MEASUREMENT OF THE MASS OF THE ELECTRON USING COMPTON SCATTERING

1. INTRODUCTION AND OBJECTIVES

Arthur Compton (1892 - 1962) investigated the scattering of X-ray[1] photons from electrons in carbon atoms in graphite. The reason for choosing X-rays is that their wave frequency is high, and so is their energy per photon, which is precisely what he needed: electromagnetic radiation with enough energy to make an electron escape from its atom. He observed that the X-rays scattered from the electrons had a lower frequency than before the collision (a longer wavelength), and that frequency was directly related to the scattering angle. That frequency difference was key. The study of the momentum before and after the collision was the answer. Here the photoelectric[2] effect arises, involving Einstein's suggestions of Planck's *quanta*, now called photons. Both scientists won a Nobel Prize for their work[3,4].

Compton, awarded with the Nobel in Physics in 1927[5], confirmed that photons are not only waves, but particles. Electromagnetic radiation has what is called wave-particle properties.

The aim of this work is to measure the mass of the electron, emulating the investigation carried out by Compton, called Compton Scattering or simply the Compton Effect after him. For this purpose, a beam of X-rays is directed towards a target. In order to measure the energy of the photons of the X-rays as they are scattered, a detector is placed at a specific angle. To ensure to collect as much information as possible, the operation is repeated changing the detector angle each time. Knowing the energy of the incident photons, plus the energy and angle of the scattered photons, gives enough data to work out the mass of the electron thanks to a formula worked out by Compton.

2. METHODS AND RESULTS

2.1 The Compton Effect and the Compton formula

The X-rays have a lower wavelength λ/m than visible light[6] and, accordingly, a higher frequency f/Hz. The energy a photon carries[7] is $E = {h \cdot c}/{\lambda}$, where h is the Planck's constant and c is the speed of light. Consequently, X-ray photons have more energy than visible light photons. That is key for Compton when trying to eject an electron from an atom, and here is where his investigation begins. When an X-ray photon collides with an electron at rest, with enough energy

[1] X-rays and the Compton Effect: http://web.pdx.edu/~egertonr/ph311-12/xraycomp.htm
[2] The Photoelectric Effect: http://physics.info/photoelectric
[3] The Nobel Prize in Physics 1918: Max Planck:
http://www.nobelprize.org/nobel_prizes/physics/laureates/1918/planck-facts.html
[4] The Nobel Prize in Physics 1921: Albert Einstein:
http://www.nobelprize.org/nobel_prizes/physics/laureates/1921/einstein-facts.html
[5] The Nobel Prize in Physics 1927: Arthur H. Compton:
http://www.nobelprize.org/nobel_prizes/physics/laureates/1927/compton-facts.html
[6] X-rays and visible light: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=1.1
[7] Energy of photon: http://www.pveducation.org/pvcdrom/properties-of-sunlight/energy-of-photon

to liberate it from the atom, the electron behaves basically as a free particle[8], gaining energy and escaping at a velocity $\boldsymbol{v}$. At this time, conservation of energy and momentum applies to both particles involved, the photon and the electron (Figure 1[9]). Therefore, the photon has mass and can behave not only as a wave, but also as a particle.

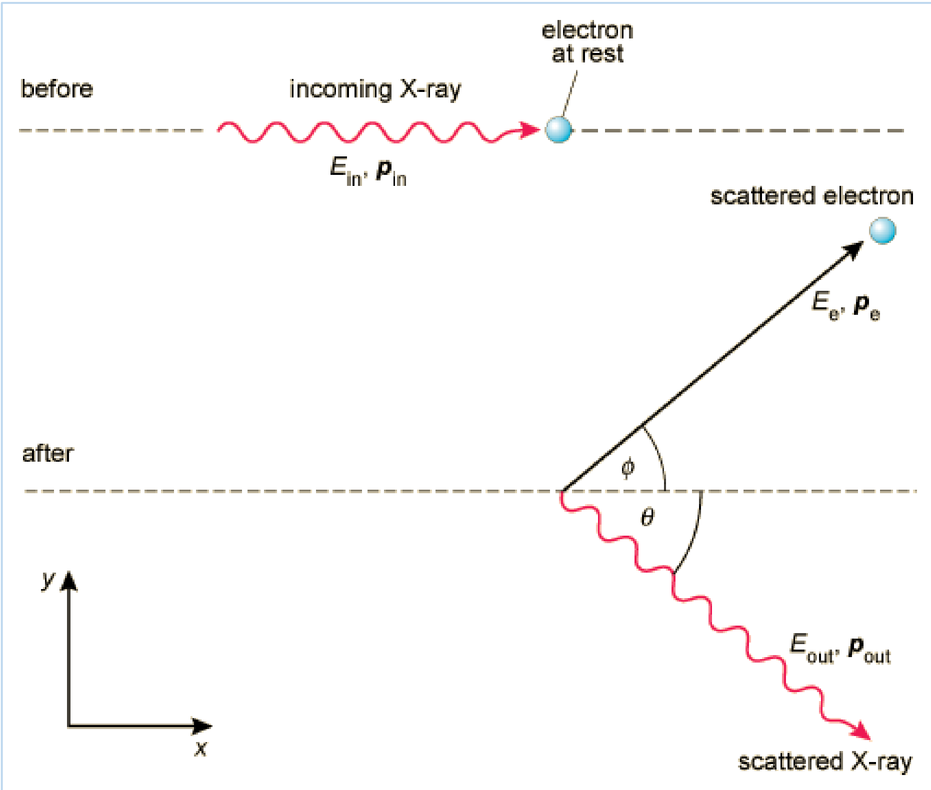


Figure 1 Schematic of the collision between X-rays and electrons in the Compton scattering.

