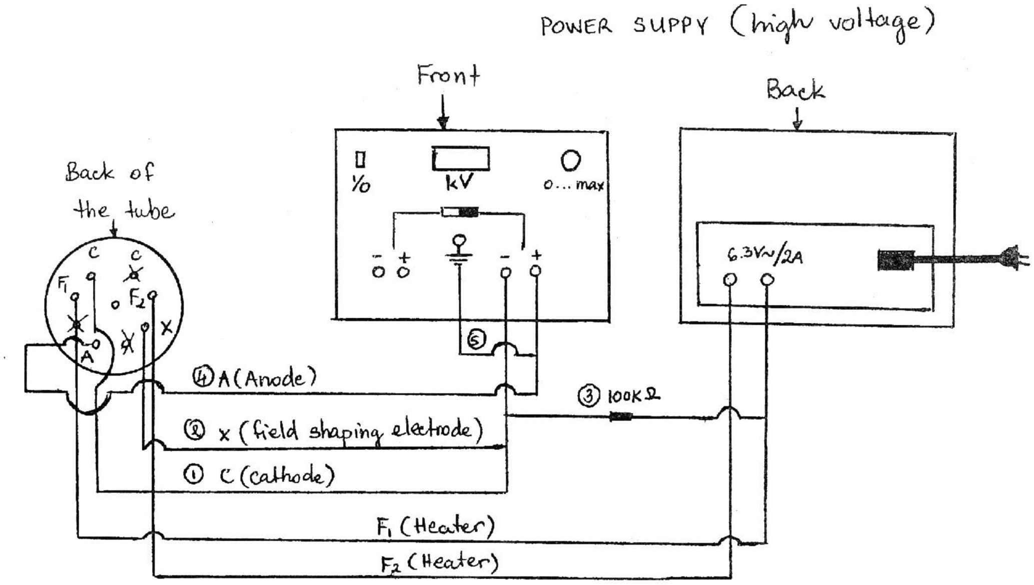
sure. Plan how you will do this.

P2 If the voltage difference across which the electrons are accelerated is V, and the known mass and charge of the electron are m and e, what are the electrons' kinetic energy and momentum, in terms of V, m, and e? (As a numerical check on your results, you should find that $V = 5700$ V gives $KE = 9.1 \times 10^{-16}$ J and $p = 4.1 \times 10^{-23}$ kg·m/s.)

P3 All you're trying to do based on your graphs is judge which one could be a graph of a proportionality, i.e., a line passing through the origin. Because of this, you can omit any constant factors from the equations you found in P2. When you do this, what do your expressions turn out to be?

P4 Why is it not logically possible for the wavelength to be proportional to both $1/p$ and $1/KE$?

P5 On each graph, you will have two data-points for each voltage, corresponding to two different measurements of the same wavelength. The two wavelengths will be almost the same, but not exactly the same because of random errors in measuring the rings. Should you get the wavelengths by combining the smaller angle with d_1 and the larger angle with d_2, or vice versa?



– : ① C (cathode) , ② X (field shaping electrode) , ③ 100KΩ (resistor) - goes to the back of the power supply

+ : ④ A (Anode) , ⑤ goes to Ground

The circuit for the new setup.