Compton worked out a formula to relate the incoming and outgoing energies of the photon after the collision with the electron, E_{in} and E_{out}. This formula involves its mass, m_e which is the main aim of this investigation, and the photon scattering angle θ. Different angles are tested during the experiment in order to measure different values of energies so as to get, as close as possible, to the actual mass of the electron.

$$E_{out} = \frac{E_{in}}{1 + \frac{E_{in}}{m_e \cdot c^2} \cdot (1 - \cos\theta)}$$

This is known as the Compton formula[10]. It is worth noticing that for an angle $\theta = 0°$ (no scattering at all) the outgoing energy would be, as expected, the incoming energy.

[8] The electron on the Compton Effect: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=2.1
[9] Schematic of the collision in Compton Scattering:
https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=2.3
[10] Derivation of the Compton formula:
https://learn5.open.ac.uk/pluginfile.php/3906/mod_resource/content/1/S288_Compton_theorysupplement.pdf

2.2 Equipment

The equipment used remotely via web is an LD DIDACTIC 554 800[11] X-ray apparatus. Its main parts[12] and the process is shown in Figure 2. Instead of using graphite as Compton did, the target is made of *Perspex*, a trademark for *poly(methyl 2-methylpropenoate)*[13,14], which also contains carbon. *Perspex* takes the role of the graphite since the binding energy of core electrons in carbon atoms is, in both cases, similar.

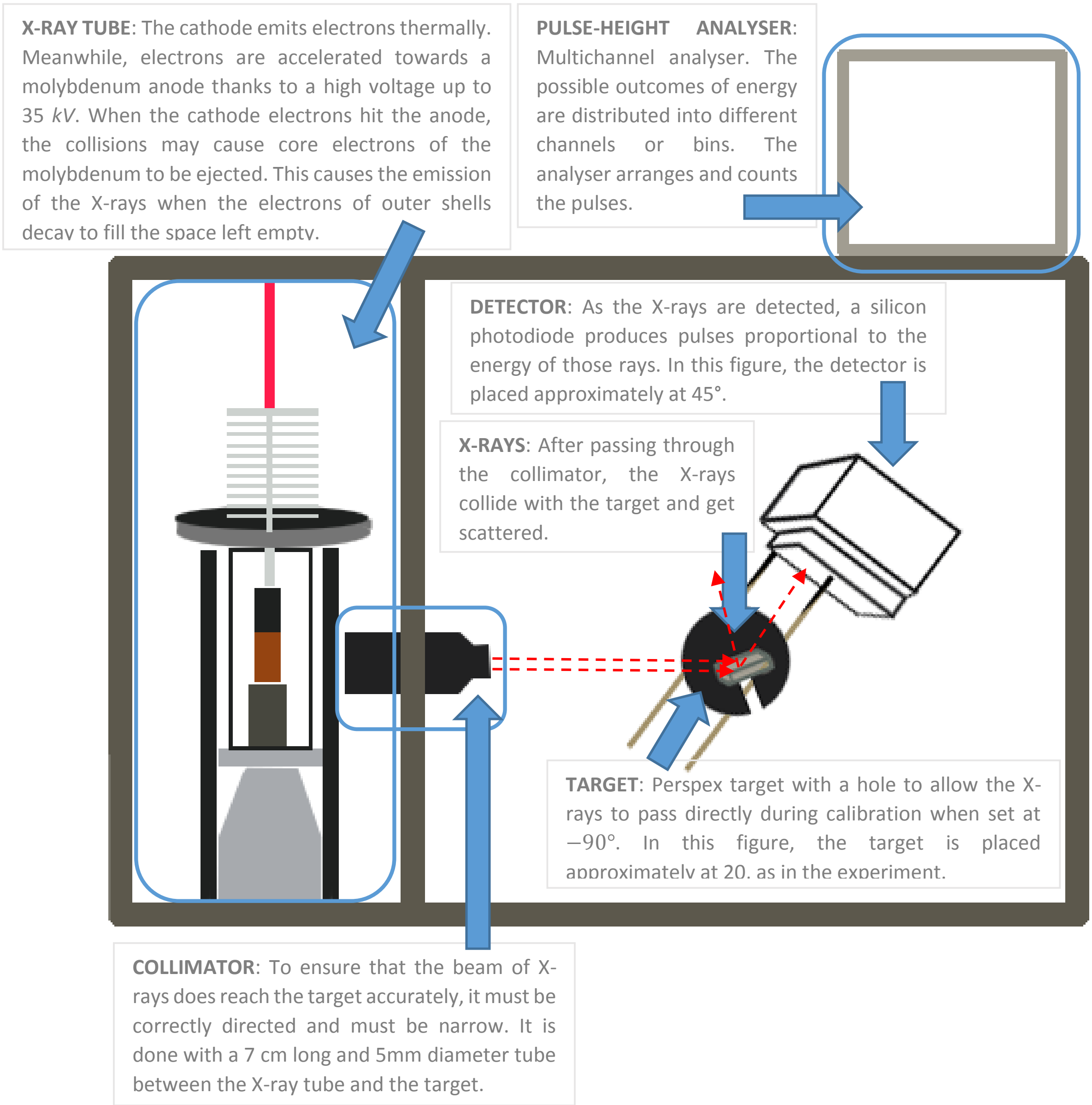


Figure 2 Schematic of the apparatus used with the main parts involved in the process.

---

[11] LD DIDACTIC 554 800 Instruction sheet: http://www.ld-didactic.de/documents/en-US/GA/GA/5/554/554800e.pdf
[12] Short guide to the LD DIDACTIC 554 800 components: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=3.2
[13] Poly(methyl 2-methylpropenoate): http://www.essentialchemicalindustry.org/polymethyl-2-methylpropenoate.html
[14] PMMA - Polymethyl Methacrylate: What it is, properties and applications: http://www.pmma-online.eu

2.3 Collecting the data

A previously developed plan was followed, with some parameters established by the procedure guide[15]. The apparatus was used initially during one session (55 minutes) using the software *Compton Scattering Controller* (Figure 3), and a user guide[16] provided. In order to contrast some hypotheses and improve the investigation, six more sessions were done in days/hours with low activity.

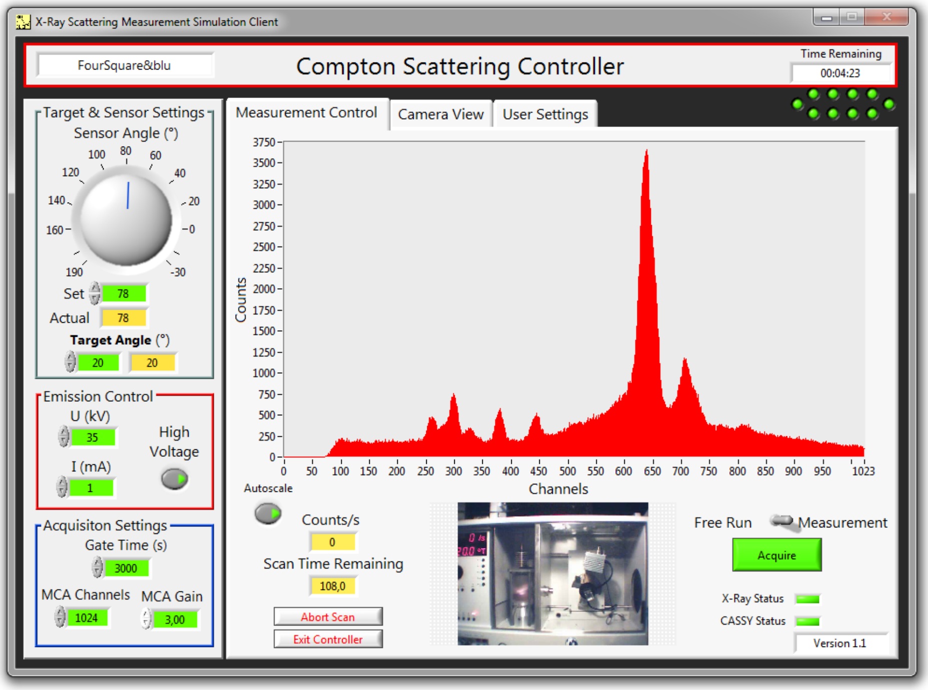


Figure 3 The interface of the software *Compton Scattering Controller* during a measurement. $[U = 35\ kV;\ I = 1\ mA;\ t = 3000\ s;\ MCA = 1024;\ \theta = 78°]$.

The process consists of two stages. The first one, the calibration, is indispensable to analyse further data. In this stage, a little amount of X-rays passes through a hole made in the target and the holder so the detector can receive the X-rays with their initial energy, E_{in}, with no scattering. The pulse-height analyser stores the results with two specific peaks[17], K_α and K_β, of known energies (main X-ray energy in molybdenum: $K_\alpha = 17.40\ KeV$, $K_\beta = 19.60\ KeV$). Once those peaks are assigned to specific channels, a linear interpolation gives the value of all the other channels.

- CALIBRATION DATA: $U = 35\ kV; I = 0.1\ mA;\ t = 300\ s;\ target\ angle = -90°;\ MCA = 1024;\ \theta = 0.5°$

During the second stage the detector is placed at different angles detecting scattered photons. The angle is selected following two recommendations[18], one angle between 30° and 40° and

[15] Experimental procedure guide: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=5
[16] *Compton scattering controller* user guide: https://learn5.open.ac.uk/pluginfile.php/3907/mod_resource/content/1/Compton_interface_instructions_13B.pdf
[17] The peaks and their interpretation: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=3.2.1
[18] Angle recommendations: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=5.5

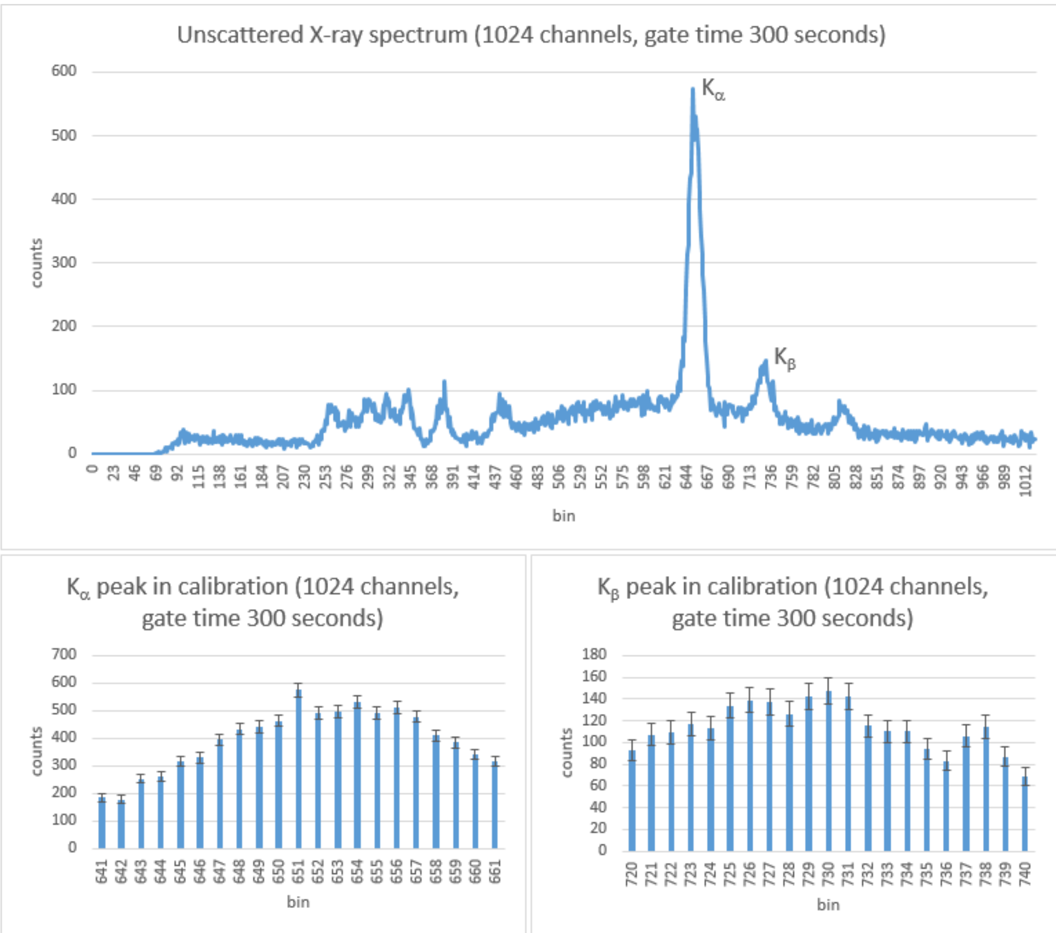
another greater than 130°. With a maximum angle of 150° and a minimum of 30°, they are split into six measurements (corresponding to the maximum time available using a gate time of 300 seconds each), obtaining a scale from 150° to 30° in steps of 24°: 150°, 126°, 102°, 78°, 54° and 30°. This gate time is chosen with the premise, thanks to previous tests at the simulator *Compton scattering planning tool*[19], that 360 seconds is a good value to obtain accurate data (i.e. sufficient signal to noise). For better coverage of the whole range of angles, it is decided to reduce the gate time to 300 seconds, gaining time for a sixth measurement.

- MEASUREMENTS DATA: $U = 35\ kV; I = 1\ mA; t = 300\ s; target\ angle = 20°; MCA = 1024\ [\theta_1 = 150°; \theta_2 = 126°; \theta_3 = 102°; \theta_4 = 78°; \theta_5 = 54°; \theta_6 = 30°]$

2.4 Results

2.4.1 Calibration

With the target at -90° (perpendicular to the X-rays) and the detector at $\theta = 0°$, θ was adjusted in steps of 0.1° to ensure a number of counts per second between 100 and 200, as required in the procedure guide. By 0.5° the value was considered appropriate. With the parameters detailed in section 2.3 (the low current prevents saturation of the detector by reducing the x-ray flux), the calibration took place during 5 minutes resulting in the graphs shown in Figure 4.



[19] *Compton Scattering planning tool* and parameters: https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080&section=4.2

Figure 4 Unscattered X-ray spectrum for calibration purposes with the peaks K_α and K_B detailed.

With the standard deviation on every channel plotted as error bars, K_α is between bins (channels) 651 and 654 (although bins 652 and 653 have too low values and are unlikely the peak), whilst K_β is between bins 725 and 731. Zooming in on their surroundings, K_α appears more clearly than K_β, and the fact K_β has very few counts so generates a greater relative uncertainty in this peak as a result.

To avoid this uncertainty in the calibration, another session was done with a gate time of 3000 seconds, resulting in K_α between bins 651 and 653 and K_β exactly at bin 726 (Figure 5). Also considering the information from the previous calibration, K_α is located at bin 651.

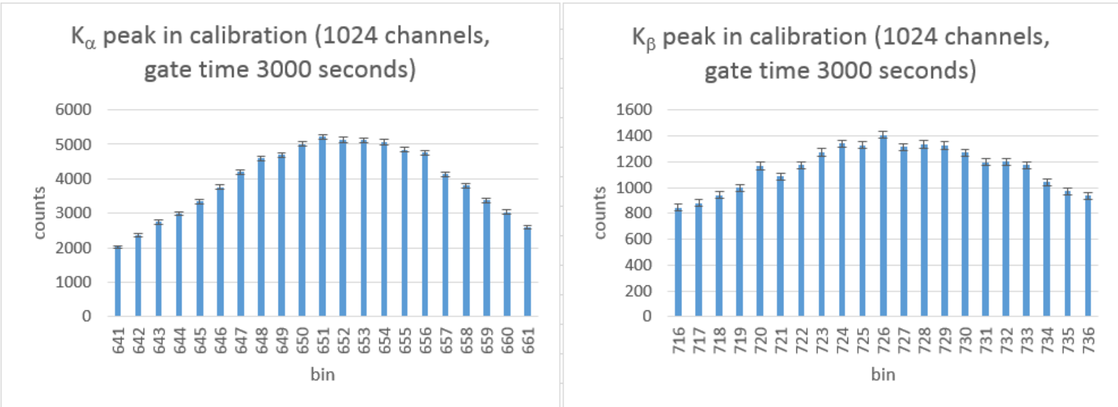


Figure 5 Detail of K_α and K_B peaks calibrating with a gate time of 3000 seconds instead of 300 seconds.

With the premises that $K_\alpha = 17.40\ KeV$ and $K_\beta = 19.60\ KeV$, and the location of the peaks through the calibration, the straight-line $bin = 34.09 \cdot E_{out} + 57.82$ is obtained. With it, the energy of each bin can be known from now on using linear interpolation.

2.4.2 Raw results

Having set up the parameters for measurements data detailed in section 2.3, the measurements for the six angles were done. The results obtained are plotted in Figure 6, where is noticeable which angle results in the highest number of scattered photons (30°) and the expected relation between the angle of scattering and the energy is observed: as the angle increases, the energy of the scattered X-rays is lower (peaks move to lower bin values). With the calibration and these results, the E_{out} on each angle is obtained (Tables 1, 2).

The 30° angle is definitely the one with the clearest and best defined peaks, in contrast to 126° and 102°, which have the highest uncertainties (Figure 7).

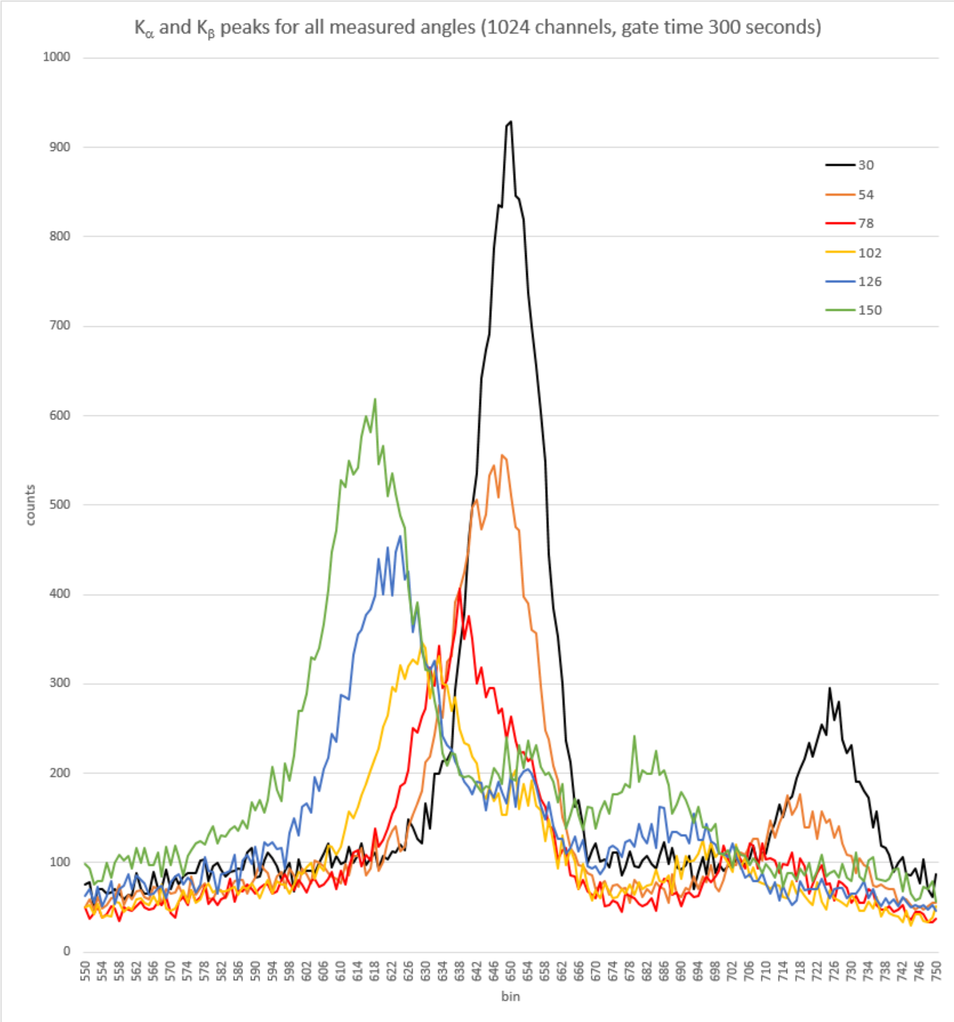


Figure 6 Spectrum for the six angles measured with their peaks K_{α} and K_{B}.

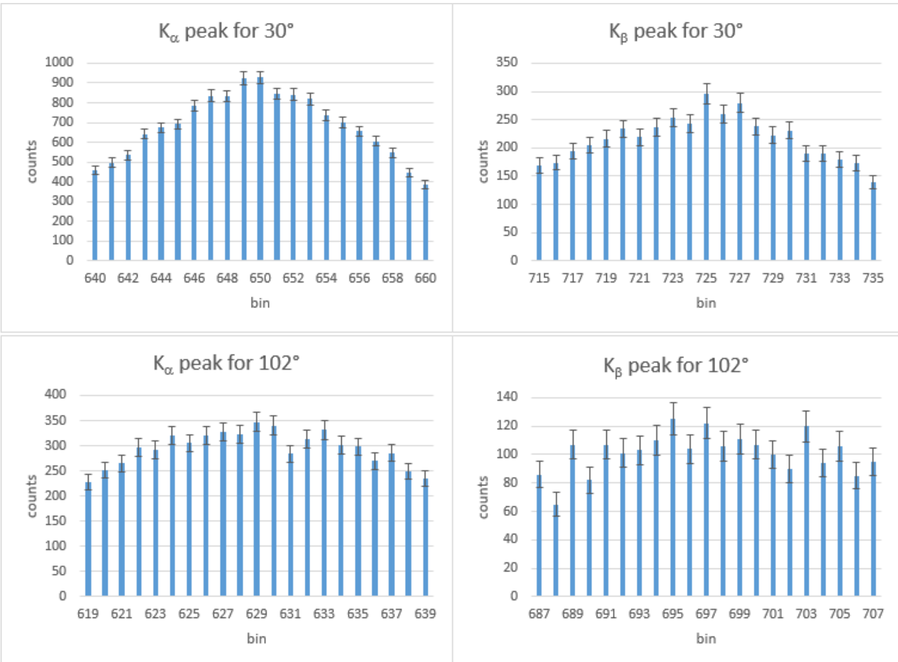


Figure 7 Comparison between peaks width for 30° and 102° angles.

3. DISCUSSION AND INTERPRETATION

3.1 Mass of the electron

The Compton formula can be rearranged to plot a straight-line (i.e. into the form y = mx +c):

$$\frac{1}{E_{out}} = \frac{1}{E_{in}} + \frac{1}{m_e c^2} \cdot (1 - \cos\theta)$$

The gradient is related to m_e and the axes with θ and E_{out}. With the data already obtained, the values for the graph are specified in Table 2, which is plotted in Figure 8, where it can be seen that every uncertainty contains the theoretical value inside its range.

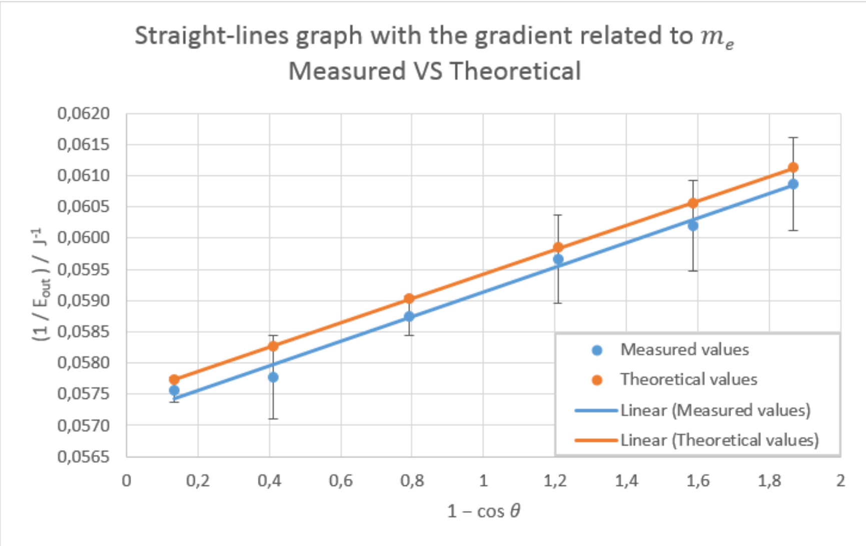


Figure 8 Comparison between theoretical and measured values according to a rearrangement of the Compton formula.

With this gradient, $m_e c^2 = 0.509\ MeV \rightarrow m_e = 9.08\ x\ 10^{-31}\ kg$, close to the actual value[20,21,22] of $m_e c^2 = 0.511\ MeV \rightarrow m_e = 9.11\ x\ 10^{-31}\ kg$. Nonetheless, if the uncertainty of the gradient is considered, the results go from $m_e c^2 = 0.288\ MeV$ to $m_e c^2 = 2.16\ MeV$, and therefore $m_e c^2 = 1.22 \pm 0.94\ MeV \rightarrow m_e = 2.17\ x\ 10^{-30} \pm 1.7\ x\ 10^{-30}\ kg$, with a relative uncertainty of 78%.

3.2 Uncertainty

When dealing with the gradient (0.001964), its uncertainty (0.0015) causes the relative uncertainty of 78%. Nevertheless, such a large uncertainty comes from joining the different $\delta(^1/_{E_{out}})$, that only have a relative uncertainty mean of 0.94% (the same as δE_{out}).

[20] Electron mass (kg): http://physics.nist.gov/cgi-bin/cuu/Value?me
[21] Electron mass energy (eV): http://physics.nist.gov/cgi-bin/cuu/Value?mec2mev
[22] Electron volt-kilogram relationship: http://physics.nist.gov/cgi-bin/cuu/Value?evkg

One method to reduce the uncertainty would be by reducing even more δE_{out}. Taking the 150° angle as an example, it has $E_{out} = 16.43\ KeV$ and $\delta E_{out} = 0.2\ KeV$. This value comes from the fact that K_α is at bin 618 ± 3, and the energy at that point has two uncertainties: the uncertainty caused by the width of the peak, and the previous uncertainty in the calibration. To reduce this uncertainty, one option is to try to be more accurate with more channels (Figure 9).

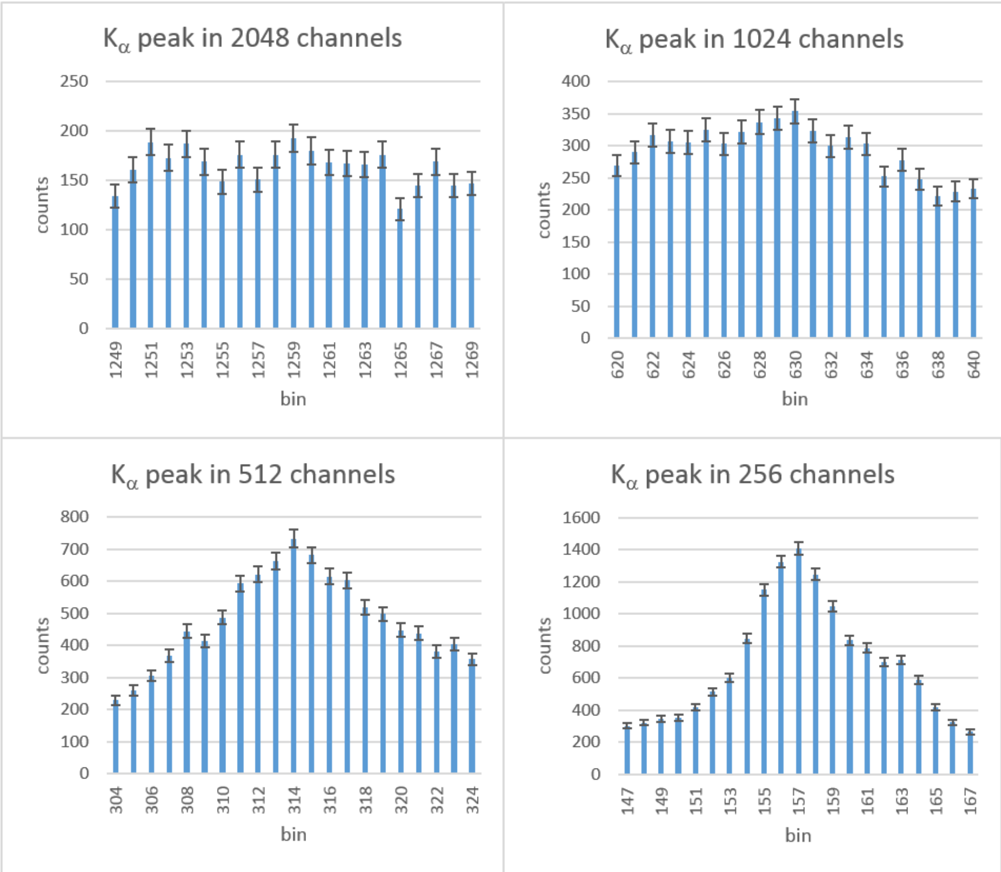


Figure 9 Comparison between K_α peaks width depending on the number of channels used.

Nonetheless, modifying only that parameter, the best accurate and recognisable shapes are also uncertain: the bigger the bin is, the wider the energy it contains. Hence, the energy resolution does not improve the results if taken as the only solution.

Another method can consist of increasing the gate time as it was done in the calibration, increasing the counts N per bin and reducing the ratio σ/N as a consequence. With this procedure, three angles are tested (Table 3), one of them twice. The standard deviation is lower, but not necessarily the uncertainty in the width. In this case, the uncertainty of the gradient lowers only in the third significant figure, which is not considered an improvement.

Hypothetically, doing it to all the angles with an exponentially higher count gate could improve it, but it has not been tested (the maximum available gate time per session $\cong 3000\ s$).

Hence, in order to get a more precise result, it could be appropriate to repeat the experiment many times with a much higher count gates (lowering, hypothetically, the energy resolution uncertainties), evaluating first whether using 2048 channels on those conditions would mean an improvement. Additionally, unlike in this work, the angular resolution uncertainty should be considered.

4. CONCLUSIONS / SUMMARY

As Compton already disclosed, scattered photons do lose energy in the collision (energy at K_α in calibration is higher than energy at K_α in every other measurement at angle θ).

Having measured the energy lost at some angles, you can reliably work out the m_e value with the Compton formula (not all the measurements are necessarily equally close to the theoretical, but the trend-line should be similar to the actual gradient line).

If the uncertainties involved are considered, the result gets further away from the accepted value with a huge relative uncertainty. Nevertheless, as a first approximation, it seems reasonable.

REFERENCES

Bowden, S., Honsberg, C. (2014) *Energy of Photon* [Online]. Available at http://www.pveducation.org/pvcdrom/properties-of-sunlight/energy-of-photon (Accessed 29 December 2014).

CIEC Promoting Science (2013) *Poly(methyl 2-methylpropenoate)* [Online]. Available at http://www.essentialchemicalindustry.org/polymethyl-2-methylpropenoate.html (Accessed 3 January 2015).

Egerton, R. (2000) *X-rays and the Compton Effect* [Online]. Available at http://web.pdx.edu/~egertonr/ph311-12/xraycomp.htm (Accessed 29 December 2014).

Elert, G. (2015) *Photoelectric Effect* [Online]. Available at http://physics.info/photoelectric (Accessed 2 January 2015).

LD Didactic GmbH (2015) *Instruction sheet 554 800* [Online], Available at http://www.ld-didactic.de/documents/en-US/GA/GA/5/554/554800e.pdf (Accessed 24 January 2015).

Nobel Media AB (2015a) *The Nobel Prize in Physics 1918. Max Planck* [Online]. Available at http://www.nobelprize.org/nobel_prizes/physics/laureates/1918/planck-facts.html (Accessed 2 January 2015).

Nobel Media AB (2015b) *The Nobel Prize in Physics 1921. Albert Einstein* [Online]. Available at http://www.nobelprize.org/nobel_prizes/physics/laureates/1921/einstein-facts.html (Accessed 2 January 2015).

Nobel Media AB (2015c) *The Nobel Prize in Physics 1927. Arthur H. Compton* [Online]. Available at http://www.nobelprize.org/nobel_prizes/physics/laureates/1927/compton-facts.html (Accessed 2 January 2015).

PMMA Acrylic Sustainable Solutions (2015) *PMMA - Polymethyl Methacrylate* [Online]. Available at http://www.pmma-online.eu (Accessed 3 January 2015).

The National Institute of Standards and Technology (2011a) *Electron mass* [Online], Available at http://physics.nist.gov/cgi-bin/cuu/Value?me (Accessed 10 January 2015).

The National Institute of Standards and Technology (2011b) *Electron mass energy equivalent in MeV* [Online], Available at http://physics.nist.gov/cgi-bin/cuu/Value?mec2mev (Accessed 10 January 2015).

The National Institute of Standards and Technology (2011c) *Electron volt-kilogram relationship* [Online], Available at http://physics.nist.gov/cgi-bin/cuu/Value?evkg (Accessed 10 January 2015).

The Open University (2013a) *Compton Remote Control Handbook* [Online], Available at https://learn5.open.ac.uk/pluginfile.php/3907/mod_resource/content/1/Compton_interface_instructions_13B.pdf (Accessed 19 December 2014).

The Open University (2013b) *Compton Scattering Theory Supplement* [Online], Available at https://learn5.open.ac.uk/pluginfile.php/3906/mod_resource/content/1/S288_Compton_theorysupplement.pdf (Accessed 24 January 2015).

The Open University (2014c) *Guide to experimental uncertainties* [Online], Available at https://learn2.open.ac.uk/pluginfile.php/1345172/mod_resource/content/1/Guide%20to%20Experimental%20Uncertainties.pdf (Accessed 18 January 2015).

The Open University (2014d) *Investigation 3: Compton scattering* [Online], Available at https://learn2.open.ac.uk/mod/oucontent/view.php?id=625080 (Accessed 24 January 2015).

TABLES

Table 1 Location for the peaks at the calibration position and at the six angles measured using 1024 channels. From the calibration, a linear interpolation is done to find out the energy associated to any other bin chosen. Equation from calibration: $bin = 34.09 \cdot E_{out} + 57.82$

The uncertainty for every bin is half its width ($\cong \mathbf{0.015\ KeV}$). The uncertainty for every angle is the combination of the uncertainties in the bin and the calibration ($\cong \mathbf{0.03\ KeV}$).

Calibration: $\boldsymbol{K_\alpha = bin\ 651 = 17.40 \pm 0.02\ KeV}$ **|** $\boldsymbol{K_\beta = bin\ 726 = 19.60 \pm 0.02\ KeV}$

- $\boldsymbol{\theta = 150°: K_\alpha = 618 \pm 3\ bins = 16.43 \pm (0.09 + 0.03)\ KeV = 16.43 \pm 0.20\ KeV}$
- $\boldsymbol{\theta = 126°: K_\alpha = 624 \pm 5\ bins = 16.61 \pm 0.20\ KeV}$ *Already rounded up
- $\boldsymbol{\theta = 102°: K_\alpha = 629 \pm 5\ bins = 16.76 \pm 0.20\ KeV}$ *Already rounded up
- $\boldsymbol{\theta = 78°: K_\alpha = 638 \pm 2\ bins = 17.02 \pm (0.06 + 0.03)\ KeV = 17.02 \pm 0.09\ KeV}$
- $\boldsymbol{\theta = 54°: K_\alpha = 648 \pm 3\ bins = 17.31 \pm (0.09 + 0.03)\ KeV = 17.31 \pm 0.20\ KeV}$
- $\boldsymbol{\theta = 30°: K_\alpha = 650 \pm 1\ bin = 17.37 \pm (0.03 + 0.03)\ KeV = 17.37 \pm 0.06\ KeV}$

Table 2 First table: the K_α channel for every angle measured, the outgoing energy calculated through linear interpolation with the calibration data, the uncertainty of this energy and automatic calculations to plot the data as a straight-line graph. Second table: the theoretical values according to the Compton formula.

θ	Bin	E_{out}	δE_{out}	$1-\cos\theta$	$1/E_{out}$	$\delta(1/E_{out})$	Gradient
150 °	618	16,43 KeV	0,2 KeV	1,8660	0,06086 KeV	0,0003334 KeV	**0.001964**
126 °	624	16,61 KeV	0,2 KeV	1,5878	0,06020 KeV	0,0007249 KeV	
102 °	629	16,76 KeV	0,2 KeV	1,2079	0,05967 KeV	0,0007120 KeV	**δGradient**
78 °	638	17,02 KeV	0,09 KeV	0,7921	0,05875 KeV	0,0002071 KeV	
54 °	648	17,31 KeV	0,20 KeV	0,4122	0,05777 KeV	0,0003004 KeV	**0.0015**
30 °	650	17,37 KeV	0,06 KeV	0,1340	0,05757 KeV	0,0000994 KeV	

θ	E_{out}	$1/E_{out}$	Gradient
150 °	16,36 KeV	0,06112 KeV	**0.001957**
126 °	16,51 KeV	0,06057 KeV	
102 °	16,71 KeV	0,05984 KeV	**for known value**
78 °	16,94 KeV	0,05903 KeV	
54 °	17,16 KeV	0,05828 KeV	$m_e c^2 = 0.511\ MeV$
30 °	17,32 KeV	0,05774 KeV	

Table 3 Comparison between the channel for K_α (maximum value for N) varying the gate time for three angles. All cases use 1024 channels. The peak is marked in red colour and the potential range, according to the counts and standard deviations, in orange.

$\theta = 150°$						$\theta = 54°$					
$t = 300\ s$			$t = 3000\ s$			$t = 300\ s$			$t = 3000\ s$		
Bin	N	σ	Bin	N	σ	Bin	N	σ	Bin	N	σ
610	528	23,0	**610**	4987	70,6	**641**	497	22,3	**641**	5351	73,2
611	520	22,8	**611**	5304	72,8	**642**	506	22,5	**642**	5599	74,8
612	549	23,4	**612**	5346	73,1	**643**	473	21,7	**643**	5780	76,0
613	534	23,1	**613**	5588	74,8	**644**	490	22,1	**644**	5578	74,7
614	542	23,3	**614**	5741	75,8	**645**	533	23,1	**645**	5728	75,7
615	576	24,0	**615**	5815	76,3	**646**	545	23,3	**646**	5746	75,8
616	599	24,5	**616**	5783	76,0	**647**	509	22,6	**647**	5766	75,9
617	581	24,1	**617**	5790	76,1	**648**	556	23,6	**648**	5422	73,6
618	618	24,9	**618**	5660	75,2	**649**	551	23,5	**649**	5375	73,3
619	546	23,4	**619**	5451	73,8	**650**	511	22,6	**650**	5267	72,6
620	566	23,8	**620**	5522	74,3	**651**	476	21,8	**651**	4897	70,0

$\theta = 78°$								
$t = 300\ s$			$t = 3000\ s$ (1)			$t = 3000\ s$ (2)		
Bin	N	σ	Bin	N	σ	Bin	N	σ
633	342	18,5	**633**	3549	59,6	**633**	3586	59,9
634	295	17,2	**634**	3676	60,6	**634**	3528	59,4
635	304	17,4	**635**	3740	61,2	**635**	3589	59,9
636	337	18,4	**636**	3700	60,8	**636**	3689	60,7
637	358	18,9	**637**	3671	60,6	**637**	3751	61,2
638	406	20,1	**638**	3668	60,6	**638**	3752	61,3
639	350	18,7	**639**	3679	60,7	**639**	3803	61,7
640	376	19,4	**640**	3599	60,0	**640**	3659	60,5
641	350	18,7	**641**	3375	58,1	**641**	3587	59,9
642	300	17,3	**642**	3290	57,4	**642**	3506	59,2
643	318	17,8	**643**	3118	55,8	**643**	3226	56,8